Measurements of moments in $B \to X_c l\nu$ and fits for $|V_{cb}|$

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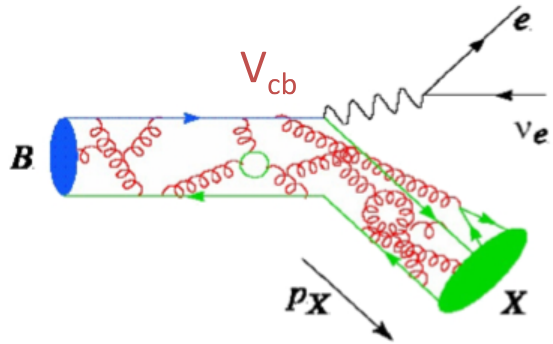
Outline of the talk



- The theoretical framework
  - As seen by an experimentalist
- Review of recent moment measurements
  - BaBar 2010 [PRD81, 032003]
  - Belle 2007 [PRD75, 032001; PRD75, 032005]
- $|V_{cb}|$ and m_b from the global fit

The semileptonic width

- Semileptonic decays b → c are a weak transition plus QCD corrections
- The theoretical tool is the operator product expansion (OPE)

Perturbative corrections
(α_s expansion)

$$\Gamma(\bar{B} \to X_c \ell \bar{\nu}) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 (1+A_{\rm ew}) A^{\rm pert}(r,\mu) F(r, \frac{\mu_\pi^2}{m_b^2}, \frac{\mu_{\rm G}^2}{m_b^2}, \frac{\rho_{\rm D}^3}{m_b^3}, \frac{\rho_{\rm LS}^3}{m_b^3}, \mu)$$

$$r = \frac{m_c}{m_b}$$

Leading order
(free quark)

Non-perturbative corrections
(Λ/m_b expansion)

- Perturbative corrections known to $O(\alpha^2_s)$ for the leading terms
- Non-perturbative corrections to $O(1/m^3_b)$ typically used in analyses
- However, this prediction depends on the quark masses (m_b, m_c) and the heavy quark parameters (μ^2_π, μ^2_G, ρ^3_D, ρ^3_{LS}) which are poorly constrained by theory

Moments of the E_l and M^2_X spectrum

Also other observables in B → Xlν can be expanded into an OPE with the same heavy quark parameters, e.g.,

- The n[th] moment of the (truncated) lepton energy spectrum

$$R_n(E_{\rm cut},\mu) = \int_{E_{\rm cut}} (E_\ell - \mu)^n \frac{{\rm d}\Gamma}{{\rm d}E_\ell}\,{\rm d}E_\ell\,, \quad \langle E_\ell^n \rangle_{E_{\rm cut}} = \frac{R_n(E_{\rm cut},0)}{R_0(E_{\rm cut},0)}$$

- The n[th] moment of the (truncated) M^2_X spectrum

$$\langle m_X^{2n} \rangle_{E_{\rm cut}} = \frac{\int_{E_{\rm cut}} (m_X^2)^n \frac{{\rm d}\Gamma}{{\rm d}m_X^2}\,{\rm d}m_X^2}{\int_{E_{\rm cut}} \frac{{\rm d}\Gamma}{{\rm d}m_X^2}\,{\rm d}m_X^2}$$

Master plan:

- Measure the quark masses and heavy quark parameters using moments
- Substitute them in the formula of the semileptonic width
- Determine $|V_{cb}|$ from the semileptonic branching fraction

Two sets of theoretical calculations

- Kinetic running mass
  - [P.Gambino, N.Uraltsev, Eur.Phys.J. C34, 181 (2004)]
  - [D.Beson, I.Bigi, N.Uraltsev, Nucl.Phys. B710, 371 (2005)]
- 1S mass
  - [C.Bauer, Z.Ligeti, M.Luke, A.Manohar, M.Trott, Phys.Rev. D70, 094017 (2004)]
- Non-perturbative parameters in the $1/m_b$ expansion

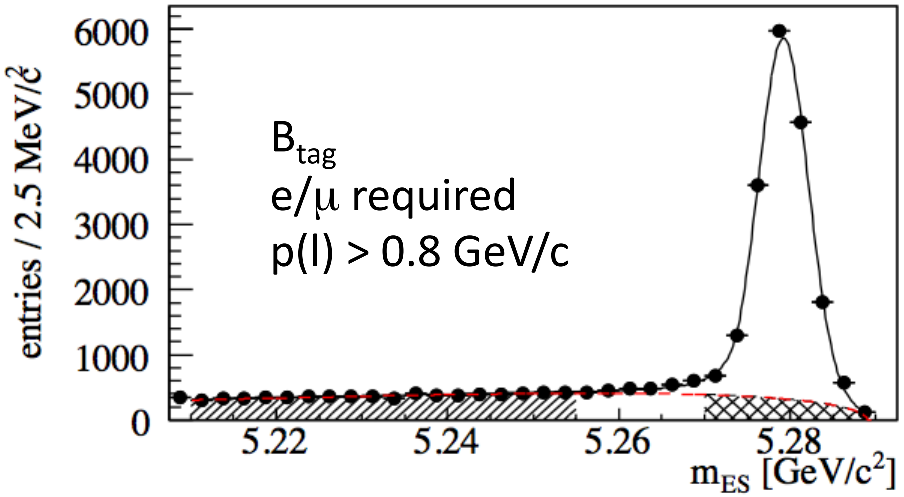
	Kinetic scheme	1S scheme
O(1)	m_b, m_c	m_b
$O(1/m^2_b)$	μ^2_π, μ^2_G	λ_1, λ_2
$O(1/m^3_b)$	ρ^3_D, ρ^3_{LS}	ρ_1, $\tau_{1\text{-}3}$

