THEORY OF HADRONIC B DECAYS: TREE AMPLITUDES

[GUIDO BELL]

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OUTLINE

THEORY

BRIEF REVIEW OF QCDF / SCET / PQCD

PERTURBATIVE CALCULATION IN QCDF

PHENOMENOLOGY

$B \to \pi\pi \;/\; \pi\rho \;/\; \rho\rho$

KEY HADRONIC PARAMETER λ_B

TREE-DOMINATED B_s DECAYS

Hadronic two-body decays

SM parametrization of decay amplitudes

$$\mathcal{A}(\bar{B} \to M_1 M_2) \;=\; e^{-i\gamma}\, T_{M_1 M_2} \;+\; P_{M_1 M_2}$$

- A_{CP} from interference of T and P with $\delta_T \neq \delta_P$
- very few examples where only T or P is relevant $(B_d \to J/\Psi K_S)$
- often crucial to estimate penguin ($B_d \to \pi\pi$) or tree ($B_d \to \eta' K_S$) "pollution"

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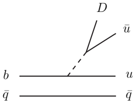
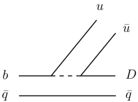
Decompose into topological amplitudes: $T \sim |V_{ub}V_{uD}| \;\to\; \alpha_1, \alpha_2, \alpha_4^u, \beta_i^u, \ldots$

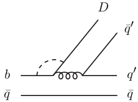
$$P \sim |V_{cb}V_{cD}| \;\to\; \alpha_4^c, \beta_i^c, \ldots$$



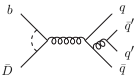
colour-allowed tree α_1



colour-suppressed tree α_2



QCD penguins α_4^p



weak annihilation β_i^p

I will focus here on α_1 and α_2 (penguins + annihilation → talk by S. Jäger)

$\Rightarrow$ test understanding of QCD dynamics with tree-dominated observables

Theory approaches

Hadronic matrix elements factorize in heavy quark limit $m_b \gg \Lambda_{QCD}$

- QCD factorization [Beneke, Buchalla, Neubert, Sachrajda 99]

$$\langle M_1 M_2 | Q_i | B \rangle \simeq F^{BM_1}(0) \int du \, T_i^I(u) \, \phi_{M_2}(u) + \int d\omega \, du \, dv \, T_i^{II}(\omega, u, v) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

"soft-overlap" in F^{BM_1}, strong phases perturbative, corrections of $\mathcal{O}(\frac{\Lambda}{m_b})$

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"soft-overlap" in F^{BM_1}, strong phases perturbative, corrections of $\mathcal{O}(\frac{\Lambda}{m_b})$

- Soft-collinear effective theory [Bauer, Pirjol, Rothstein, Stewart 04]

SCET = QCDF: simply EFT vs diagrammatical formulation

BPRS ≠ BBNS: phenomenological implementation for $B \to M_1 M_2$ quite different

issues: F^{BM_1} traded for ξ^{BM_1}, $\alpha_s(\sqrt{\Lambda m_b})$ non-perturbative, ... (minor)

long-distance charm loops, zero-bin subtractions (major)

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- perturbative QCD [Keum, Li, Sanda 00]

$$\langle M_1 M_2 | Q_i | B \rangle \simeq \int d\omega \, du \, dv \, dk_{i\perp} \, T_i(\omega, u, v, k_{i\perp}) \, \phi_B(\omega, k_{1\perp}) \, \phi_{M_1}(v, k_{2\perp}) \, \phi_{M_2}(u, k_{3\perp})$$

recently substantially modified: soft factor from Glauber gluons? [Li, Mishima 09]

QCDF / SCET / pQCD

	BBNS (QCDF)	BPRS (SCET)	pQCD
$\alpha_s(\sqrt{\Lambda m_b})$	perturbative	non-perturbative	perturbative
charm loops	perturbative (small phase)	non-perturbative (large phase from fit to data)	perturbative (small phase)
weak annihilation (power correction)	non-perturbative (crude model, arbitrary phase)	perturbative (with zero bins, small phase)	perturbative (large phase)
strong phases	generically small ($\sim \alpha_s, 1/m_b$)	can be sizeable (charm loops)	can be sizeable (annihilation, Glaubers)
perturbative calculation	partially NNLO	NLO	partially NLO
hadronic input	from lattice + QCD sum rules	from QCD sum rules + data, model $\xi_J^{BM}(z)$	from QCD sum rules + data, model $\phi_B(x, b)$

