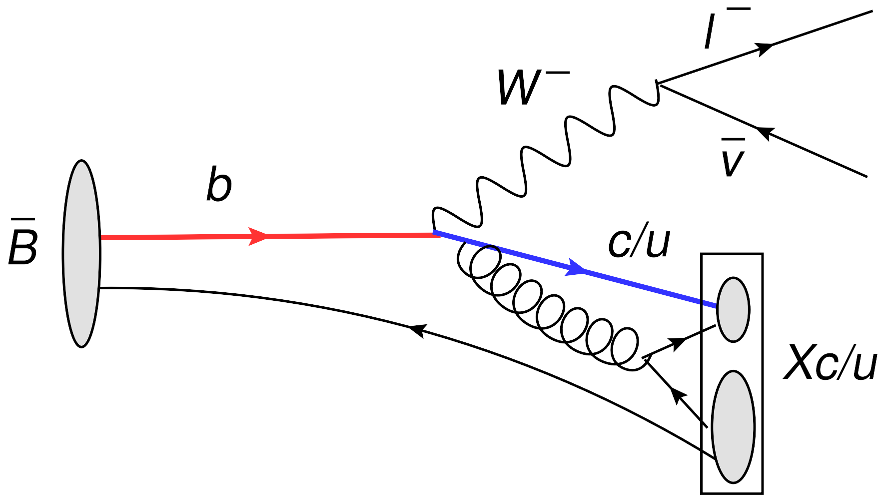
Inclusive determination of $|V_{\rm ub}|$: theory status

Einan Gardi (Edinburgh)

Inclusive vs. Exclusive semileptonic B decays

Inclusive final state



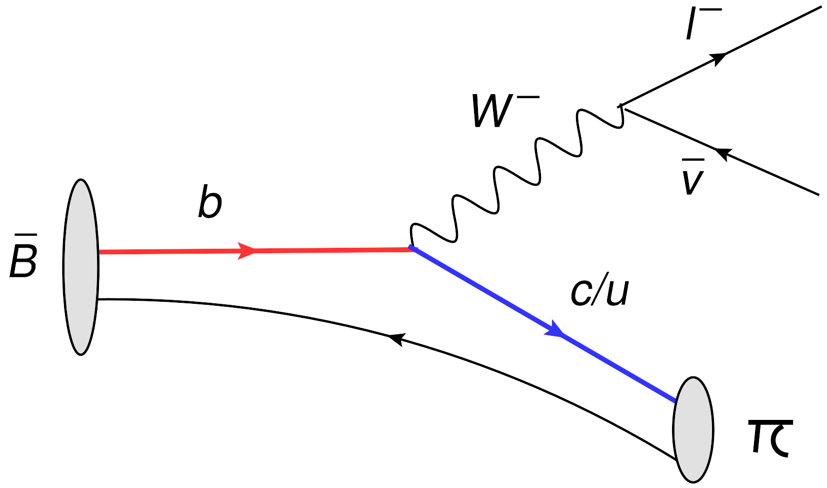
$$\Gamma = \frac{G_F^2 |V_{\rm qb}|^2}{192\pi^3} m_b^5 (1 + \cdots)$$

Can compute in pert. QCD:

confinement is $\mathcal{O}(\Lambda^2/m_b^2)$

but for $b \to u$ most measurements have stringent cuts

Exclusive final state



$$d\Gamma/dq^2 = \frac{G_F^2 |V_{\rm qb}|^2}{192\pi^3} \left| f_+(q^2) \right|^2$$

Experimentally: Good S/B

but — proportional to form factor:

confinement is $\mathcal{O}(1)$ — Lattice or QCD sum rules

Inclusive and Exclusive have different strengths — complementarity!

Semileptonic decay into charm: heavy–quark expansion

- Easy experimentally: large BF ($\gtrsim 10\%$)
- Easy theoretically: confinement effects in moments appear through a few non-perturbative matrix elements of local operators

$$\Gamma(\bar{B} \to X_c l\bar{\nu}) = \underbrace{\Gamma(b \to X_c l\bar{\nu}; \mu)}_{\text{on-shell b-quark decay with IR cutoff}} + \frac{C_1 \mu_\pi^2(\mu) + C_2 \mu_G^2(\mu)}{m_b^2} + \frac{(\ldots)}{m_b^3}$$

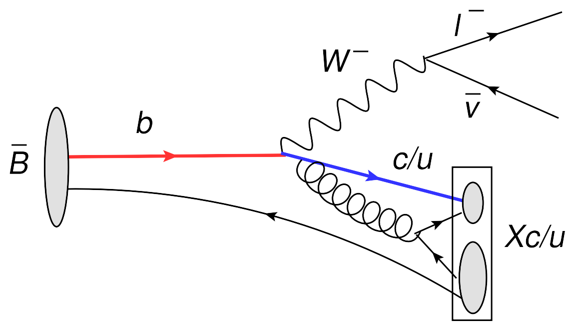
where the kinetic energy $\mu_\pi^2(\mu) \equiv \left\langle \bar{B} \left| \bar{b}\,(i\vec{D})^2\, b \right| \bar{B} \right\rangle_\mu / \, (2M_b)$

- cutoff (μ) dependence cancels order–by–order.
- Yields good fits: determination of $|V_{cb}|$ at $\pm 1\%$ accuracy, as well as very useful constraints on m_b, m_c, and μ_π^2.

Inclusive semileptonic $b \to u$ decays

- Inclusive $b \to u$ has *an overwhelming* charm background:

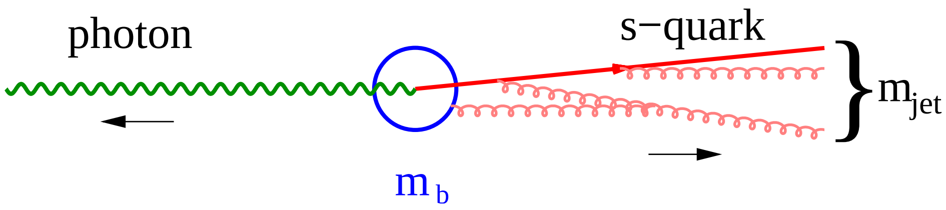
$$\frac{\Gamma(b \to u l^- \bar{\nu})}{\Gamma(b \to c l^- \bar{\nu})} = \frac{|V_{\rm ub}|^2}{|V_{\rm cb}|^2} \simeq \frac{1}{50}$$



- $b \to c$ events always have $M_X > 1.7$ GeV — cuts distinguish them!
- Many experimental analyses; measured branching fraction varies: 20%– 70% of the total (recently $\sim 90\%$)
  $\Longrightarrow$ To extract $|V_{\rm ub}|$ we need to compute the spectrum.
- OPE does not apply in a restricted kinematic region. For small M_X there are large corrections...
- Major progress on the theory side. Different approaches:
  - Expansion in shape functions, matched with OPE (BLNP)
  - Resummed perturbation theory $+$ power corrections (DGE)
  - OPE–based structure–function parametrization (GGOU)

$\bar{B} \longrightarrow X_s\gamma$: jet kinematics and the momentum distribution function

The decay: a large energy release

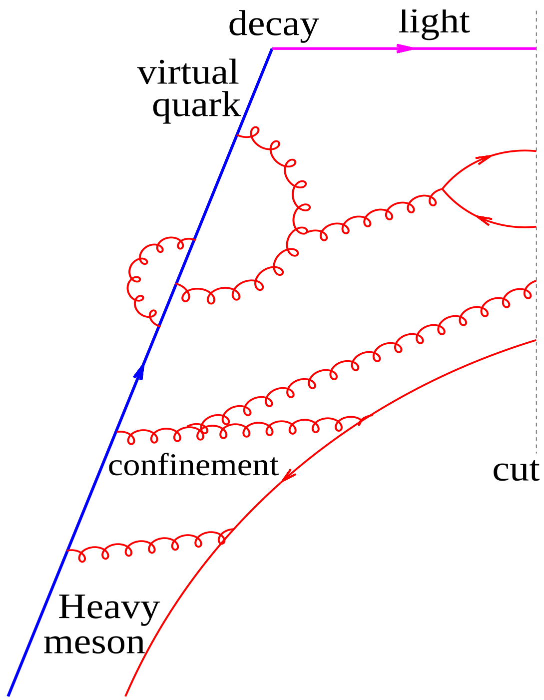


- Collimated jet of particles recoiling against the photon:

$$\frac{d\Gamma}{dE_\gamma} \sim \delta\left(E_\gamma - m_b/2\right)$$

- This spectral line is smeared due to the motion of the decaying b quark, which can be understood as Fermi motion or as a result of soft QCD radiation, gluon momenta $k^+ \ll m_b$.

