

Super γ : sensitivity with future experiments.

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In many cases, the answer given by the experiment does not really match the question asked by theory.

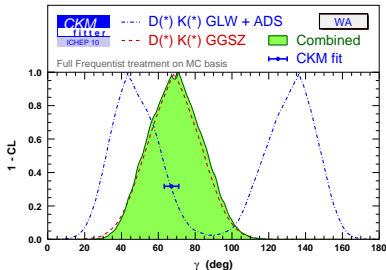
“They say the Ultimate Answer or whatever is Forty-two, how am I supposed to know what the question is?”

Douglas Adams

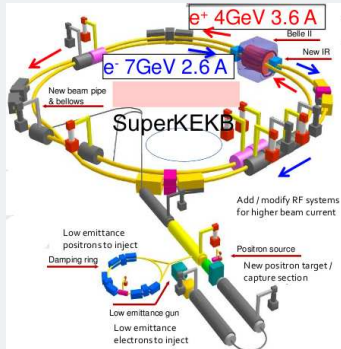
The Hitchhiker's Guide to the Galaxy

This is not the case for γ (up to 10^{-6} precision).

And by the way, $\gamma = 42^\circ$ is not ruled out yet...



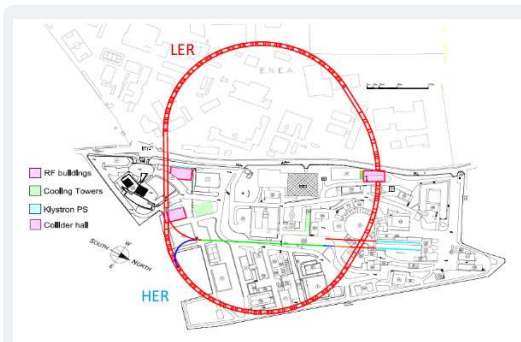
Super B factories



SuperKEKB and Belle II

$$L = 8 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$$

$$\int L dt = 50 \text{ ab}^{-1}$$



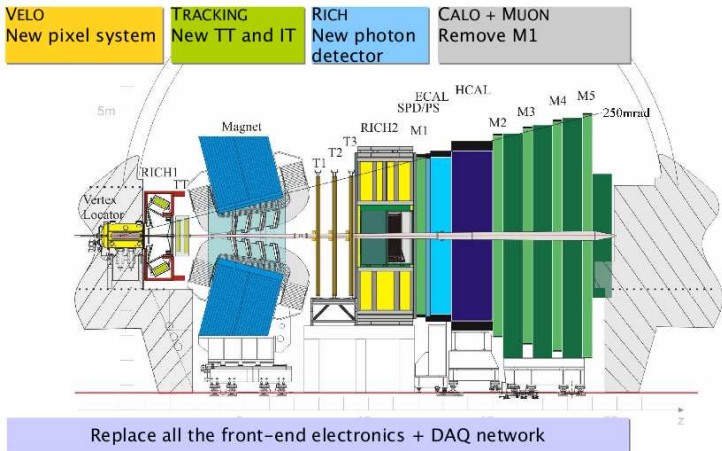
SuperB project

$$L = 10^{36} \text{ cm}^{-2} \text{ s}^{-1}$$

$$\int L dt = 75 \text{ ab}^{-1}$$

Will refer to both projects as SuperB.

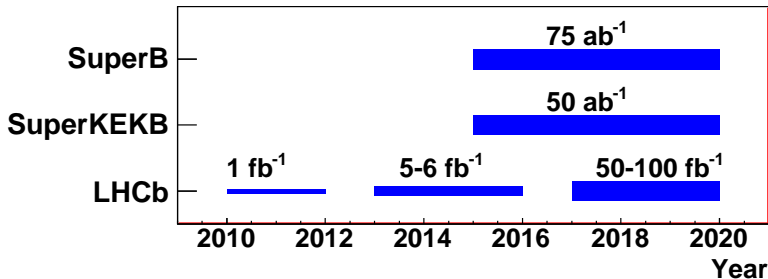
LHCb upgrade



Luminosity $2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$

$\int L dt = 50 - 100 \text{ fb}^{-1}$

Increased hadronic trigger efficiency ($\times 2 - 3$).



