Rare K-Decays

CKM Conference 2010
Warwick

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TU Munich

9 September 2010

Goal

The Goal

- find New Physics (NP)
- understand the NP

Kaon FCNC processes $\nearrow$ suppressed within SM

Kaon FCNC processes $\searrow$ sensitive to NP

- $K_L \to \mu^+\mu^-$ $\sim 40\%$ SD
- $K_L \to \pi^0 e^+ e^-$ $\sim 40\%$ SD
  $K_L \to \pi^0 \mu^+\mu^-$
- $K^+ \to \pi^+ \nu\bar{\nu}$ $\sim 98\%$ SD
  $K_L \to \pi^0 \nu\bar{\nu}$

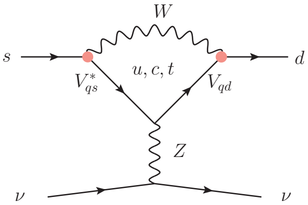
How precise?
Are Long Distance Efects under control?

What can rare K-decays tell us about NP?

General SM Structure

FCNC processes | loop-induced within SM

an example:



CKM factors

- $\lambda \approx \sin\theta_{\text{Cabibbo}}$
- $V_{qs}^* V_{qd} \equiv \lambda_q$
- $\lambda_u + \lambda_c + \lambda_t = 0$

$$CPC: \quad \lambda_t \sim \lambda^5 \quad \lambda_c \sim \lambda$$
$$\not{CP}: \quad \text{Im}\lambda_t \sim \lambda^5 \quad \text{Im}\lambda_c \sim \lambda^5$$

Loop Functions

- charm-loop $\propto \frac{m_c^2}{M_W^2}$
- top-loop $\propto \frac{m_t^2}{M_W^2}$

Hard Quadratic GIM

High sensitivity to SD Physics

Combination

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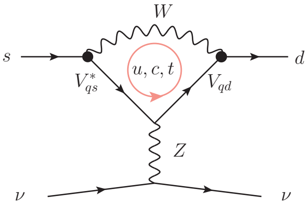

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Combination

Motivations

Master-Amplitude

$$\mathcal{A}_{\text{decay}} = \sum_i \langle Q_i \rangle \quad V^i_{\text{CKM}} \quad F_i(\text{high energy parameters})$$

valid for
B,D,K -Decays

F_i	:	Loop Functions (SD)	:	different in BSM
V^i_{CKM}	:	CKM factors	:	departure from MFV
$\langle Q_i \rangle$	:	Matrix elements of operators (LD)		

Additional operators can be generated by NP

NP can hide everywhere

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Motivations for K-Physics

Master-Amplitude

$$\mathcal{A}_{\text{decay}} = \sum_i \langle Q_i \rangle \quad V^i_{\text{CKM}} \quad F_i$$

F_i: F_i **universal** (B,D,K-Decays) in MFV models

$\Downarrow$

- Constrained from different observables
- Correlations between B,D,K-Physics

  (see talk by Straub)
- To which extend does this universality hold?

Motivations for K-Physics

Master-Amplitude

$$\mathcal{A}_{\text{decay}} = \sum_i \langle Q_i \rangle \; V^i_{\text{CKM}} \; F_i$$

V^i_{CKM}:

B-Physics:	$b \to s$	$\propto \lambda^2$	
	$b \to d$	$\propto \lambda^3$	
K-Physics:	$d \to s$	$\propto \lambda^5$	(top)

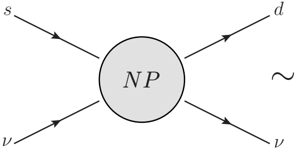
Extreme Suppression

$\Downarrow$

High Sensitivity to deviations from MFV

Kaons and New Physics

What are Kaons telling us about NP?



$$\sim \frac{C_{s\to d}}{\Lambda_{NP}^2}$$

Kaons

$$\frac{C_{s\to d}}{\Lambda_{NP}^2} \sim 10^{-10}$$

Low Energy NP

$\Lambda_{NP} \sim 1$TeV

Kaons: $C_{s\to d} < \lambda^5$

Generic NP

$C_{s\to d} \sim 1$

Kaons: $\Lambda_{NP} > 75$TeV

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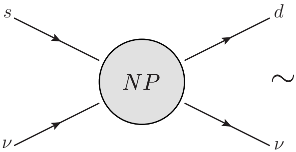