Analogy with Deep Inelastic Scattering

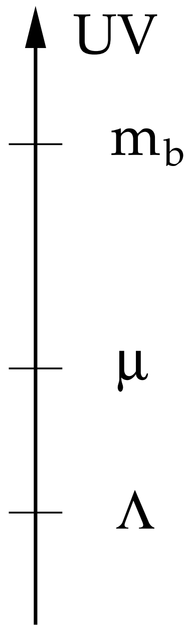


Decay with jet kinematics probes the momentum carried by the b quark field Ψ in the B meson

[Neubert; Bigi *et al.* ('93)]

$$S(k^+;\mu) = \int_{-\infty}^{\infty} \frac{dy^-}{4\pi} \mathrm{e}^{-ik^+y^-} \langle B|\, \bar{\Psi}(y)[y,0]\gamma_+\Psi(0)\, |B\rangle$$

S is the momentum distribution function,
or “shape function"



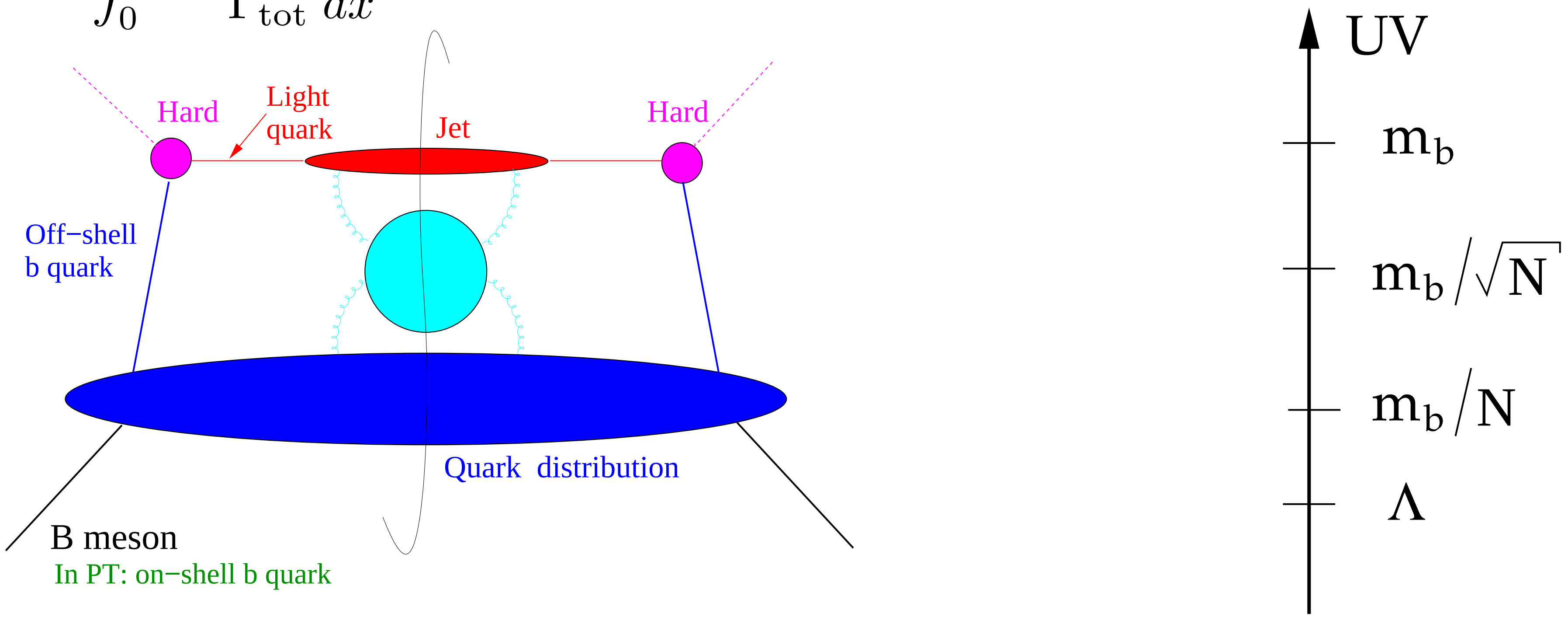
The decay rate (near the end point) is a convolution:

$$\Gamma(P^+) \simeq \int dk^+ C(P^+ - k^+;\mu)\, S(k^+;\mu) \,+\, \mathcal{O}(1/m_b)$$

Factorization in inclusive decays (Korchemsky & Sterman '94)

Define N such that large N probes jet kinematics $x = 1 - p^+/p^- \to 1$:

$$\Gamma_N \equiv \int_0^1 dx \frac{1}{\Gamma_{\rm tot}} \frac{d\Gamma}{dx} x^{N-1} = H(m_b) J(m_b^2/N;\mu) S(m_b/N;\mu) + \mathcal{O}(1/N)$$



Hierarchy of scales $\Longrightarrow$ Factorization $\Longrightarrow$ Sudakov Resummation:

	Hard:		**Jet:**		**Quark Distribution — Soft:**
	m_b	$\gg$	$m_{\rm jet} = m_b\sqrt{1-x}$	$\gg$	$p^+_{\rm jet} \equiv E_{\rm jet} - \lvert\vec{p}_{\rm jet}\rvert = m_b(1-x)$
Moments	m_b	$\gg$	$m_b/\sqrt{N}$	$\gg$	m_b/N

The OPE hard–cutoff approach (GGOU)

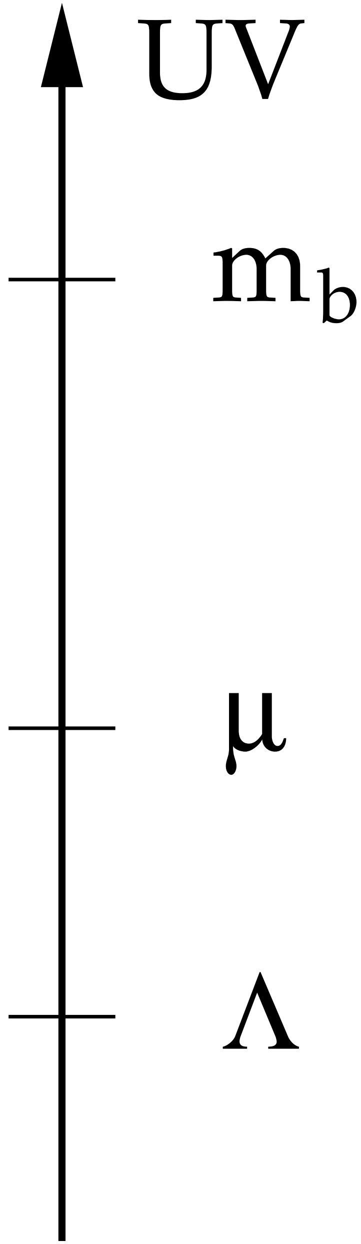
- Gambino, Giordano, Ossola & Uraltsev write each structure function as a convolution:

$$W_i(P^+, q^2) = \int dk^+ F_i(k^+, q^2; \mu) W_i^{\text{pert}}(P^+ - k^+, q^2; \mu)$$

  A hard cutoff $\mu = 1$ GeV is implemented in the 'kinetic scheme'. $F_i(k^+, q^2; \mu)$ are non-perturbative functions, parametrized subject to constrains on the moments of W_i computed by OPE.

- Advantages: simple and prudent! Perturbation theory is used in a safe regime above 1 GeV; the infrared is parametrized.

- Limitations:
  - Extensive parametrization: the unknown functions $F_i(k^+, q^2; \mu)$ depends on *two* kinematic variables.
  - Known structure of infrared singularities not used.

