





Introduction

Recent kaon physics results came from many

Experimental (ISTRA+, KTeV, KLOE, NA48...)

and Theoretical (Lattice groups, CHPT groups...) efforts

A KAON WG to

Officially combine results including preliminaries, suggest new measurements, compare parametrizations, compare M C generators, update theoretical inputs Incomplete list

semi-leptonic related: Vus, Vus/Vud, Form Factor slopes, universality of lepton coupling, test of r.h. Currents...

CP, CPT related: ε_{κ} , Bell-Steinberger Test, ...

Others: δ_0 - δ_2 ,....

A WG on Precise SM tests in K decays M.Antonelli LNF-INFN - FLAVIANET meeting Barcelona



• Review of leptonic and semileptonic kaon decay data including recent results (determined by experiments with very different techniques):

E865@BNL: rare K⁺ decays in flight; π^0 Dalitz decay in final states.

KLOE@**DaΦne**: pure K beams, lifetimes, absolute BR

NA48@CERN: intense K⁰, K⁺ beams from SPS proton beam, ratio of BR's

KTeV@FermiLab: intense K_L beam from Tevatron proton beam, ratio of BR's

ISTRA+@IHEP (Protvino): ratio of K+13 BR's

- Substantial progress made in lattice calculations of the hadronic matrix elements
- Precise analytic calculations (ChPT) of radiative correction and isospin-breaking effects.
- Very precise determination of Vus
- Stringent tests of the Standard Model
- First outcome: FlaviaNet Kaon WG note (0801.1817[hep-ph]).



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Regular Article - Theoretical Physics

(1005.2323[hep-ph])

An evaluation of $|V_{us}|$ and precise tests of the Standard Model from world data on leptonic and semileptonic kaon decays

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All results presented are from this paper

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Leptonic and semileptonic K decays

• Within the SM leptonic and semileptonic K decays can used to obtain the most accurate determination of the element Vus of the CKM matrix

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{G_F^2 m_K^5}{192\pi^3} C_K S_{\text{ew}} |V_{us}|^2 f_+(0)^2 I_K^{\ell}(\lambda_{+,0}) \left(1 + \delta_{SU(2)}^K + \delta_{\text{em}}^{K\ell}\right)^2$$

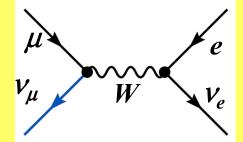
$$\frac{\Gamma(K_{\ell 2(\gamma)}^{\pm})}{\Gamma(\pi_{\ell 2(\gamma)}^{\pm})} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K}{f_{\pi}^2 m_{\pi}} \left(\frac{1 - m_{\ell}^2 / m_K^2}{1 - m_{\ell}^2 / m_{\pi}^2}\right)^2 \times (1 + \delta_{\text{em}})$$

• Test unitarity of the quark mixing matrix (V_{CKM}) :

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \epsilon_{NP}$$

$$\epsilon_{
m NP} \sim M_W^2/\Lambda_{
m NP}^2$$

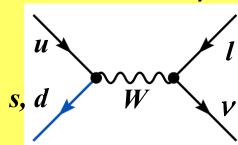
μ decay



$$(g_{\mu}g_{e})^{2}/M_{W}^{4}=G_{F}^{2}$$

?

K, π and nuclear β decays



$$(g_q g_1)^2 (V_{ud})^2 + |V_{us}|^2 / M_W^4 = G_{CKM}^2$$

Study within a **model-independent effective theory approach** the implications of precise measurements of Kl2 and Kl3 decays for SM extension [Cirigliano, Gonzalez-Alonso, and Jenkins, arXiv:0908.1754 hep-ph]

Phenomenology in U(3)⁵ flavor symmetry limit

• Taking into account all the Precision Electroweak constraints, the maximal deviation of $|\Delta_{CKM}|$ allowed is:

$$-9.5 \times 10^{-3} \le \Delta_{\text{CKM}} \le 0.1 \times 10^{-3}$$
;

- \rightarrow deviation from CKM unitarity at -1% level not ruled out by PEW tests.
- Even a % level test of CKM unitarity would provide information not available through other precision tests at low- and high-energy.
- $\delta Vus=0.5\%$ combined with $\delta Vud=0.02\%$ (nuclear beta decays) allow to probe NP effective scales of the order of 10 TeV.



net NP test from (semi)leptonic K decays

Study within a model-independent effective theory approach the implications of precise measurements of Kl2 and Kl3 decays for SM extension [Cirigliano, Gonzalez-Alonso, and Jenkins, arXiv:0908.1754 hep-ph]

Beyond U(3)⁵ limit.

