

# CKM 2010

Measurement of  $\phi_2$  in  $B \rightarrow \pi\pi, \rho\pi$  and  $\rho\rho$

Jeremy Dalseno

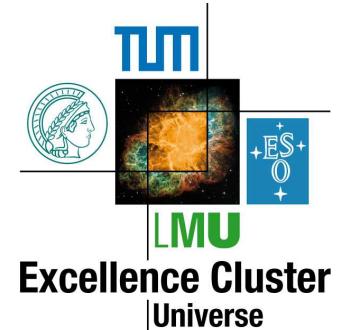


Max-Planck-Institut für Physik  
(Werner-Heisenberg-Institut)

Max-Planck-Institut für Physik  
Excellence Cluster Universe

jdalseno [@] mpp.mpg.de

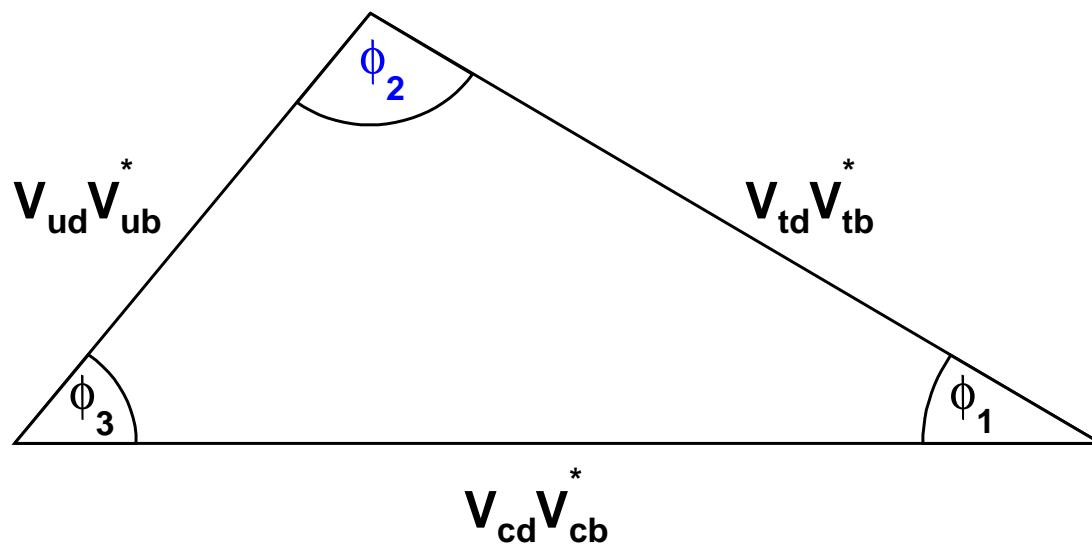
08 September 2010



# Outline

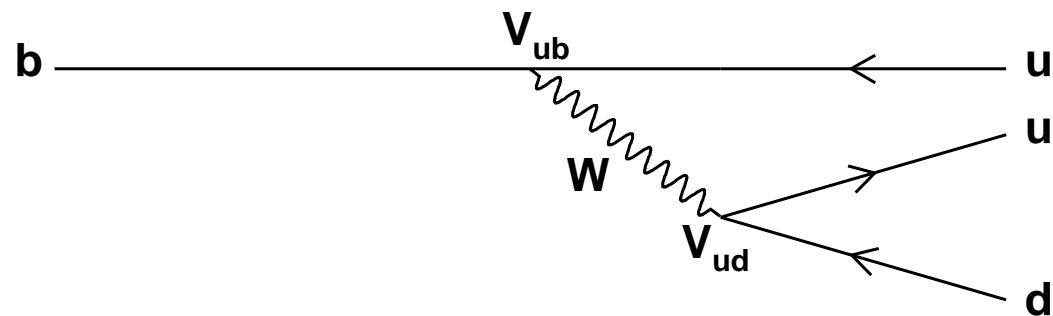
1.  $B \rightarrow \pi\pi$
2.  $B \rightarrow \rho\rho$
3.  $B^0 \rightarrow (\rho\pi)^0$

From unitarity of the CKM matrix,

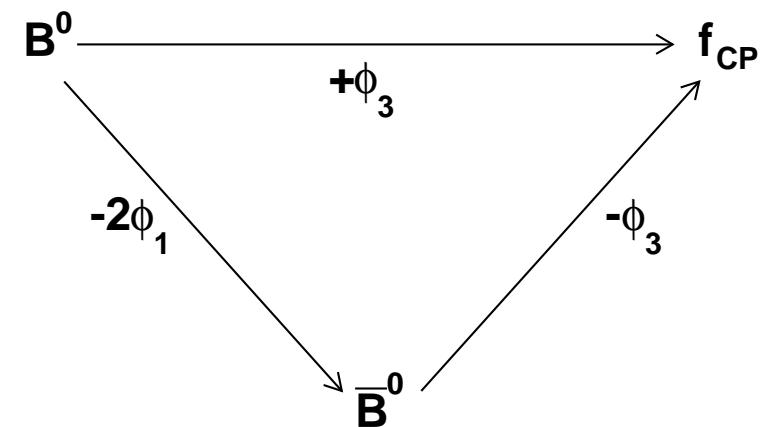


# Introduction

Tree-level  $b \rightarrow u\bar{u}d$  transitions sensitive to  $\phi_2$



$V_{ub}$  carries the phase  $e^{-i\phi_3}$



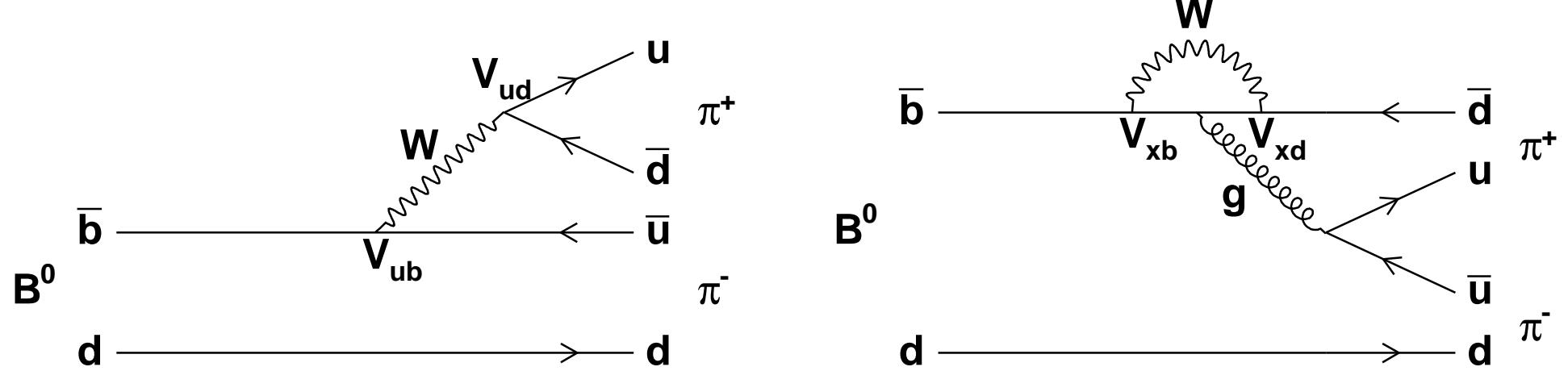
Measure a time difference between  $B$  decays,  $\Delta t$ , and the flavour of the tag-side  $B$ ,  $q$

$$\mathcal{P}(\Delta t, q) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left[ 1 + q(\mathcal{A}_{CP} \cos \Delta m_d \Delta t + \mathcal{S}_{CP} \sin \Delta m_d \Delta t) \right]$$

If only the tree amplitude is present we expect,  $\mathcal{A}_{CP} = 0$ ,  $\mathcal{S}_{CP} = \sin 2\phi_2$

# Introduction

Both tree and penguin amplitudes may contribute to the final state



Tree and penguin amplitudes carry different strong and weak phases

Direct  $CP$  violation,  $\mathcal{A}_{CP} \neq 0$ , is possible

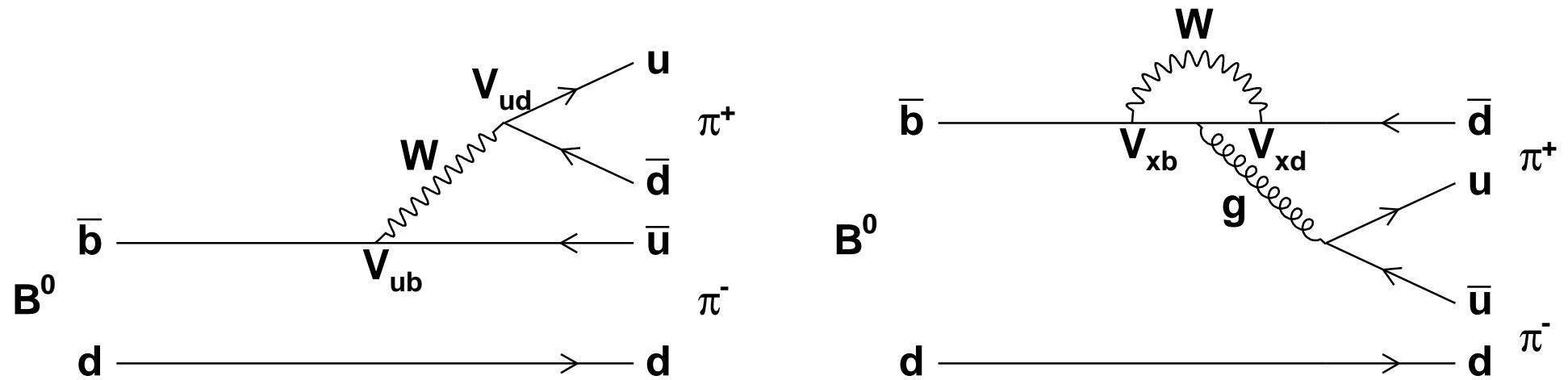
Measure an effective  $\phi_2$

$$\mathcal{S}_{CP} = \sqrt{1 - \mathcal{A}_{CP}^2} \sin(2\phi_2 - 2\Delta\phi_2) = \sqrt{1 - \mathcal{A}_{CP}^2} \sin 2\phi_2^{\text{eff}}$$

