# Theory of hadronic B decays: penguin amplitudes 

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Part of this is based on chapter 9.1 in the CKM 2008 proceedings, Phys.Rept. 494 (2010) 197-414, arXiv:0907.5386v2

## Physical amplitudes

- Any SM 2-light-hadron amplitude can be written

$$
\begin{gathered}
\mathcal{A}\left(\bar{B} \rightarrow M_{1} M_{2}\right)=e^{-i \gamma} T_{M_{1} M_{2}}+P_{M_{1} M_{2}} \\
T_{M_{1} M_{2}}=V_{u D}\left|V_{u b}\right|\left[C_{1}\left\langle Q_{1}^{u}\right\rangle+C_{2}\left\langle Q_{2}^{u}\right\rangle+\sum_{i=3}^{12} C_{i}\left\langle Q_{i}\right\rangle\right] \quad \text { "tree" } \\
P_{M_{1} M_{2}}=V_{c D}\left|V_{c b}\right|\left[C_{1}\left\langle Q_{1}^{c}\right\rangle+C_{2}\left\langle Q_{2}^{c}\right\rangle+\sum_{i=3}^{12} C_{i}\left\langle Q_{i}\right\rangle\right]
\end{gathered}
$$

Qi: operators in weak hamiltonian
$\mathrm{C}_{\mathrm{i}}$ QCD corrections from short distances (<hc/mb) \& new physics $\left\langle Q_{i}\right\rangle=\left\langle M_{1} M_{2}\right| Q_{i}|B\rangle$ : QCD at distances $>h c / m_{b}$, strong phases

## Relevance of penguins

- angle measurements in tree-dominated modes ( $\pi \pi, \pi \rho, \rho \rho$ ):
$S_{+-}=\sin (2 \alpha)$ in no-penguin limit
knowledge of P/T "pollution" determines $\alpha(\mathrm{y})$, without need for isospin constructions, SU(3), etc.
- $b \rightarrow s$ decays penguin-dominated in SM

$\sim \lambda^{4}$



$\sim \lambda^{2}$
sensitive to loops \& new physics mK puzzle, etc


## Tonolooiceirnortures

- matrix elements are contained in correlation functions (LSZ)

$$
\left\langle M_{1} M_{2}\right| Q_{i}|B\rangle \sim \int d A \int d \bar{\psi} d \psi j_{B}^{\mu}(x) j_{M_{1}}^{\nu}(y) j_{M_{2}}^{\rho}(z) Q_{i}(w) e^{i\left(S_{Q C D+Q E D}\right)}
$$

which can be represented as "Wick contractions"
[Buras\&Silvestrini hep-ph/9812392]


- RG invariant combinations of these define "topological" amplitudes - nonperturbatively!
- represent the contribution to the matrix element of $Q_{i}$ as
(still no perturbation expansion)



## Penguin anatomy I

[figures from Buras\&Silvestrini]

$\beta_{3}$ "mixes" with $\alpha_{4}^{c}$ under RG (above $\mathrm{m}_{\mathrm{b}}$ ) but useful to keep separate for heavy-quark limit it has traditionally been considered a part of the (topological) penguin amplitude

Example: $\quad P_{\bar{B}^{0} \rightarrow \pi^{+} K^{-}}=\left|V_{c s} V_{c b}\right| A_{\pi K}\left(\alpha_{4}^{c}+\beta_{3}^{c}+\alpha_{4, \mathrm{EW}}^{c}-\frac{1}{2} \beta_{3, \mathrm{EW}}^{c}\right)$
hadronic normalization
(form factor \& decay constants) - a convention

## Therory aporodanes

- $1 / \mathrm{N}_{\mathrm{c}}$ expansion e.g.