LHCb:

20×10^{12} produced $B\bar{B}$ pairs
 detection efficiency $< 1\%$.
 B/S typically $O(1)$

SuperB:

50×10^9 produced $B\bar{B}$ pairs
 detection efficiency $O(100\%)$
 clean signals

Benchmark modes

Mode	SuperB (50 ab^{-1})	LHCb (50 fb^{-1})
$B \rightarrow D(K\pi)K$ allowed	200K	4M
$B \rightarrow D(K_S\pi\pi)K$	100K	300K
$B_S \rightarrow D_S(KK\pi)K$	-	700K

- Main "golden" modes ($B \rightarrow DK$ with $D \rightarrow hh'$, $D \rightarrow K_S hh$) accessible to both LHCb and SuperB.
- SuperB can reconstruct modes with neutrals such as $K_S\pi^0$. Not possible or very hard at LHCb.
- B_S , B_C , Λ_b decays are available at LHCb.

- Expect to reach $O(1^\circ)$ sensitivity with SuperB and LHCb.
- The methods used to extract γ need to have little or no theoretical uncertainty.
- Controlling experimental systematics can be tricky. Expect that most of it should also scale with statistics (depends on control samples).
- Statistical errors should be much easier to calculate: Gaussian regime.

Use simple likelihood estimates; compare SuperB and LHCb side-by-side.

Methods involving $D \rightarrow hh$: SuperB

$$R_{GLW\pm} = 1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma$$

$$A_{GLW\pm} = \pm 2r_B \sin \delta_B \sin \gamma / R_{GLW\pm}$$

$$R_{ADS} = r_B^2 \pm r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \gamma$$

$$A_{ADS} = 2r_B r_D \sin(\delta_B + \delta_D) \sin \gamma / R_{ADS}$$

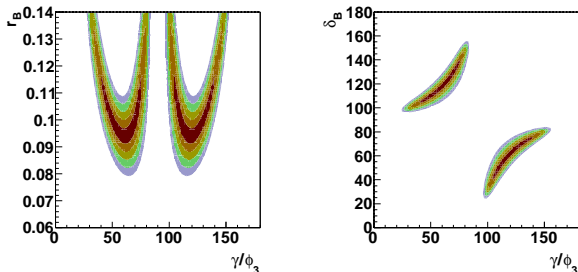
CP-even: $D \rightarrow hh$

CP-odd: $K_S \pi^0, K_S \omega$

6 observables

4 parameters:

$\gamma, r_B, \delta_B, \delta_D$



GLW alone, poor sensitivity in practice.

Methods involving $D \rightarrow hh$: SuperB

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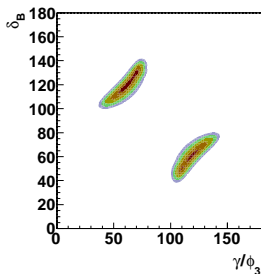
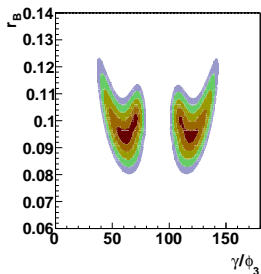
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GLW+ADS, no δ_D constraint — still ambiguous, non-Gaussian tail

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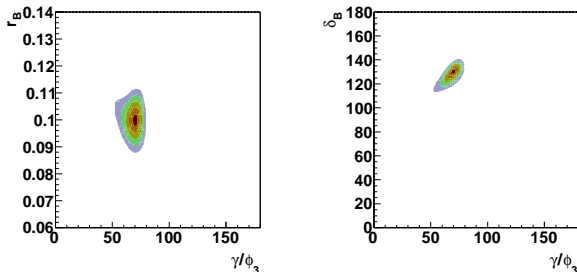
Sensitivity with

$$r_B = 0.1$$

$$\gamma = 70^\circ$$

$$\delta_B = 130^\circ$$

$$\delta_D = 22^\circ - 180^\circ:$$



GLW+ADS, $\sigma(\delta_D) = 2^\circ$.