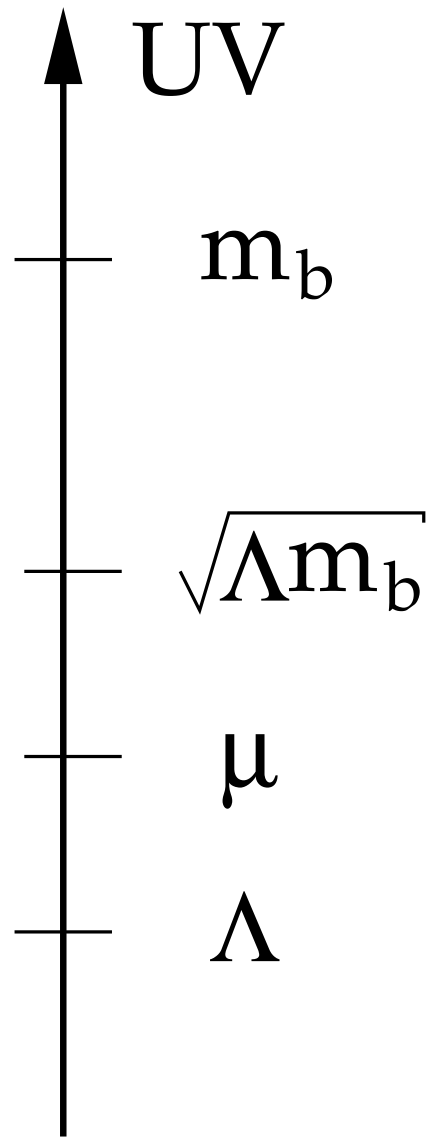


The shape function approach (BLNP)

- For jet kinematics $P^+ \ll P^- \simeq m_b$ one has

$$\frac{d\Gamma}{dP^- dP^+ dE_l} = HJ \otimes S(k^+, \mu) + \frac{\sum H_n J_n \otimes S_n(k^+, \mu)}{m_b} + \cdots$$

- The shape function approach by Bosch, Lange, Neubert & Paz combines a P^+/m_b expansion, valid for jet kinematics, with the local OPE.
- Advantages: elaborate use of theoretical tools. Sudakov resummation of jet logs.
- Limitations:
  - starting at $\mathcal{O}(1/m_b)$ more unknowns than observables
  - Even the first $S(k^+, \mu)$ cannot be computed non-perturbatively. It is parametrized based on known center (m_b) and width (μ_π^2) alone.



NNLO corrections in the shape-function region (BLNP)

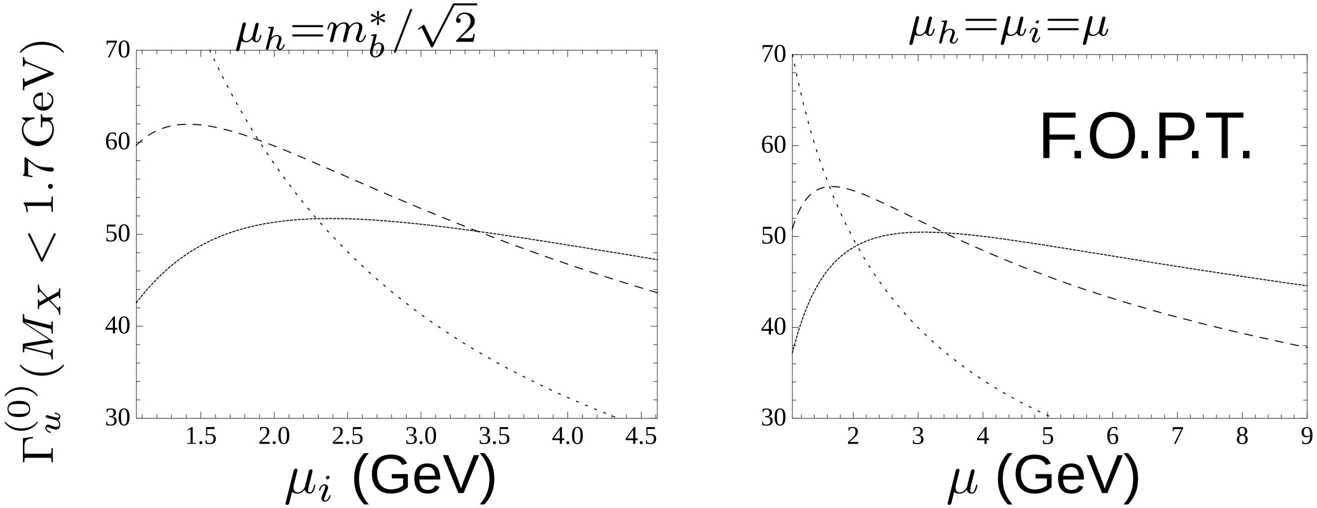
- Recent progress (2008-2009) in computing NNLO corrections to the hard function (two loop virtual diagrams)

  [Bonciani and Ferroglia; Asatrian, Greub and Pecjak; Beneke Huber and Li; Bell]

- The impact of these corrections within the BLNP framework was studied by Greub, Neubert, Pecjak (2009)

$$\frac{d\Gamma}{dP^- dP^+ dE_l} = H(P^-, \mu_h, \mu) J(\sqrt{P^- P^+}, \mu_i, \mu) \otimes S(k^+, \mu) + \mathcal{O}(P^+/m_b)$$

  - for $\mu_i = 1.5$ GeV (default, so far): $\sim 8\%$ upwards shift of $|V_{\rm ub}|$.
  - large μ_i dependence (better do fixed order?!)



Infrared safety

The moments of inclusive decay spectra are infrared and collinear safe - they have finite expansion coefficients to any order in perturbation theory!

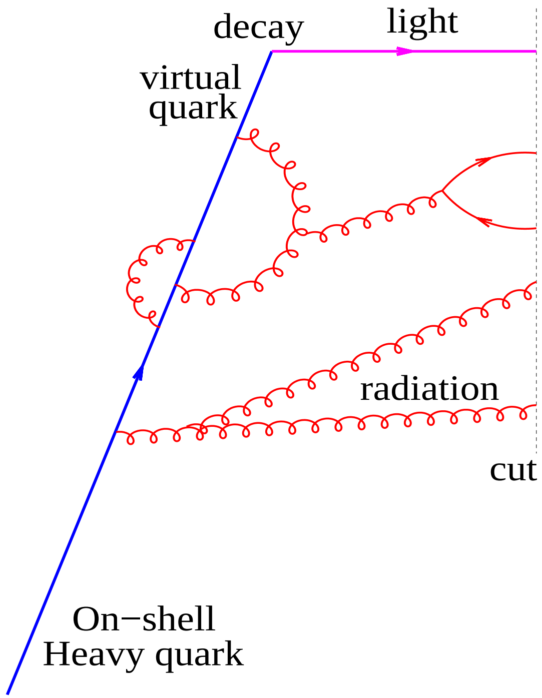
Why use a cutoff?

The perturbative part of the momentum distribution function

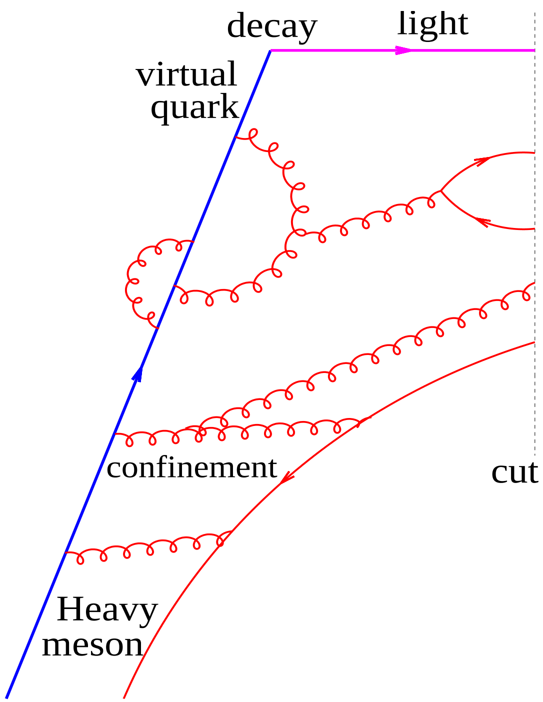
- The momentum distribution of the heavy quark in the meson is a non-perturbative object. However, it has a perturbative analog, the momentum distribution in an on-shell b-quark. It's infrared safe!
- Their moments differ by power corrections $(N\Lambda/m_b)^k \ll 1; \;\; k \geq 3$.