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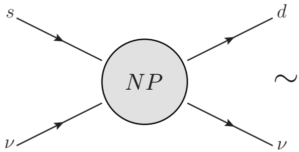

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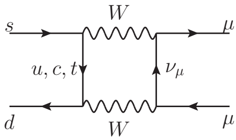
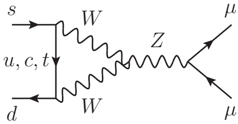
$K_L \to \mu^+\mu^-$

CP conserving leptonic mode

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_W}\left(\lambda_t\, Y(x_t) + \lambda_c\, Y_{NL}\right)(\bar{s}\gamma_\mu\gamma_5 d)(\bar{\mu}\gamma^\mu\gamma_5\mu) + h.c.$$

$Y(x_t)$: SD top contribution
Y_{NL} : SD charm contribution





- perturbative calculation
- Penguin, Box diagrams
- top- and charm-quarks relevant

→ NLO QCD on top [Buchalla, Buras 93]
→ NNLO QCD on charm [Gorbahn, Haisch 06]

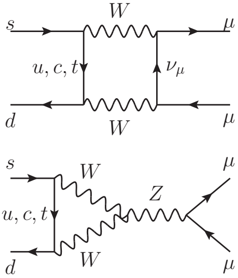
under control

$K_L \to \mu^+\mu^-$

CP conserving leptonic mode

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \left(\lambda_t \, Y(x_t) \; + \lambda_c \, Y_{NL} \right) (\bar{s}\gamma_\mu\gamma_5 d)(\bar{\mu}\gamma^\mu\gamma_5\mu) + h.c.$$

$\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K^0(P)\rangle$: matrix element, LD contribution

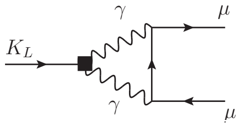


- non-perturbative
- use isospin symmetry
- extracted from $K^+ \to \mu^+\nu_\mu$

under control

$K_L \to \mu^+\mu^-$

LD effect



- absoptive (Re) & dispersive part (Im)
- large absorptive part
  $\rightarrow$ saturates the rate
- dispersive part cannot be predicted in ChPT
- divergent in ChPT at LO
- $+$ interference

$\approx 17\%$ uncertainty

$\rightarrow$ [D'Ambrosio, Isidori, Portoles 98]

$\rightarrow$ [Isidori, Unterdorfer 03]

Difficult to test the SD contribution

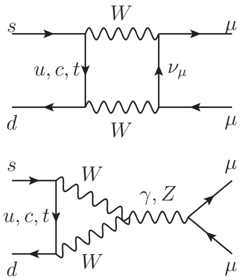
$\Longrightarrow$ $K_L \to \mu^+\mu^-$ has seen better days...

$K_L \to \pi^0 l^+ l^-$

1. Contribution : direct $\not{CP}$: DCPV

$$\mathcal{H}_{\text{eff}}^{V,A} = -\frac{G_F \alpha}{\sqrt{2}} \lambda_t \left[y_{7V} \; (\bar{s}\gamma_\mu d)(\bar{l}\gamma^\mu l) \; + y_{7A} \; (\bar{s}\gamma_\mu d)(\bar{l}\gamma^\mu \gamma_5 l) \right] + h.c.$$

y_{7V}, y_{7A}: SD contribution



- perturbative calculation
- Penguin, Box diagrams
- $+ \gamma$-Penguin
- RGE analysis

$\rightarrow$ NLO QCD [Buras, Lautenbacher, Misiak, Münz 94]

$\approx \pm 5\%$ uncertainty

$K_L \to \pi^0 l^+ l^-$

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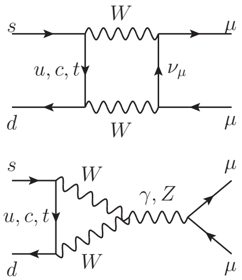
Matrix Elements

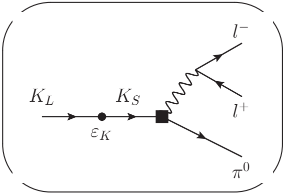


- extract from $K \to \pi l \nu$ (K_{l3}) decays
- $\langle \pi^0 e^+ e^- | Q_A | K^0(P) \rangle$ suppressed by m_e
- isospin breaking effects included

few $^0\!/_{00}$ uncertainty

[Mescia, Smith 07]