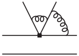
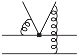
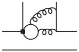
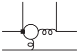
- theory predictions for direct CP asymmetries can differ a lot!
- measurements (even bounds) of pure annihilation decays highly appreciated: $B_d \to K^- K^+, B_s \to \pi\pi/\pi\rho/\rho\rho$

Perturbative calculation in QCDF

Ongoing effort to compute NNLO corrections in QCDF

$$\langle M_1 M_2 | Q_i | B \rangle \simeq F^{BM_1} \, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

strong phases $\sim \mathcal{O}(\alpha_s)$ $\Rightarrow$ NNLO is first correction for direct CP asymmetries!

Status	2-loop vertex corrections (T_i^I)	1-loop spectator scattering (T_i^{II})
Trees	 [GB 07, 09] [Beneke, Huber, Li 09]	 [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	 [in progress]	 [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

- ▶ factorization found to hold (as expected) at highly non-trivial order
- ▶ direct CP asymmetries not yet available at NNLO
- ▶ first NNLO results for CP-averaged branching ratios of tree-dominated decays

[GB, Pilipp 09; Beneke, Huber, Li 09]

Tree amplitudes

Perturbative structure in QCDF to NNLO

$$
\begin{aligned}
\alpha_1(\pi\pi) \;&=\; [\,1.008\,]_{V_0} + [\,0.022 + 0.009\,i\,]_{V_1} + [\,0.024 + 0.026\,i\,]_{V_2} \\
&\quad - [\,0.014\,]_{S_1} - [\,0.016 + 0.012\,i\,]_{S_2} - [\,0.008\,]_{1/m_b} \\
&=\; 1.015^{+0.020}_{-0.029} + (0.023^{+0.015}_{-0.015})\,i \\
\alpha_2(\pi\pi) \;&=\; [\,0.224\,]_{V_0} - [\,0.174 + 0.075\,i\,]_{V_1} - [\,0.029 + 0.046\,i\,]_{V_2} \\
&\quad + [\,0.084\,]_{S_1} + [\,0.037 + 0.022\,i\,]_{S_2} + [\,0.052\,]_{1/m_b} \\
&=\; 0.194^{+0.130}_{-0.095} - (0.099^{+0.057}_{-0.056})\,i
\end{aligned}
$$

- PT well-behaved, individual NNLO corrections quite significant but cancellations
- α_1: stable under radiative corrections, precise prediction
- α_2: real part dominated by spectator scattering $\sim \lambda_B^{-1} = \int \frac{d\omega}{\omega}\phi_B(\omega)$

  $\Rightarrow$ substantial hadronic uncertainties, but $\arg(\alpha_2/\alpha_1)$ small
- pQCD: similar up to 2009, now large α_2 with large $\arg(\alpha_2/\alpha_1)$ from $B \to \pi\pi$ data

  SCET: very large α_2 from $B \to \pi\pi$ data, but negligible $\arg(\alpha_2/\alpha_1)$

Confronting data: $B \to \pi\pi/\pi\rho/\rho\rho$

[GB, Pilipp 09]

CP-averaged branching ratios in units of 10^{-6}

Mode	QCDF	B	Experiment
$\pi^-\pi^0$	$6.22^{+2.37}_{-2.01}$	5.46	$5.59^{+0.41}_{-0.40}$
$\rho_L^-\rho_L^0$	$21.0^{+8.5}_{-7.3}$	21.3	$22.5^{+1.9}_{-1.9}$
$\pi^-\rho^0$	$9.34^{+4.00}_{-3.23}$	10.4	$8.3^{+1.2}_{-1.3}$
$\pi^0\rho^-$	$15.1^{+5.7}_{-5.0}$	11.9	$10.9^{+1.4}_{-1.5}$
$\pi^+\pi^-$	$8.96^{+3.78}_{-3.32}$	5.21	$5.16^{+0.22}_{-0.22}$
$\pi^0\pi^0$	$0.35^{+0.37}_{-0.21}$	0.63	$1.55^{+0.19}_{-0.19}$
$\pi^+\rho^-$	$22.8^{+9.1}_{-8.0}$	13.2	$15.7^{+1.8}_{-1.8}$
$\pi^-\rho^+$	$11.5^{+5.1}_{-4.3}$	8.41	$7.3^{+1.2}_{-1.2}$
$\pi^\pm\rho^\mp$	$34.3^{+11.5}_{-10.0}$	21.6	$23.0^{+2.3}_{-2.3}$
$\pi^0\rho^0$	$0.52^{+0.76}_{-0.42}$	1.64	$2.0^{+0.5}_{-0.5}$
$\rho_L^+\rho_L^-$	$30.3^{+12.9}_{-11.2}$	22.3	$23.6^{+3.2}_{-3.2}$
$\rho_L^0\rho_L^0$	$0.44^{+0.66}_{-0.37}$	1.33	$0.69^{+0.30}_{-0.30}$

