On the consistency between CP violation in the K- vs. B_d-systems within the SM

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Summary

- The ϵ_K formula
  - Going beyond lowest order in the OPE: the κ_ϵ correction
- The ϵ_K – $\sin 2\beta$ correlation within the SM: full consistency ?
- Some speculations about new-physics contributions

Based on:

Buras, DG (PRD 08 & PRD 09)
Buras, DG, Isidori (PLB 10)

Short statement of the problem

☑ **CPV discussed here:** CPV in (meson-antimeson) mixing

***K*-system**	:	ϵ_K	*Very well measured & theoretically controlled* → *Most stringent test of CPV within the SM*
***B_d*-system**	:	$\sin 2\beta$	
***B_s*-system**	:	$\sin 2\beta_s$	

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☑ **Main point of the talk**

ϵ_K is of the form: $\epsilon_K = \# \cdot B_K \cdot \sin 2\beta$

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- B_K: **input from LQCD**
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Buras, DG, PRD08:

$$\epsilon_K = \kappa_\epsilon \cdot \epsilon_K^{(\text{approx})}$$

κ_ϵ **= 0.92**

Then:

(a) take $\sin 2\beta = S_{\psi K s} \sim 0.68$ $\Longrightarrow$ $|\epsilon_K| \simeq 1.8 \cdot 10^{-3}$ (vs. $|\epsilon_K|^{\text{exp}} = 2.2 \cdot 10^{-3}$)

(b) take $|\epsilon_K| = |\epsilon_K|^{\text{exp}}$ $\Longrightarrow$ $\sin 2\beta \simeq 0.8$

From the central values, agreement looks at no better than 20 % level

CPV in *K*-physics: intro to formalism

The d and s quarks can form, through strong interactions, the following bound states

$$|K^0\rangle \sim \begin{pmatrix} d \\ \bar{s} \end{pmatrix}$$

$$|\overline{K}^0\rangle \sim \begin{pmatrix} \bar{d} \\ s \end{pmatrix}$$

and phase conventions can be defined so that

$$\begin{cases} CP\,|K^0\rangle = -\,|\overline{K}^0\rangle \\ CP\,|\overline{K}^0\rangle = -\,|K^0\rangle \end{cases}$$

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K^0 and $\overline{K}^0$ mix into each other because of weak interactions.
However, if *CP* were a good symmetry, one would end up with the physical states:

$$|K\rangle_{\rm even} = \frac{|K^0\rangle - |\overline{K}^0\rangle}{\sqrt{2}}$$

[*CP = +1 admixture: decays into* $|\pi\pi\rangle$]

$$|K\rangle_{\rm odd} = \frac{|K^0\rangle + |\overline{K}^0\rangle}{\sqrt{2}}$$

[*CP = -1 admixture: has to decay into* $|\pi\pi\pi\rangle$]

Intro 2: what is ϵ_K (experimentally)

However, the actual physical admixtures are (slightly) different:

$$|K_S\rangle \;\propto\; |K\rangle_{\rm even} + \overline{\epsilon}\,|K\rangle_{\rm odd}$$
$$|K_L\rangle \;\propto\; |K\rangle_{\rm odd} + \overline{\epsilon}\,|K\rangle_{\rm even}$$

small parameter

Reflecting the experimental fact that mixing (slightly) violates CP

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(*Reflecting the experimental fact that mixing (slightly) violates CP*)

The magnitude of this *CP* violation is accessed experimentally by measuring the amplitude ratios:

$$\eta_{+-} \;=\; \frac{\langle \pi^+\pi^-|K_L\rangle}{\langle \pi^+\pi^-|K_S\rangle} \qquad\qquad \eta_{00} \;=\; \frac{\langle \pi^0\pi^0|K_L\rangle}{\langle \pi^0\pi^0|K_S\rangle}$$

(**Note:** K_L can decay to $\pi\pi$ either directly or indirectly, namely via mixing into K_S)

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It turns out that the corresponding types of CP violation can be disentangled by the following quantities:

$$\epsilon_K \;=\; \frac{1}{3}\,(\eta_{00} + 2\eta_{+-}) \qquad \epsilon' \;=\; \frac{1}{3}\,(\eta_{+-} - \eta_{00})$$

"Indirect" CP violation (through mixing)

"Direct" CP violation (directly in the decay)

Intro 3: how to get the ϵ_K theory formula

Experiments deal with states of $K_{L,S}$, and with charged or neutral π's.