$K_L \to \pi^0 l^+ l^-$



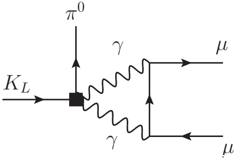
2. Contribution : indirect $\not{CP}$: ICPV

- $K_L \to \varepsilon_K K_1 [\to \pi^0 l^+ l^-]$
- 2 relevant parameters
  $\to \varepsilon_K$
  $\to |a_S|$ (main theory error, only absolute value)
  (const., destr. interference with DCPV ?)
  $\approx 20\%$ uncertainty

[Buchalla, D'Ambrosio, Isidori 03; Friot, Greynat, De Rafael 04]

- extract sign from lepton asymmetry

[Mescia, Smith, Trine 06]

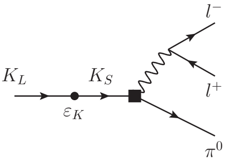


3. Contribution : CP conserving : CPC

- NOT divergent as in $K_L \to \mu^+ \mu^-$
- NO amplitude saturation
- helicity suppressed (0 for e-mode)
- 30% uncertainty

[Isidori, Smith, Unterdorfer 04]

$K_L \to \pi^0 l^+ l^-$



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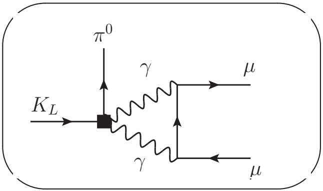

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[Isidori, Smith, Unterdorfer 04]

$K_L \to \pi^0 l^+ l^-$

DCPV : ICPV : CPC compete

$$\text{Br}^{theo}(K_L \to \pi^0 e^+ e^-) = 3.54^{+0.98}_{-0.85} \quad (1.56^{+0.62}_{-0.49}) \quad \times 10^{-10}$$
$$\text{Br}^{theo}(K_L \to \pi^0 l^+ l^-) = 1.41^{+0.28}_{-0.26} \quad (0.95^{+0.22}_{-0.21}) \quad \times 10^{-10}$$

[Mescia, Smith, Trine, 06]

$$\text{Br}^{exp}(K_L \to \pi^0 e^+ e^-) < 28 \times 10^{-11}$$ [KTeV 04]
$$\text{Br}^{exp}(K_L \to \pi^0 l^+ l^-) < 38 \times 10^{-10}$$ [KTeV 04]

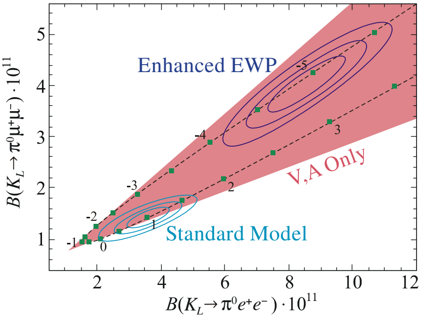
What can we do with $K_L \to \pi^0 l^+ l^-$?

$K_L \to \pi^0 l^+ l^-$

Goal: entangling NP

$\mathcal{B}(K_L \to \pi^0 \mu^+ \mu^-) / \mathcal{B}(K_L \to \pi^0 e^+ e^-)$ -plane ideal for discriminating NP models

[Isidori, Smith, Unterdorfer 04; Mescia, Smith, Trine 06]



[Mescia, Smith, Trine 06]

Model Independent Analysis

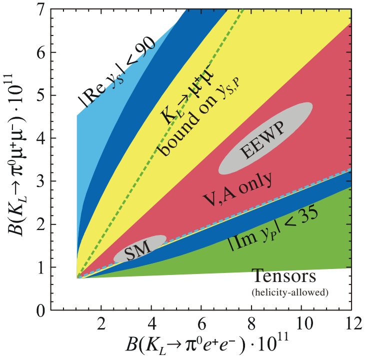
<u>Case 1</u>

- NP changes only y_{7A}, y_{7V}
  (SD Wilson Coefficients)
- No new operators
- e^+e^- VS $\mu^+\mu^-$
  $\to$ different phase-space
  $\to$ $K_L \to \gamma\gamma$ contribution