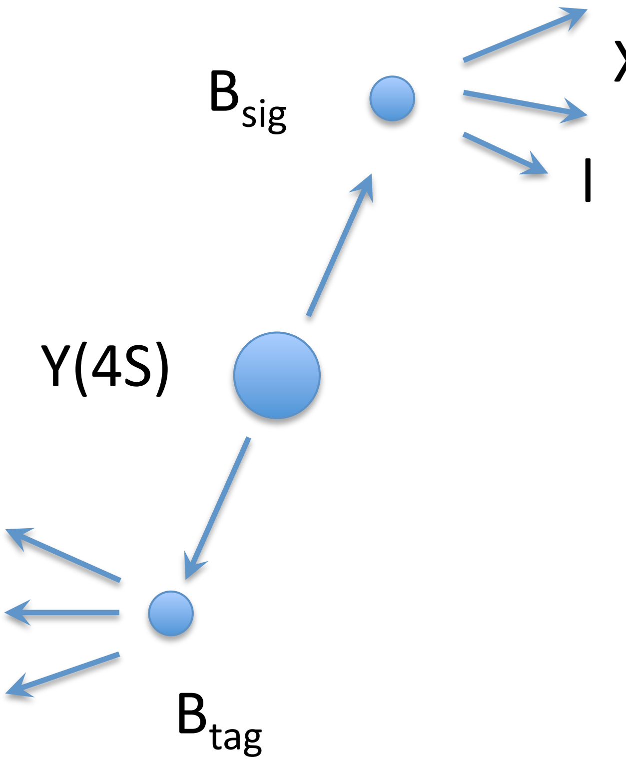
PRD 81, 032003 (2010)

BaBar hadronic moments

232M BB

- Fully reconstruct the hadronic decay of one B in Y(4S) → BB (efficiency ~0.4%, purity ~80%)





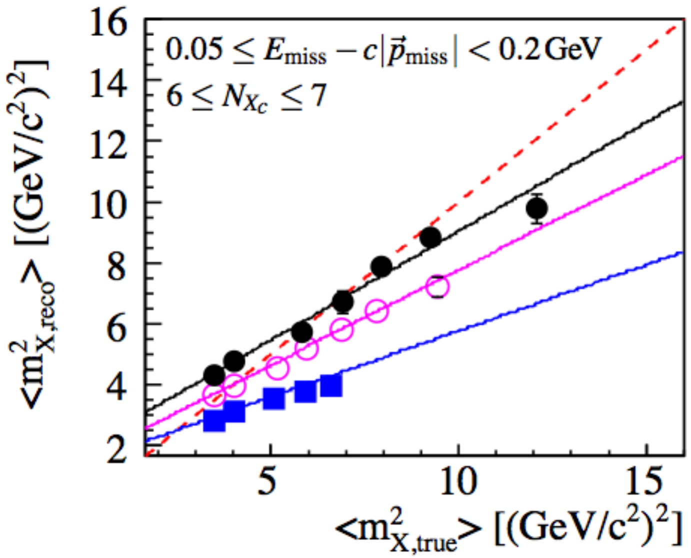
- Require one identified lepton amongst the signal-side particles (p > 0.8 GeV/c)
- Combine all remaining particles to the X system and do a kinematic fit
  - 4-momentum conservation
  - Missing mass consistent with zero mass neutrino