# Introduction

Can recover  $\phi_2$  with an SU(2) isospin analysis

M. Gronau and D. London, PRL **65**, 3381 (1990)



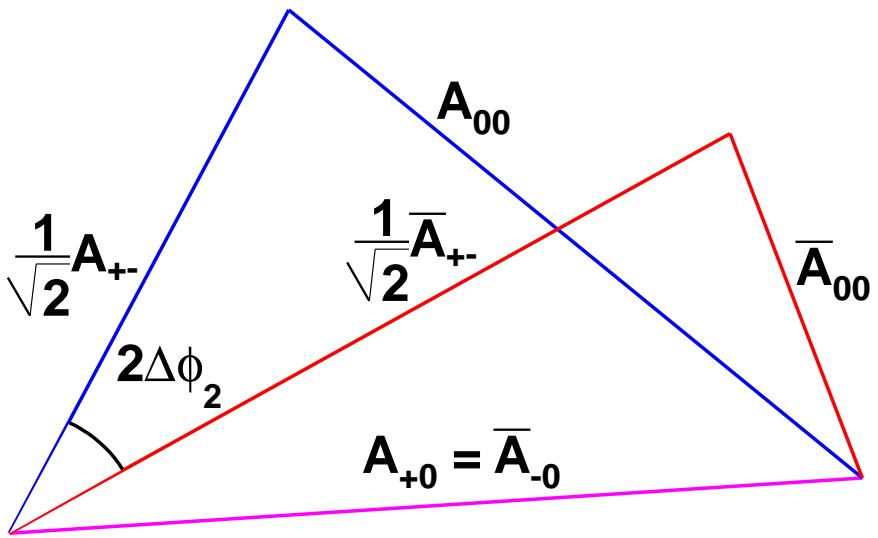
Consider  $B^+ \rightarrow \pi^+ \pi^0$  which has  $I_3 = 1 \Rightarrow$  Isospin,  $I = 1, 2$

Gluon in penguin carries  $I = 0 \Rightarrow$  penguin has  $I = 0, 1$

Bose-Einstein statistics forbids anti-symmetric state,  $I = 1$

$\Rightarrow I = 2$  and therefore  $B^+ \rightarrow \pi^+ \pi^0$  is a pure tree mode

# Introduction



2-fold ambiguity of  $\phi_2$  in measured  $S_{CP}$ , therefore 8-fold ambiguity in  $\phi_2$  in isospin analysis

Fully determined from 6 physical observables

$$\begin{aligned} & \mathcal{B}(B^0 \rightarrow \pi^+\pi^-), \mathcal{B}(B^0 \rightarrow \pi^0\pi^0), \mathcal{B}(B^+ \rightarrow \pi^+\pi^0) \\ & \mathcal{A}_{CP}(\pi^+\pi^-), \mathcal{S}_{CP}(\pi^+\pi^-), \mathcal{A}_{CP}(\pi^0\pi^0) \end{aligned}$$

Consider the set of 3  $B \rightarrow \pi\pi$  modes

$$A_{+0} = \frac{1}{\sqrt{2}}A_{+-} + A_{00}$$

$$\bar{A}_{-0} = \frac{1}{\sqrt{2}}\bar{A}_{-+} + \bar{A}_{00}$$

$A_{ij}$ : Amplitude of  $B \rightarrow \pi^i\pi^j$

Neglecting electroweak penguins,  $A_{+0} = \bar{A}_{-0}$

4-fold ambiguity in  $2\Delta\phi_2$

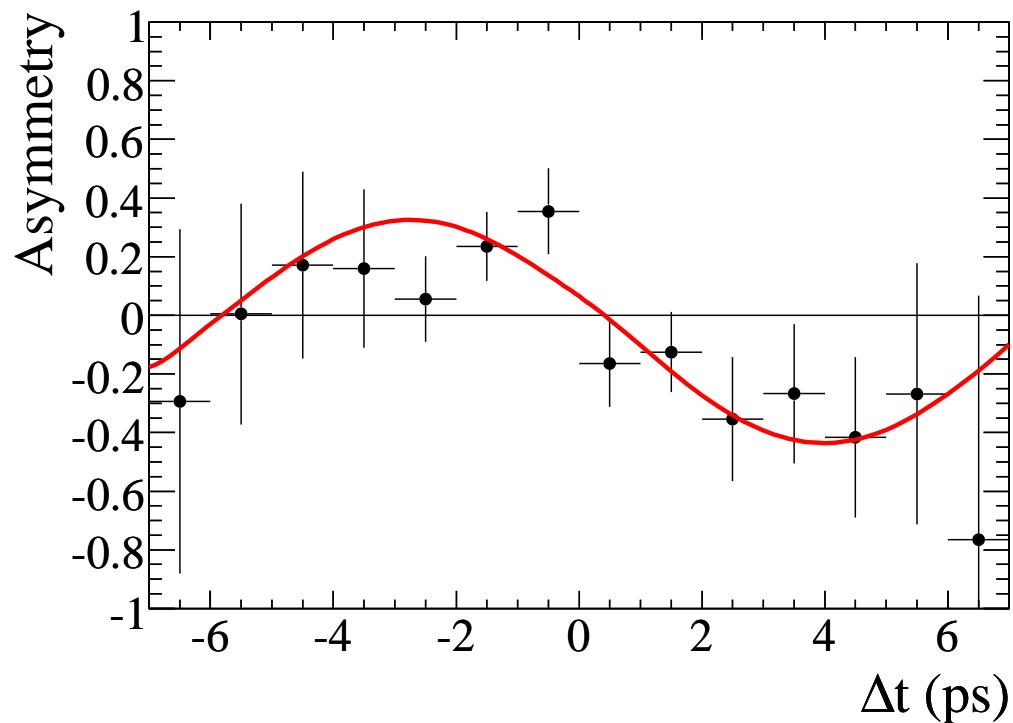
# $B \rightarrow \pi\pi$

BaBar

arXiv:0807.4226 (2008)

467 million  $B\bar{B}$  pairs

$$a_{CP}(\Delta t) \equiv (N_{B^0} - N_{\bar{B}^0})/(N_{B^0} + N_{\bar{B}^0})$$



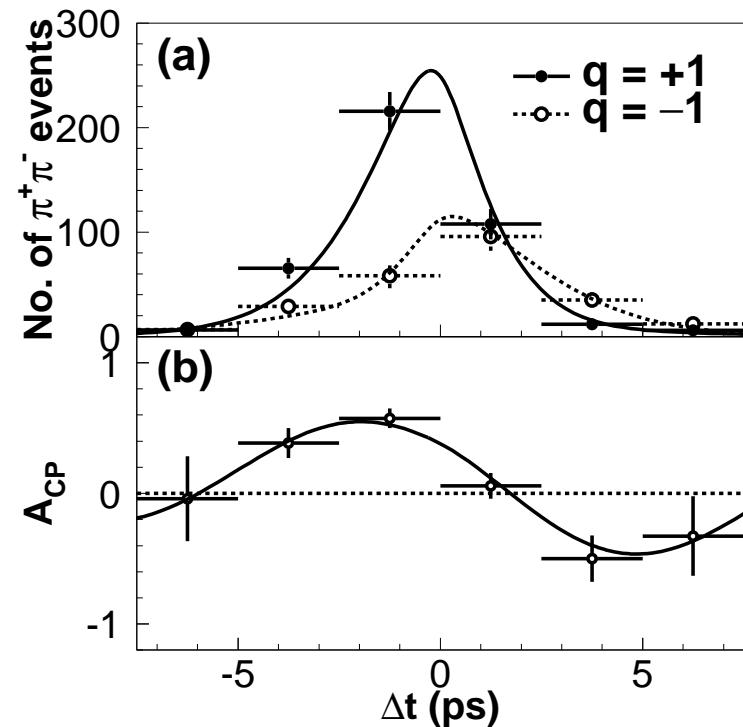
Clear mixing-induced asymmetry

Belle

PRL 98, 211801 (2007)

535 million  $B\bar{B}$  pairs

$\Delta t$  distribution and asymmetry



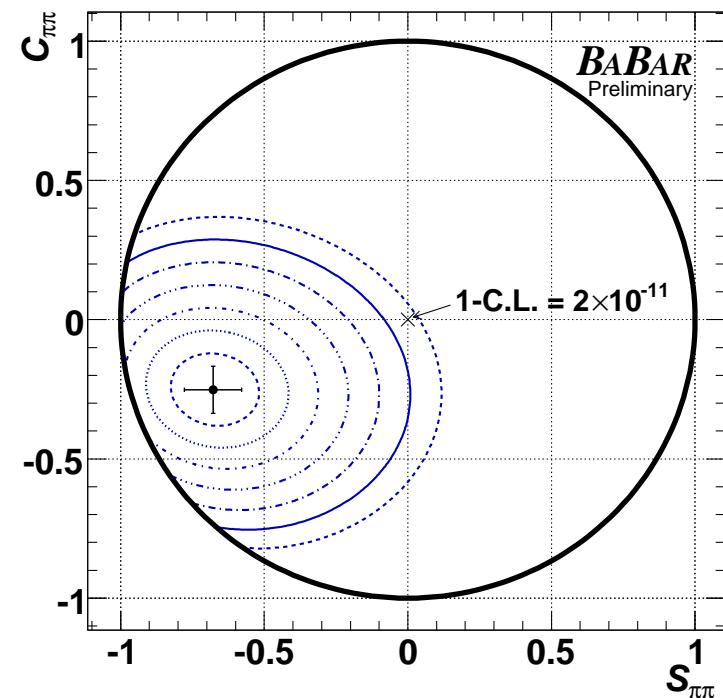
Height difference shows direct  $CP$  asymmetry

# $B \rightarrow \pi\pi$

BaBar

$$\mathcal{A}_{CP} = +0.25 \pm 0.08 \pm 0.02 \text{ (3.0}\sigma)$$

$$\mathcal{S}_{CP} = -0.68 \pm 0.10 \pm 0.03 \text{ (6.3}\sigma)$$



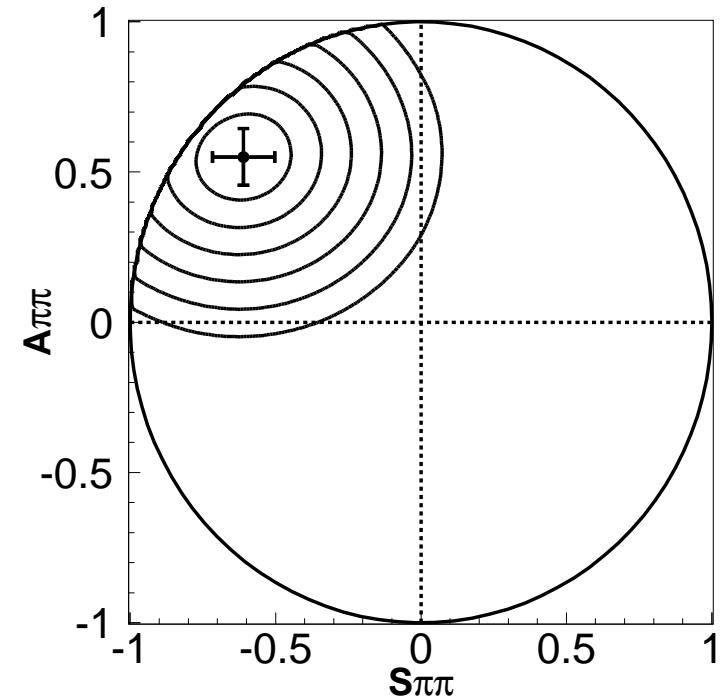
$$\mathcal{C}_{CP} = -\mathcal{A}_{CP}$$

