Standard Model Theory of Neutron & Nuclear Beta Decay

(Electroweak Radiative Corrections)

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Refs: A. Sirlin RMP 50, 573 (1978) + earlier work

WJM & A. Sirlin, PRL 56, 22 (1986); ibid 96, 032002 (2006)

A. Czarnecki, WJM, A. Sirlin, PRD 70, 093006 (2004)

$SU(2)_L x U(1)_Y$ Standard Model Electroweak Radiative Corrections to $\mu \rightarrow e \bar{\nu}_e \nu_\mu$ and $n \rightarrow p e \bar{\nu}_e$ both Infinite but renormalized using ($G_F^0 \rightarrow G_\mu$)

Quark mixing divergences absorbed in $V^0_{ud} \rightarrow V_{ud}$ maintaining Unitarity

The CKM Quark Mixing Matrix:

$$V^{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$ 3x3 Unitary Matrix

Unitarity $\rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Any "Apparent" Deviation from 1 Implies "New Physics" at the tree or quantum loop level

Short-distance behavior of μ and β decays differ due to weak hypercharges of μ_L(Y=-1) and $d_{L.}$(Y=+1/3)

Nevertheless, the ratio of τ_n and τ_μ is finite and calculable

- Muon Decay $\Gamma_0(\mu \rightarrow e\nu\nu)=F(m_e^2/m_\mu^2)G^0{}_F{}^2 m_\mu{}^5/192\pi^3= 1/\tau_\mu{}^0$
- Neutron Decay $\Gamma_0(n \rightarrow pe\nu)=\mathbf{f}G^0{}_F{}^2|V^0{}_{ud}|^2 m_e{}^5(1+3g_A{}^2)/2\pi^3=1/\tau_n{}^0$

$F(x)=1-8x+8x^3-x^4-12x^2\ln x$ Phase Space Factor

f=1.6887 phase space factor, including Fermi function(~5.6%), proton recoil, finite nucleon size… Uncertainty<10^{-4}

Other Effects: Weak Magnetism, Induced Pseudoscalar etc. negligible

g_A and τ_n important for Unitarity test, solar neutrino flux, primordial abundances, spin content of proton, Goldberger-Treiman/Muon Capture, Bjorken Sum Rule, lattice benchmark…

Must be precisely determined!

Electroweak Radiative Corrections to Muon Decay

Virtual One Loop Corrections + Inclusive Bremsstrahlung

Absorb Ultraviolet divergences and some finite parts in

$G^0_F = g_0^2/4\sqrt{2}m^2_{W0} \rightarrow G_\mu$

$\boldsymbol{\tau_\mu^{-1} = \Gamma(\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu(\gamma)) \equiv F(m_e^2/m_\mu^2) G_\mu^2 m_\mu^5 [1+RC]/192\pi^3}$

$RC = \alpha/2\pi(25/4-\pi^2)(1+\alpha/\pi[2/3\ln(m_\mu/m_e)-3.7)\ldots]$ Fermi Th.

Other SM and **"New Physics"** radiative corrections absorbed into G_μ. Eg. Top Mass, Higgs Mass, Technicolor, Susy,W*…