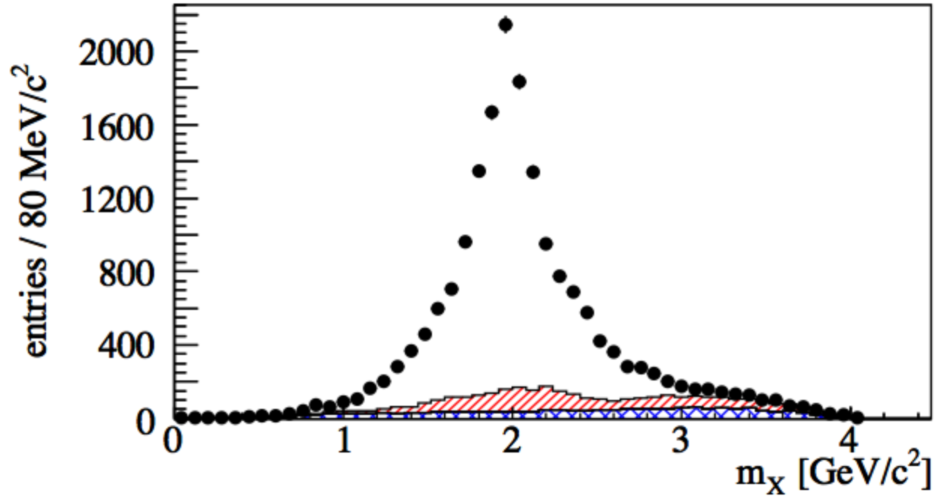
Moment measurement

PRD 81, 032003 (2010)

232M BB

- Hadronic mass spectrum after kinematic fit





- Linear correction of the measured moments in bins of X multiplicity, E_{miss}-cp_{miss} and lepton momentum

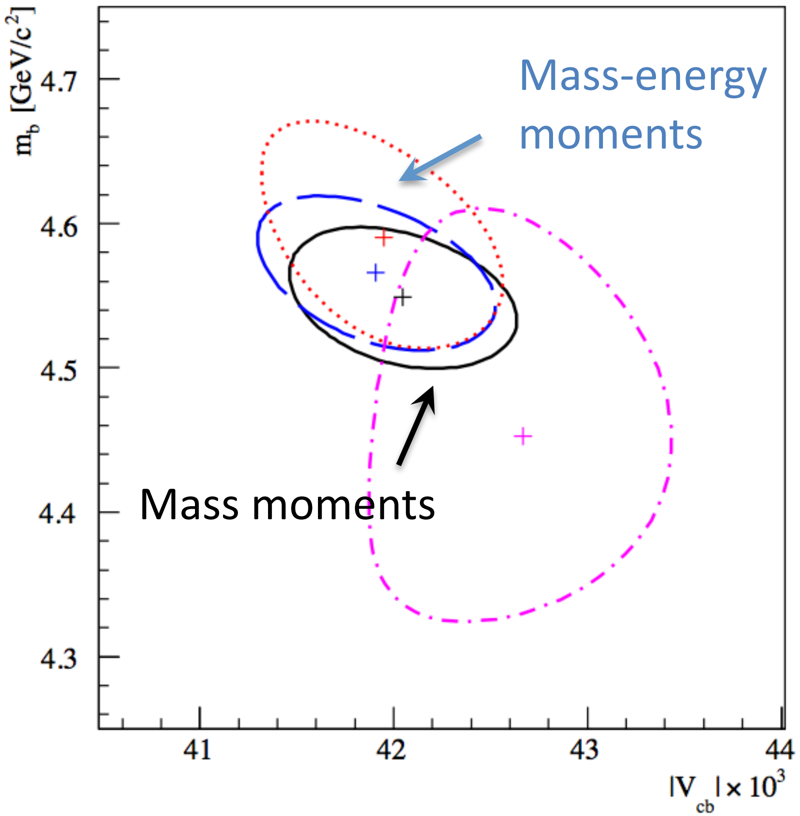
- Moments of the hadronic mass spectrum up to M^6_X for E_{cut} between 0.8 and 1.9 GeV are measured
- Also mixed mass-energy moments are determined and the electron energy moments from [PRD69, 111104] are re-evaluated

BaBar only HQE fit

PRD 81, 032003 (2010)

232M BB

- Two OPE fits in the kinetic scheme to
  - 12 hadronic mass moments, or
    12 combined mass-energy moments
  - 13 energy moments (including partial branching fractions)
  - 3 photon energy moments in $B \to X_s \gamma$



	Hadronic moments	Mass-energy moments
$\lvert V_{cb}\rvert$ (10^{-3})	42.05 +/- 0.45 +/- 0.70	41.91 +/- 0.48 +/- 0.70
m_b (GeV^2/c^2)	4.549 +/- 0.031 +/- 0.038	4.556 +/- 0.034 +/- 0.041
Br($B \to X_c l \nu$) (%)	10.64 +/- 0.17 +/- 0.06	10.64 +/- 0.17 +/- 0.06
χ^2/ndf	10.9 / 28	8.2 / 28