Both experiments have observed  $CP$  violation

Belle

$$\mathcal{A}_{CP} = +0.55 \pm 0.08 \pm 0.05 \text{ (5.5}\sigma)$$

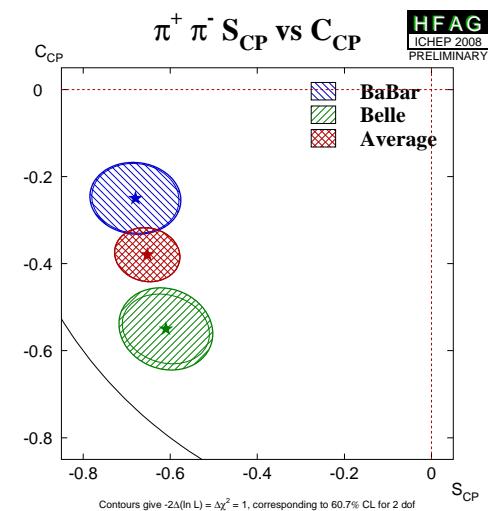
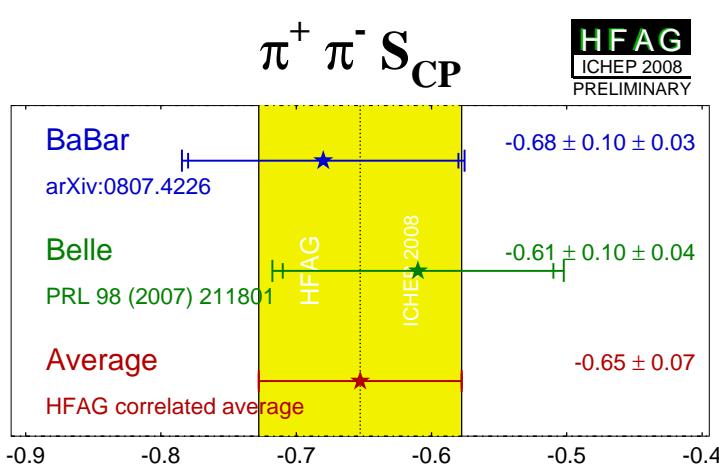
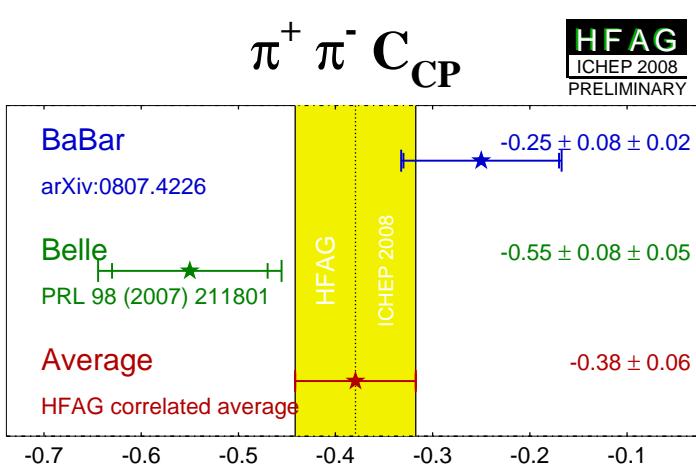
$$\mathcal{S}_{CP} = -0.61 \pm 0.10 \pm 0.04 \text{ (5.3}\sigma)$$



$CP$  violation observed in individual parameters

# $B \rightarrow \pi\pi$

World average



$$\mathcal{C}_{CP} = -\mathcal{A}_{CP}$$

Difference between BaBar and Belle measurements down to  $1.9\sigma$

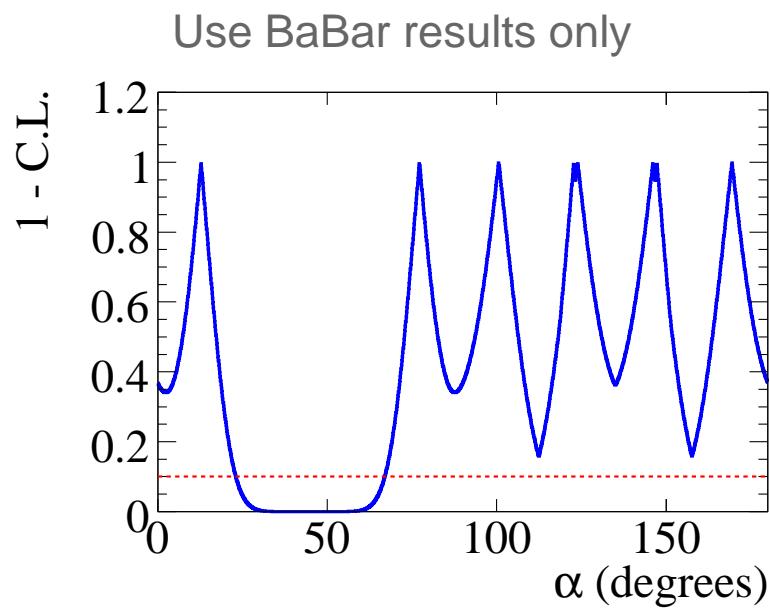
Both experiments demonstrate that more than a tree amplitude is present

# $B \rightarrow \pi\pi$

Construct  $\chi^2$  for 6 physical observables from 5 constraining isospin amplitudes and  $\phi_2$   
 $(A_{+0}, A_{+-}, A_{00}, \bar{A}_{+-}, \bar{A}_{00})$

Scan  $\phi_2$ , minimise  $\chi^2$  and convert to CL for  $6 - 5 = 1$  degree of freedom

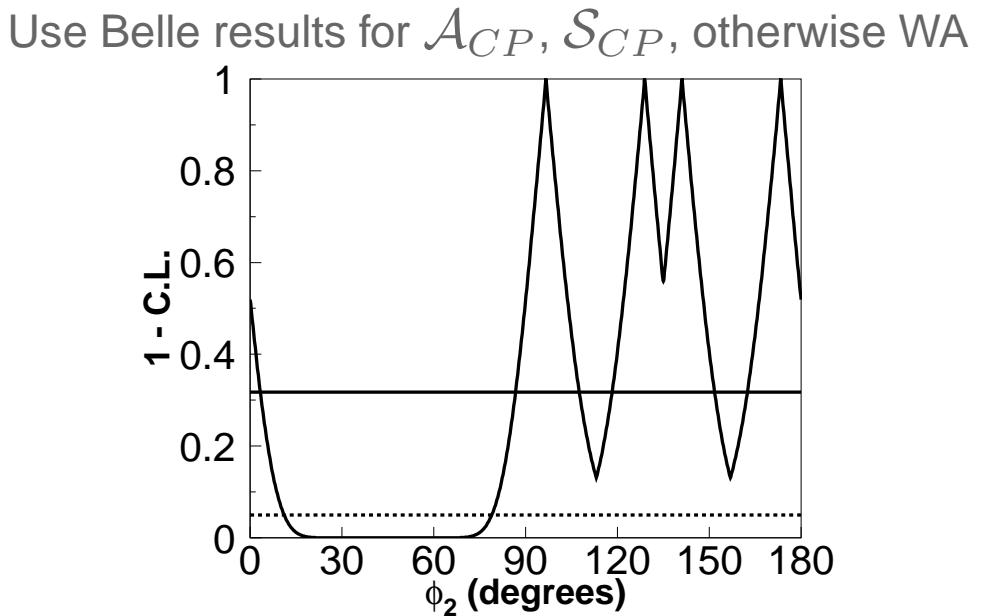
BaBar



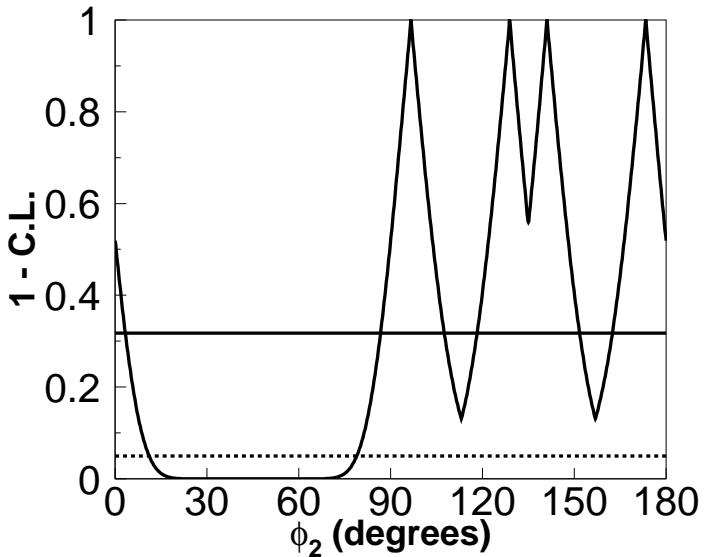
Use BaBar results only

$[23^\circ, 67^\circ]$  excluded at 90% CL

Belle



Use Belle results for  $\mathcal{A}_{CP}$ ,  $\mathcal{S}_{CP}$ , otherwise WA



$[11^\circ, 79^\circ]$  excluded at 95% CL

$$B \rightarrow \rho\rho$$

$S \rightarrow VV$ , Spin  $0 \rightarrow 1 + 1$

$J_{\rho\rho} = 0$  and orbital angular momentum,  $L$ , has no component along the decay axis

Final state a superposition of 3 possible polarisation amplitudes: 1 longitudinal and 2 transverse

