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Status of $|V_{us}|$ and g_A on the lattice

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These two talks review recent progress in lattice QCD simulations of kaon leptonic and semi-leptonic decays for the determination of the CKM-matrix element $|V_{us}|$ as well as simulations of the nucleon axial charge g_A.

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1 Introduction

Simulations of lattice QCD can provide predictions for the non-perturbative sector of the Standard Model. In principle, once the free parameters of the QCD-Lagrangian, the quark masses and the gauge coupling, have been fixed model independent predictions for spectra and matrix elements of QCD are possible. In practice, results of lattice QCD simulations are still affected by a number of systematic uncertainties due to approximations which have to be understood and controlled reliably before being used for stringent tests of the Standard Model (SM). The FLAVIANet Lattice Averaging Group (FLAG) [1] constituted itself in order to collect results of state-of-the art lattice simulations and to provide the non-expert reader with an overview over their quality in an easily accessible way (a colour code for various quality criteria illustrates FLAG's results: ★ for satisfactory, ● for should be improved, ■ for unsatisfactory attempt to satisfy systematics). Where appropriate FLAG provides an average over the most advanced results or just a range. It should be born in mind that lattice QCD is evolving continuously. The quality of results and therefore the criteria set up by FLAG will be subject to change.
In the following I will first review lattice results for the ratio of the kaon and pion decay constant f_K/f_π and the $K \to \pi$ semi-leptonic decay form factor at vanishing momentum transfer, $f_+^{K\pi}(0)$. In combination with experimental results for the decays, SM tests and a prediction for the CKM-matrix element $|V_{us}|$ are possible. This first part of the presentation is based on the FLAG group's findings. The remainder of the talk is dedicated to recent developments in the computation of the nucleon axial charge, g_A.

2 $|V_{us}|$

The determination of $|V_{us}|$ proceeds as follows: On the one hand, one experimentally measures the rate of a flavour changing process $s \to u$, where s is the strange quark and u the up quark. On the other hand one computes the SM prediction for the same process whose amplitude is proportional to the CKM-matrix element $|V_{us}|$ and which receives contributions from the electromagnetic, the weak and the strong interactions. While the former two can be treated conveniently in perturbation theory for the processes considered here, the contribution from the latter needs to be treated non-perturbatively. Since we envisage testing the SM it is clear that the computation should be model-independent and lattice QCD is the method of choice. Eventually, $|V_{us}|$ is determined by equating the experimental result with the SM-prediction.
For kaon/pion leptonic decays the relation between experiment and theory in the SM was computed by Marciano [2] and using the latest analysis of experimental results [3]

Collaboration	Ref.	N_f	pub. status	χ extrapol.	finite volume errors	cont. extrapolation	$f_+(0)$
RBC/UKQCD 10	[4]	2+1	A	●	★	■	$0.9599(34)(^{+31}_{-47})(14)$
RBC/UKQCD 07	[5]	2+1	A	●	★	■	0.9644(33)(34)(14)
ETM 09A	[6]	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	[7]	2	C	■	★	■	$0.9647(15)_{stat}$
RBC 06	[8]	2	A	■	★	■	0.968(9)(6)
JLQCD 05	[9]	2	C	■	★	■	0.967(6), 0.952(6)
							f_K/f_π
BMW 10	[10]	2+1	A	★	★	★	$1.192(7)(6)^\dagger$
JLQCD/TWQCD 09B	[11]	2+1	C	●	■	■	$1.210(12)_{\rm stat}$
MILC 09A	[12]	2+1	C	●	★	★	$1.198(2)(^{+6}_{-8})$
MILC 09	[13]	2+1	A	●	★	★	$1.197(3)(^{+6}_{-13})$
ALVdW 08	[14]	2+1	C	●	●	●	1.191(16)(17)
PACS-CS 08, 08B	[15, 16]	2+1	A	★	■	■	1.189(20)
HPQCD/UKQCD 08	[17]	2+1	A	●	●	★	1.189(2)(7)
RBC/UKQCD 08	[18]	2+1	A	●	★	■	1.205(18)(62)
NPLQCD 06	[19]	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
ETM 09	[20]	2	A	●	●	★	1.210(6)(15)(9)
QCDSF/UKQCD 07	[21]	2	C	●	★	●	1.21(3)

Table 1: FLAG's quality assessment of lattice results for $f_+(0)$ and f_K/f_π [1].

it yields the correlation

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.2758(5)\,. \tag{1}$$

Since lattice QCD can provide f_K/f_π one obtains a prediction for $|V_{us}/V_{ud}|$. For semi-leptonic kaon decays the latest summary of experimental results together with SM-contributions yields [3]

$$|V_{us}| f_+^{K\pi}(0) = 0.2163(5)\,, \tag{2}$$

and $|V_{us}|$ is readily extracted provided the prediction of $f_+^{K\pi}(0)$ from simulations of lattice QCD.