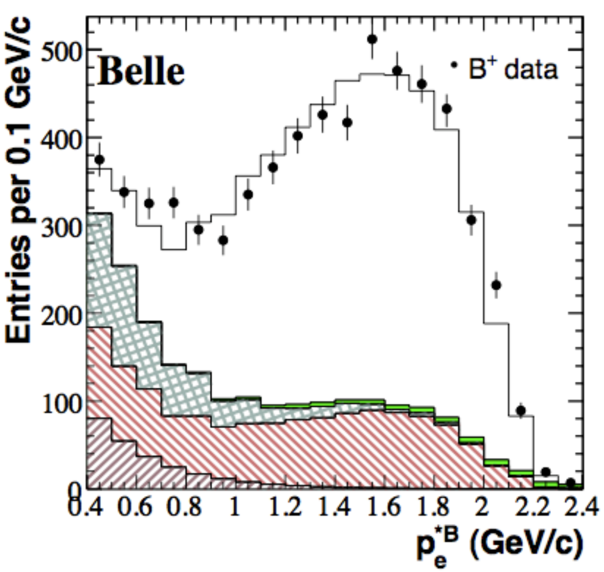
- First error is experimental, second theoretical

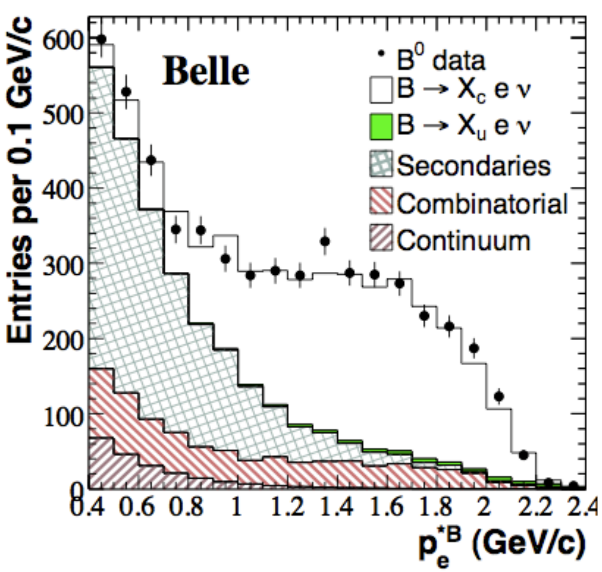
Belle E_l and M^2_X moments

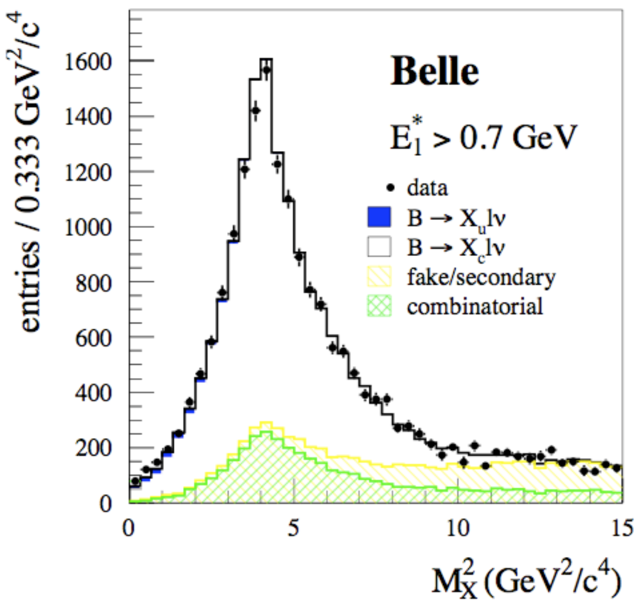
[PRD 75, 032001 (2007)]
[PRD 75, 032005 (2007)]

152M BB

- For both the E_l and M^2_X measurements, similar experimental method using fully reconstructed events
- The finite detector resolution is unfolded with SVD algorithm [NIM A372, 469 (1996)]
- $<E^n_e>$ measured for n=0,…,4 and E_{cut}=0.4-2.0 GeV
- $<M^{2n}_X>$ measured for n=1,2 and E_{cut}=0.7-1.9 GeV