- Corrections to the U(3)⁵ limit can be introduce within MFV and via generic flavor structures (pseudoscalar and tensor structures).
- A high sensitive probe of U(3)⁵ violating structures is provided by comparing the Vus value extracted by the helicity suppressed $K\mu 2$ decays and the helicity allowed K13 modes, using the ratio

$$R_{\mu 23} = \left(\frac{f_K/f_{\pi}}{f_{+}(0)}\right)^{-1} \left(\left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_{\pi}}\right)_{\mu 2} \frac{|V_{ud}|_{0^+ \to 0^+}}{[|V_{us}|f_{+}(0)]_{\ell 3}}$$
 (minimize impact of f_K and e.m. corrections)

Within SM, $R_{\mu 23}=1$; the inclusion of Higgs-mediated scalar currents leads to

$$R_{\mu 23} pprox \left| 1 - \frac{m_{K^+}^2}{m_{H^+}^2} \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta} \right|$$

Plavi A Determination of V_{us} from Kl2 decays

Within SM, the ratio of photon inclusive K_{12} to π_{12} decay rates is:

$$\frac{\Gamma(K_{\mu 2(\gamma)})}{\Gamma(\pi_{\mu 2(\gamma)})} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{|f_K|^2}{|f_{\pi}|^2} \times \frac{M_K (1 - m_{\mu}^2 / M_K^2)^2}{m_{\pi} (1 - m_{\mu}^2 / m_{\pi}^2)^2} \times (1 + \delta_{em})$$

Obtain $|V_{\mu s}|$ from:

- measurements of the inclusive K_{12} and π_{12} decay widths;
- $|V_{ud}|$ =0.97425(22) from super-allowed 0+ \rightarrow 0+ nuclear beta decays [Hardy and Towner, Phys. Rev. C79(2009) 055502]

Use precise evaluation of long-distance e.m. corrections $\delta_{em} = -0.0070(18)$.

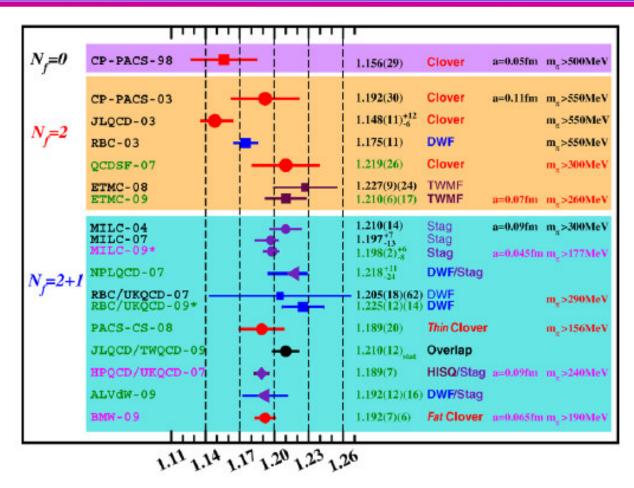
 $f_{\rm K}/f_{\pi}$ not protected by the Ademollo-Gatto theorem: only lattice.

(lattice calculation of $f_{\rm K}/f_{\pi}$ and radiative corrections benefit of cancellations).



Theoretical estimate of f_{κ}/f_{π}

Summary of lattice QCD determinations of $f_{\rm K}/f_{\pi}$.



 $f_{\rm K}/f_{\pi}$ (our choice): average (stat. error only + smallest syst. error) of results with analysis of all systematics [BMW, MILC09, HPQCD/UKQCD]: 1.193(6) - consistent with other choices, e.g.: 1.196(10) [LAT2009] and 1.190(10) [FLAG].