Theory calculates with K^0, $\overline{K}^0$ and with π states of definite isospin.

Expand the one set of states in terms of the other set as

$$|K_{S(L)}\rangle = N_{\bar{\epsilon}}\left[(1+\bar{\epsilon})|K^0\rangle \mp (1-\bar{\epsilon})|\overline{K}^0\rangle\right]$$

$$|\pi^+\pi^-\rangle = \sqrt{\frac{2}{3}}|(\pi\pi)_{I=0}\rangle + \sqrt{\frac{1}{3}}|(\pi\pi)_{I=2}\rangle \qquad |\pi^0\pi^0\rangle = \sqrt{\frac{1}{3}}|(\pi\pi)_{I=0}\rangle - \sqrt{\frac{2}{3}}|(\pi\pi)_{I=2}\rangle$$

and plug into $\epsilon_K = \frac{1}{3}(\eta_{00} + 2\eta_{+-})$

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☑ and plug into $\epsilon_K = \frac{1}{3}(\eta_{00} + 2\eta_{+-})$

One gets: $\epsilon_K = \bar{\epsilon} + i\,\xi$

[Important formula #1]

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Important formula #1

Determined from the off-diagonal entries of the mixing Hamiltonian:

$$H_W = \begin{pmatrix} M - i\Gamma/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M - i\Gamma/2 \end{pmatrix}$$

Can be computed in pert. theory (with some caveats)

Weak phase of $K^0 \to \pi\pi$ (I = 0)

$$\xi \equiv \mathrm{Im}(A_0)/\mathrm{Re}(A_0)$$

where $A_0 e^{i\delta_0} = \langle(\pi\pi)_{I=0}|H_W|K^0\rangle$

Needs to be calculated non-pert.
OR extracted from ϵ'/ϵ (as in Buras, DG, 08)

Intro 3: how to get the ϵ_K theory formula

- Solving the eigenvalue problem (namely solving $\bar{\epsilon}$ in terms of the H_w entries) one arrives at:

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12}^K)}{\Delta M_K} + \xi \right)$$

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Recap: The quantities relevant to this formula are

$$\Delta M_K \equiv m_{K_L} - m_{K_S} \simeq 3.5 \times 10^{-15}\mathrm{GeV}$$

$$\Delta\Gamma_K \equiv \Gamma_{K_L} - \Gamma_{K_S} \simeq -7.4 \times 10^{-15}\mathrm{GeV}$$

Note: $\Delta\Gamma_K \approx -2\Delta M_K$ → **numerical accident!**

$$\phi_\epsilon \equiv \arctan\left(-\frac{\Delta M_K}{\Delta\Gamma_K/2} \right) = (43.5 \pm 0.7)^\circ$$

$$M_{12}^K \equiv \langle K^0 | \mathcal{H}_{\Delta S=2} | \overline{K}^0 \rangle$$

Amplitude for *K*-mixing:
sensitive to non-SM contributions

$$\xi = \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}$$

with A_0 the amplitude for the decay $K^0 \to \pi\pi$ (0-isospin)

Usual approximations in the ϵ_K formula

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12}^K)}{\Delta M_K} + \xi \right)$$

Note

The formula typically adopted in phenomenology takes

- $\xi \to 0$
- $\phi_\epsilon = 45°$

Since both the deviations of ϕ_ϵ from 45° and ξ from zero are corrections, one can rewrite the general formula for ϵ_K as

$$\epsilon_K = \kappa_\epsilon \times \epsilon_K(\xi = 0,\ \phi_\epsilon = 45°)$$

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How close is κ_ϵ to unity?

In the Standard Model, we estimated
[See: Buras, DG, PRD08]

$\kappa_\epsilon = 0.92 \pm 0.02$

Note: the corrections from $\xi \neq 0$ AND $\phi_\epsilon \neq 45°$ have like sign.