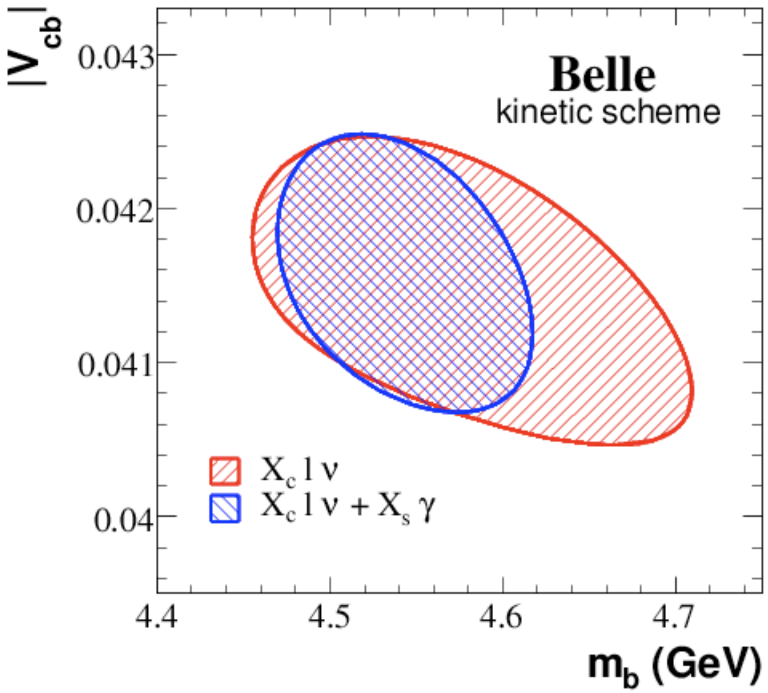


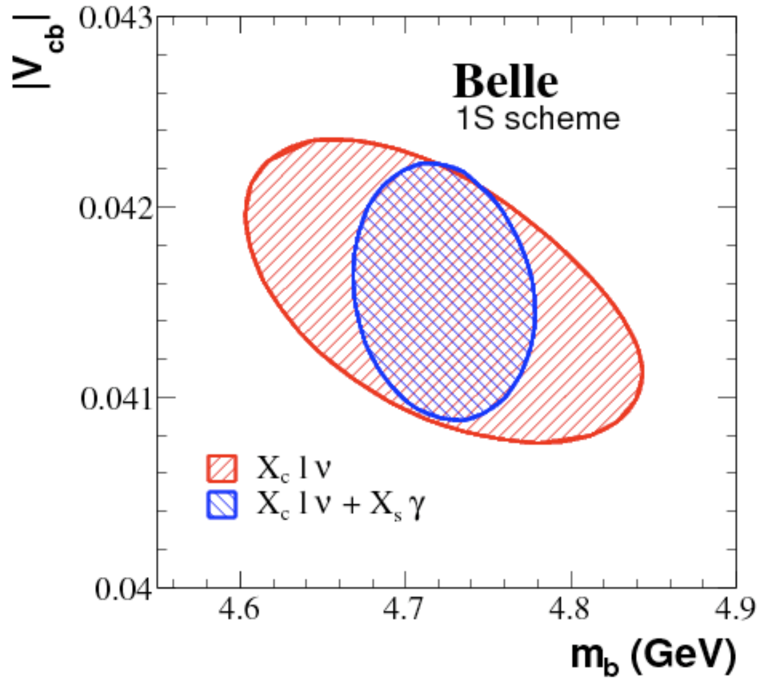
[PRD 78, 032016 (2008)]

Belle only HQE fit

152M BB

- 25 moments used
  - 14 energy moments
  - 7 mass moments
  - 4 photon energy moments in B → $X_s\gamma$
- Fits in kinetic and 1S schemes





	Kinetic scheme	1S scheme
$\lvert V_{cb}\rvert$ (10^{-3})	41.58 +/- 0.90	41.56 +/- 0.68
Br(B → $X_c l\nu$) (%)	10.49 +/- 0.23	10.60 +/- 0.28
χ^2/ndf	4.7 / 18	7.3 / 18