Gardi '04

quark distribution in an on-shell heavy quark



quark distribution in the B meson



Factorization in inclusive decays

Define N such that large N probes jet kinematics $x = 1 - p^+/p^- \to 1$:

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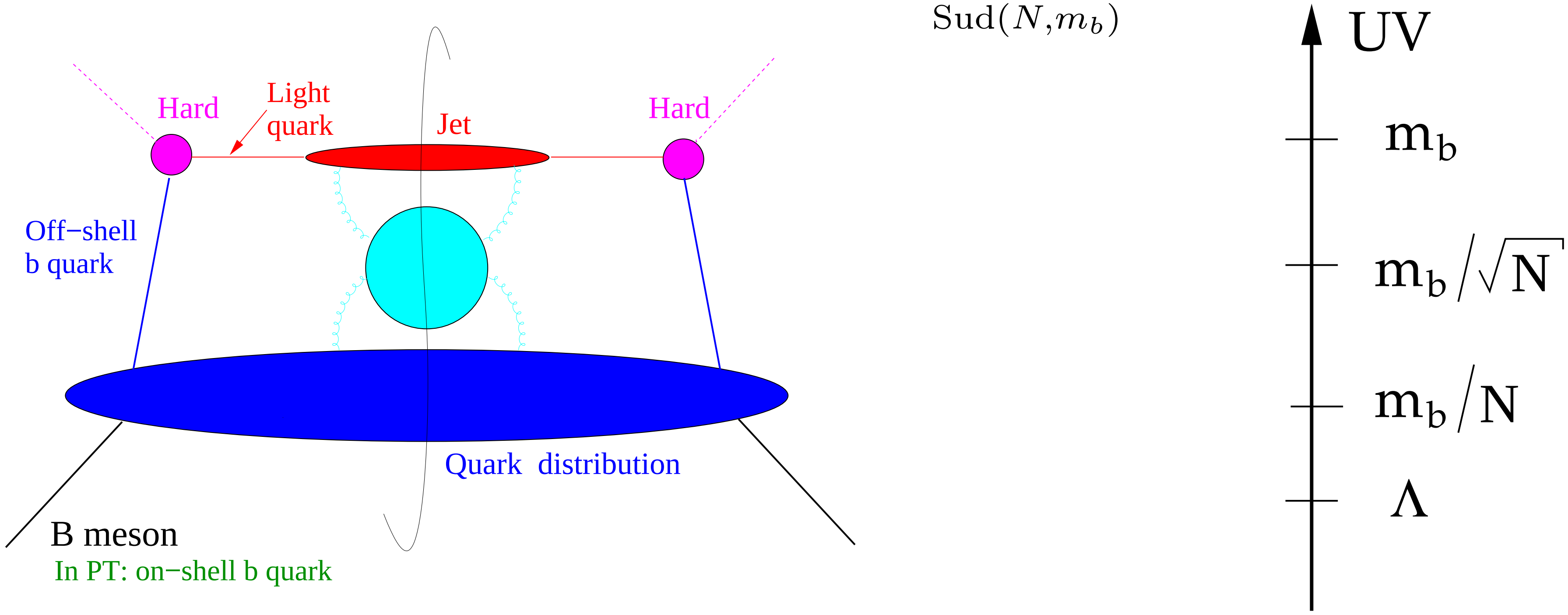

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Identifying and resumming large corrections

Renormalon resummation:

running–coupling corrections,

which dominate the large–order

asymptotics of the series, $n \to \infty$

$\sum_n n!\, \alpha_s{}^n \longrightarrow$ soft dynamics

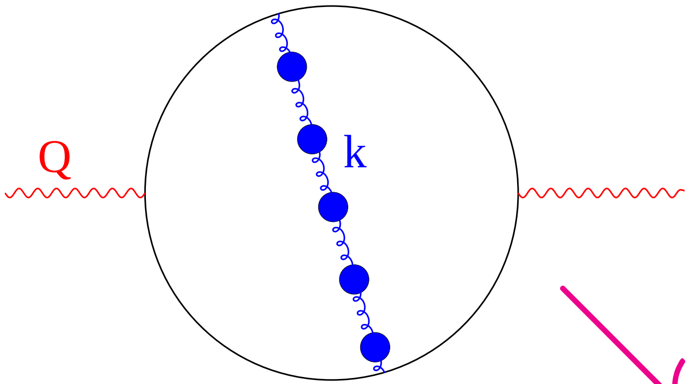
Sudakov resummation:

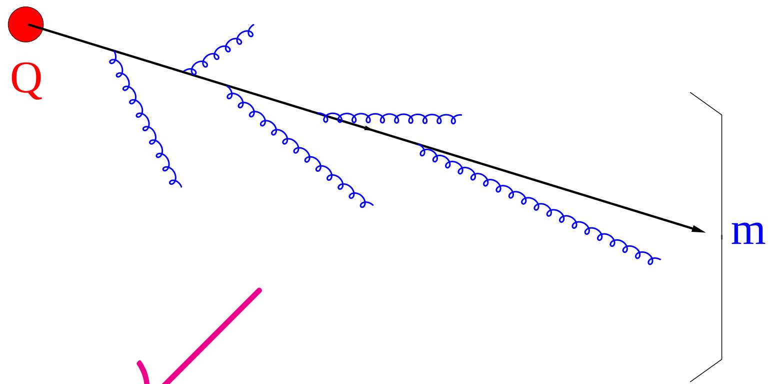
multiple soft and collinear radiation,

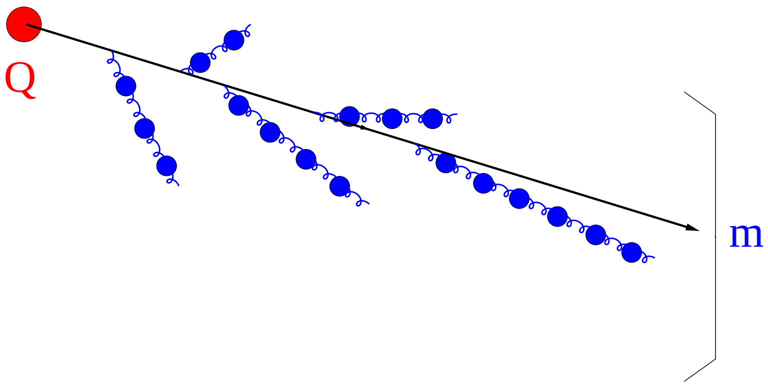
which dominate the dynamics

near threshold $\mathrm{m} \to 0$

$\sum_n \alpha_s{}^n \ln^{2n}(\mathrm{m}/\mathrm{Q})$







Dressed Gluon Exponentiation

Dressed Gluon Exponentiation (DGE)

- Resummed perturbation theory (on-shell heavy quark) yields:

$$\frac{1}{\Gamma_{\rm tot}}\frac{d\Gamma}{dP^+dP^-dE_l} = \int_{-i\infty}^{i\infty}\frac{dN}{2\pi i}\left(1-\frac{P^+-\bar{\Lambda}}{P^--\bar{\Lambda}}\right)^{-N} H(N,P^-,E_l)\,\overline{\rm Sud}(P^-,N)$$

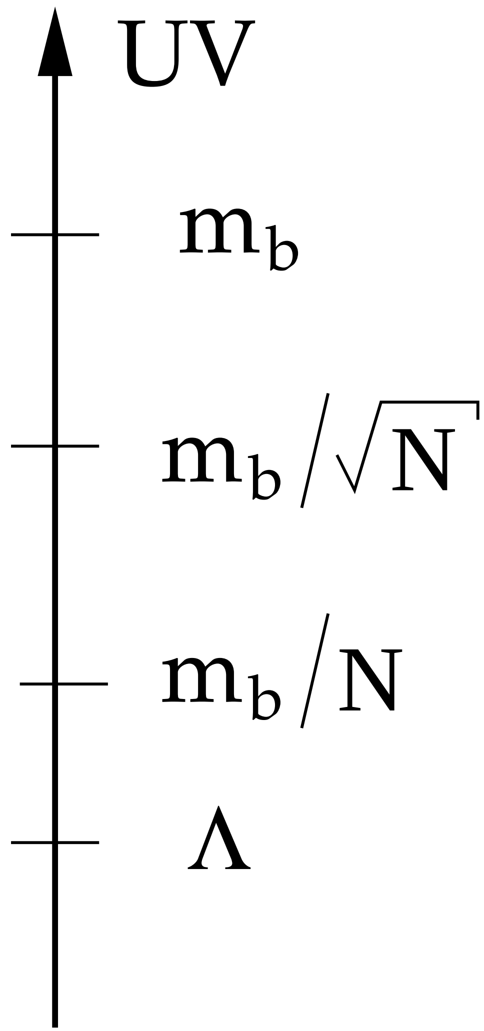
soft and collinear radiation is summed into a Sudakov factor

$$\overline{\rm Sud}(p^-,N) = \exp\left\{\frac{C_F}{\beta_0}\int_0^\infty \frac{du}{u}\,T(u)\,\left(\frac{\Lambda}{p^-}\right)^{2u}\right.$$
$$\left.\left[\underbrace{B_{\mathcal{J}}(u)\Gamma(-u)\,(1-N^u)}_{\text{Jet}} - \underbrace{B_{\mathcal{S}}(u)\Gamma(-2u)\,(1-N^{2u})}_{\text{Quark Distribution}}\right]\right\}$$

- Renormalon resummation indicates the presence of specific power corrections $(N\Lambda/p^-)^k$ in the exponent!
  - $u=1/2$ ambiguity cancels with the pole mass renormalon.
  - $u=1$ renormalon is missing ($B_{\mathcal{S}}(1)=0$).
  - $u\geq 3/2$ ambiguities are present in the on-shell spectrum.