$\sigma(\delta_D)$	$\sigma(\gamma)$
-	5.1°
11°	4.7°
1°	3.9°

Methods involving $D \rightarrow hh$: SuperB

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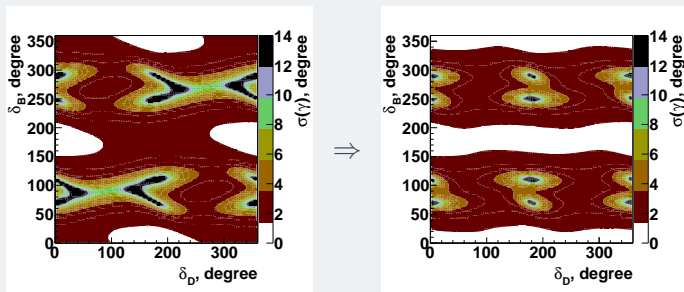
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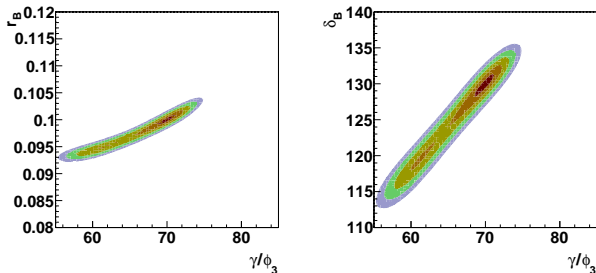
$\sigma(\gamma)$ depends on strong phases \rightarrow external constraints on r_B help (e. g. from $B \rightarrow (K\pi\pi\pi)_D K$).

Methods involving $D \rightarrow hh$: LHCb

$$\begin{aligned} N(B^\pm \rightarrow D(K^\pm \pi^\mp)K^\pm) &= N_{K\pi} [1 + r_B^2 r_D^2 + 2r_B r_D \cos(\delta_B - \delta_D \pm \gamma)] \\ N(B^\pm \rightarrow D(K^\mp \pi^\pm)K^\pm) &= N_{K\pi} [r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D \pm \gamma)] \\ \hline N(B^\pm \rightarrow D(h^+ h^-)K^\pm) &= N_{hh} [1 + r_B^2 + 2r_B \cos(\delta_B \pm \gamma)] \end{aligned}$$

Only CP-even states are available at LHCb.

GLW+ADS modes provide enough constraints if $N_{K\pi}/N_{hh}$ is fixed.



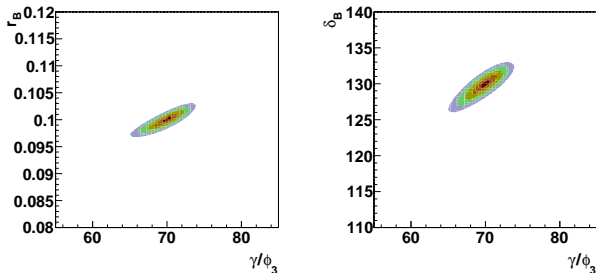
$B \rightarrow D(hh')K$, no δ_D constraint.

Methods involving $D \rightarrow hh$: LHCb

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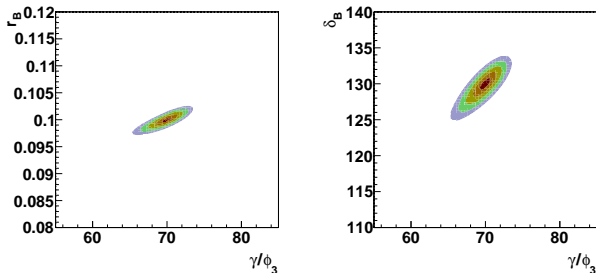
$$B \rightarrow D(hh')K, \sigma(\delta_D) = 2^\circ.$$

Methods involving $D \rightarrow hh$: LHCb

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$B \rightarrow D(hh')K$, $B \rightarrow D(K\pi\pi\pi)K$, no δ_D constraint.

Methods involving $D \rightarrow hh$: summary

- Combination of both ADS and GLW modes is essential to reach a good γ precision.
- External δ_D constraint is necessary to resolve ambiguities and to reach optimal precision: need $\sigma(\delta_D) \sim 1^\circ$!
- Sensitivity strongly depends on strong phase values δ_D and δ_B . In some unlucky cases up to $\times 10$ worse and more. Additional constraints may help in most cases. SuperB looks safer since more modes with potentially different δ_D and δ_B can be used.

Sensitivity for $r_B = 0.1$, $\gamma = 70^\circ$, $\delta_B = 130^\circ$, $\delta_D = 22^\circ - 180^\circ$:

	SuperB (50 ab^{-1})	LHCb (50 fb^{-1})
$D \rightarrow hh, D \rightarrow K\pi$	5.1°	1.4°
$D \rightarrow hh, D \rightarrow K\pi, \sigma(\delta_D) = 1^\circ$	3.9°	1.0°
$D \rightarrow hh, D \rightarrow K\pi, D \rightarrow K\pi\pi\pi$	2.4°	0.8°

- Other modes available at SuperB: $B^\pm \rightarrow D^*(D\pi^0)K^\pm$,
 $B^\pm \rightarrow D^*(D\gamma)K^\pm$, $B^\pm \rightarrow DK^{*\pm}$ can double the precision.