HFAG global fit

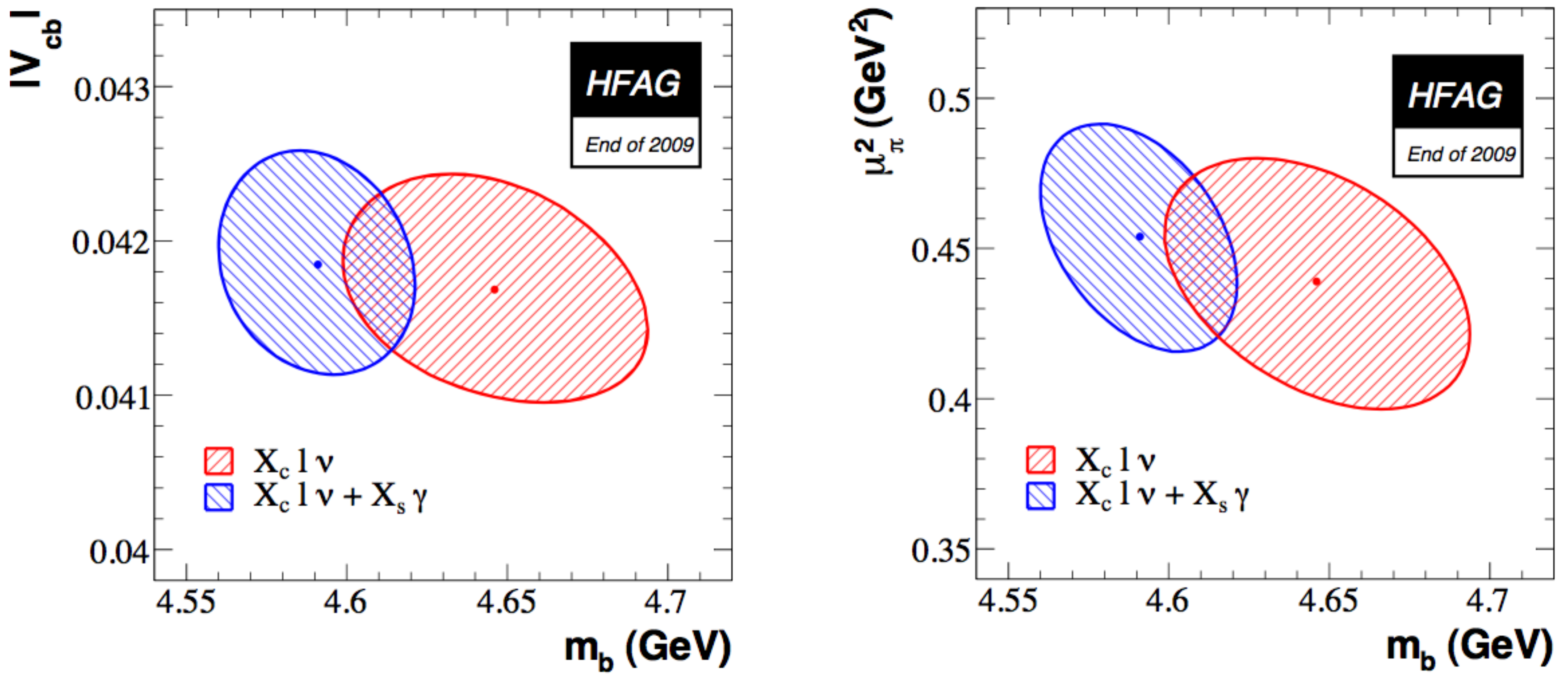
- Combine moment measurements of different experiments (BaBar, Belle, CLEO, CDF, DELPHI)
- Kinetic scheme fit
  - 7 free parameters: the quark masses m_b and m_c, the HQ parameters μ^2_π, μ^2_G, ρ^3_D and ρ^3_{LS} and the $b \rightarrow c$ branching fraction
  - $|V_{cb}|$ calculated using [Nucl. Phys. B665, 367 (2003)]
- 1S scheme fit
  - 7 free parameters: m_b, λ_1, ρ_1, τ_1, τ_2, τ_3 and $|V_{cb}|$
- Only external input: B lifetime

Dataset

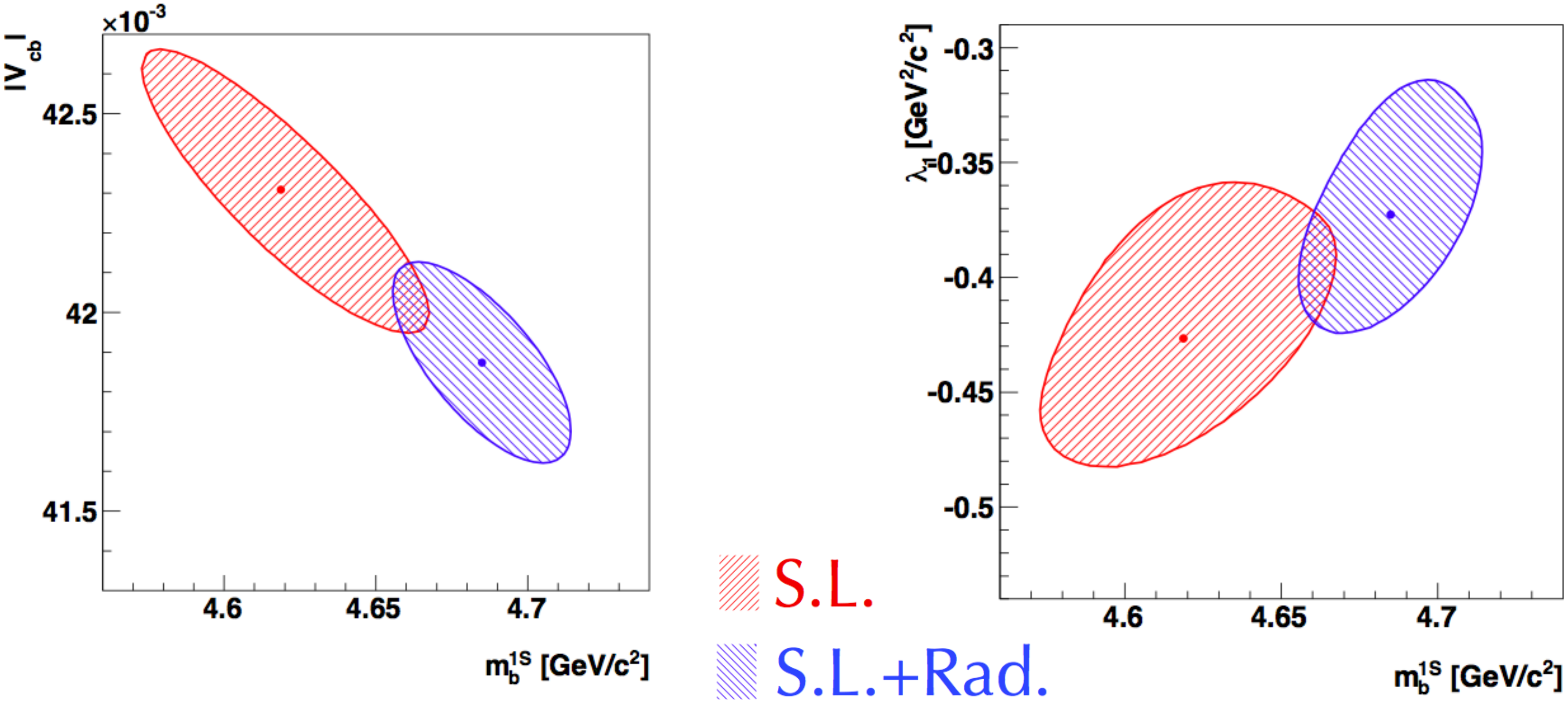
- 66 measurements used
  (29 from BaBar, 25 from Belle and 12 from other expts)