Table 1 compiles currently available lattice results with 2 and 2+1 flavours of dynamical fermions for both $f_+^{K\pi}(0)$ and f_K/f_π. Representative error budgets by BMW for f_K/f_π and by RBC+UKQCD for $f_+^{K\pi}(0)$ are provided in the table in figure 1. The chiral extrapolation is the dominant systematic uncertainty in both cases and in the case of f_K/f_π also discretisation effects are of almost the same magnitude. As lattice simulations at the physical point now seem feasible (in fact, some collaborations are already simulating directly at the physical point [22, 23]) the uncertainty in the chiral extrapolation is likely to become sub-dominant in the near future. Concerning the continuum extrapolation it should be noted that standard lattice simulation algorithms recently were found to be affected by critical slowing down. That is, as one removes the lattice spacing and thereby increases the cut-off, the known algorithms turned out not to sample all of phase within reasonable simulation-lengths [24, 25, 26]. To date it is not clear to which extent observables like f_K/f_π and $f_+^{K\pi}(0)$ are affected (biased) by this effect. This effect has to be understood and controlled before trusting lattice data at very small lattice spacings.

The results for f_K/f_π and for $f_+^{K\pi}(0)$ for $N_f = 2$ and $N_f = 2+1$, respectively, are all mutually compatible. Given that the simulation and analysis techniques that lead to all these results differ significantly amongst the quoted collaborations this should cause confidence in the approach. Comparing results with $N_f = 2$ and $N_f = 2+1$ flavours of dynamical sea quarks, the effect of adding the dynamical strange quark does not lead to any visible effects beyond the current level of precision.

An in-depth review of current lattice results for f_K/f_π and $f_+^{K\pi}(0)$ is provided in [1] and the following averages/recommended values are taken from this reference: $f_K/f_\pi|_{N_f=2+1} = 1.193(6)$ (average over BMW, MILC and HPQCD/UKQCD), $f_K/f_\pi|_{N_f=2} = 1.210(18)$ (ETM) and, $f_+^{K\pi}(0)|_{N_f=2+1} = 0.960(5)$ (RBC+UKQCD) and $f_+^{K\pi}(0)|_{N_f=2} = 0.956(8)$ (ETM).

On the one hand these results can be used once for making predictions for $|V_{ud}|$, $|V_{us}|$, $f_+^{K\pi}(0)$ and f_K/f_π based on the experimental results (1) and (2) and on the assumption of CKM first row unitarity $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 = 1$ (at the current level of precision $|V_{ub}|$ is too small to play any significant role). On the other hand, when using only the experimental result as input the first row unitarity can be tested. This is summarised in the plot in figure 1.

Given that KLOE-2 is aiming at reducing the uncertainty in their experimental determination for $|V_{us}|f_+^{K\pi}(0)$ by a factor of about two in the next three years [27, 28] it is fair to ask about prospects on the theory side. Recent progress for $f_+^{K\pi}(0)$ [29, 4] has allowed to remove one of the two most dominant uncertainties (momentum resolution in lattice simulations). The remaining dominant uncertainty is the one due to the chiral extrapolation. It is therefore conceivable that in particular the uncertainty in $f_+^{K\pi}(0)$, where lattice artefacts are a sub-dominant effect, will see a considerable improvement in the near future: For $f_+^{K\pi}(0)$ cut-off effects will remain a sub-dominant uncertainty for a while: Flavour symmetry implies that if the average light quark mass m_q is set equal to the strange quark mass m_s, the lattice data yield $f_+^{K\pi}(0) = 1$, irrespective of the lattice spacing or the size of the box and for any value of m_s. Cut-off effects can therefore only affect the difference $1 - f_+^{K\pi}(0)$, which turns out to be

source	$\delta(f_K/f_\pi)$
statistics	0.6%
chiral extrapolation	
- functional form	0.3%
- pion mass range	0.3%
cont. extrapolation	0.3%
exited states	0.2%
scale setting	0.1%
finite volume	0.1%
total	0.8%
source	$\delta f_+^{K\pi}(0)$
statistical	0.3%
chiral extrapolation	0.4%
cont. extrapolation	0.1%
total	0.5%