In the helicity basis,

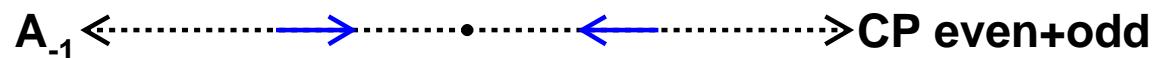


$$CP = (-1)^L$$

$$L = 0, 2 \rightarrow CP = +1$$



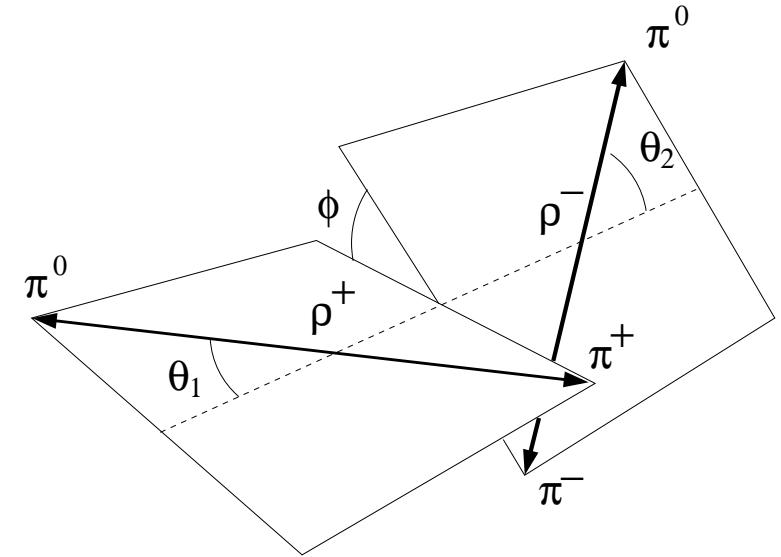
$$L = 0, 1, 2 \rightarrow CP = \text{mixed}$$



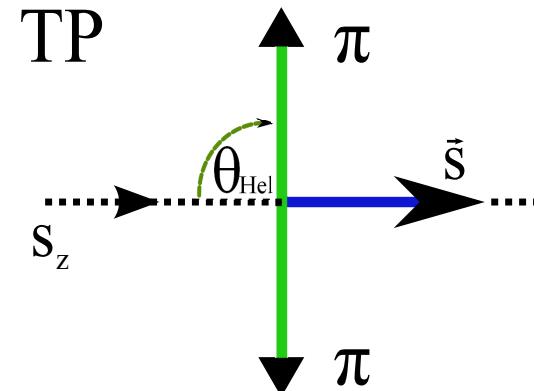
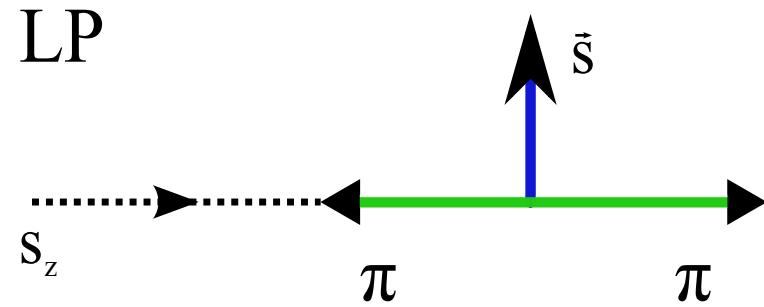
$$B \rightarrow \rho\rho$$

Decompose the longitudinal contribution

Integrating over  $\phi$ , angular decay rate



$$\frac{d^2 N}{d \cos \theta_1 d \cos \theta_2} \propto 4 f_L \cos^2 \theta_1 \cos^2 \theta_2 + (1 - f_L) \sin^2 \theta_1 \sin^2 \theta_2, \quad f_L \equiv \frac{|A_0|^2}{|A_0|^2 + |A_{+1}|^2 + |A_{-1}|^2}$$



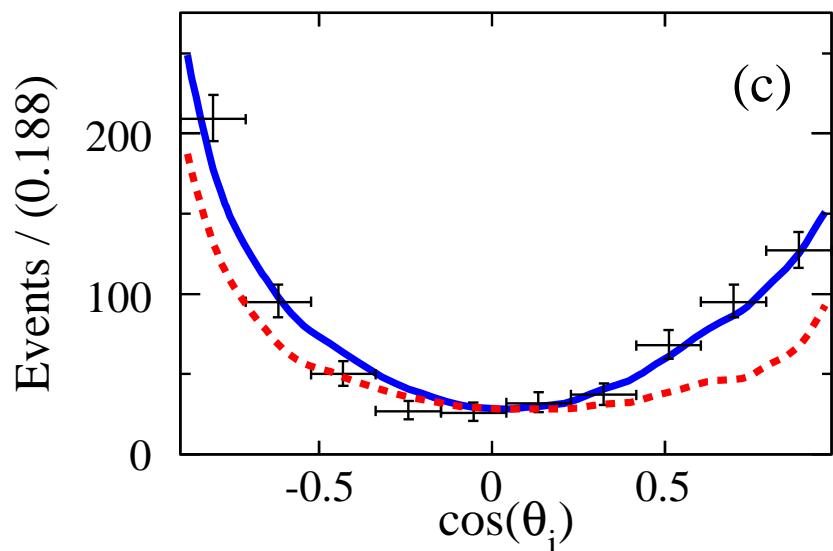
$$B^+ \rightarrow \rho^+ \rho^-$$

BaBar

PRD **76** 052007 (2007)

384 million  $B\bar{B}$  pairs

Helicity



$$\mathcal{B}(B^+ \rightarrow \rho^+ \rho^-) = (25.5 \pm 2.1^{+3.6}_{-3.9}) \times 10^{-6}$$

$$f_L = 0.992 \pm 0.024 \text{ (stat)} {}^{+0.026}_{-0.013} \text{ (syst)}$$

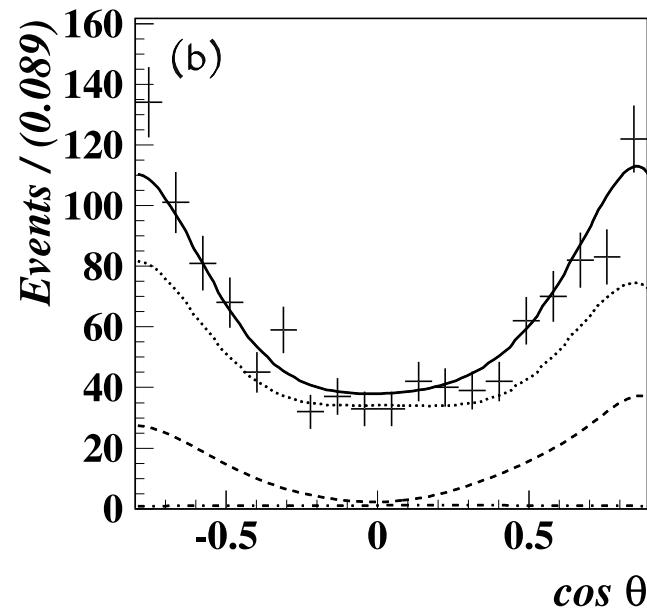
Longitudinal polarisation dominates

Belle

PRL **96** 171801 (2006)

275 million  $B\bar{B}$  pairs

Helicity



$$\mathcal{B}(B^+ \rightarrow \rho^+ \rho^-) = (22.8 \pm 3.8^{+2.3}_{-2.6}) \times 10^{-6}$$

$$f_L = 0.941^{+0.034}_{-0.040} \text{ (stat)} \pm 0.030 \text{ (syst)}$$

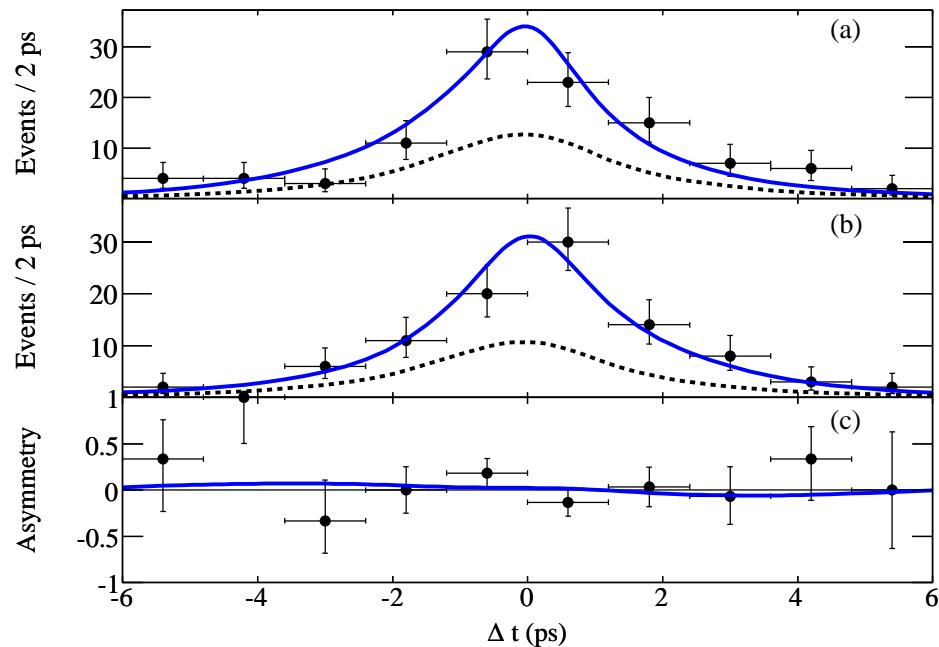


BaBar

PRD 76 052007 (2007)

384 million  $B\bar{B}$  pairs

$\Delta t$  distribution and asymmetry



$$\mathcal{A}_{CP} = -0.01 \pm 0.15 \pm 0.06$$

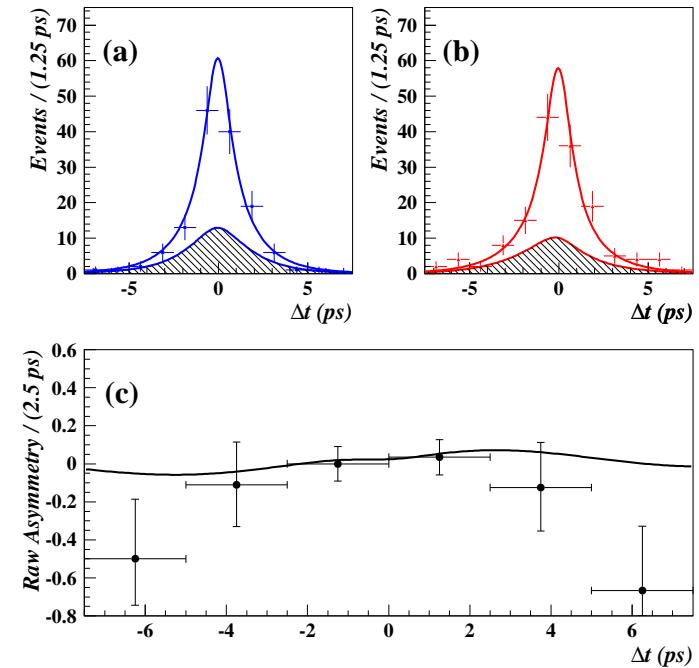
$$\mathcal{S}_{CP} = -0.17 \pm 0.20^{+0.05}_{-0.06}$$