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_B \to \lambda_B/2$ and smaller form factors)

- theo. uncertainties highly correlated (F^{BM_1}, $|V_{ub}|$)
- colour-suppressed modes $\pi^0\pi^0/\pi^0\rho^0/\rho^0\rho^0$ rather uncertain (λ_B and $1/m_b$)
- overall preference for enhanced colour-suppressed amplitude
- $\rho^0\rho^0$ and $\pi^0\rho^0$ (with smaller penguins) fit better than $\pi^0\pi^0$

[for a similar analysis cf. Beneke, Huber, Li 09]

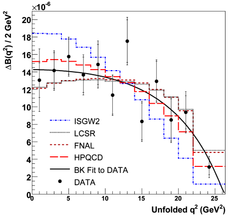
Testing the QCD dynamics

For colour-allowed modes can eliminate dependence on F^{BM_1} and $|V_{ub}|$ via

$$\mathcal{R}_{M_3}(M_1 M_2) = \frac{\Gamma(B \to M_1 M_2)}{d\Gamma(B \to M_3 \ell\nu)/dq^2|_{q^2=0}}$$

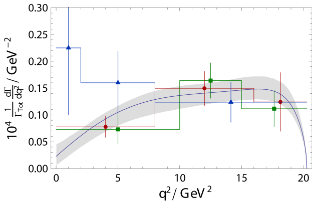
$\Rightarrow$ requires measurement of semileptonic decay spectrum and extrapolation to $q^2 = 0$

$B \to \pi\ell\nu$



[Babar 06]

$B \to \rho\ell\nu$



[Babar 05; Belle 07; CLEO 07; figure from Flynn et al 08]

$\Rightarrow \quad |V_{ub}| F_+^{B\pi}(0) = (9.1 \pm 0.7) \cdot 10^{-4}$

[Ball 06]

currently insufficient to extract $|V_{ub}| A_0^{B\rho}(0)$

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[GB, Pilipp 09]

Mode	QCDF	B	Experiment
$\mathcal{R}_\pi(\pi^- \pi^0)$	$0.70^{+0.12}_{-0.08}$	0.95	$0.81^{+0.14}_{-0.14}$
$\mathcal{R}_\rho(\rho_L^- \rho_L^0)$	$1.91^{+0.32}_{-0.23}$	2.38	n.a.
$\mathcal{R}_\rho(\pi^- \rho^0)$	$0.85^{+0.22}_{-0.14}$	1.16	n.a.
$\mathcal{R}_\pi(\pi^0 \rho^-)$	$1.71^{+0.27}_{-0.24}$	2.07	$1.57^{+0.32}_{-0.32}$
$\mathcal{R}_\pi(\pi^+ \pi^-)$	$1.09^{+0.22}_{-0.20}$	0.97	$0.80^{+0.13}_{-0.13}$
$\mathcal{R}_\pi(\pi^+ \rho^-)$	$2.77^{+0.32}_{-0.31}$	2.46	$2.43^{+0.47}_{-0.47}$
$\mathcal{R}_\rho(\pi^- \rho^+)$	$1.12^{+0.20}_{-0.14}$	1.01	n.a.
$\mathcal{R}_\rho(\rho_L^+ \rho_L^-)$	$2.95^{+0.37}_{-0.35}$	2.68	n.a.
$R(\rho_L^- \rho_L^0 / \rho_L^+ \rho_L^-)$	$0.65^{+0.16}_{-0.11}$	0.89	$0.89^{+0.14}_{-0.14}$
$R(\pi^- \pi^0 / \pi^+ \pi^-)$	$0.65^{+0.19}_{-0.14}$	0.98	$1.01^{+0.09}_{-0.09}$

- theoretical uncertainties largely reduced
- satisfactory description of clean observables
- colour-allowed amplitudes seem to be under control

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_B \to \lambda_B/2$ and smaller form factors)

[for a similar analysis cf. Beneke, Huber, Li 09]

What can we learn about α_2?