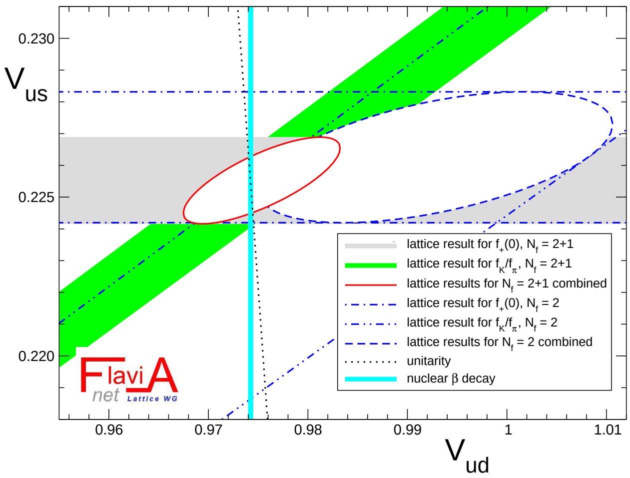

Figure 1: Left: Error budgets for state-of-the art lattice computations for f_K/f_π (BMW [10]) and for $f_+^{K\pi}(0)$ (RBC+UKQCD [5, 4]); Right: FLAG's illustration of lattice results in the $|V_{us}|$-$|V_{ud}|$-plane [1]. The ellipse represent the combined unitarity analysis for $N_f = 2+1$ flavours (solid red) and $N_f = 2$ flavours (dashed blue) while the black dashed line represents SM-unitarity. According to this analysis all results are compatible with first row unitarity.

about 0.04. For f_K/f_π the error due to the chiral extrapolation will disappear once all collaborations simulate directly at the physical point. The statistical error can be reduced by simulating longer (naively it reduces with $1/\sqrt{N}$ where N is proportional to the Monte Carlo time). However, as mentioned above, the uncertainty due to the continuum extrapolation is currently causing concern and will very likely stay at the current level until the algorithmic problems have been solved.

3 g_A

The systematic effects in computations of the nucleon axial charge g_A are much less well understood than the ones for the observables in the previous section. The two major difficulties are: i) in lattice QCD properties of a ground state with specified quantum numbers are extracted from the time-dependence of Euclidean n-point correlation functions. For 2-point functions for example, the ground state properties can be extracted from a window in Euclidean time, where on the one hand excited states are exponentially suppressed and on the other hand the statistical properties of the correlation functions are still good. For pseudo scalar mesons the variance of 2-point functions remains constant as a function of time while for neutron- and proton 2-point functions the variance increases exponentially. It is therefore often impossible to decide with certainty whether a window for the ground state exists for the latter

class of 2-point functions, and hence, whether it can be determined reliably. To this end the Mainz group [30] has implemented a summation method [31] which seems promising in subtracting excited states.
While this approach may be the method of choice for future calculations further systematic effects that one is only starting to understand better arise from finite size effects (typical lattice volumes extend only over 3-4fm in the spatial directions). Moreover, as long as the simulations are for unphysically heavy quark masses, the data has to be extrapolated to the physical point. One typically uses ansätze motivated by baryon chiral perturbation theory of which the convergence properties still remain to be understood.
Current lattice simulations are not able to reproduce the experimental prediction for g_A (for an overview see [32]). Given the difficulties involved in its calculation this should not cause excitement - on the contrary, g_A is currently not a prediction of lattice QCD but rather a candle for assessing the quality of nucleon matrix element calculations on th lattice.

4 Conclusions

These two talks briefly summarise the status of lattice simulations relevant for the SM prediction of the CKM-matrix element $|V_{us}|$ via simulations of leptonic and semi-leptonic meson decays. Also efforts for the determination of the nucleon axial charge g_A are addressed. The prediction of $|V_{us}|$ is already very advanced and it will continue improving steadily as simulations are pushed towards the physical point. There are currently no known conceptional issues in going to this limit. Conceptional problems will however have to be addressed before taking the continuum limit.
While these considerations do in principle also apply to nucleon matrix elements like the one defining the nucleon axial charge g_A, the discrepancy between lattice results and the experimental value turns out to increase when reducing the quark mass. This effect is currently attributed to finite size effects and excited states contributions. It remains to be seen if the apparent discrepancy between theory predictions and the experimental result persists as simulations with larger lattice volumes are carried out and the new field theoretic ideas to subtract excited states contributions are applied.
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