Belle

PRD 76 011104 (2007)

Update to 535 million  $B\bar{B}$  pairs

$\Delta t$  distribution and asymmetry

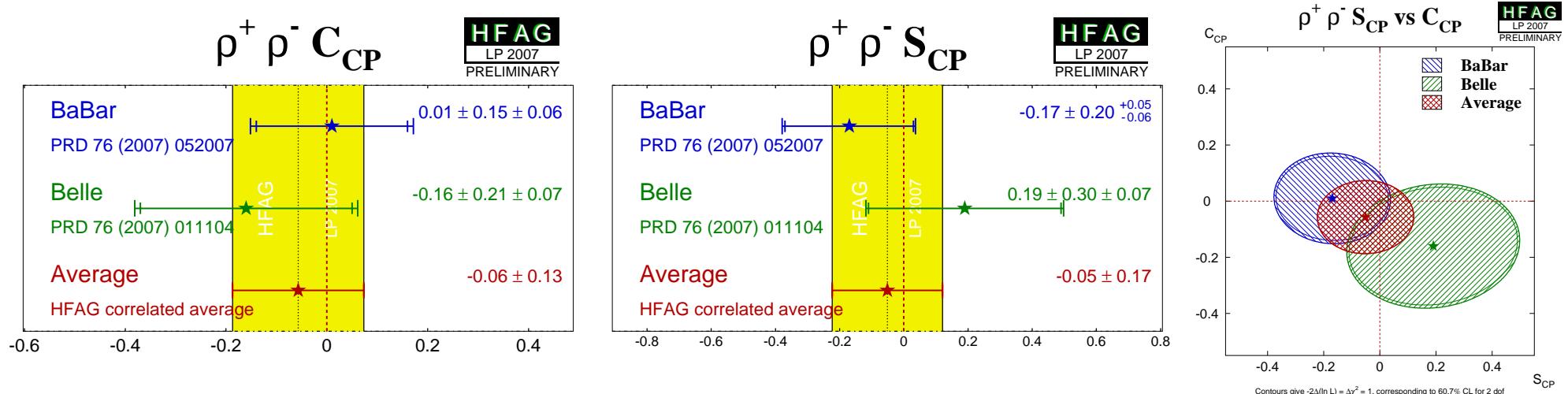


$$\mathcal{A}_{CP} = +0.16 \pm 0.21 \pm 0.07$$

$$\mathcal{S}_{CP} = +0.19 \pm 0.30 \pm 0.07$$

$$B^+ \rightarrow \rho^+ \rho^-$$

World average



$$\mathcal{C}_{CP} = -\mathcal{A}_{CP}$$

Good agreements between experiments

$\mathcal{A}_{CP} \approx 0$ , small penguin contribution

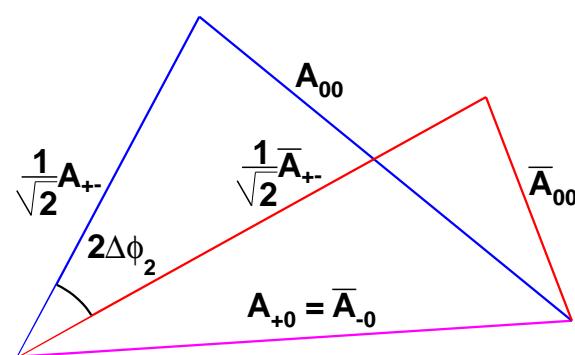
$$B^+ \rightarrow \rho^+ \rho^0$$

Recent results from BaBar

PRL 102 141802 (2009)

$$\mathcal{B}(B^+ \rightarrow \rho^+ \rho^0) = (23.7 \pm 1.4 \pm 1.4) \times 10^{-6}$$

Precise measurement of isospin triangle base



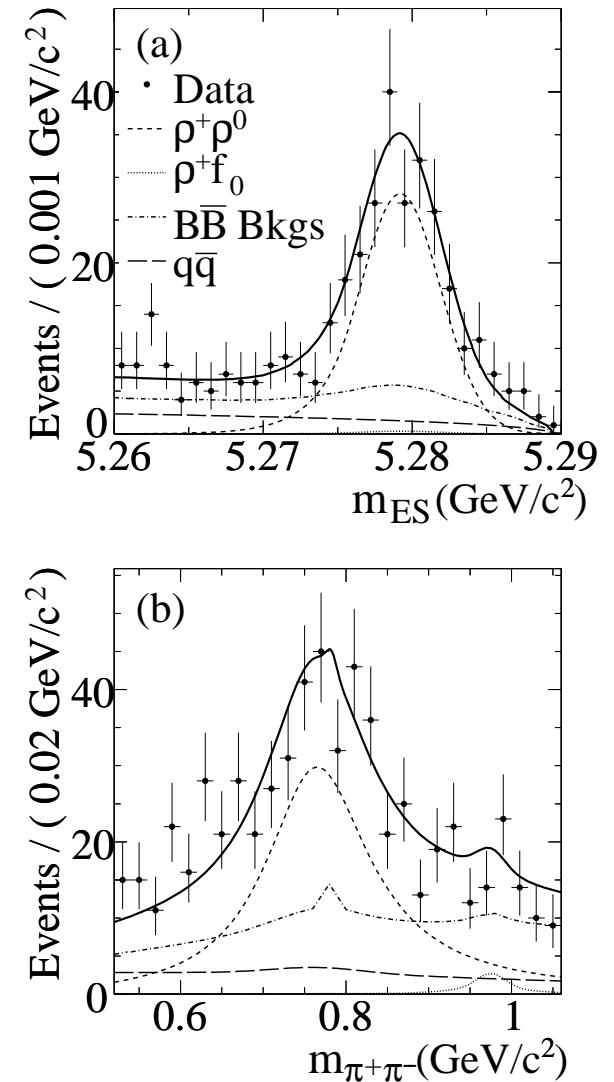
$$f_L = 0.950 \pm 0.015 \pm 0.006$$

Dominantly longitudinally polarised

$$\mathcal{A}_{CP} = 0.054 \pm 0.055 \pm 0.010$$

No evidence for electroweak penguins

Belle will update this analysis soon with  $\sim 10$  times statistics



$$B^0 \rightarrow \rho^0 \rho^0$$

Time-dependent measurement removes 4-fold ambiguity of  $\Delta\phi_2$

Difficult to isolate  $\rho^0 \rho^0$  in presence of other 4-body signals,  $a_1\pi, \rho\pi\pi, 4\pi, f_0\rho^0, f_0f_0, f_0\pi\pi$

BaBar

PRD **78** 071104(R) (2008)

465 million  $B\bar{B}$  pairs

$$N(B^0 \rightarrow \rho^0 \rho^0) = 99^{+35}_{-34} \pm 15$$

$$\mathcal{B}(B^0 \rightarrow \rho^0 \rho^0) =$$

$$(0.92 \pm 0.32 \pm 0.14) \times 10^{-6}$$

$3.1\sigma$  evidence

$$f_L = 0.75^{+0.11}_{-0.14} \text{ (stat)} \pm 0.04 \text{ (syst)}$$

$$\mathcal{A}_{CP} = -0.2 \pm 0.8 \pm 0.3$$

$$\mathcal{S}_{CP} = +0.3 \pm 0.7 \pm 0.2$$

Belle

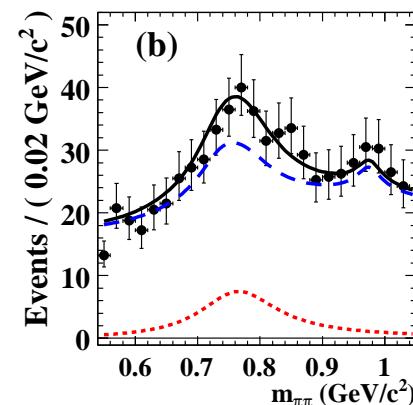
PRD **78** 111102(R) (2008)

657 million  $B\bar{B}$  pairs

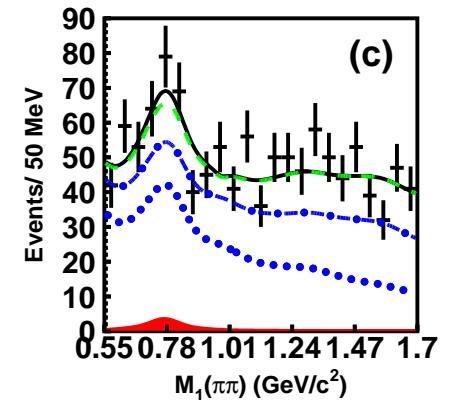
$$N(B^0 \rightarrow \rho^0 \rho^0) = 24.5^{+23.6+10.1}_{-22.1-16.2}$$

$$\mathcal{B}(B^0 \rightarrow \rho^0 \rho^0) < 1.0 \times 10^{-6} \text{ at 90% CL}$$