Plavi A Determination of V_{us.} from Kl3 decays

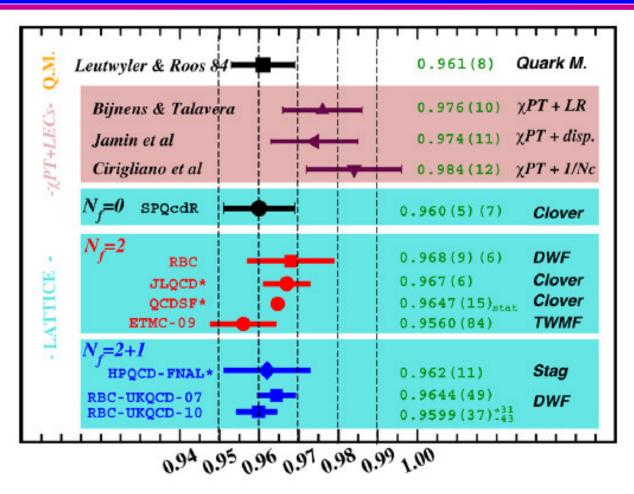
$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+(0)|^2 I_{K,l}(\lambda) (1 + \delta_K^{SU(2)} + \delta_{K,l}^{EM})^2$$
(with $K = K^+, K^0$; $l = e, \mu$ and $C_K^2 = 1/2$ for $K^+, 1$ for K^0)

	Theory			Experiment		
Decay Rate			$\Gamma(K_{13(\gamma)})$ BR and lifetimes			
Form Factor	$f_{+}(0)$ hadronic matrix element at zero momentum transfer			$I_{K,l}(\lambda)$ Phase space: λ param. form factor dependence on t		
Corrections	S _{EW} short distance EW	δ _K SU(2) strong SU(2) breaking	δ _{K,l} EM \ long distance EM		Mode K_{e3}^{0} K_{e3}^{\pm}	$\delta_{\text{EM}}^{K\ell}$ (%) 0.495 ± 0.110 0.050 ± 0.125
δ _{SU(2)} =0.029(4) EPJC 04, 006 (2008)					$K^{0}_{\mu 3} \\ K^{\pm}_{\mu 3}$	0.700 ± 0.110 0.008 ± 0.125 $11,006(2008)$



Theoretical estimate of $f_{+}(0)$

Present determinations of $f_{+}(0)=f_{+}^{K0\pi}-(0)$ from analytical or semianalytical approaches, and lattice QCD,



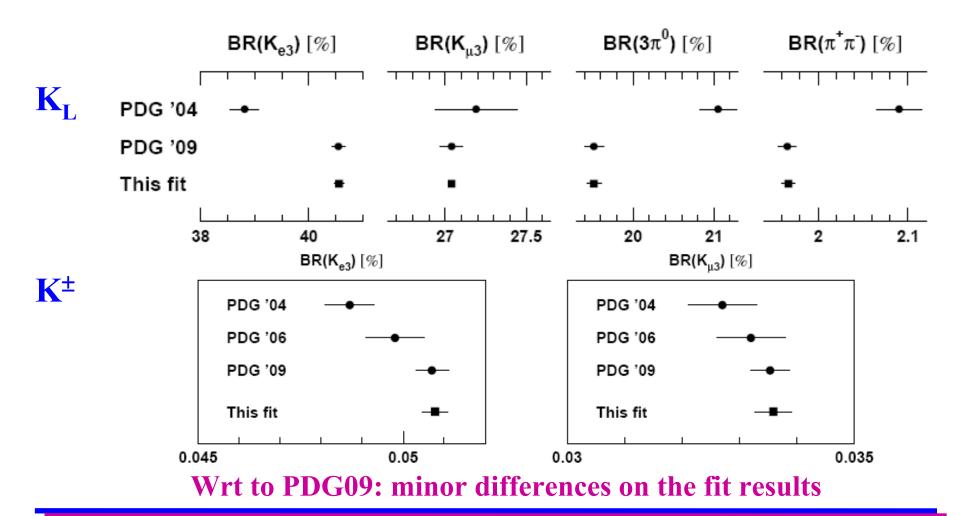
 $f_{+}(0)$ (our choice): 0.959(5), our error symmetrization of RBC/UKQCD-10

- fairly representative of results and spread of values
- consistent with $N_f=2$ and $N_f=2+1$ average: **0.962(5)** [PoS LAT2009, 013(2009)].