This accident builds up a **– 8 % total correction!**

How to estimate κ_ϵ

As we saw before,
κ_ϵ is defined by the relation

$$\epsilon_K = \kappa_\epsilon \times \epsilon_K(\xi = 0,\ \phi_\epsilon = 45^\circ) \quad \Longrightarrow \quad \kappa_\epsilon = \frac{\sin\phi_\epsilon}{1/\sqrt{2}} \times (1 + \Delta_\epsilon)$$

- Parameterizes the effect of $\xi \neq 0$.
- It is dominated by QCD-penguin operator contributions to the process $K \to \pi\pi$, that are very hard to compute directly.

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However, κ_ϵ can be estimated indirectly, through ϵ' / ϵ, using the relation

$$\frac{\epsilon'}{\epsilon} = -\omega\Delta_\epsilon\,(1 - \Omega)$$

- $\omega = Re(A_2)/Re(A_0) = 0.045$ is known very precisely ("$\Delta I = ½$ rule")
- Ω represents the ratio between EW-penguin and QCD-penguin contributions to ϵ' / ϵ
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$$\Delta_\epsilon = -\frac{1}{\omega(1-\Omega)}\left[\frac{\epsilon'}{\epsilon}\right]_{\text{exp}}$$

$$= -\ 0.054\ (1 \pm 25\%)$$

Using

- $\epsilon' / \epsilon = (1.66 \pm 0.26) \times 10^{-3}$
- $\Omega = 0.33\ (1 \pm 20\%)$ [within the SM]

See the analysis by: **Buras-Jamin, JHEP04**

A closer look at the OPE

- We arrived at $|\epsilon_K| = \sin\phi_\epsilon \left[\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \xi \right]$

$$\mathrm{Im}(M_{12}) = \mathrm{Im}(M_{12}^{(6)}) + \ldots$$

What about this?

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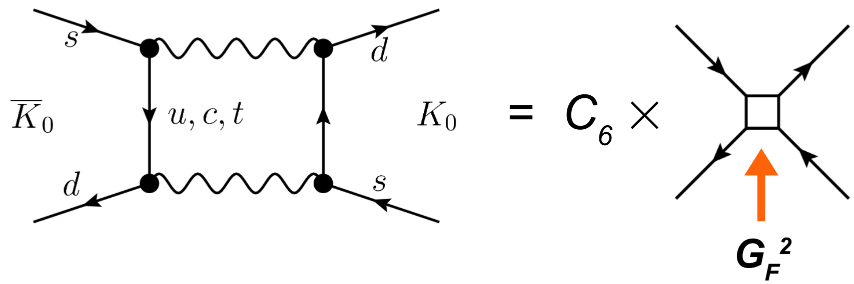
- ☑ **Dominant contributions to M_{12}**

- $M_{12}^{(6)}$:



$Q_6 = (\bar{d}\,s)_{V-A}\,(\bar{d}\,s)_{V-A}$

(ΔS = 2 operators)

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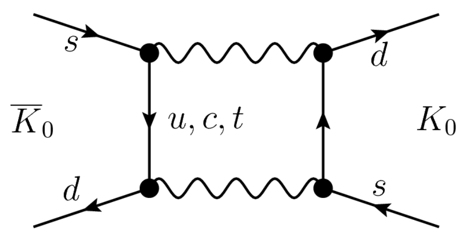
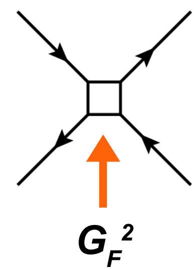


$= C_6 \times$



← $Q_6 = (\bar{d}\,s)_{V-A}\,(\bar{d}\,s)_{V-A}$

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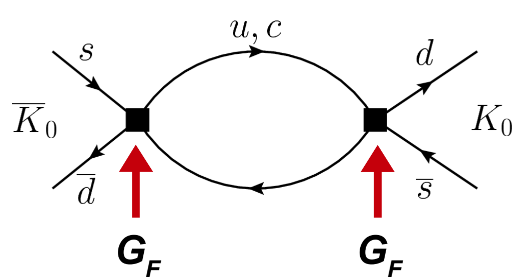
■ $(M_{12})_{\text{long-dist.}}$: two insertions of ΔS = 1 operators



need to "hadronize" the internal lines

⇒ *Use Chiral Perturbation Theory*

Main point about this diag.: *Its absorptive part is exactly the leading contribution to ξ.*
So, if we keep ξ, we have to include the dispersive part as well, i.e. $(M_{12})_{\text{long-dist.}}$