BaBar	$\langle E^n_l \rangle$: n=0,1,2,3 [PRD 69, 111104 (2004), PRD 81, 032003 (2010)] $\langle M^{2n}_X \rangle$: n=1,2, 3 [PRD 81, 032003 (2010)] $\langle E^n_\gamma \rangle$: n=1,2 [PRL 97, 171803 (2006), PRD 72, 052004 (2005)]
Belle	$\langle E^n_l \rangle$: n=0,1,2,3 [PRD 75, 032001 (2007)] $\langle M^{2n}_X \rangle$: n=1,2 [PRD 75, 032005 (2007)] $\langle E^n\gamma \rangle$: n=1,2 [PRL 103, 241801 (2009)]
CDF	$\langle M^{2n}_X \rangle$: n=1,2 [PRD 71, 051103 (2005)]
CLEO	$\langle M^{2n}_X \rangle$: n=1,2 [PRD 70, 032002 (2004)] $\langle E^n_\gamma \rangle$: n=1 [PRL 87, 251807 (2001)]
DELPHI	$\langle E^n_l \rangle$: n=1,2,3 $\langle M^{2n}_X \rangle$: n=1,2 [EPJ C45, 35 (2006)]

P.S. The 1S analysis still uses the 2004 BaBar hadronic moments
[PRD 69, 111103 (2004)]



Input	$\|V_{cb}\|$ (10^{-3})	m^{kin}_b (GeV)	μ^2_π (GeV^2)	χ^2/ndf
All moments	41.85+/-0.42(fit)+/-0.09 (τ_B)+/-0.59(th)	4.591+/-0.031	0.454+/-0.038	29.7/59
$X_c l\nu$ only	41.68+/-0.44(fit)+/-0.09 (τ_B)+/-0.58(th)	4.646+/-0.047	0.439+/-0.042	24.2/48



Input	$\lvert V_{cb}\rvert$ (10^{-3})	m^{1S}_b (GeV)	λ_1 (GeV^2)	χ^2/ndf
All moments	41.87 +/- 0.25	4.685 +/- 0.029	-0.373 +/- 0.052	32.0/57
$X_c l\nu$ only	42.31 +/- 0.36	4.619 +/- 0.047	-0.427 +/- 0.057	24.2/46

Discussion

- $|V_{cb}|$ from the global fit

	$\lvert V_{cb}\rvert$ (10^{-3})	χ^2/ndf
Kinetic scheme	41.85 +/- 0.73	29.7/59
1S scheme	41.87 +/- 0.25	32.0/57

- $|V_{cb}|$ from $B \rightarrow D^*l\nu$
  - $F(1)|V_{cb}|$ = (36.04 +/- 0.52) x 10^{-3} [HFAG end of 2009]
  - F(1) = (0.921 +/- 0.013 +/- 0.020) [PRD 79, 014506 (2009)]
  - $|V_{cb}|$ = (38.9 +/- 0.7(exp) +/- 1.0(th)) x 10^{-3}
  - ~2.3 σ difference between inclusive and exclusive

- Is there anything in the global fit that can account for this discrepancy?