BaBar



Belle



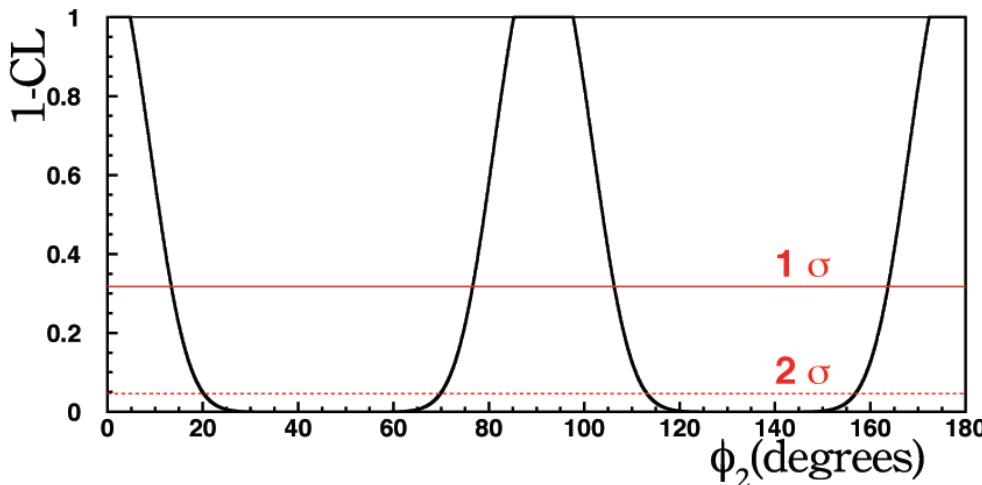
# $B^0 \rightarrow \rho\rho$

Branching fraction of  $B^+ \rightarrow \rho^+ \rho^0$  large compared to  $B^0 \rightarrow \rho^0 \rho^0$

Nearly flat isospin triangles  $\Rightarrow$  4 solutions of  $\Delta\phi_2$  nearly degenerate

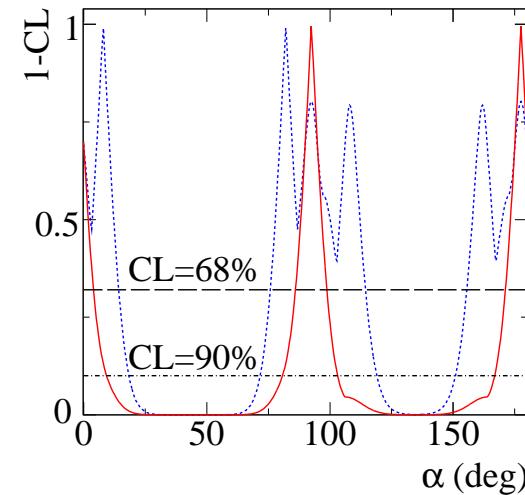
Belle

Use Belle results for  $\mathcal{B}(\rho^0 \rho^0)$ , otherwise WA  
Before BaBar's  $B^+ \rightarrow \rho^+ \rho^0$  update



BaBar

PRL 102 141802 (2009)  
Use BaBar results only



Plateau due to no constraint on  $\mathcal{A}_{CP}(\rho^0 \rho^0)$   
 $\phi_2 = (91.7 \pm 14.9)^\circ$

Blue: Before  $\mathcal{B}(B^+ \rightarrow \rho^+ \rho^0)$  increase  
 $\phi_2 = (92.4^{+6.0}_{-6.5})^\circ$

$B \rightarrow \rho\rho$  currently the best environment for constraining  $\phi_2$  because of relatively small penguins

$$B^0 \rightarrow (\rho\pi)^0$$

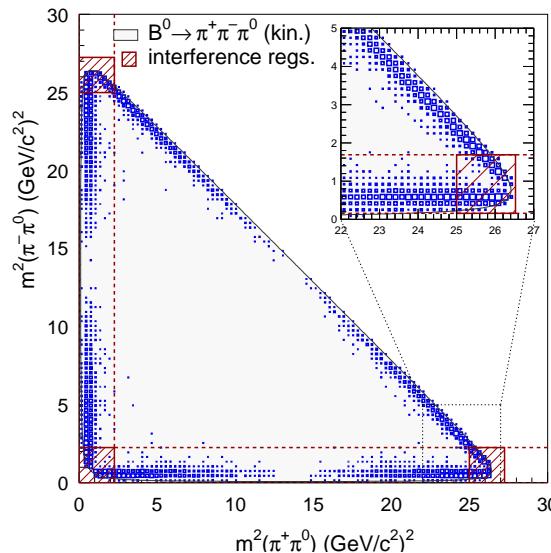
Not a  $CP$  eigenstate, need to consider the 4 flavour-charge configurations

Corresponding isospin analysis has 12 unknowns compared to 6 for  $CP$  eigenstates

However, can constrain  $\phi_2$  without ambiguity explicitly in the analysis

A. Snyder and H. Quinn, PRD **48** 2139 (1993)

Include variation of the strong phases of the interfering  $\rho$  resonances in the Dalitz Plot



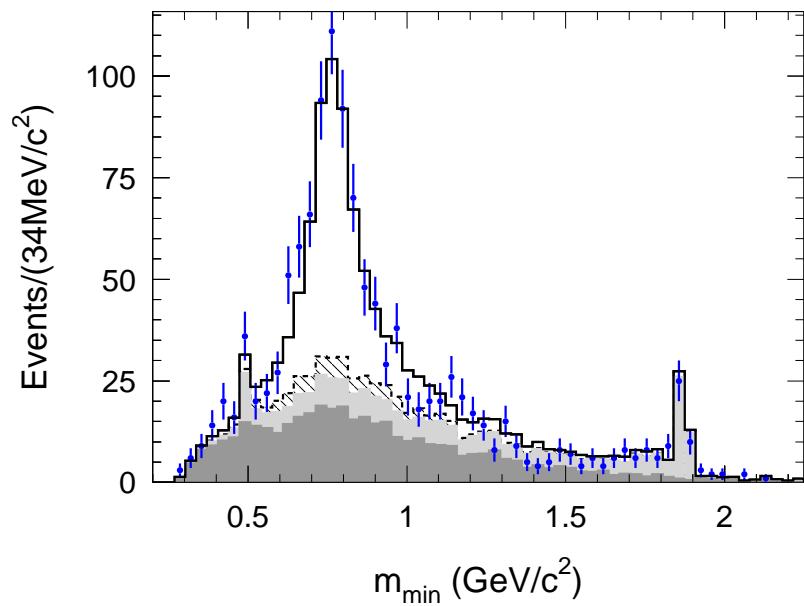
$$B^0 \rightarrow (\rho\pi)^0$$

BaBar

PRD **76** 012004 (2007)

375 million  $B\bar{B}$  pairs

Mass projections



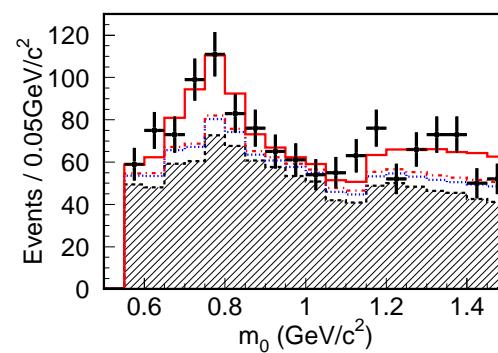
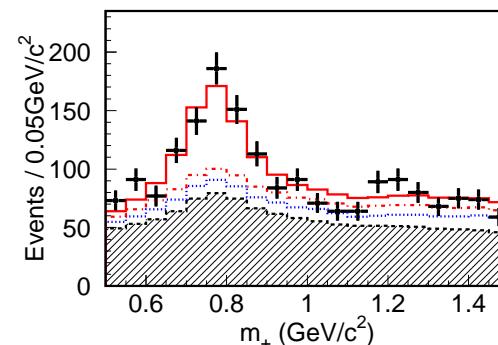
Plot minimum of  $m_+, m_-, m_0$

Belle

PRL **98** 221602 (2007)

449 million  $B\bar{B}$  pairs

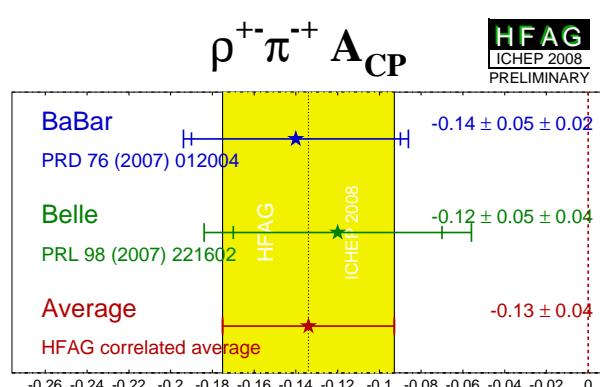
Mass projections



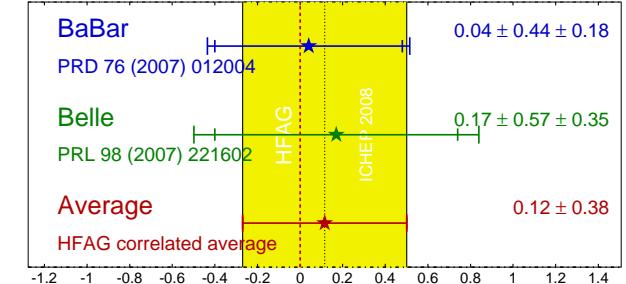
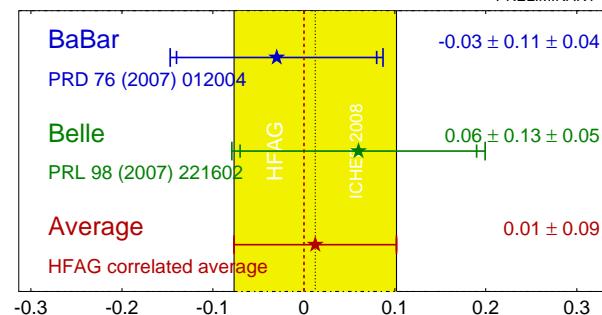
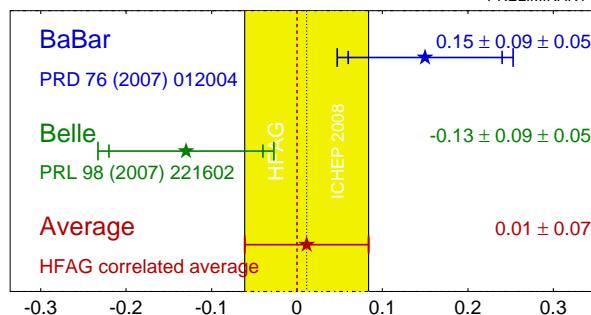
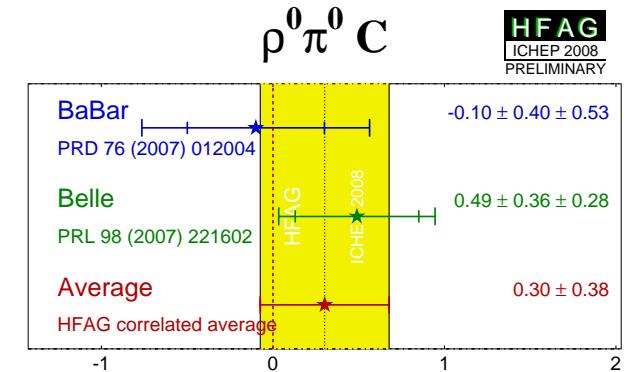
$$B^0 \rightarrow (\rho\pi)^0$$