Note on BR and lifetime data set

Careful reading of the original papers \rightarrow definition of different data set and/or parameters wrt to PDG





Parameterization of form factors

$|V_{us}f_{+}(0)|$ extraction needs calculation of the phase space integrals:

$$I_K^\ell = \int_{m_\ell^2}^{t_0} dt \; \frac{1}{m_K^8} \; \lambda^{3/2} \; \left(1 + \frac{m_\ell^2}{2t}\right) \; \left(1 - \frac{m_\ell^2}{2t}\right)^2 \left(\bar{f}_+^2(t) + \frac{3m_\ell^2 \Delta_{K\pi}^2}{(2t + m_\ell^2)\lambda} \bar{f}_0^2(t)\right)$$

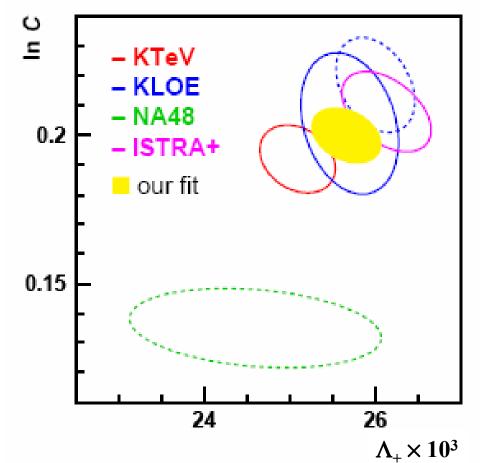
- Class II: based on a systematic mathematical expansion (e.g. Taylor, "z-par.")
- freedom to determine high-order terms from data
- strong par. correlation \rightarrow no sensitivity to high order terms ($\lambda_0^{\prime\prime}$) [PoS 2008(KAON)002]
- accurate description in physical region needs at least 2nd Taylor exp. [PLB638(2009)480]
- test of low-energy dynamics involving Callan-Treiman th. needs orders>2nd.
- Class I: to reduce the number of parameters, impose additional physical constraints
- **pole**: dominance of single resonance $M_{V,S}$ (one free parameter) vector: K*(892) ok; scalar: no obvious dominance.
- dispersive: ff analytic (except real t> $(m_K + m_{\pi})^2$) functions in the complex t-plane. vector: numerically similar to pole (K*(892) dominance);

scalar: necessary without dominant one-particle intermediate state.



Form factors: dispersive approach

Results from KTeV KLOE ISTRA+ NA48 This fit



Dashed lines show **NA48*** and **preliminary KLOE** data **not in fit**

$\Lambda_+ \times 10^3$	=	25.66 ± 0.41
ln C	=	0.2004(91)
$\rho(\Lambda_+, \ln C)$	=	-0.33
χ^2/ndf	=	5.6/5 (34%)

Integrals						
Mode	Quad-lin	Disp				
K^0_{e3}	0.15457(20)	0.15476(18)				
K^+_{e3}	0.15894(21)	0.15922(18)				
$K^0_{~\mu 3}$	0.10266(20)	0.10253(16)				
$K^+_{\mu 3}$	0.10564(20)	0.10559(17)				

Maximum change 0.2% if same data used as for quad-lin fits

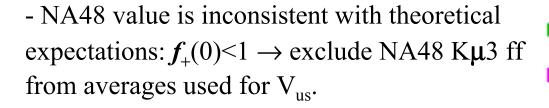
Kμ3 scalar ff: test of χPT

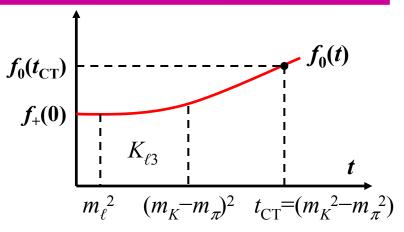
Dispersive parameterization for $f_0(t)$ plus Callan-Treiman relation

$$C \equiv \tilde{f}_0(\Delta_{K\pi}) = \frac{f_K}{f_\pi} \frac{1}{f_+(0)} + \Delta_{CT}$$