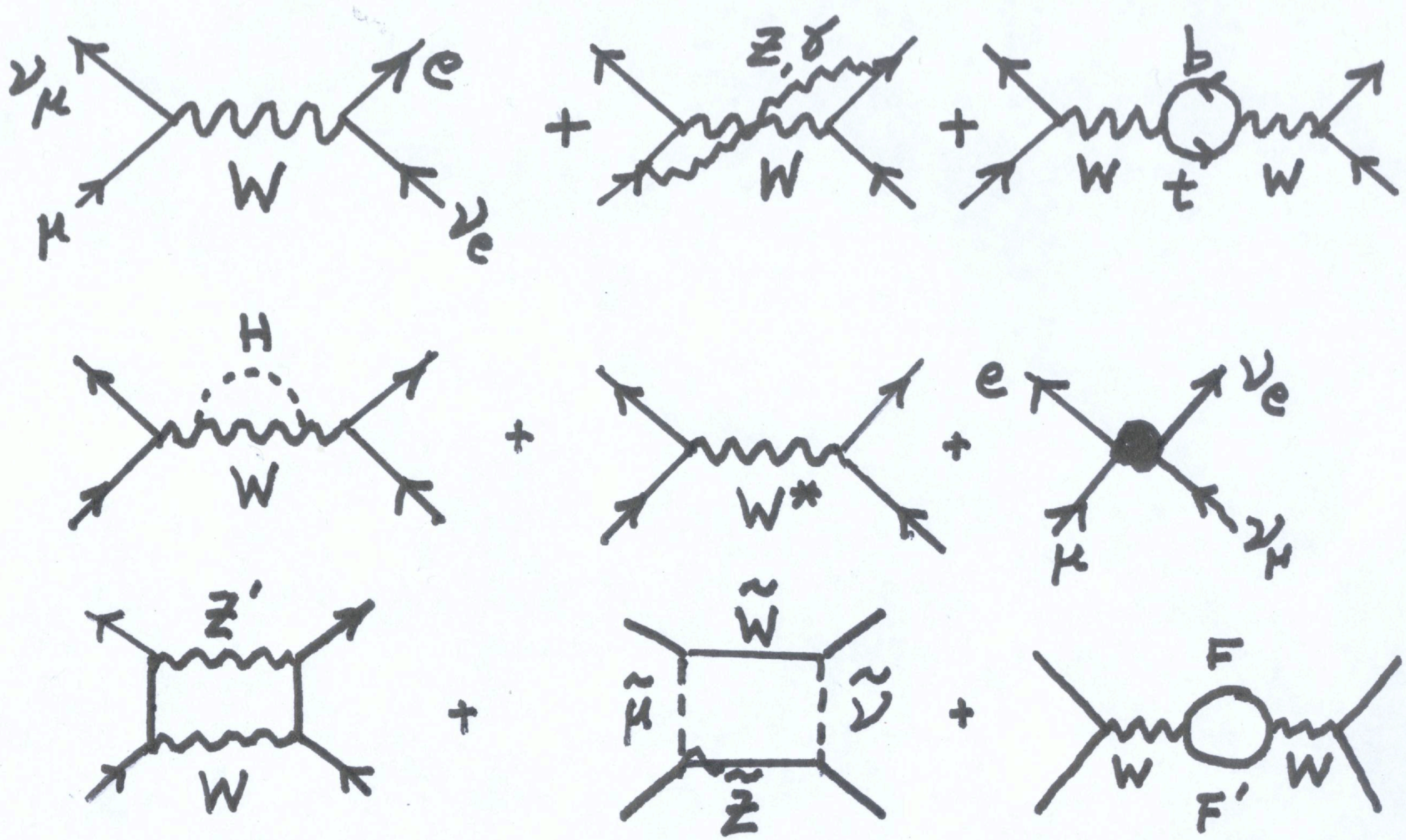
MuLAN & FAST experiments at PSI:

New World Ave. $\tau_{\mu+}$ 1ppm! gives

$G_\mu = 1.166361 \times 10^{-5}\,\mathrm{GeV}^{-2}$ precise & important

(see P. Debevec's talk)

Loop and Tree Level Corrections to Muon Decay



Z' Boson SUSY Technicolor

+ . . .

Electroweak Radiative Corrections to Neutron Beta Decay

Include Virtual Corrections + Inclusive Bremsstrahlung

Normalize using G_μ from the muon lifetime

Absorbs Ultraviolet Divergences & some finite parts

$$1/\tau_n = f G_\mu^2 |V_{ud}|^2 m_e^5 (1+3g_A^2)(\underline{1+RC})/2\pi^3$$

f=1.6887 (Includes Fermi Function)

RC calculated for (Conserved) Vector Current since it is not renormalized by strong interaction at zero momentum transfer.

Same RC used to **define** g_A: $[A(g_A)=(1.001)A^{exp}]$

$$RC = \alpha/2\pi[\overline{g}(E_m) + 3\ln(m_Z/m_p) + \ln(m_Z/m_A) + 2C + A_{QCD}]$$

+ higher order

$g(E_e)$=Universal Sirlin Function (1967) from Vector Current

$\alpha/2\pi\ \bar{g}(E_m=1.292579\text{MeV})=0.015056$ long distance loops and brem. averaged over the decay spectrum. Independent of Strong Int. up to $O(E_e/m_P)$

$g(E_e)$ also applies to Nuclei A. Sirlin (1967)

$3\alpha/2\pi\ln(m_Z/m_p)$ short-distance (Vector) log ***not*** renormalized by strong int.

$[\alpha/2\pi[\ln(m_Z/m_A)+2C+A_{QCD}]$ Induced by axial-current loop

Includes hadronic uncertainty

$m_A=1.2$GeV long/short distance matching scale (factor 2 unc.)