For  $B^0 \rightarrow \rho^\pm \pi^\mp$

$\mathcal{A}_{CP}$  is time and flavour-integrated  $CP$  asymmetry



For  $B^0 \rightarrow \rho^0 \pi^0$



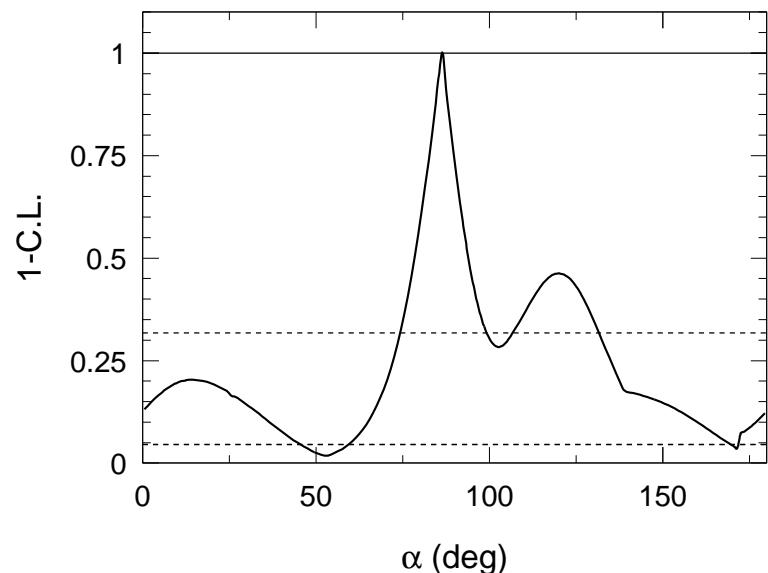
Good agreement between experiments

$$B^0 \rightarrow (\rho\pi)^0$$

Perform  $\phi_2$  scan

BaBar

Use  $B^0 \rightarrow (\rho\pi)^0$  results only



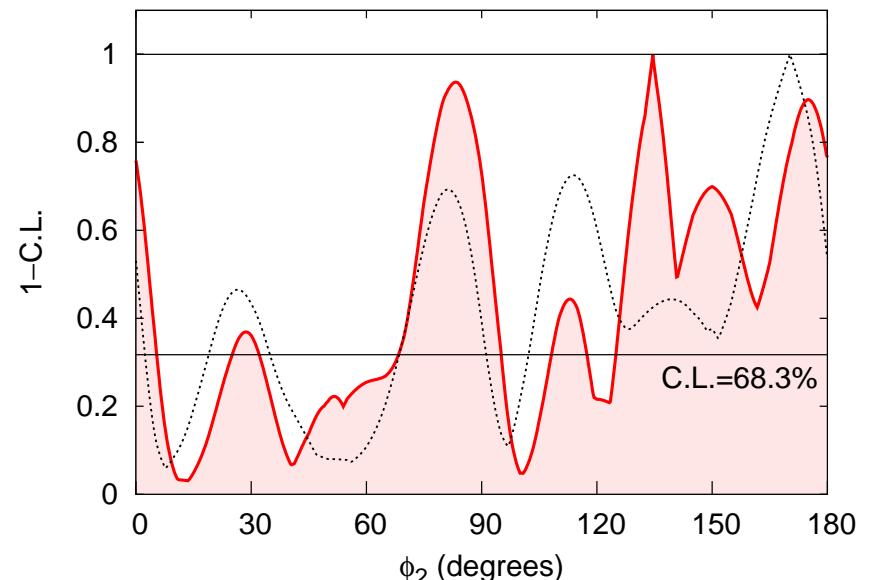
$$\phi_2 = (87^{+45}_{-13})^\circ$$

for entire  $\phi_2$  range

Difficult to pin down  $\phi_2$  with  $B^0 \rightarrow (\rho\pi)^0$

Belle

Also include  $\mathcal{B}$  and  $\mathcal{A}_{CP}$  of  $B^+ \rightarrow \rho^+\pi^0, \rho^0\pi^+$



Dotted line: Use  $B^0 \rightarrow (\rho\pi)^0$  results only

$$68^\circ < \phi_2 < 95^\circ \text{ at } 68.3\% \text{ CL}$$

for solution consistent with SM

# Summary

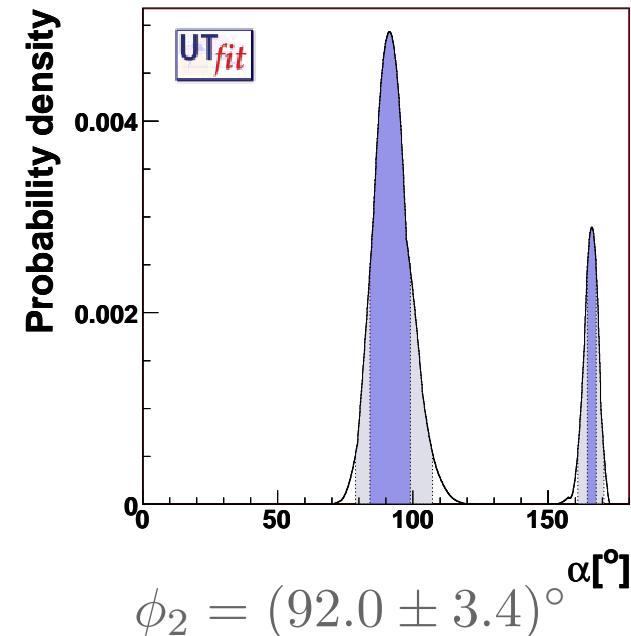
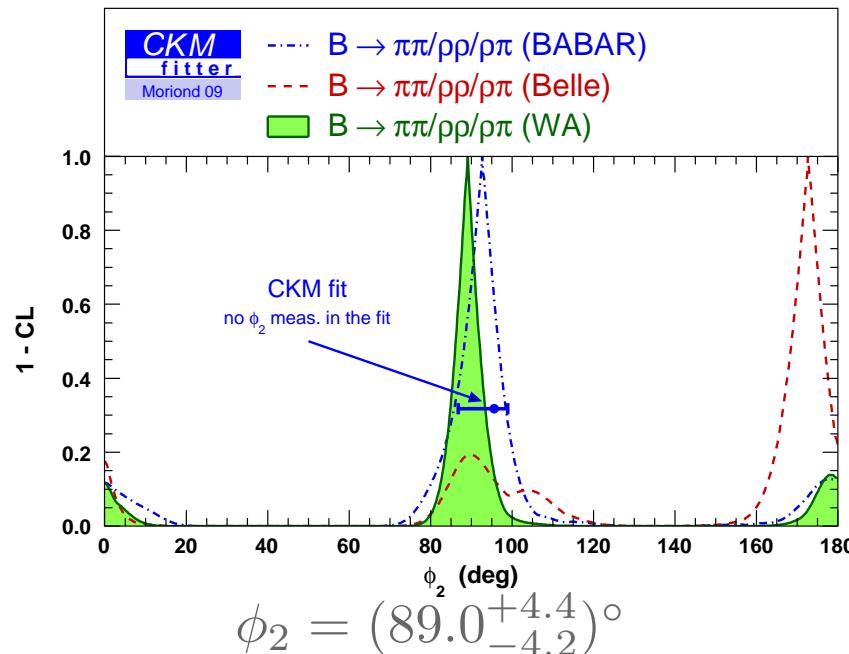
Many measurements of  $\phi_2$  performed by the  $B$  factories

$B \rightarrow \rho\rho$  currently gives tightest constraint on  $\phi_2$

But only  $B^0 \rightarrow (\rho\pi)^0$  can ultimately constrain  $\phi_2$  without ambiguity

Both experiments have final data sets taken at  $\Upsilon(4S)$  resonance

Many final results from the  $B$  factories still anticipated



# Backup

# $B \rightarrow \pi\pi$ Observables

BaBar

467 million  $B\bar{B}$  pairs

$$\mathcal{B}(B^0 \rightarrow \pi^0\pi^0) = (1.83 \pm 0.21 \pm 0.13) \times 10^{-6}$$

$$\mathcal{A}_{CP}(B^0 \rightarrow \pi^0\pi^0) = +0.43 \pm 0.26 \pm 0.05$$

arXiv:0807.4226 (2008)

383 million  $B\bar{B}$  pairs

$$\mathcal{B}(B^+ \rightarrow \pi^+\pi^0) = (5.02 \pm 0.46 \pm 0.29) \times 10^{-6}$$

$$\mathcal{A}_{CP}(B^+ \rightarrow \pi^+\pi^0) = +0.03 \pm 0.08 \pm 0.01$$

PRD 76 091102 (2007)

Belle

535 million  $B\bar{B}$  pairs

$$\mathcal{B}(B^0 \rightarrow \pi^0\pi^0) = (1.1 \pm 0.3 \pm 0.1) \times 10^{-6}$$

$$\mathcal{A}_{CP}(B^0 \rightarrow \pi^0\pi^0) = +0.44^{+0.73+0.04}_{-0.62-0.06}$$

arXiv:hep-ex/0610065 (2006)

449 million  $B\bar{B}$  pairs

$$\mathcal{B}(B^+ \rightarrow \pi^+\pi^0) = (6.5 \pm 0.4^{+0.4}_{-0.5}) \times 10^{-6}$$