Pattern of colour-suppressed modes not conclusive

Mode	QCDF	B	Experiment
$\pi^0\pi^0$	$0.35^{+0.37}_{-0.21}$	0.63	$1.55^{+0.19}_{-0.19}$
$\pi^0\rho^0$	$0.52^{+0.76}_{-0.42}$	1.64	$2.0^{+0.5}_{-0.5}$
$\rho^0_L\rho^0_L$	$0.44^{+0.66}_{-0.37}$	1.33	$0.69^{+0.30}_{-0.30}$

- penguins in $\pi^0\pi^0$ non-negligible
- dominated by different uncertainties
  $\Rightarrow$ cannot construct any clean ratio

Consider instead pure tree decays $\pi^-\pi^0/\rho^-\rho^0$

Mode	QCDF	B	Experiment
$\mathcal{R}_\pi(\pi^-\pi^0)$	$0.70^{+0.12}_{-0.08}$	0.95	$0.81^{+0.14}_{-0.14}$
$\mathcal{R}_\rho(\rho^-_L\rho^0_L)$	$1.91^{+0.32}_{-0.23}$	2.38	n.a.
$R(\rho^-_L\rho^0_L/\rho^+_L\rho^-_L)$	$0.65^{+0.16}_{-0.11}$	0.89	$0.89^{+0.14}_{-0.14}$

- no QCD penguins, no weak annihilation
- very clean access to $|\alpha_1+\alpha_2|$

$\Rightarrow$ I consider this as the strongest support for an enhancement of α_2 (smaller λ_B?)

- consistent with overall pattern in $\pi\pi/\pi\rho/\rho\rho$ data
- measurement of $B\to\rho\ell\nu$ spectrum would be helpful

Key hadronic parameter: λ_B

Spectator-scattering $\sim \lambda_B^{-1}$ can potentially enhance the QCDF prediction of α_2

What do we know about $\lambda_B^{-1}(\mu) = \int \frac{d\omega}{\omega} \phi_B(\omega;\mu)$?

- $\lambda_B(1\text{GeV}) = \begin{cases} (460 \pm 110)\text{ MeV} & \text{QCD sum rules} \\ (480 \pm 120)\text{ MeV} & \text{OPE + shape model} \end{cases}$ [Braun, Ivanov, Korchemsky 03] [Lee, Neubert 05]
- OPE with dim 5 operators $\Rightarrow \sim 370$ MeV (sensitive to λ_E^2, λ_H^2 !) [Kawamura, Tanaka 08]
- $\pi\pi/\pi\rho/\rho\rho$ numbers based on (400 $\pm$ 150) MeV, but data seem to prefer $\sim$ 200 MeV ?

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- $\pi\pi/\pi\rho/\rho\rho$ numbers based on (400 ± 150) MeV, but data seem to prefer ~ 200 MeV ?

Experiments can help to constrain λ_B from $B \to \gamma\ell\nu$!

- requires energetic photon, expected branching ratio $\sim 10^{-6}$
- Babar 09: $\text{Br}(B \to \gamma\ell\nu) < 15.6 \cdot 10^{-6} \quad \Rightarrow \quad \lambda_B(\mu) > 300$ MeV ?
- issues: cut on photon energy?

  requires NLO analysis (known!) to assign scale dependence

Tree-dominated B_s decays

[GB, preliminary]