$C=0.8g_A(\mu_N+\mu_P)=0.891$ (long distance γW Box diagram) WJM&A.Sirlin(1986)

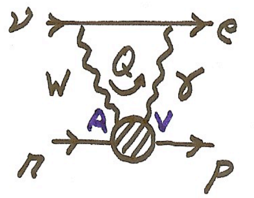
$A_{QCD}= -\alpha_s/\pi(\ln(m_Z/m_A)+\text{cons})=-0.34$ QCD Correction

$[\alpha/\pi\ln(m_Z/m)]^n$ leading logs summed via renormalization group,
Next to leading short distance logs~-0.0001,
and $-\alpha^2\ln(m_p/m_e)=-0.00043$ estimated (for neutron decay)
Czarnecki, WJM, Sirlin (2004) 1+RC=1.0390(8) main unc. from m_A
matching short and long distance γW (VA) Box

γW Box Diagram

Weak Axial-Vector Induced Radiative Corrections

AV Loop → V → Superallowed β-decays

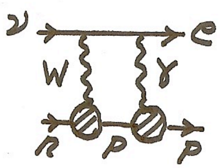


$\longrightarrow \underbrace{\bar{u}\gamma_\mu d}_{\text{Vector}}\, \bar{e}_L \gamma^\mu \nu_L + QCD$

$$RC = \frac{\alpha}{4\pi}\int_0^\infty dQ^2 \frac{m_W^2}{Q^2+m_W^2} F(Q^2)$$

Large Q^2 $\quad F(Q^2) = \frac{1}{Q^2}\left[1 - \frac{\alpha_s(Q^2)}{\pi} + \dots\right] + O\left(\frac{1}{Q^4}\right)$

Small Q^2 → Nucleon Form Factors



$\rightarrow \frac{\alpha}{2\pi}\left(0.8 g_A (\mu_p + \mu_n)\right)$

$$\frac{\alpha}{2\pi}\left\{ \ln\frac{m_Z}{m_A} + \underbrace{A_g}_{\text{QCD}} + \underbrace{2C}_{\text{Long Distance}} \right\}$$

m_A = matching

2006 Improvement WJM & A. Sirlin

1.) Use large N_{QCD} Interpolator to connect long-short distances

2.) Relate neutron beta decay to Bjorken Sum Rule ($N_F=3$)

$1-\alpha_s/\pi \rightarrow 1-\alpha_s(Q^2)/\pi-3.583(\alpha_s(Q^2)/\pi)^2-20.212(\alpha_s(Q^2)/\pi)^3$

$-175.7\,(\alpha_s(Q^2)/\pi)^4$ (Recent Baikov,Chetyrkin and Kuhn)

Negligible Effect

The extra QCD corrections lead to a matching between short and long distance corrections at about $Q^2=(0.8\text{GeV})^2$

Very little change in size of RC, but uncertainties reduced by at least a factor of 2!

(Both Prescriptions Agree)

1+RC= 1.0390(8)→1.03886(39) for Neutron Beta Decay

Reduction by 1.4×10^{-4} (Same for $0^+\rightarrow0^+$ beta decays)

RC Error Budget

1) Neglected Two Loop Effects: **±0.0001** conservative

2) Long Distance $\alpha/\pi C \sim \alpha/\pi\ (0.75 g_A(\mu_N+\mu_P))=0.0020$

Assumed Uncertainty ±10%→**±0.0002** reasonable?

3) Long-Short Distance Loop Matching: 0.8GeV<Q<1.5GeV

±*100*% → **±0.0003** conservative

Total RC Error **±0.00038** →ΔV_{ud}=**±0.00019**

More Aggressive Analysis →ΔV_{ud}=±0.00013

(1/2 conservative)

Superallowed ($0^+ \to 0^+$) Beta Decays & V_{ud}

RC same as in Neutron Decay but with $\bar{g}(E_m)$ averaged Nuclear decay spectrum, C modified by Nucleon-Nucleon Interactions and $+Z\,\alpha^2 \ln(m_p/m_e)$ corrections (opposite sign from neutron)

$$ft = |V_{ud}|^2(2984.5s)(1+RC)(1+NP\ corr.)$$

Nuclear Physics (NP) isospin breaking effects

(Hardy & Towner Calculations: See later critique)

ft values + RC for 13 precisely measured nuclei found to be consistent with CVC: Average $\to V_{ud}$

Superallowed Nuclear Beta Decays

RC Uncertainty-Same as Neutron Decay

Nuclear Unc. - Significantly Reduced (2006-08)

Nuclear Coulomb Corrections Improved

$$|V_{ud}| = \underline{0.97425}(11)_{Nuc}(19)_{RC}$$

(2008 Hardy and Towner Update)

(0.97418((13)(14)(19) in PDG08)

(0.97377(11)(15)(19) in PDG06)

(0.97340(80) in 2004) Factor of 3 worse

The Kaon Revolution of 2004-2005

(Starting with BNL E865) +FNAL, Frascati & CERN

BR($K \to \pi e \nu$) increased by ≈6%!