Goal: entangling NP

$\mathcal{B}(K_L \to \pi^0 \mu^+ \mu^-)/\mathcal{B}(K_L \to \pi^0 e^+ e^-)$ -plane ideal for discriminating NP models

[Isidori, Smith, Unterdorfer 04; Mescia, Smith, Trine 06]



[Mescia, Smith, Trine 06]

Model Independent Analysis

Case 2

New Operators generated

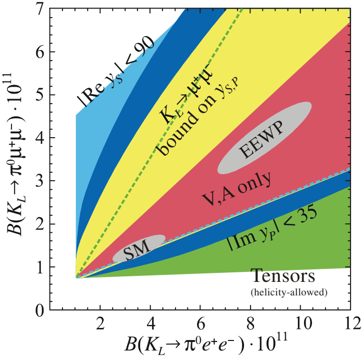
- Scalar- Pseudoscalar Operators
  - $(\bar{s}d)(\bar{l}l)$
  - $(\bar{s}d)(\bar{l}\gamma_5 l)$
- Tensor- Pseudotensor- Operators
  - $(\bar{s}\sigma_{\mu\nu}d)(\bar{l}\sigma^{\mu\nu}l)$
  - $(\bar{s}\sigma_{\mu\nu}d)(\bar{l}\sigma^{\mu\nu}\gamma_5 l)$
- With & Without Helicity Suppression

$K_L \to \pi^0 l^+ l^-$

Goal: entangling NP

$\mathcal{B}(K_L \to \pi^0\mu^+\mu^-)/\mathcal{B}(K_L \to \pi^0 e^+e^-)$ -plane ideal for discriminating NP models

[Isidori, Smith, Unterdorfer 04; Mescia, Smith,Trine 06]



[Mescia, Smith,Trine 06]

Model Independent Analysis

Case 2

Pick your favourite Model:

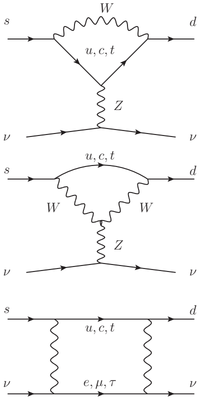
- MSSM large $\tan\beta$
- SUSY without R-parity
- leptoquarks
- ...

disentangled on the

$\mathcal{B}(K_L \to \pi^0\mu^+\mu^-)/\mathcal{B}(K_L \to \pi^0 e^+e^-)$

$K \to \pi\nu\bar{\nu}$

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{q=u,c,t} \lambda_q \; X(x_q) \cdot (\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_{lL}\gamma^\mu \nu_{lL}) \; + h.c.$$



- $x_t = \frac{m_t^2}{M_W^2}$
- 2 modes: $K^+ \to \pi^+\nu\bar{\nu}$ & $\underbrace{K_L \to \pi^0\nu\bar{\nu}}_{\not{CP}}$
- one dominant dim-6 operator
- no γ Penguin
- no $\gamma\gamma$ contribution

$K \to \pi\nu\bar{\nu}$

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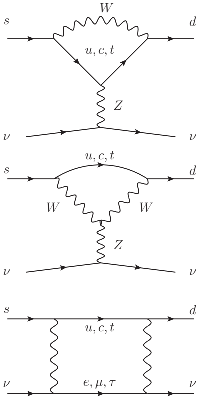


Matrix Elements

- extracted from K_{l3} decays
- isospin breaking effects included

[Mescia, Smith 07]

- κ_ν^+ and κ_ν^L
- only 8% and 21% of total theory error!

$\Longrightarrow$ NLO Perturbative Calculations Relevant

$K \to \pi\nu\bar{\nu}$: EW corrections

Why EW corrections on X_t?

$$\mathcal{H}_{\text{eff}} = \frac{2}{\pi\sqrt{2}} G_F \frac{\alpha}{\sin^2\theta_W} \lambda_t \, X_t\Big(\frac{m_t^2}{M_W^2}\Big) + \ldots \text{charm} \ldots + h.c.$$

$$X_t = \underbrace{X^{(0)}}_{\text{LO}} + \underbrace{\frac{\alpha_s}{4\pi} X^{(1)}}_{\text{NLO QCD}} + \underbrace{\frac{\alpha_s^2}{(4\pi)^2} X^{(2)}}_{\text{NNLO QCD}} + \underbrace{\frac{\alpha_s^3}{(4\pi)^3} X^{(3)}}_{\text{NNNLO QCD}} + \ldots$$