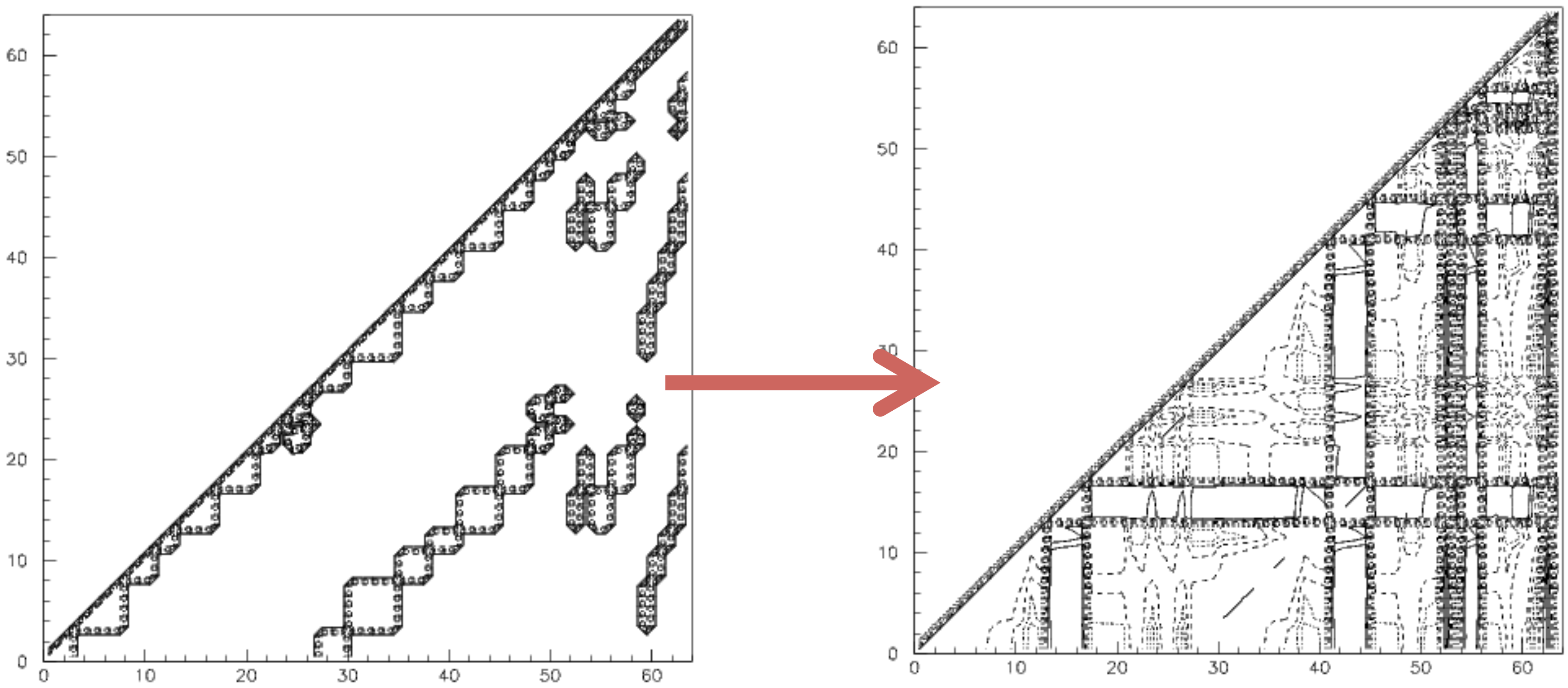
Theory error correlations in the kinetic scheme fit

- For many moments used in the fit, the theory error is larger than the experimental uncertainty
  - Theory errors and their correlations thus strongly influence the result of the fit
- The default fit assumes that
  - The theory error is 100% correlated for the same moment at different E_{cut}
  - The theory errors on different moments (e.g., $<E_l>$ and $<E^2_l>$, $<E_l>$ and $<M^2_X>$, etc.) are uncorrelated

An alternative model for theory correlations

- Having the FORTRAN codes for the theory expressions, I can derive the theory error correlations using a toy MC approach

default | alternative



Results for $|V_{cb}|$, m_b and μ^2_π

	$\|V_{cb}\|$ (10^{-3})	m^{kin}_b (GeV)	μ^2_π (GeV^2)	χ^2/ndf
Default	41.67 +/- 0.73	4.601 +/- 0.034	0.440 +/- 0.040	29.7/57
Alternative	40.85 +/- 0.89	4.605 +/- 0.031	0.312 +/- 0.060	54.2/57
Alternative without correlations	41.95 +/- 0.81	4.589 +/- 0.042	0.446 +/- 0.063	25.9/57

- 'Alternative without correlations' means that in the alternative scenario, correlations between different moments are artifically set to zero
- In these three fits, only the theory error correlations are different
  - Experimental measurements and theory central values and errors are identical

Short message on theory error correlations

- As long as theory correlations between different moments are set to zero, fit results are very similar in terms of $|V_{cb}|$
- Introducing correlations between different moments lowers the value of $|V_{cb}|$ and increases the χ^2 of the fit

Summary

- The OPE can describe inclusive observables in $B \rightarrow X_c l\nu$ and provides a reliable way of extracting $|V_{cb}|$ from inclusive decays
- High precision measurements of the partial $B \rightarrow X_c l\nu$ width and the moments of the lepton energy and hadronic mass spectrum are available, mainly from the B factories
- This allows to determine $|V_{cb}|$ with a global fit analysis

	$\|V_{cb}\|$ (10^{-3})	χ^2/ndf
Kinetic scheme	41.85 +/- 0.73	29.7/59
1S scheme	41.87 +/- 0.25	32.0/57