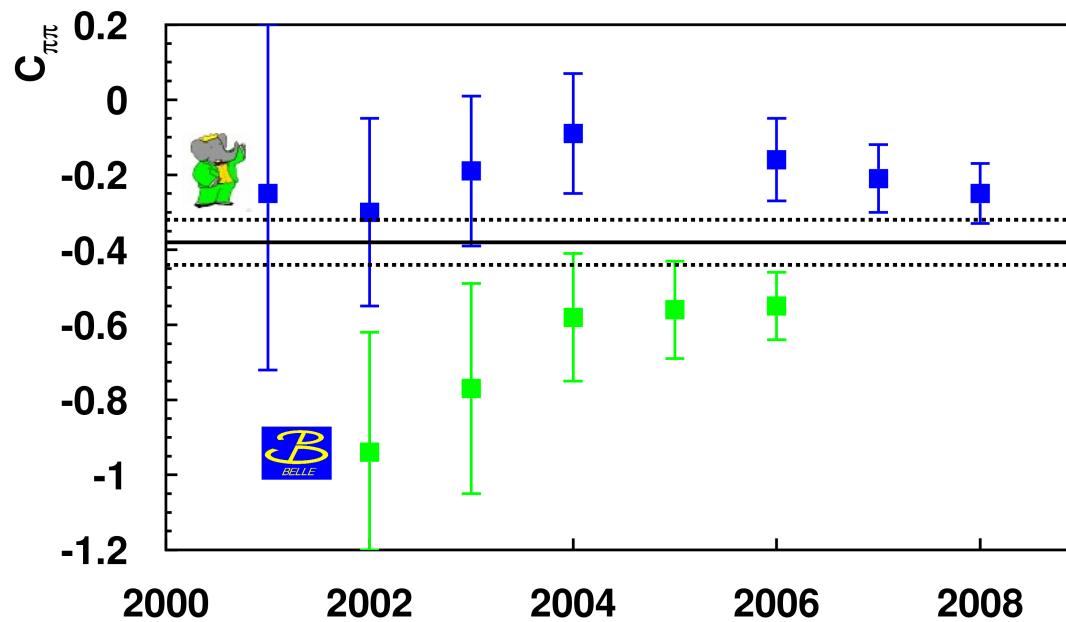
PRL 99 121601 (2007)

535 million  $B\bar{B}$  pairs

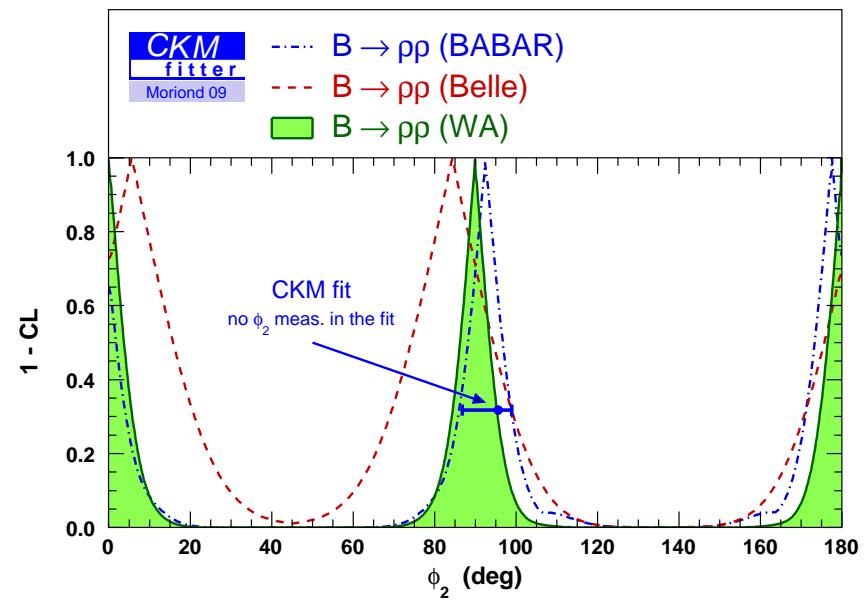
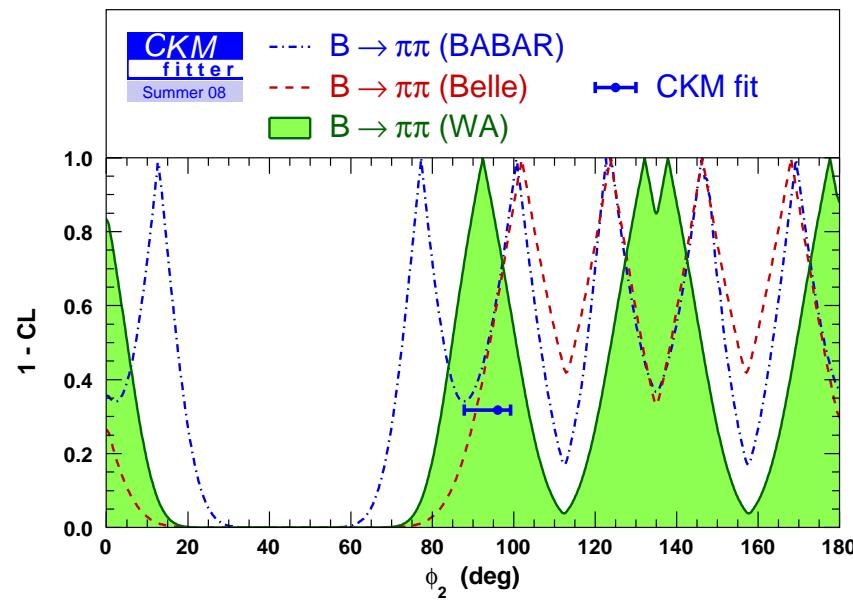
$$\mathcal{A}_{CP}(B^+ \rightarrow \pi^+\pi^0) = +0.07 \pm 0.06 \pm 0.01$$

Nature 452 332 (2008)

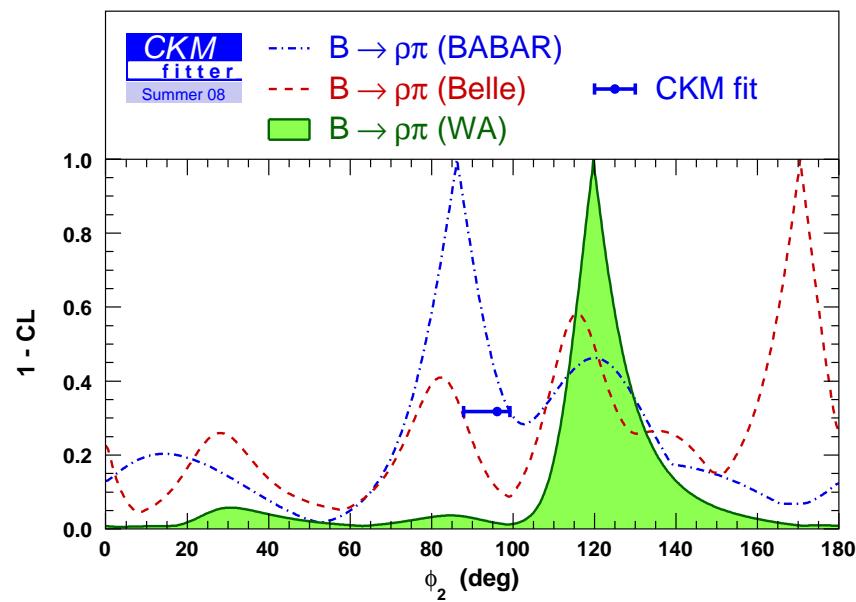
# History of $B \rightarrow \pi\pi$



# $\phi_2$ Averages



# $\phi_2$ Averages



$$B^0 \rightarrow (\rho\pi)^0$$

Time and amplitude differential decay rate,

$$\frac{d^3\Gamma}{d\Delta t ds_+ ds_-} \propto e^{-|\Delta t|/\tau_{B^0}} \left\{ (|A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2) - q(|A_{3\pi}|^2 - |\bar{A}_{3\pi}|^2) \cos \Delta m_d \Delta t + 2q \Im \left[ \frac{q}{p} A_{3\pi}^* \bar{A}_{3\pi} \right] \sin \Delta m_d \Delta t \right\}$$

$$|A_{3\pi}|^2 \pm |\bar{A}_{3\pi}|^2 = \sum_{\kappa \in \{+, -, 0\}} |f_\kappa|^2 U_\kappa^\pm + \sum_{\kappa < \sigma \in \{+, -, 0\}} 2(\Re[f_\kappa f_\sigma^*] U_{\kappa\sigma}^{\pm, \Re} - \Im[f_\kappa f_\sigma^*] U_{\kappa\sigma}^{\pm, \Im})$$

$$\Im \left[ \frac{q}{p} A_{3\pi}^* \bar{A}_{3\pi} \right] = \sum_{\kappa \in \{+, -, 0\}} |f_\kappa|^2 I_\kappa + \sum_{\kappa < \sigma \in \{+, -, 0\}} (\Re[f_\kappa f_\sigma^*] I_{\kappa\sigma}^{\Im} + \Im[f_\kappa f_\sigma^*] I_{\kappa\sigma}^{\Re})$$

27 coefficients  $U, I$  determined from a fit to data

$f$ : Form factors and line shapes

$$B^0 \rightarrow (\rho\pi)^0$$

Convert to Quasi-two-body parameters

For  $B^0 \rightarrow \rho^\pm \pi^\mp$

$$U_\kappa^\pm = |A_\kappa|^2 \pm |\bar{A}_\kappa|^2$$

$$U_{\kappa\sigma}^{\pm,\Re} = \Re[A_\kappa A_\sigma^* \pm \bar{A}_\kappa \bar{A}_\sigma^*]$$

$$U_{\kappa\sigma}^{\pm,\Im} = \Im[A_\kappa A_\sigma^* \pm \bar{A}_\kappa \bar{A}_\sigma^*]$$

$$I_\kappa = \Im[\bar{A}_\kappa A_\kappa^*]$$

$$I_{\kappa\sigma}^{\Re} = \Re[\bar{A}_\kappa A_\sigma^* - \bar{A}_\sigma A_\kappa^*]$$

$$I_{\kappa\sigma}^{\Im} = \Im[\bar{A}_\kappa A_\sigma^* + \bar{A}_\sigma A_\kappa^*]$$

$$e^{+2i\phi_2} = \frac{\bar{A}_+ + \bar{A}_- + 2\bar{A}_0}{A_+ + A_- + 2A_0}$$

$$\mathcal{A}_{CP} = \frac{U_+^+ - U_-^+}{U_+^+ + U_-^+}$$

$$\mathcal{C}_{CP} = \frac{1}{2} \left( \frac{U_+^-}{U_+^+} + \frac{U_-^-}{U_-^+} \right), \quad \mathcal{S}_{CP} = \frac{I_+}{U_+^+} + \frac{I_-}{U_-^+}$$

$$\Delta\mathcal{C} = \frac{1}{2} \left( \frac{U_+^-}{U_+^+} - \frac{U_-^-}{U_-^+} \right), \quad \Delta\mathcal{S} = \frac{I_+}{U_+^+} - \frac{I_-}{U_-^+}$$

For  $B^0 \rightarrow \rho^0 \pi^0$

$$\mathcal{A}_{CP} = -\frac{U_0^-}{U_0^+}, \quad \mathcal{S}_{CP} = \frac{2I_0}{U_0^+}$$