O(1)


- "naive factorization" for $\mathrm{N}_{\mathrm{c}}$-> infinity
- strong phase of penguin is $O\left(1 / N^{2}\right)$
- main drawback: can't compute
- QCD light-cone sum rules evaluate correlation function off shell; OPE \& lightcone expansion
- express hadronic matrix elements
 in terms of simpler objects (form factors etc.) and a perturbatively evaluated dispersion integral.
- works also for form factors themselves (and other objects)
- main drawback: uncertainty due to "continuum threshold" is difficult to quantify


## Theory approaches II

[Beneke, Buchalla, Neubert, Sachrajda (BBNS); Bauer et al]

- heavy-quark expansion in $\Lambda_{Q C D} / \mathrm{m}_{\mathrm{B}}$ (talk G Bell) [QCDF / SCET; pQCD approach]

$T^{1}, T^{\text {II }}$ computable in perturbation theory in strong coupling
- "naive factorization" for $m_{B}$-> infinity
- strong phases are $\mathrm{O}\left(\alpha_{\mathrm{s}}\right)$ or $\mathrm{O}\left(\Lambda_{Q C D} / \mathrm{m}_{\mathrm{b}}\right)$
- annihilation power suppressed altogether
- hierarchies of penguin amplitudes between final states containing pseudoscalars and vectors
- main drawback: $\mathrm{O}\left(\Lambda_{\mathrm{QCD}} / \mathrm{m}_{\mathrm{B}}\right)$ power corrections don't factorize, in general, and hard to estimate
- flavour $\operatorname{SU}(3)$ - relate $b \rightarrow s$ and $b \rightarrow d$; eliminate amplitudes from data. Good if redundant observables ( $\gamma$ in SM), less powerful for NP search; $\mathrm{SU}(3)$ breaking not controlled


## Penguin anatomy II: $1 / \mathrm{m}_{b}$

like quark chiralities
opposite quark chiralities

$\alpha_{4}^{c}=$

("scalar penguin")

However: $\quad r_{\chi}^{\pi}(\mu)=\frac{2 m_{\pi}^{2}}{m_{b}(\mu)\left(m_{u}+m_{d}\right)(\mu)} \sim \frac{\Lambda_{\mathrm{QCD}}}{m_{b}}$
but ~ 1 numerically
"chiral enhancement"
no chiral enhancement present for vector $M_{2}$-> much smaller penguin amplitudes

$\mathrm{O}\left(1 / \mathrm{m}_{\mathrm{b}}\right)$, does not factorize modeled by naively factorized expression with IR cutoff by BBNS large and complex in pQCD approach
[Keum, Li, Sanda 2000] very small in light-cone sum rules

## Status of perturbative kernels

- $a_{4}$ computed to $O\left(\alpha_{s}\right)$ (vertex kernel $\left.T^{\prime}\right)$

Beneke et al 1999-2001
$\mathrm{O}\left(\alpha_{\mathrm{s}}{ }^{2}\right)$ (spectator scattering kernel $\mathrm{T}^{\mathrm{II}}$ ) Beneke, SJ 2006
Jain, Rothstein, Stewart 2007
For the latter one needs to compute (besides simpler terms)

penguin loop could be large because $\mathrm{C}_{1} \sim 1$ first appears at this order however, due to a (not understood) cancellation it gives almost no contribution
proves factorization \& perturbative stability but leaves NLO results intact. Hence instead of quoting numbers I refer to the comprehensive phenomenology in [Beneke\&Neubert 03]

- $a_{6}$ computed to $O\left(\alpha_{s}\right)$ (vertex kernel $T^{\prime}$ ) spectator scattering vanishes at this order
At $\mathrm{O}\left(\alpha_{\mathrm{s}}{ }^{2}\right)$ one needs to compute the same diagrams as above could potentially be large contribution for PP and VP final states


## Comparison to data I

$$
P_{M_{1} M_{2}} /\left(C_{\pi \pi}+T_{\pi \pi}\right) \sim \hat{\alpha}_{4}^{c}\left(M_{1} M_{2}\right) /\left(\alpha_{1}(\pi \pi)+\alpha_{2}(\pi \pi)\right)
$$

can be fit to $B R$, $A_{C P}\left(\pi^{+} K^{-}\right)$and $B R\left(\pi^{+} \pi^{-}\right)$using one $S U(3)$ relation