CP-averaged branching ratios in units of 10^{-6}

Mode	QCDF	B	Experiment
$\pi^- K^+$	$8.73^{+5.77}_{-4.60}$	4.88	$5.0^{+1.1}_{-1.1}$
$\pi^0 K^0$	$0.50^{+0.71}_{-0.35}$	1.12	n.a.
$\pi^- K^{*+}$	$15.4^{+8.6}_{-7.0}$	11.0	n.a.
$\pi^0 K^{*0}$	$0.39^{+0.58}_{-0.26}$	0.90	n.a.
$\rho^- K^+$	$22.4^{+14.7}_{-11.6}$	12.5	n.a.
$\rho^0 K^0$	$0.73^{+1.28}_{-0.58}$	2.24	n.a.
$\rho_L^- K_L^{*+}$	$40.7^{+22.4}_{-18.3}$	29.1	n.a.
$\rho_L^0 K_L^{*0}$	$0.70^{+1.07}_{-0.54}$	1.87	n.a.
$\rho^- K^+ / \pi^- K^+$	$2.57^{+0.31}_{-0.26}$	2.57	n.a.
$\rho_L^- K_L^{*+} / \pi^- K^{*+}$	$2.64^{+0.31}_{-0.33}$	2.65	n.a.

B: mimics enhanced colour-suppressed amplitude (with $\lambda_{B_s} \to \lambda_{B_s}/2$ and smaller form factors)

- hadronic parameters less well known (λ_{B_s}, $F_+^{B_s K}$, $A_0^{B_s K^*}$)
- much simpler pattern of annihilation contributions
- $\pi^- K^+$ again seems to point at smaller form factors
- clean ratios of colour-allowed modes $\sim f_\rho^2 / f_\pi^2$; may be used to test charming penguins [Zhu 10]

Conclusion

Tree amplitudes have recently been determined to NNLO in QCDF

- colour-allowed tree α_1: precise prediction, supported by data
- colour-suppr. tree α_2: suffers from hadronic uncertainties

  data seem to prefer larger values (smaller λ_B?)

Further refinements possible with experimental input

- λ_B from $B \to \gamma \ell \nu$ with energetic photon
- $B \to \rho \ell \nu$ spectrum to determine $|V_{ub}| A_0^{B\rho}(0)$
- tree-dominated B_s decays
- pure annihilation decays $B_d \to K^- K^+, B_s \to \pi\pi/\pi\rho/\rho\rho$

Backup slides

Long-distance charm loops

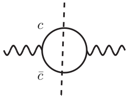
Old charming penguin story – two different questions:

- power-suppressed but numerically important [Colangelo et al. 89; Ciuchini et al. 97+]
  - $\Rightarrow$ not supported by light-cone sum rule estimate [Khodjamirian, Mannel, Melic 03]
- leading power spoiling factorization [BPRS vs BBNS 04,05]

  does the threshold region with a non-relativistic $c\bar{c}$ pair require a special treatment?

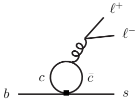
Recent work addresses second question [BBNS 09]

(a) $e^+e^- \to$ hadrons



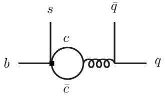
$$\int dq^2 \dots \operatorname{Im} \Pi(q^2)$$

(b) $B \to X_s \ell^+ \ell^-$



$$\int dq^2 \dots \left|\Pi(q^2)\right|^2$$

(c) charming penguins



$$\int dq^2 \dots \Pi(q^2)$$

$\Rightarrow$ global quark-hadron duality holds in (a) and (c), but breaks down in (b)

$\Rightarrow$ no special treatment required in (c), long-distance charm loops are power-suppressed

Glauber gluons in pQCD

It has been realized recently that k_T-factorization breaks down in $pp \to h_1 h_2 X$ at high p_T

[Collins, Qiu 07]

- problem related to a peculiar mode: Glauber gluons
- effect is a non-universal long-distance contribution $\Rightarrow$ ruins k_T-factorization
- problem not present in collinear factorization

Important for pQCD approach to non-leptonic B decays

- confirmed that problem exists $\Rightarrow$ modification of pQCD approach [Li, Mishima 09]
- claimed that it leads to a universal soft factor e^{iS} $\Rightarrow$ k_T-factorization still holds
- e^{iS} from fit to $\text{Br}(\pi^0\pi^0)$ $\Rightarrow$ large complex C $\Rightarrow$ "solves" $\pi\pi/\pi K$ puzzles

Issues:

- operator definition of soft factor?
- if universal why associated to π but not to ρ? $\Rightarrow$ would worsen $\text{Br}(\rho^0\rho^0)$
- at present I consider this rather as a fit than as a dynamical explanation