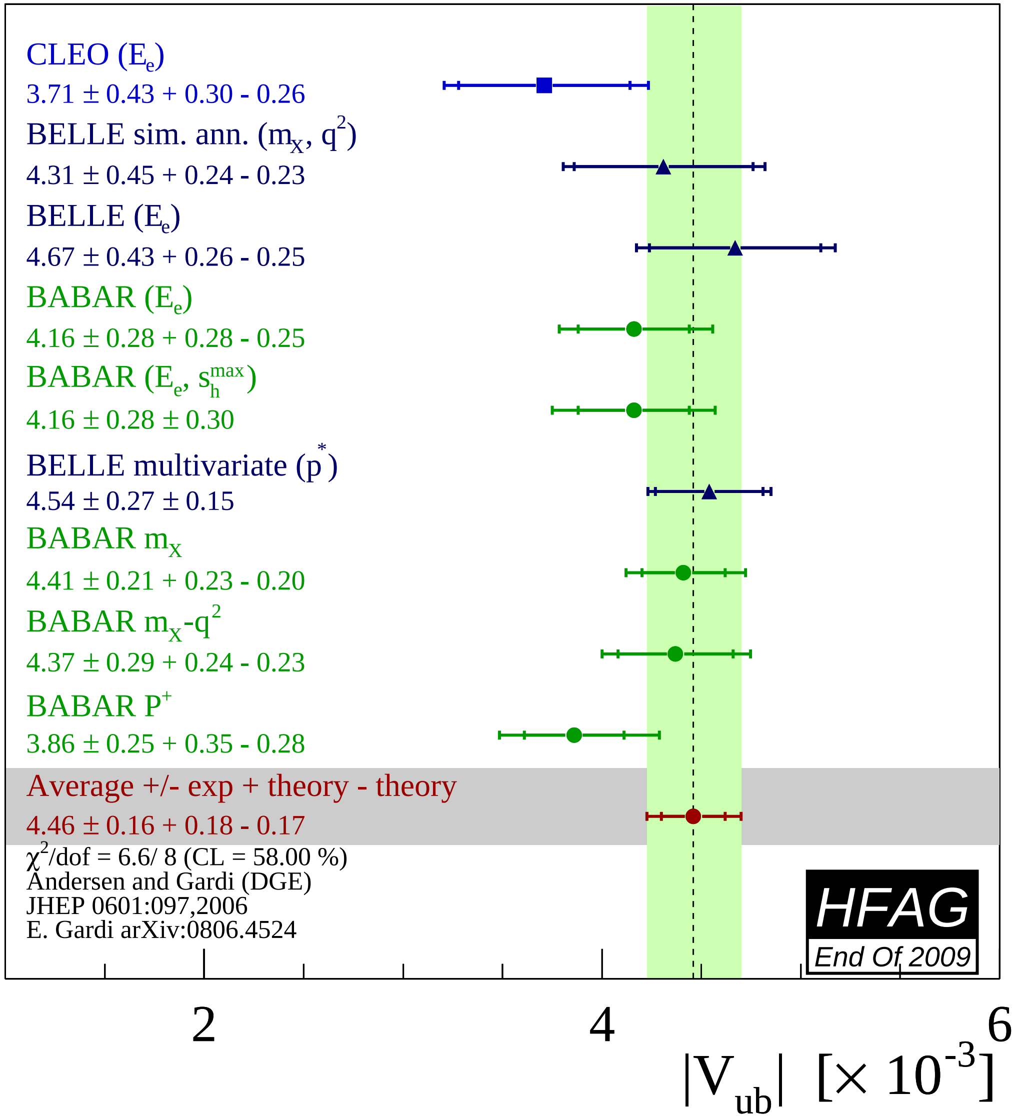
Dressed Gluon Exponentiation (DGE)

- Resummed on-shell calculation in moment space, with no cutoff! resummation includes:
  - Sudakov logs of both jet and quark–distribution — both currently at NNLL accuracy!
  - Renormalon resummation in the exponent.
- Parametrization of power corrections in moment space
- Advantages: Ultimate use of resummed perturbation theory; minimal parametrization.
- Limitations: difficult to relate the magnitude of power corrections to conventional cutoff based definitions.



World Average $|V_{\rm ub}|$ using DGE — HFAG compilation

- The results of different cuts are all consistent. $\chi^2/\mathrm{dof} = 6.6/8$
- Smallest uncertainty: $|V_{ub}| = (4.46 \pm 0.16 \pm 0.18) \cdot 10^{-3}$
- Would average m_b is used, $m_b^{\overline{\rm MS}}(m_b) = 4.222 \pm 0.051$ GeV. m_b: the largest source of error!



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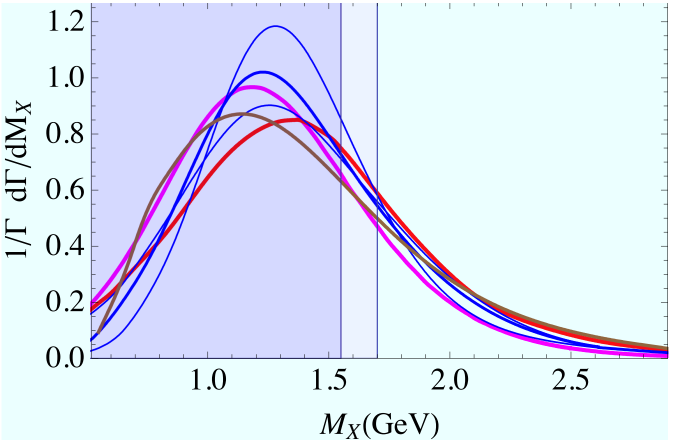
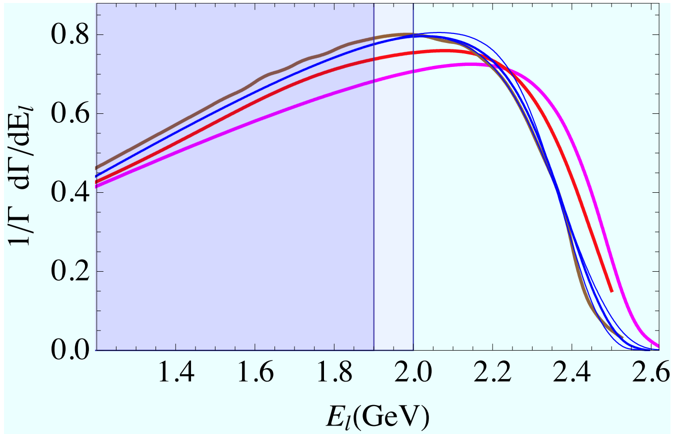
Inclusive $\bar{B} \to X_u l \bar{\nu}$ — theoretical approaches

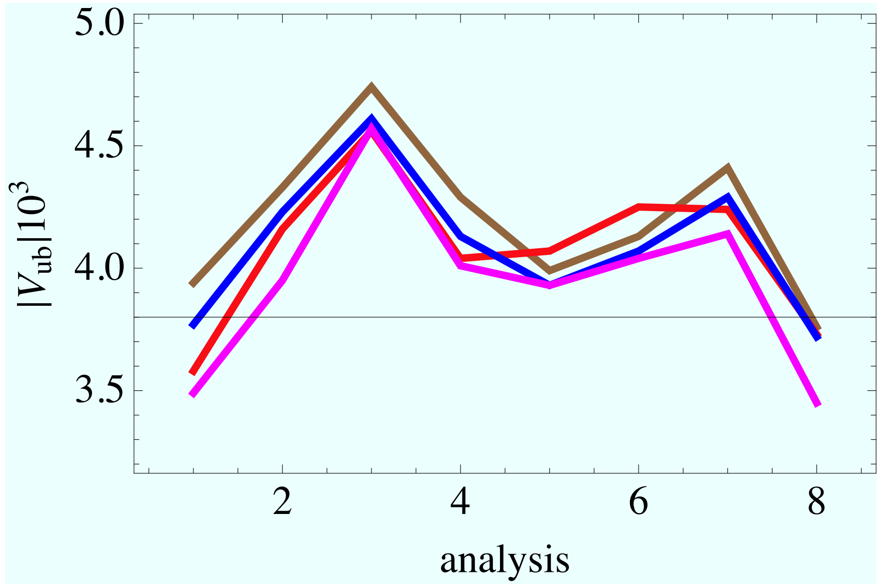
- OPE hard–cutoff approach: parametrization of the contribution to the structure functions below $\mu \sim 1$ GeV (kinetic scheme) convoluted with perturbation theory above μ, constrained by OPE results for their first few moments.
- Shape–Function approach: special treatment of shape function region using dim. reg. cutoff $\mu < \sqrt{m_b \Lambda}$ with Sudakov resummation of jet logs above μ and parametrization of leading and subleading $\mathcal{O}(\Lambda/m_b)$ shape functions below μ; matching with local OPE
- Resummation–based approach: resummed on-shell calculation with no cutoff, supplemented by parametrization of power corrections in moment space.
  DGE combines Sudakov resummation of both jet and quark–distribution logs with PV renormalon resummation.