M_{12} within ChPT

Within ChPT, the only $\Delta S = 1$ operator relevant for our calculation at $O(p^2)$ is:

$$L^{(2)}_{\Delta S=1} = F^4 G_8 (\partial_\mu U^+ \partial^\mu U)_{23} + \text{h.c.}$$

In particular: $A_0 = A[K^0 \to (\pi\pi)_{I=0}] \propto G_8$

implying

$$(M_{12})_{G_8^2} \propto (G_8^*)^2 \quad \Rightarrow \quad \frac{\text{Im}(M_{12})_{G_8^2}}{\text{Re}(M_{12})_{G_8^2}} = -2\xi$$

$(\mathbf{8}_L, \mathbf{1}_R)$ operator under $SU(3)_L \times SU(3)_R$

⇒ responsible for the "$\Delta I = ½$ rule"

Two consequences:

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(b) its coupling can be determined from exp.

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Buras, DG, Isidori, PLB10

☑ Therefore
$$\mathrm{Im}(M_{12}) = \mathrm{Im}(M_{12}^{(6)}) + \mathrm{Im}(M_{12})_{G_8^2} + \{\text{non-}G_8^2\}$$
$$\mathrm{Im}(M_{12})_{G_8^2} = -\xi(\Delta m_K)_{G_8^2}$$

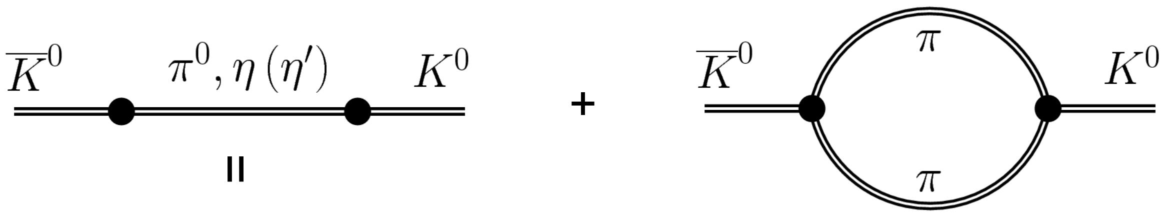
namely
$$|\epsilon_K| = \sin\phi_\epsilon \left[\frac{\mathrm{Im}(M_{12}^{(6)})}{\Delta m_K} + \xi \left(1 - \frac{(\Delta m_K)_{G_8^2}}{\Delta m_K} \right) \right]$$

Important formula #2

M_{12} within ChPT: continued

Long-distance contrib's to M_{12} act as a correction to the ξ piece.
The problem is restated into that of computing the G_8^2 contributions to the mass splitting:

$(\Delta m_K)_{G_8^2}$:



=

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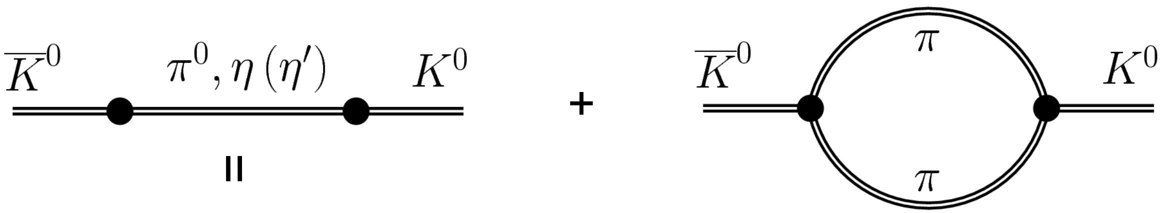

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We restricted to the ($\pi\pi$)-loop: $A^{(\pi\pi)}$

- Only $A^{(\pi\pi)}$ has an absorptive part; hence it's the only component whose weak phase can be measured, from $K^0 \to (\pi\pi)_{I=0}$
- Only contribution that survives in the $SU(2)_L \times SU(2)_R$ limit of ChPT
- Kaon loops go into the redefinition of the local terms
- There are doubts about the reliability of kaon-loops: their effective threshold lies at $2\ m_K > m_\rho$ (!)