- Assuming a f_K/f_{π} value, obtain a value for $f_+(0)$.
- Consistency test between scalar ff measurement and lattice calculations: WA for ln C (0.2004(91))

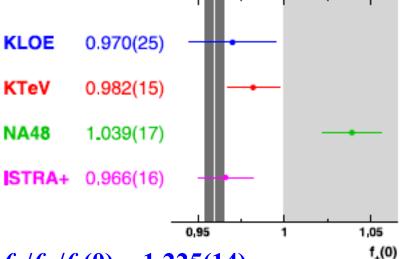
gives: $f_{+}(0) = 0.974(12)$





lattice QCD

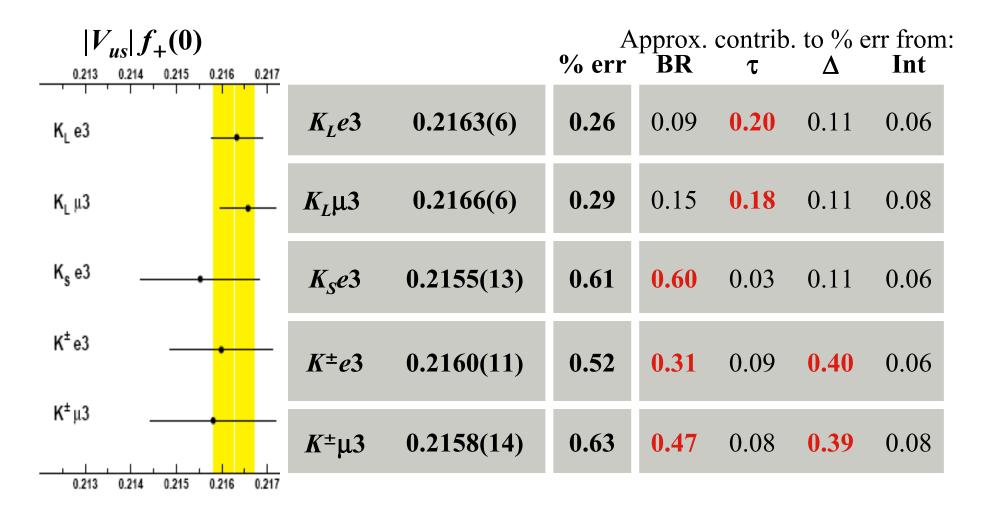
0.959(5)



WA exp. data on ln C alone gives $f_{\rm K}/f_{\pi}/f_{+}(0) = 1.225(14)$ completely independent of any information from lattice estimates



$V_{us} f_{+}(0)$ from K_{l3} data



Average: $|V_{us}| f_{+}(0) = 0.2163(5)$ $\chi^2/\text{ndf} = 0.77/4 (94\%)$

Plavi Accuracy of SU(2)-breaking corrections

Fit 5 modes with separate values of $|V_{us}| f_{+}(0)$ for K^{\pm} and $K_{L,S}$ modes; K^{\pm} modes modes are corrected for the isospin-breaking using $\delta^{SU(2)}_{theory} = 2.9(4)\%$.

When fit performed without SU(2) corrections for K^{\pm} modes; from ratio of neutral-charged-modes, obtains an **experimental estimate of** $\delta^{SU(2)}$:

$$\delta^{SU(2)}_{\rm exp} = 2.7(4)\%$$

- Check of the $\delta^{SU(2)}$ estimate from χ PT; the uncertainty on $\delta^{SU(2)}_{theory}$ contributes significantly on the overall uncertainty of $|V_{us}| f_+(0)$ from charged modes.
- Since $\delta^{SU(2)}$ can be expressed in terms of the quark mass ratio (at LO):

$$\delta_{\text{SU}(2)}^{K^{\pm}\pi^{0}} = \frac{3}{4} \frac{1}{R}$$
, with $R = \frac{m_{s} - \widehat{m}}{m_{d} - m_{n}}$

its phenomenological determination can be used to derive constraints on the ratio of quark masses.