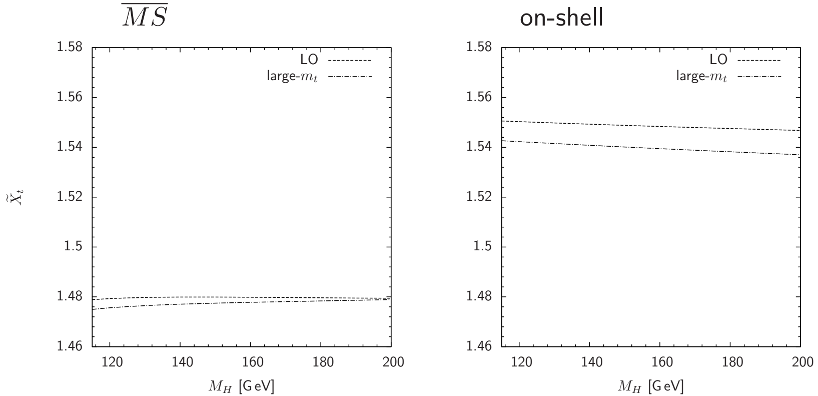
LO EW parameters still NOT fixed

- ambiguity from renormalisation scheme (on-shell VS $\overline{MS}$)

$$X_t = \underbrace{X^{(0)}}_{\text{LO}} + \underbrace{\frac{\alpha}{4\pi} X^{(EW)}}_{\text{NLO EW}} + \ldots$$

- only fixed after calculating $X^{(EW)}$

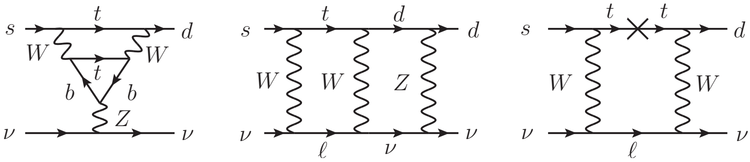
$K \to \pi\nu\bar{\nu}$: EW corrections



- $X^{(EW)}$ known for large-m_t limit
- large-m_t known to be a bad approximation

  [Buchalla, Buras 98]
- $\pm 2\%$ uncertainty in X_t scales to $\pm 4\%$ uncertainty in Branching Ratios

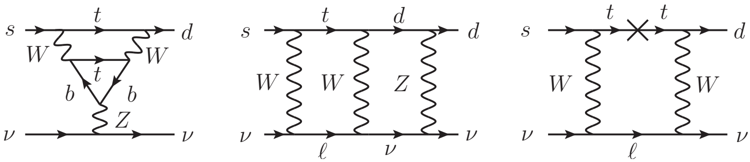
$K \to \pi\nu\bar{\nu}$: EW corrections



- full two-loop calculation $\mathcal{O}(1000)$ diagrams
- $\overline{MS}$ renormalisation for calculation
- matching on 5-quark effective theory at μ_t
- HV-scheme for diagrams with anomalies

- 2 independent calculations
- reproduced large-m_t limit

$K \to \pi\nu\bar{\nu}$: EW corrections



Error Estimation

3 schemes:

on-shell: G_F, M_Z, M_t, M_H, and α

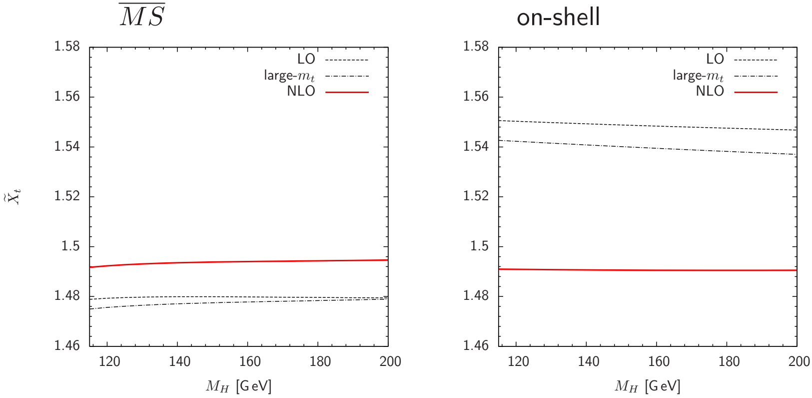
$\overline{MS}$: g_1, g_2, v, λ, and y_t