See: Donoghue, 0909.0021

M_{12} within ChPT: results

We get

$$\frac{(\Delta m_K)_{G_8^2}}{\Delta m_K} = 0.4 \pm 0.2$$

Cross-check:

$$\rho \equiv 1 - \frac{(\Delta m_K)_{G_8^2}}{\Delta m_K} = \frac{(\Delta m_K)_{\text{short-dist.}} + (\Delta m_K)_{\eta'}}{\Delta m_K}$$

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0.6 ± 0.2 **= 0.7 ± 0.1** **≈ - 0.3**

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Therefore, our final phenomenological formula for ϵ_K reads:

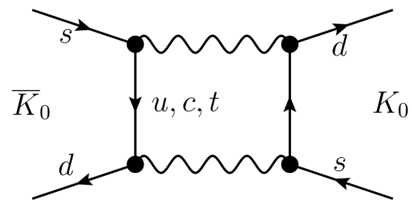
$$\epsilon_K = \sin\phi_\epsilon e^{i\phi_\epsilon}\left(\frac{\mathrm{Im}(M_{12}^{(6)})}{\Delta m_K} + \rho\,\xi\right)$$

with $\rho = 0.6 \pm 0.3$

We conservatively increase by 50% the ρ error to account for the subleading contributions from non-G_8^2 pieces

Error budget in ϵ_K: intuitive arguments for the main error components

$\epsilon_K \propto \mathrm{Im}(M_{12})$



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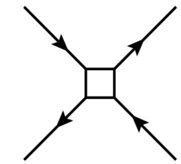

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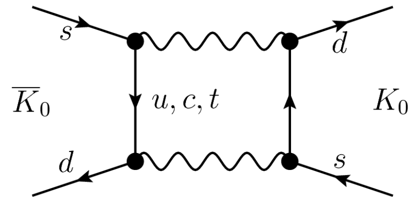

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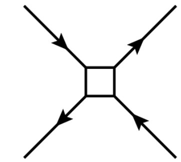

$\propto \hat{B}_K$

Main error components (building up a O(15%) total error)

(a): $\epsilon_K \propto \hat{B}_K$

$$\delta \hat{B}_K / \hat{B}_K \approx 5\%$$

Error budget in ϵ_K: intuitive arguments for the main error components

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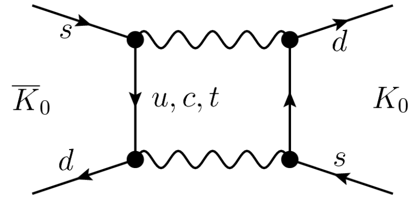
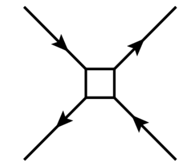


$\sim \left(V_{ts} V_{td}^*\right)^2 \cdot f(m_t, \ldots) =$



$\propto \hat{B}_K$

Main error components (building up a O(15%) total error)

(a): $\epsilon_K \propto \hat{B}_K$ $\quad$ $\delta \hat{B}_K / \hat{B}_K \approx 5\%$

(b): $\left(V_{ts} V_{td}^*\right)^2$

recall that: $\mathrm{CKM} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & V_{cb} \\ V_{td} & V_{ts} & \cdot \end{pmatrix}$ $\Rightarrow$ $\begin{cases} V_{ts} \sim V_{cb} \\ V_{td} \sim \lambda V_{cb} \end{cases}$ $\Rightarrow \left(V_{ts} V_{td}^*\right)^2 \sim \lambda^2 \, |V_{cb}|^4$ **(!)**

$4\,\delta |V_{cb}| / |V_{cb}| \approx 11\%$

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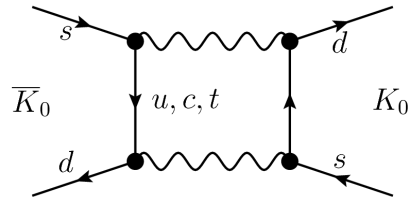

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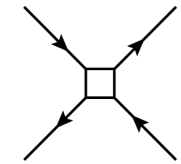