All Major K_L BRs Changed! ε_K changed by 3.7σ!

Now Based on: $\Gamma(K \to \pi l \nu)_{exp}$ & $\Gamma(K \to \mu\nu)/\Gamma(\pi \to \mu\nu)_{exp}$

\+ Lattice Matrix Elements $f_+(0)=0.960(5)$ & $f_K/f_\pi=1.193(6)$

***2010 Flavianet Analysis Currently*:**

$|V_{us}|$=0.2253(13) from $K \to \pi l \nu$ Vector

$|V_{us}|$=0.2252(13) from $K \to \mu\nu$ Axial-Vector

$|V_{us}|$=0.2253(9) Kaon Average (was ~0.220 pre 2004)

(Watch for lattice updates)

CURRENT STATUS of CKM Unitarity

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9999(4)_{Vud}(4)_{Vus}$$

$$=0.9999(6)$$

Outstanding Agreement With Unitarity

Confirms CVC & SM Radiative Corrections:

$2\alpha \ln(m_Z/m_p)/\pi + \ldots \approx +3.6\%$ at 60 sigma level!

Naively Fits m_Z=90(7)GeV vs 91.1875GeV (Direct)

Comparison of G_μ with other measurements (normalization) constrains or unveils **"New Physics"**

New Physics Constraints-Implications:

Exotic Muon Decays, W*bosons, SUSY, Technicolor,

Z' Bosons, $H^\pm$, Heavy Quark/Lepton Mixing…

Recent Superallowed Beta Decay Issue

- Isospin Breaking Coulomb Corrections of Hardy and Towner ***questioned*** by: G. Miller & A. Schwenk

  N. Auerbach

  H. Liang et al.

Hardy and Towner (1-δ_C) correction increases V_{ud}

δ_C~0.2-1.6% Correction

Recent Claims δ_C is smaller due to nuclear radial excitations

smaller V_{ud}=0.97425→0.9730 (Liang, Giai, Meng)

$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2$ reduced to (roughly) 0.9975?

Unitarity Lost?

Issue needs "complete" quantitative resolution

(see talk by I. Towner)

Neutron Decay ($n\rightarrow pe\bar{\nu}$) & V_{ud}

$$|V_{ud}|^2 = \frac{4908.7(1.9)\text{sec}}{\tau_n(1+3g_A^2)}$$ **(V&A Interactions)**

Measure τ_n and $g_A \equiv G_A/G_V$ (decay asymmetries)

2008 PDG τ_n^{ave}=885.7(8)sec, g_A^{ave}=1.2695(29)

$\rightarrow |V_{ud}|^{ave}=0.9746(4)_{\tau n}(18)_{gA}(2)_{RC}$ **reasonable but …**

More recent 878.5(8)sec? & $g_A\approx$1.2750(9)

$\rightarrow |V_{ud}|=\underline{0.9751}(4)_{\tau n}(6)_{gA}(2)_{RC}$ **also reasonable**

(Are τ_n & g_A both shifting?)

History g_A=1.18→1.23→1.25→1.26→1.27→?

$0^+\rightarrow 0^+$ Nuclear Beta V_{ud}=0.97425(22) + τ_n= 878.5(8)sec

$\rightarrow g_A\approx\underline{1.2763}(8)$!

Many New τ_n & g_A Experiments Planned

Conclusion

1) ***Current Exps & Th:*** $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9999(4)_{Vud}(4)_{Vus}$

Great Unitarity Test & Sucess→ No New Physics!

Nuclear Isospin Breaking? Needs Resolution

Radiative Corrections Stable *(Unchallenged!)*

2) ***Neutron Decay:*** $|V_{ud}|=[4908.7(1.9)s/\tau_n(1+3g_A^2)]^{1/2}$ **clean & precise**

Neutron Lifetime Controversy (6σ discrepancy)

$\tau_n^{PDG}=885.7(8)s$ **vs** $\tau_n=878.5(8)s$ **Needs Resolution**

g_A **larger? PerkeoII→1.2750(9) vs** $g_A^{PDG}=1.2695(29)$

Larger g_A & smaller τ_n → Unitarity, solar neutrino flux, primordial nuclear abundances, proton spin, Goldberger-Treiman/Muon Capture, Bjorken Sum Rule, lattice calculation benchmark…

V_{ud}, τ_n and g_A must be precisely determined!