Comparing the different theoretical approaches





- DGE-BLNP-GGOU: consistent spectra
- Consistent $|V_{\rm ub}|$ from each analysis within non-parametric theory uncertainty



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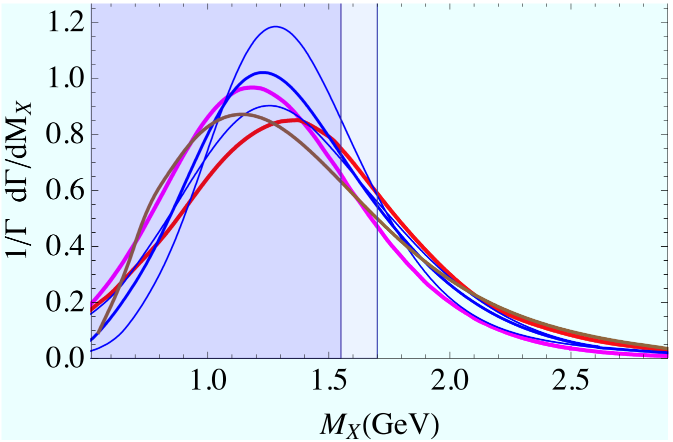

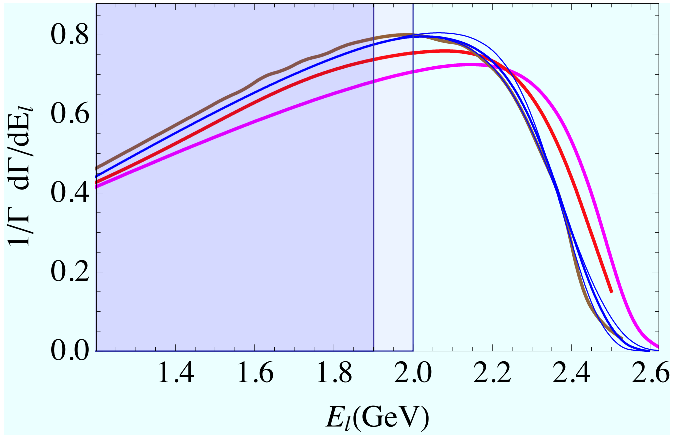

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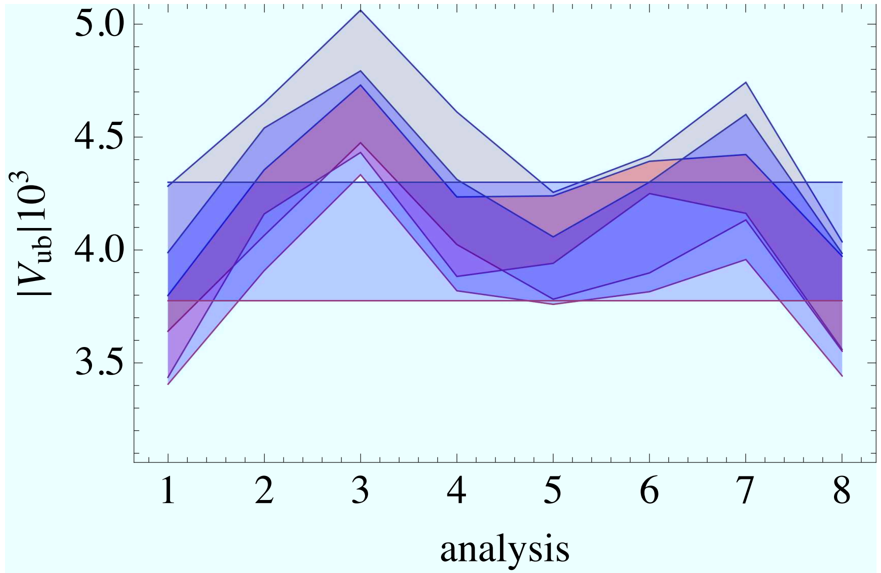


What is known at NNLO

Inclusive semileptonic $B \to X_u l\nu$:
The triple differential width is known since 1999 De Fazio & Neubert
What do we know beyond NLO?

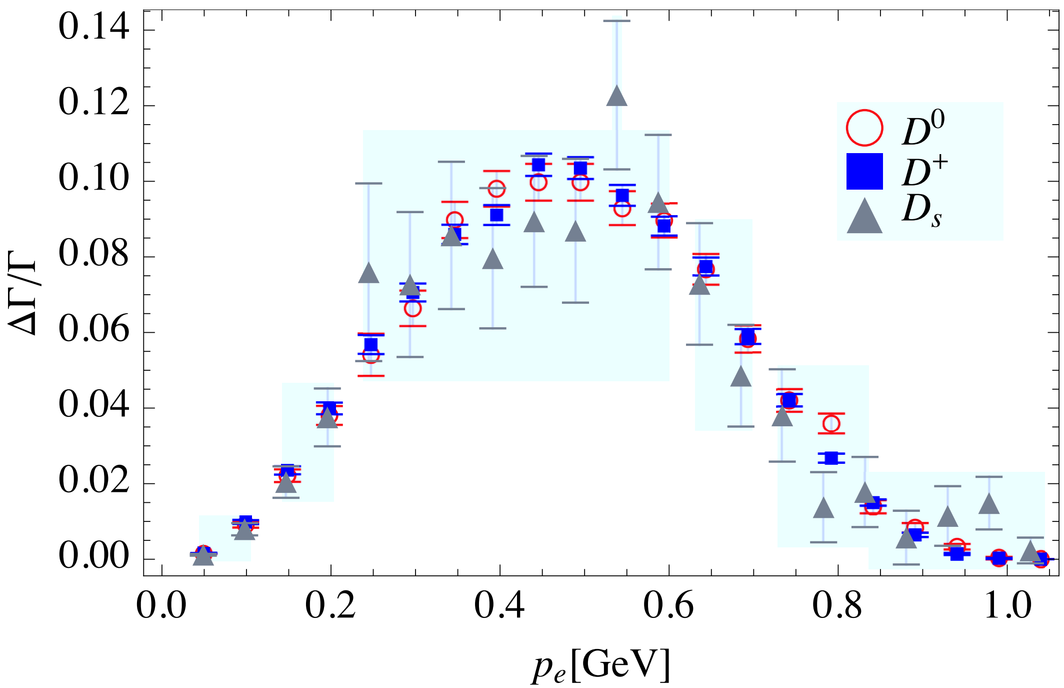
- NNLO is known in full for the total decay width [van Ritbergen (1999)]
- The triple differential width (real and virtual) is known to all orders in the large-β_0 limit [Gambino, Gardi & Ridolfi (2006)]
  Used at $\mathcal{O}(\alpha_s^2\beta_0)$ for $V_{\rm ub}$ determination (in DGE, GGOU) since 2008
- The Sudakov factor: NNLL both Jet and Soft [Gardi (2005)]
  Used in DGE, BLNP since 2005.
- Sudakov factorization: constants in jet & soft Becher & Neubert
  The Hard function [Bonciani and Ferroglia; Asatrian, Greub and Pecjak; Beneke Huber and Li; Bell (2008-9)]
  Used in BLNP since 2009 Greub, Neubert, Pecjak

Completion of NNLO is important (Super B!)