K_{l3} data and lepton universality

For each state of kaon charge, evaluate:

$$r_{\mu e} = \frac{[|V_{us}|f_{+}(0)]_{\mu 3, \text{exp}}^{2}}{[|V_{us}|f_{+}(0)]_{e 3, \text{exp}}^{2}} = \frac{\Gamma_{K\mu 3}}{\Gamma_{Ke 3}} \frac{I_{e 3}(1 + 2\delta_{\text{EM}}^{Ke})}{I_{\mu 3}(1 + 2\delta_{\text{EM}}^{K\mu})}$$

Modes	2004 BRs*	World data
$K_{L,S}$	1.040(13)	1.003(5)
K ±	1.013(12)	0.998(9)
Avg	1.034(10)	1.002(5)

*Assuming current values for form-factor parameters and $\Delta^{\rm EM}$; K_S not included

As statement on lepton universality

- compare to results from world data:

$$\pi \rightarrow lv$$
 $(r_{\mu e}) = 1.0042(33)$ Ramsey-Musolf, Su & Tulin '07

$$\tau \rightarrow l \nu \nu (r_{\mu e}) = 1.000(4)$$
Davier, Hoecker & Zhang '06

As statement on calculation of δ^{EM}

- highly successful
- results confirmed at per-mil level



Test of CKM unitarity

Determine $|V_{us}|$ and $|V_{ud}|$ from a fit to the results:

$$|V_{us}f_{+}(0)|=0.2163(5), f_{+}(0)=0.959(5);$$

 $|V_{us}|/|V_{ud}|f_{K}/f_{\pi}=0.2758(5), f_{K}/f_{\pi}=1.193(6)$

$$|V_{us}| = 0.2254(13)$$
 [$K_{\ell 3}$ only],
 $|V_{us}/V_{ud}| = 0.2312(13)$ [$K_{\ell 2}$ only]

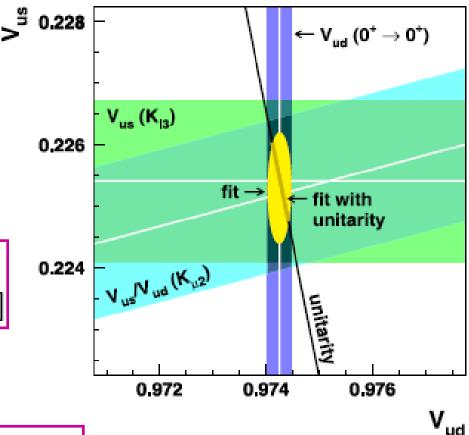
Adding $|V_{ud}|=0.97425(22)$, obtains $(\chi^2/\text{ndf}=0.014/1, P=91\%, \text{negligible}$ correlation between V_{us} and V_{ud}):

$$|V_{ud}| = 0.97425(22),$$

 $|V_{us}| = 0.2253(9) \quad [K_{\ell 3}, K_{\ell 2}, 0^+ \to 0^+]$

Including in the fit the unitairty constraint, obtains ($\chi^2/\text{ndf}=0.024/2$, P=99%):

$$|V_{us}| = \sin \theta_C = \lambda = 0.2254(6)$$
 [with unitarity]







Using the current WA value

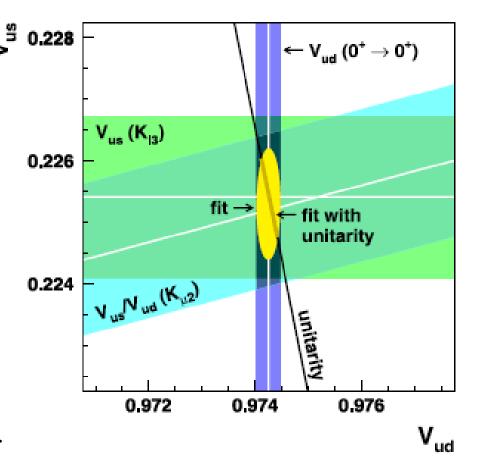
 $|V_{ub}|$ =0.00393(36), the first-row unitarity

sum is Δ_{CKM} =-0.0001(6), in striking agreement with unitarity hypothesis.