one-loop running and fit for initial conditions

$\to$ introduces residual M_H dependence

mix: on-shell masses, but $\overline{MS}$ couplings

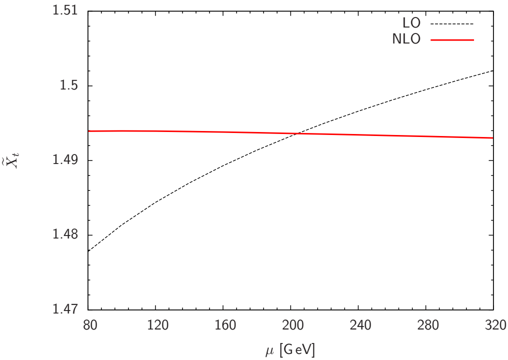
$K \to \pi\nu\bar{\nu}$: EW corrections



completely fixes the renormalisation scheme dependence
$\pm 2\% \to \pm 0.3\%$

[arXiv:1009.0947v1 [hep-ph]]

$K \to \pi\nu\bar{\nu}$: EW corrections



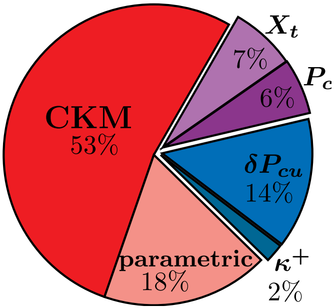
removed the remaining scale dependence

Remember: loop-functions are universal

$\Longrightarrow$ Correction applies also to:
$B \to X_{d,s}\nu\bar{\nu}$

$K \to \pi\nu\bar{\nu}$: Branching Ratios

$$\mathrm{Br}\left(K^+ \to \pi^+\nu\bar{\nu}\right) = \kappa_\nu^+ \left|\lambda_t\, X_t \; + \mathrm{Re}\lambda_c \left(P_c \; + \delta P_{c,u} \right)\right|^2$$

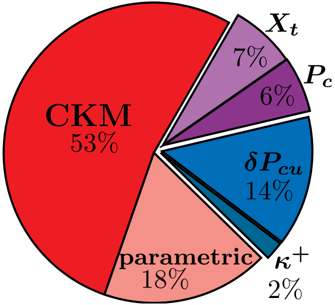


Long Distance

- κ_ν^+ with Isospin Corrections
- QED radiative corrections
  [Mescia, Smith 07]
- $\delta P_{c,u}$: dim-8 Operators below μ_c
  [Falk, Lewandowski, Petrov 01]
- $\delta P_{c,u}$: light quark contributions
  [Isidori, Mescia, Smith 05]
- Improvement by Lattice possible!
  [Isidori, Martinelli, Turchetti 06]

$K \to \pi\nu\bar{\nu}$: Branching Ratios

$$\text{Br}\left(K^+ \to \pi^+\nu\bar{\nu}\right) = \kappa_\nu^+ \left|\lambda_t X_t + \text{Re}\lambda_c \left(P_c + \delta P_{c,u}\right)\right|^2$$

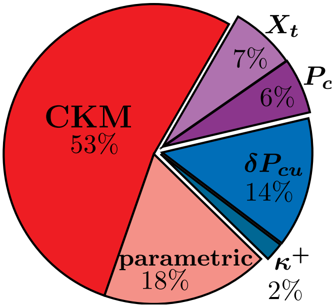


Short Distance

- P_c : NNLO QCD
  [Buras, Gorbahn, Haisch, Nierste 05]
- P_c : NLO EW
  [Brod, Gorbahn 08]
- X_t : NLO QCD
  [Misiak, Urban; Buchalla, Buras 99]
- X_t : NLO EW
  [Brod, Gorbahn, ES 10]

$K \to \pi\nu\bar{\nu}$: Branching Ratios

$$\text{Br}\left(K^+ \to \pi^+\nu\bar{\nu}\right) = \kappa_\nu^+ \left|\lambda_t\, X_t + \text{Re}\lambda_c \left(P_c + \delta P_{c,u} \right)\right|^2$$



Numbers

- 7 events at E787/949

$$\text{Br}^{exp} = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$

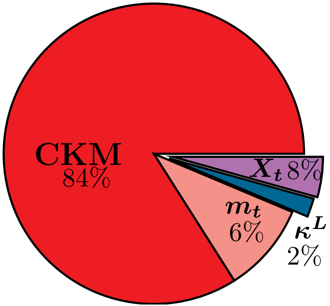
$$\text{Br}^{the} = 8.22^{+0.74}_{-0.65} \pm 0.29 \times 10^{-10}$$

[Brod, Gorbahn, ES 10]

- NA62 aims at $\mathcal{O}(100)$ events!