Weak Annihilation

Gambino & Kamenik and Ligeti, Luke & Manohar

CLEO data on semileptonic D decay:



$\Longrightarrow$ Constraint on Weak Annihilation in $B \to X_u l\nu$:

Upper bound of 2% in the total rate

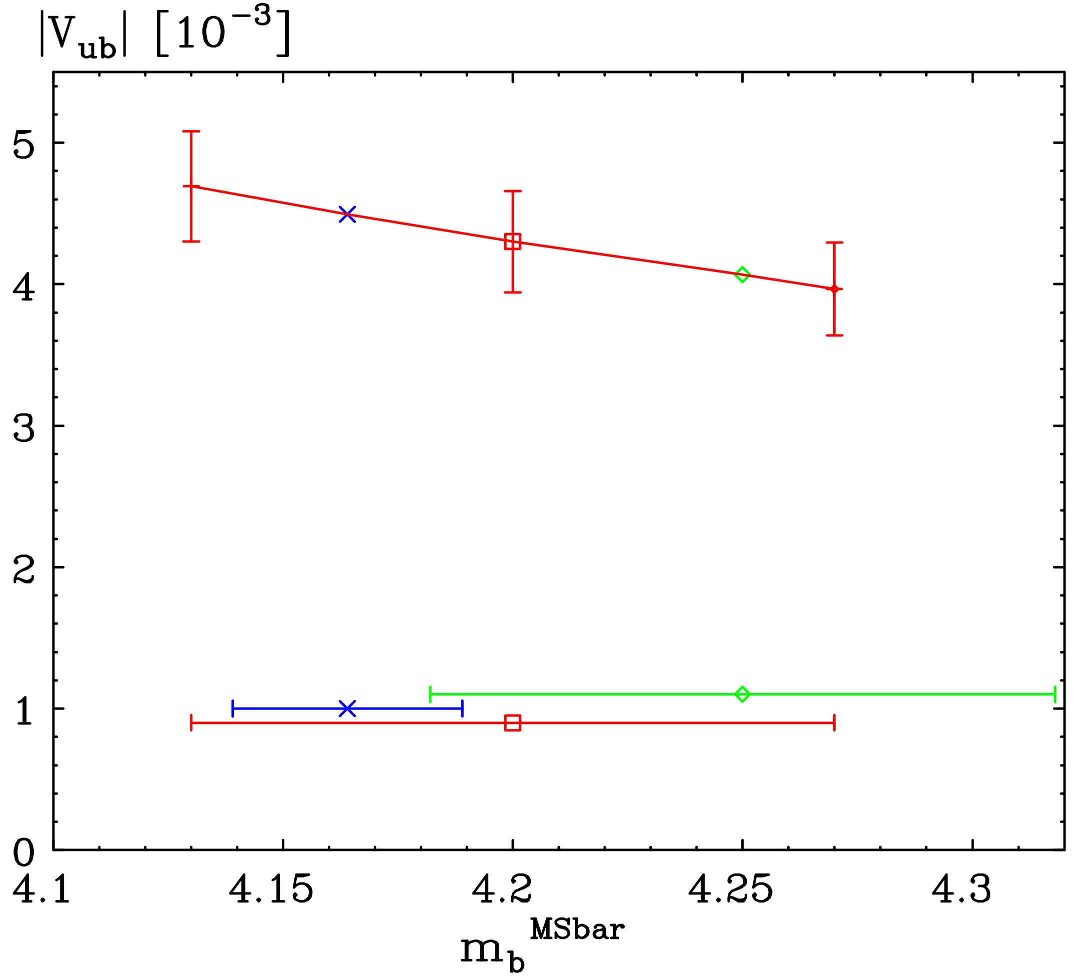
Ignoring WA can raise V_{ub} in a fully inclusive measurement by just 1%!

Up to $\sim 2\%$ when cuts are applied.

Significance of m_b

Total rate: $\Gamma_{\rm tot} \sim |V_{\rm ub}|^2\, m_b^5$

Cuts significantly enhance the m_b dependence!

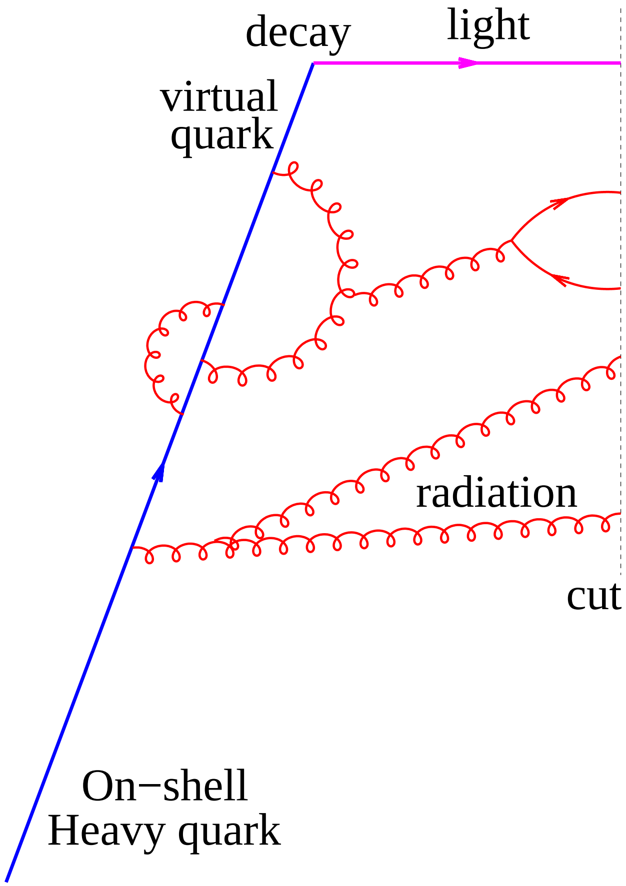


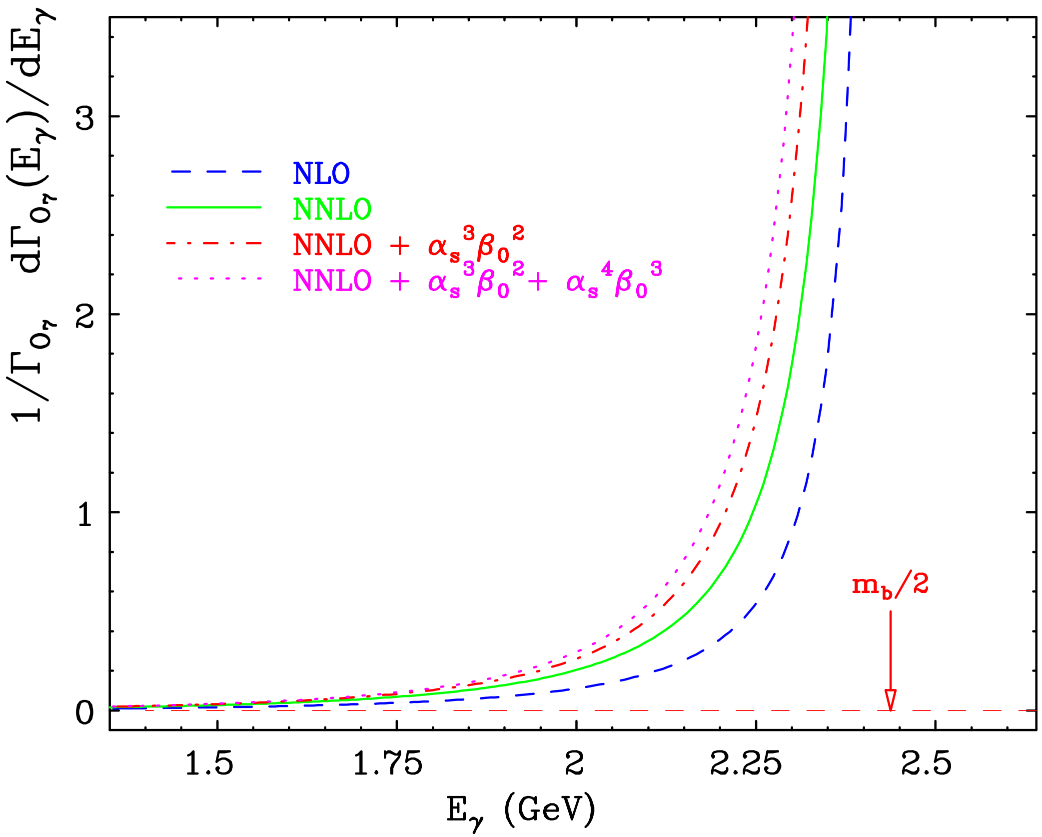
Conclusions

- We have a robust determination of $|V_{\rm ub}|$ from inclusive measurements. Different experimental cuts and different theoretical approaches agree well.
- Total error on $|V_{\rm ub}|$ is less than 10%.
  Theory and experimental errors are of similar magnitudes.
- The largest uncertainty is due to the input b-quark mass.
- Partial NNLO results are available; full NNLO would be important for Super B.