Allow to set bounds on the effective scale of the operators that parametrize NP contributions to Δ_{CKM} :

Λ> 11 TeV (90% C.L.).

For three operators (ll, ϕ l, ϕ q), constraint at the same level as Z-pole measurements; for the 4-fermion operator (lq), improves LEP2 bounds by one order of magnitude.

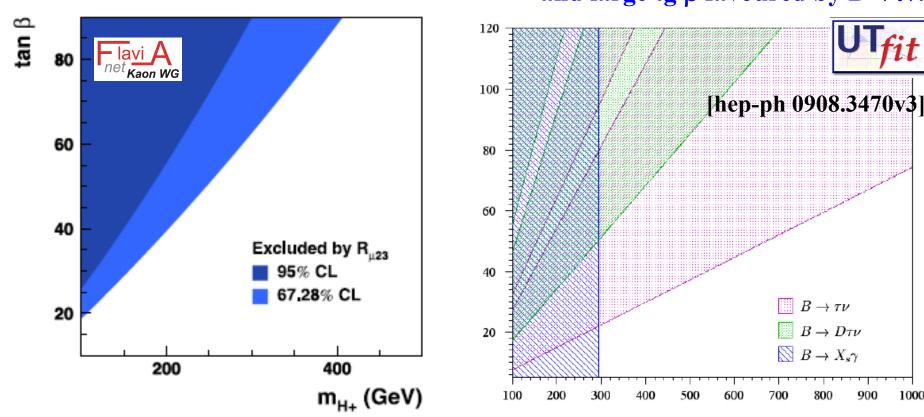


Tavi Bounds on non helicity-suppressed amps

With a 3-parameter fit (V_{us} from K13, V_{us}/V_{ud} from K μ 2, V_{ud}) with 1 constraint: $[V_{us}(K_{l3})]^2 + [V_{ud}(0^+ \rightarrow 0^+)]^2 + [V_{ub}]^2 = 1$, obtains ($\chi^2/ndf = 0.0003/1 P = 99\%$, $\rho = -0.55$):

$$|V_{us}| = 0.2254(8)$$
 $[K_{\ell 3}, 0^+ \to 0^+, \text{unitarity}],$ $R_{\mu 23} = 0.999(7)$ $[K_{\mu 2}].$

this excludes the region at low m_{H^+} and large tg β favoured by $B \rightarrow \tau \nu$.





$f_{+}(0)$ and f_{K}/f_{π} : straight calculation

$$Q_{\ell 2} = \frac{(|V_{us}|f_{+}(0))^{2}}{|V_{ud}|^{2}} \times \frac{1}{f_{+}(0)^{2}} \times \frac{f_{K}^{2}}{f_{\pi}^{2}}$$

From decay rates and rad. corr.

From nucl.

From K13

β-decay

Straight calculation from $K\mu 2/\pi\mu 2$ relation and <u>assuming SM</u>:

• Use
$$Q_{\ell 2} = \frac{\Gamma_{K_{\ell 2(\gamma)}^{\pm}}}{\Gamma_{\pi_{\ell 2(\gamma)}^{\pm}}} \frac{1}{(1 + \delta_{\text{em}})} = 0.07604(26)$$

- Obtain $f_{\rm K}/f_{\pi}/f_{+}(0) = 1.242(4)$ depends on decay rate data, radiative corrections; unitarity not assumed, although V_{us} equality in Kµ2 and Kl3 decays is.
- using $f_{+}(0) = 0.959(5)$ obtain $f_{K}/f_{\pi} = 1.192(8)$
- using $f_K/f_{\pi} = 1.193(6)$ obtain f+(0) = 0.960(6)

$- lavi A f_{+}(0)$ and f_{K}/f_{π} : fit assuming unitarity

$$Q_{\ell 2} = \frac{(|V_{us}|f_{+}(0))^{2}}{|V_{ud}|^{2}} \times \frac{1}{f_{+}(0)^{2}} \times \frac{f_{K}^{2}}{f_{\pi}^{2}}$$