## BBNS model

of annihilation

## Charming penguin

- Charm penguin loops appear as part of the penguin amplitude. Could a priori be large
[Ciuchini et al 97]
- In the HQE, they enter both $\mathrm{a}_{4}$ and $\mathrm{a}_{6}$; were argued to factorize at leading power ("hard" regions)
- Bauer et al (BPRS) 2004: One should add a nonperturbative contribution $A_{c \bar{c}}^{M_{1} M_{2}}$ for the nonrelativistic charm "threshold" region
- disagreement over power suppression of this region. It also evidently overlaps with the "hard" region.
- BBNS 2009: power counting in BPRS 2004 was wrong (error in matching onto nonrelativistic effective theory).

This paper also explains how in $B->\left.X_{s} I^{+}\right|^{-}$ the nonrelativistic charm region can account for $99 \%$ or the rate: It is not inclusive enough

part of rate

(b)
not part of rate

I consider this issue fully resolved.

## Comparison to data II

- The direct CP asymmetries come out wrong for several modes, particularly for the $\pi \mathrm{K}$ final states (see talk by S Mishima)

$$
\begin{aligned}
& A_{C P}\left(\pi^{+} K^{-}\right) \text {has opposite sign [cf above] } \\
& A_{C P}\left(\pi^{0} K^{-}\right) \neq A_{C P}\left(\pi^{0} K^{-}\right) \\
& \text {at around } 5 \sigma
\end{aligned} \quad \text { [eg Belle, Nature 2008] }
$$

- It has been argued that this implies new physics, see eg
[Buras, Fleischer, Recksiegel, Schwab 03; Baek et al 04; Lunghi, Soni 08; Arnowitt et al; Khalil, Kou; Hou; Soni et al 08; Barger et al 09; Khalil, Masiero, Murayama 09; many more ... ]
for instance through modified electroweak penguin contributions (which factorize similarly to tree amplitudes)
- and/or that the colour-suppressed tree amplitude is large and complex and/or the penguin imaginary parts are wrong in factorization (or receive large power corrections), see eg
Gronau et al; Buras, Fleischer, Recksiegel, Schwab 03; Baek et al 04, 09; Yoshikawa 03;
Ciuchini et al 08, Gronau,Pirjol,Zupan 10
Some support for the latter from $\mathrm{Acp}_{\mathrm{cp}}\left(\pi^{+} \pi^{-}\right)$via $\operatorname{SU}(3)$


## Annihilation $\beta_{3}$

- The colour-leading piece to the annihilation contribution $\beta_{3}$ to the QCD penguin amplitude has a naively factorizing structure

(where $Q_{6}$ has been "Fierzed" to colour singlet $x$ singlet form)

This is proportional to the "scalar form factor". A sum rule calculation gives a small and approximately real result, so it cannot resolve the penguin puzzles [Khodjamirian Mannel, Melcher, Melic, hep-ph/0509049]

Note that this is a relatively "simple" sum rule for a form factor, for which sum rules have a good track record (when compared with lattice or data driven determinations)

- In contrast, the pQCD approach finds a large and complex value albeit with large uncertainties. [Keum, Li, Sanda 2000]


## Summary

- Dynamical description of penguin amplitudes
- well-defined $1 / \mathrm{m}_{\mathrm{b}}$ expansion, leading terms factorize with a stable perturbation expansion
- one potentially large missing piece (in $\mathrm{a}_{6}$ )
- leading-power long-distance charm penguin dead
- Data
- clearly respects the hierarchies predicted by the HQ expansion (PP, VP versus PV, VV)
- on direct CP asymmetries doesn't fit well with theory: either higher orders in $\mathrm{a}_{6}$ are important, or annihilation terms are large, or there is new physics in some amplitudes, or a combination of these