The photon–energy spectrum in perturbation theory

Perturbation theory is badly divergent: Sudakov double logs near the endpoint; huge corections.

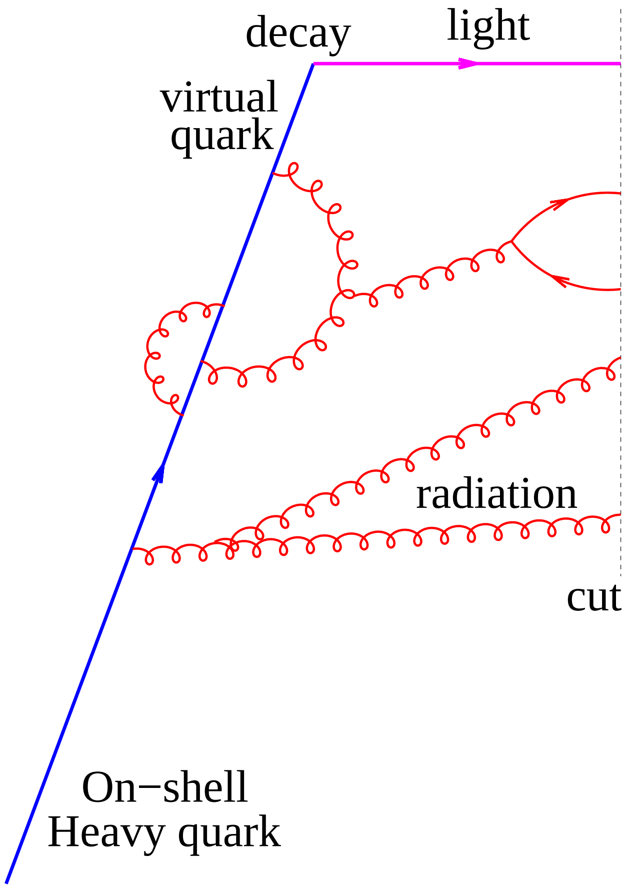


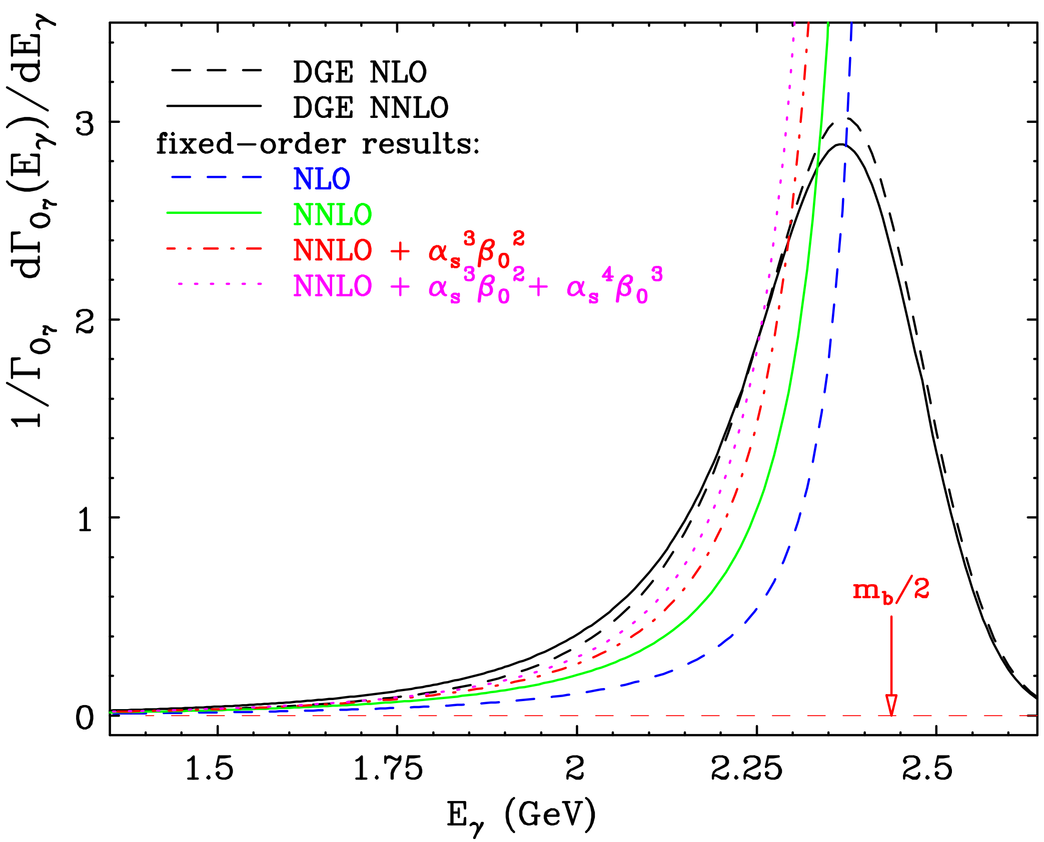


The photon–energy spectrum: resummed perturbation theory

Resummed perturbation theory is qualitatively different: Support properties; stability!

Power corrections are small: resummed perturbation theory yields a good approximation to the meson decay spectrum

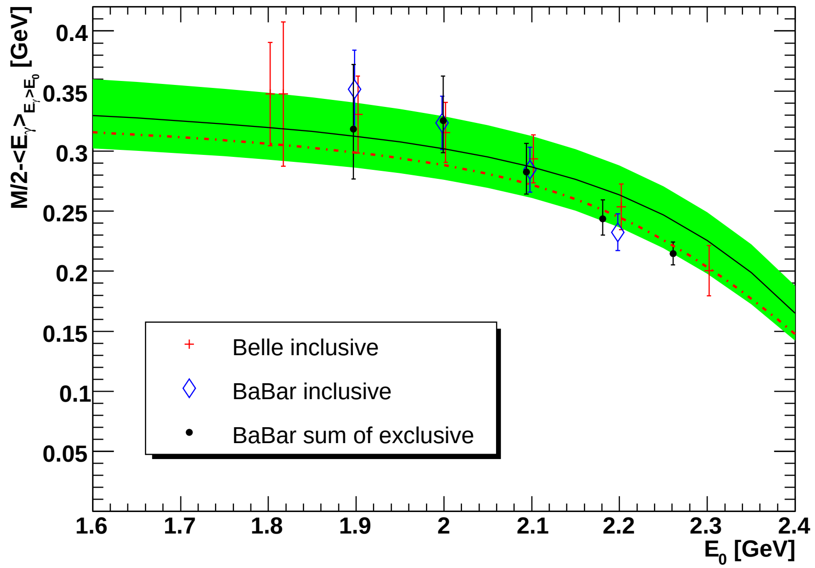


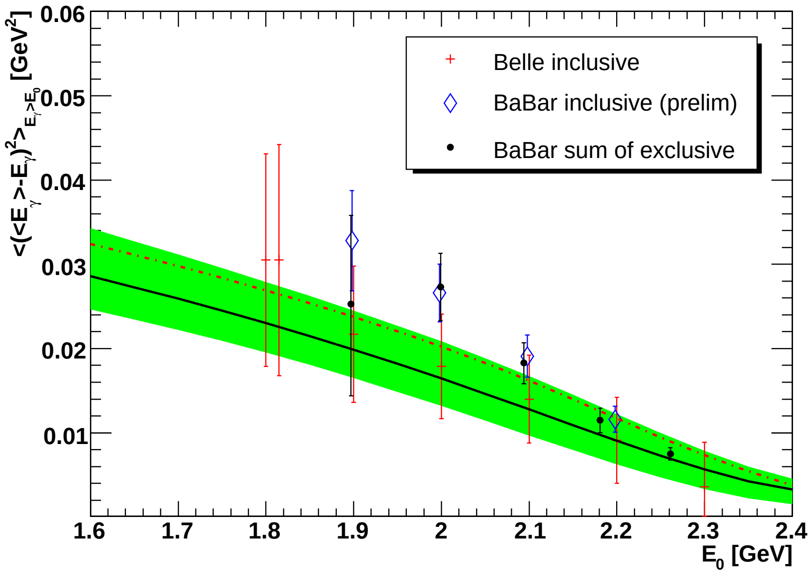


E_γ moments as a function of the cut: theory *vs.* data

$$\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \, \frac{d\Gamma(E_\gamma)}{dE_\gamma} \, E_\gamma$$

$$\left\langle \left(\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} - E_\gamma \right)^n \right\rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \, \frac{d\Gamma(E_\gamma)}{dE_\gamma} \left(\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} - E_\gamma \right)^n$$





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- good agreement between theory and data!
- prospects: determination of m_b and power corrections.