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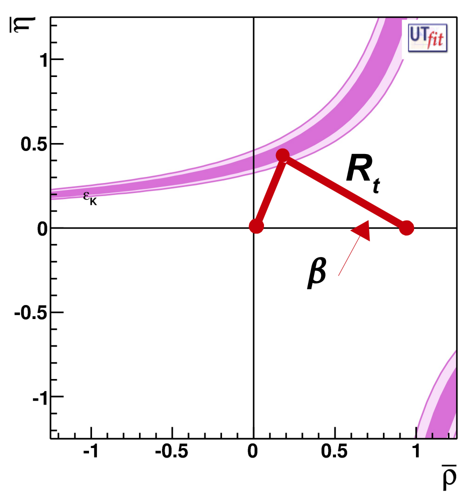

In the $(\bar{\rho}, \bar{\eta})$ plane, the ϵ_K constraint produces a hyperbola

$$\Rightarrow \quad \epsilon_K \propto \bar{\eta}\,(1 - \bar{\rho}) = R_t^2 \sin\beta \cos\beta \propto R_t^2 \sin 2\beta$$

Hence $\epsilon_K \propto \boxed{R_t^2}$ — $2\,\delta R_t / R_t \approx 8\%$

(*this component of the error will go down with a precise γ measurement*)

Status of the problem from global fits

☑ The general formula for ϵ_K discussed here has been recently included in global CKM fits by the CKMfitter and UTfit collaborations.

CKMfitter

See: Lenz + Nierste + CKMfitter, 1008.1593

- With the CKMfitter input values (that include flat components in the errors for, e.g., $|V_{cb}|$, B_K, κ_ϵ, η_{cc}, η_{ct}, η_{tt}, quark masses) & the Rfit treatment of theory errors, they find no discrepancy.

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- With Gaussian errors they find:

$$10^3|\epsilon_K| = 1.91^{+0.26}_{-0.24} \quad \text{or} \quad 10^3|\epsilon_K| = 1.77^{+0.18}_{-0.16}$$

2.4σ

depending on the lattice input (theirs or Laiho+Lunghi+Van de Water's, respectively)

- They conclude: *"The potential anomaly in $|\epsilon_K|$ cannot yet be precisely quantified independently of the theoretical inputs and therefore deserves further investigations."*

Status of the problem from global fits

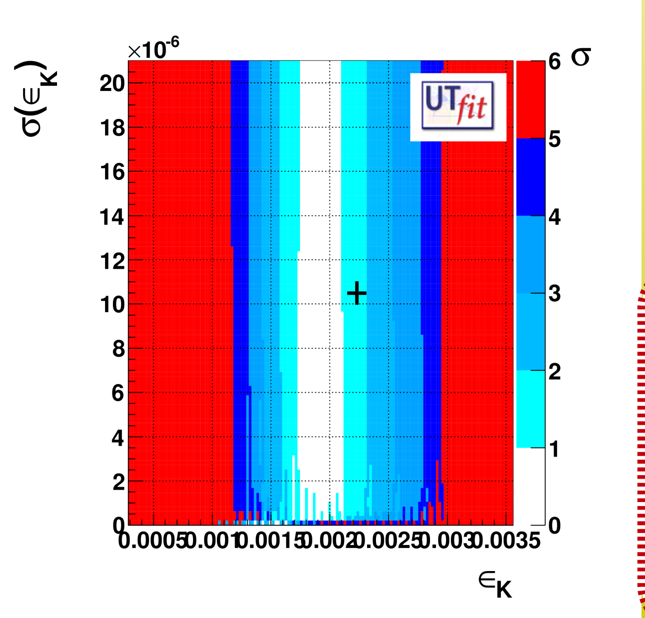
UTfit

See **C. Tarantino** (for UTfit), talk at EPS-HEP09 and plots at **www.utfit.org**

- *"We have included in ϵ_K the contributions of ξ and $\phi \neq \pi/4$.*

  We note that the new contributions in ϵ_K generate some tension in particular between the constraints provided by the experimental measurements of ϵ_K and $\sin 2\beta$. As a consequence, the indirect determination of $\sin 2\beta$ turns out to be larger than the experimental value by $\sim 2.0\sigma$."