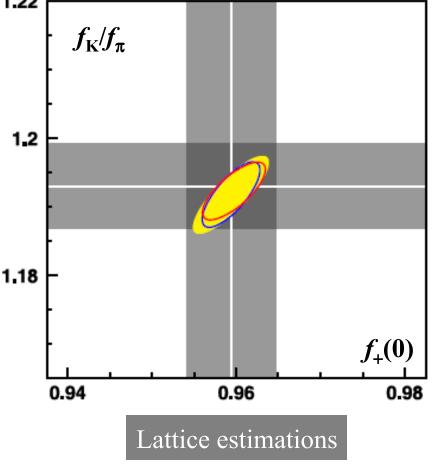
Assuming SM

From decay rates From nucl. and rad. corr. **B**-decay

- $f_{\rm K}/f_{\pi}$ and $f_{+}(0)$ values from a fit.
- 5 parameters: V_{ud} , $V_{us}f_{+}(0)$, Q_{12} , f_{K}/f_{π} , and $f_{+}(0)$.
 - 3 inputs: V_{ud} , $V_{us}f_{+}(0)$, Q_{12}
 - 2 constraints: $\Gamma(K\mu 2)/\Gamma(\pi\mu 2)$ relation and Unitarity
- Obtain (correlation ρ =0.84):

$$f_{+}(0) = 0.959(5),$$

 $f_{K}/f_{\pi} = 1.192(6)$ [with unitarity]



$f_{\mu}(0)$ and f_{K}/f_{π} : fit assuming unitarity

$$Q_{\ell 2} = \frac{(|V_{us}|f_{+}(0))^{2}}{|V_{ud}|^{2}} \times \frac{1}{f_{+}(0)^{2}} \times \frac{f_{K}^{2}}{f_{\pi}^{2}}$$

From decay rates From nucl. and rad. corr. **β**-decay

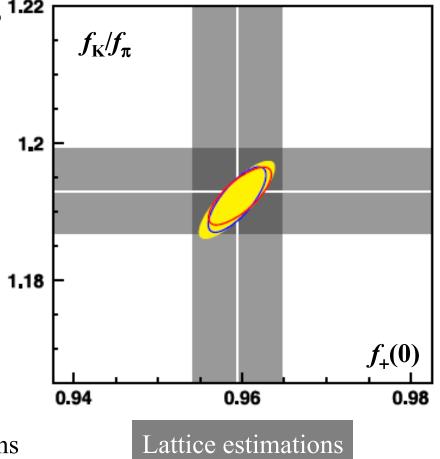
With either of reference values of $f_{\rm K}/f_{\pi}$ or $f_{+}(0)$ as an **additional** input:

- with input $f_{+}(0)=0.959(5)$, obtain $f_{+}(0)=0.9594(34)$ and $f_{K}/f_{\pi}=1.192(5)$

- with input $f_{K}/f_{\pi} = 1.193(6)$, obtain $f_{+}(0)=0.960(4)$ and $f_{K}/f_{\pi}=1.192(4)$.

Reference values are a near-perfect match with experimental data and SM assumptions

Assuming SN



Conclusions

Experimental precision on K leptonic and semileptonic decays nicely matched below per cent level by theoretical precision

- perform very precise measurements of SM parameters
- results: $|V_{us}f_{+}(0)|=0.2163(5)$ and $|V_{us}|/|V_{ud}|f_{K}/f_{\pi}=0.2758(5)$
- set stringent bounds on beyond-SM physics

In FlaviaNet Kaon WG (1005.2323 [hep-ph]) updated analysis of:

- overall determination of V_{us}, with and w/o imposing CKM unitarity
- $V_{us}(K (3))$ vs $V_{us}(K \mu 2)$: constraints on deviation from V-A structure
- test of lepton universality in Kl3 decays

Moreover:

- CKM matrix unitarity tested at 0.06%: O(10TeV) bound on NP-scale represents one of the most stringent constraints on beyond-SM physics.
- cross-checks of $\delta_{\mathrm{SU}(2)}$ and $f_+(0)$ theoretical results (within SM only)

Kaons can push fundamental principles at severe test, continuing to shed light on physics on and beyond SM