$K \to \pi\nu\bar{\nu}$: Branching Ratios

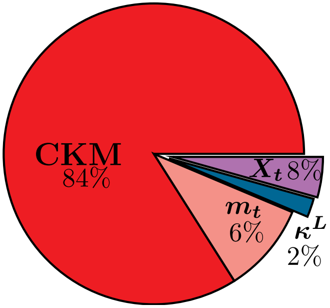
$$\text{Br}\left(K_L \to \pi^0\nu\bar{\nu}\right) = \kappa_\nu^L \left(1 - \sqrt{2}|\epsilon_K|\frac{1+P_c(X)/A^2 X_t - \rho}{\eta}\right) \left(\frac{\text{Im}\lambda_t}{\lambda^5} X_t\right)^2$$



- CPC contribution negligible
  [Buchalla, Isidori 98]
- small 1% effect from ICPV
  [Buchalla, Buras 96]
- $\not{CP}$: only top contribution
- very clean and sensitive to SD

$K \to \pi\nu\bar{\nu}$: Branching Ratios

$$\text{Br}\left(K_L \to \pi^0\nu\bar{\nu}\right) = \kappa_\nu^L \left(1 - \sqrt{2}|\epsilon_K|\frac{1+P_c(X)/A^2X_t-\rho}{\eta}\right)\left(\frac{\text{Im}\lambda_t}{\lambda^5}\,X_t\right)^2$$



Numbers

- upper bound from E391a

$$\text{Br}^{exp} < 6.7 \times 10^{-8}$$

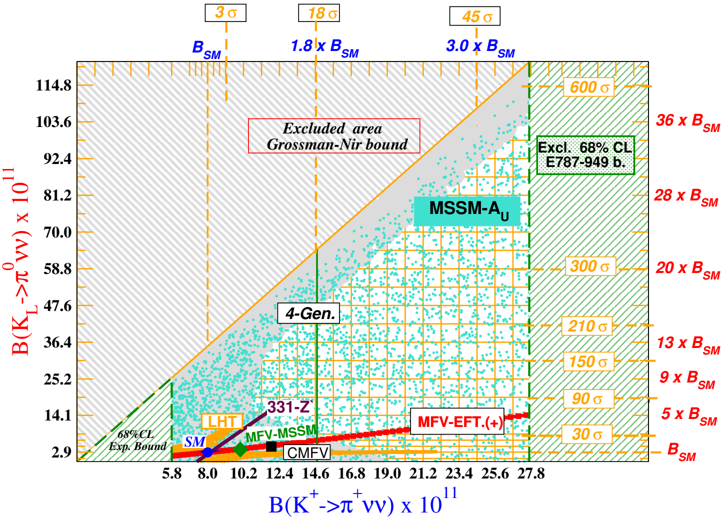
$$\text{Br}^{the} = 2.57^{+0.38}_{-0.36} \pm 0.04 \times 10^{-11}$$

[Brod, Gorbahn, ES 10]

- KOTO $\mathcal{O}(100)$ events ?

$K \to \pi\nu\bar{\nu}$: New Physics

The $\mathcal{B}(K_L \to \pi^0 \mu^+ \mu^-) / \mathcal{B}(K_L \to \pi^0 e^+ e^-)$-plane



[http://www.lnf.infn.it/wg/vus/content/Krare.html]

Conclusions

Even after 60 years of intensive Kaon studies (and 2 Nobel prices)

Kaons still excite us!

Together with ε_K, rare K-decays:

- probe high energy regimes — E $\gg$ TeV
- provide stringent bounds on BSM models
- discriminate between models BSM — (e.g. $K_L \to \pi^0 l^+ l^-$)
- can measure NP parameters — (e.g. $K \to \pi\nu\bar{\nu}$)

Interesting times are ahead!