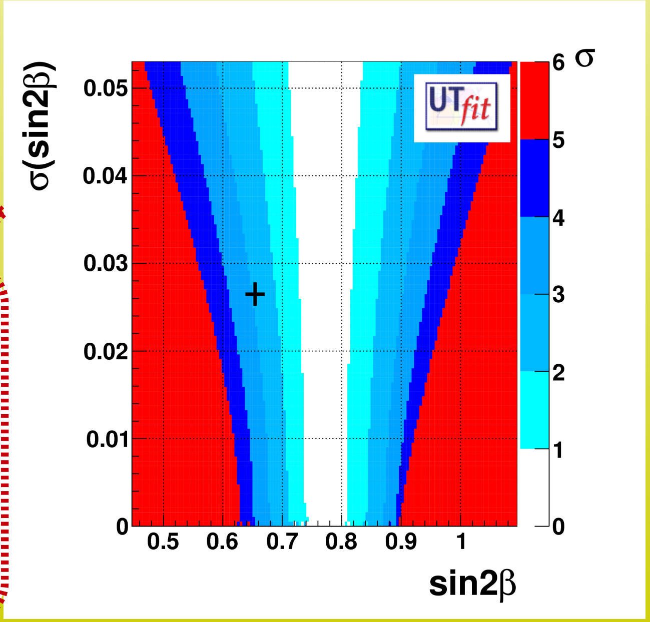
- The situation is best summarised by the UTfit compatibility plots for the relevant quantities



Global SM fit, with $|\epsilon_K|$ **not** in the fit

Global SM fit, with $\sin 2\beta$ **not** in the fit

Note: potential shift due to DCS peng's (**see Faller, Fleischer, Jung, Mannel, 08**), not yet included here.



Speculations on new physics

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- **New physics**

  At this stage, the possibility of new physics entering the problem is of course only speculative.

  One can, however, imagine two extreme scenarios for the ϵ_K - $\sin 2\beta$ correlation:

Scenario 1: **$\sin 2\beta$ is SM-like.**

ϵ_K^{SM} is lower than the exp value.
New physics adds constructively to the SM contribution.

Scenario 2: **ϵ_K is SM-like.**

$\sin 2\beta$ is higher than the exp value (taken e.g. from $S_{\psi K_s}$).
Hence $S_{\psi K_s} = \sin 2(\beta+\phi_d)$, with ϕ_d a new, *negative*, phase.

More on scenario 1:

$\sin 2\beta = S_{J/\psi\ K_S}$

In this case one gets $|\epsilon_K^{SM}| = 1.85\ (1 \pm\ 15\%) \times 10^{-3}$

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Since the SM formula for ϵ_K goes as

$$|\epsilon_K|^{\rm SM} = [\text{const.fact.}] \times \left(\underbrace{[\text{CKM}] \cdot S_0(m_t^2/m_W^2)}_{\approx 75\% \text{ of the total}} + \ ... \right)$$

The simplest solution is a positive shift in the ϵ_K loop function

- *This solution is of MFV type. In fact, the CKM structure is preserved and non-SM physics only enters the short-distance S-function*

  [Buras et al., 00]

- *Barring non-SM operators mediating mixing, the above shift would be universal, i.e. also affect B_d and B_s mass differences (and cancel in their ratio)*

More on scenario 2:

$\sin 2\beta_{J/\psi\ Ks} = \sin 2(\beta + \phi_d)$

See also:
Lunghi, Soni, PLB08

In this case the phase β cannot be accessed directly from the $J/\psi\ K_s$ mode

However, one possible way to determine β is by using ϵ_K, Δm_d and Δm_s only.

An indicative figure, obtained with the CKMfitter package, is

$$\sin 2\beta = 0.88^{+0.11}_{-0.12}$$

to be compared with

$$\sin 2\beta_{J/\psi\ Ks} = 0.681 \pm 0.025$$

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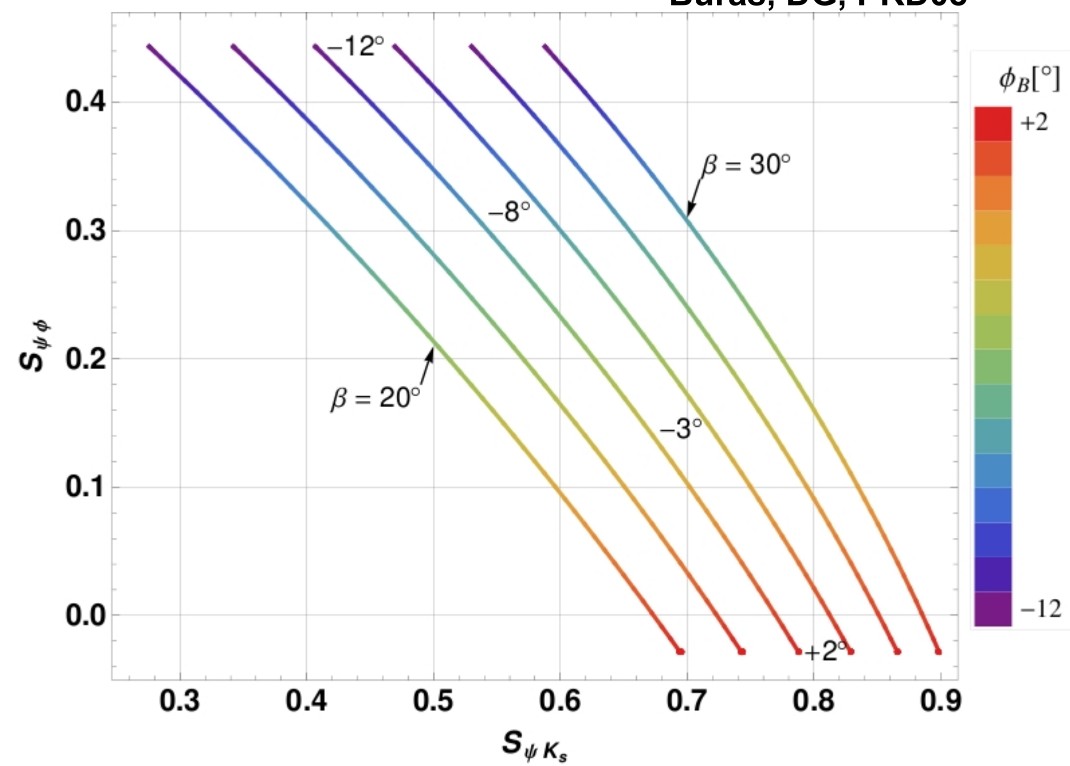
The B_d new-physics phase, negative, may be correlated (even in size) with the negative new phase in B_s hinted at by Fermilab

In fact, it could even be:

$$\phi_B = \phi_d \approx \phi_s \approx -9^\circ$$

$$\begin{cases} \beta_{\psi K_s} < \beta \approx 30^\circ \\ S_{\psi\phi} \approx 0.4 \end{cases}$$

Buras, DG, PRD08



Conclusions

- *The correlation ϵ_K – sin2β is a fundamental consistency check of SM CP violation. With regards to CP violation, it is the only one available at present.*
- *Our analysis shows that a (more) accurate SM formula for ϵ_K implies a non-negligible downward shift in the central value.*
- *Looking at the entailed prediction for sin2β, the above shift hints at a tension. While, with present errors, no statement above 2 sigma can be made, the issue warrants further investigation.*

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$$|\epsilon_K|^{\rm SM} \propto \kappa_\epsilon \, \hat{B}_K \, |V_{cb}|^4 \, |V_{us}|^2 \, R_t^2 \, \sin 2\beta$$

$$\Rightarrow \quad \frac{\delta|\epsilon_K|^{\rm SM}}{|\epsilon_K|^{\rm SM}} \approx \sqrt{\underbrace{\left(\frac{\delta \hat{B}_K}{\hat{B}_K}\right)^2}_{5\%} + \underbrace{\left(4\frac{\delta|V_{cb}|}{|V_{cb}|}\right)^2}_{11\%} + \underbrace{\left(2\frac{\delta R_t}{R_t}\right)^2}_{8\%}} \approx 15\%$$

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- ☑ *Important is also the effort towards a NNLO calculation of the* η_{ct} (**Brod+Gorbahn, 2010**) *and* η_{cc} *coefficients. Note in fact that:*

$$|\epsilon_K^{\rm SM}| = \{\text{t}-\text{t contrib.}\} + \{\text{c}-\text{t contrib.}\} + \{\text{c}-\text{c contrib.}\}$$

+72.6% **+40.9%** **-13.5%**

Partial cancellations at work