Giri, Grossman, Soffer, Zupan, PRD 68, 054018 (2003)

Bondar, Belle Dalitz analysis meeting (2002)

Observable — D decay Dalitz plot density (different for B^+ and B^-):

$$\rho(m_+^2, m_-^2) = |A_\pm|^2 = \left| \text{img} + r_B e^{i\delta_{B\pm} i\gamma} \text{img} \right|^2$$

Convenient fit parameters: $x_\pm + iy_\pm = r_B e^{i(\delta_{B\pm} + \gamma)}$.

Model-dependent analysis — a few degree model error. Binned model-independent approach is a solution:

$$M_i^\pm = h\{K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(x_\pm c_i - y_\pm s_i)\},$$

 $c_i = \langle \cos(\Delta\delta_D) \rangle$, $s_i = \langle \sin(\Delta\delta_D) \rangle$ obtained by CLEO-c.In the binned fit, can reach $\sim 90\%$ precision compared to the unbinned by choosing proper binning. With increased charm data — finer binning, closer to the unbinned precision.

$B^+ \rightarrow DK^+$, $D \rightarrow 3$ body Dalitz analysis

- Sensitivity practically independent of δ_B and γ .
- Errors scale proportional to $1/r_B$. Expect Gaussian errors with high statistics.
- In LHCb, $B/S \sim 1$ is expected \Rightarrow suppression of sensitivity.

	Event sample	$\sigma(\gamma)$
SuperB (50 ab^{-1})	100K	$1.5 - 2^\circ$
LHCb (50 fb^{-1})	300K	$1 - 1.5^\circ$

- CLEO-c error of c_i , s_i translates to $2 - 3^\circ$ error in γ .
- There is a slight model assumption in CLEO-c analysis due to use of $K_L\pi\pi$ tags. To *completely* avoid model assumptions, use only $\sim 1/3$ of statistics \Rightarrow expect $3 - 5^\circ$ error.
- Reducing the model error to $\sim 1^\circ$ requires $\sim 10 - 20 \text{ fb}^{-1}$ of $e^+e^- \rightarrow \psi(3770)$ data.
- Other D modes can be used $K_S KK$, $KK\pi\pi$ (SuperB, LHCb), $\pi\pi\pi^0$, $K_S\pi\pi\pi^0$, $K\pi\pi^0$ (SuperB, LHCb?).

$B^0 \rightarrow DK^{*0}$ can be used in the same way as $B \rightarrow DK$ for ADS, GLW analyses.

Smaller \mathcal{B} by factor 4–5

Larger $r_B = 0.3$ (0.1 for $B \rightarrow DK$).

} Expect similar sensitivity

Additional complication: other states in $B \rightarrow DK\pi$ amplitude.

Turns out to be an advantage: simultaneous Dalitz analysis of $B \rightarrow D(K\pi)K\pi$ and $B \rightarrow D_{CP}(hh)K\pi$ decays provides additional sensitivity and resolution of ambiguities.

T. Gershon, M. Williams, PRD 08 092002 (2009), arXiv:0909:1495

Inherent model dependence — can be limiting with $\sim 1^\circ$ precision.

Using binned analysis with the addition of $B \rightarrow D(K_S\pi\pi)K\pi$ state to the combined analysis allows to avoid model uncertainties. Correlations constrain $B \rightarrow DK\pi$ amplitude from $D \rightarrow K_S\pi\pi$.

T. Gershon, A.P., PRD 81 014025 (2010), arXiv:0910:5437

Time-integrated B^0 analyses

The amplitude of the $B^0 \rightarrow D(K_S \pi^+ \pi^-) K^+ \pi^-$ is $A = \bar{A}_B \bar{A}_D + e^{i\gamma} A_B A_D$ and thus the Dalitz plots $B^0 \rightarrow DK^+ \pi^-$ and $D \rightarrow K_S \pi^+ \pi^-$ are correlated (4D analysis!).

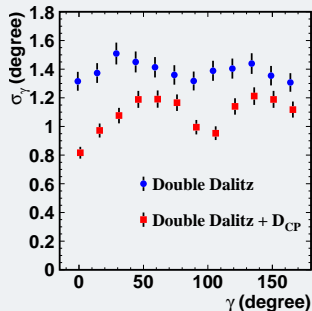
Binning ($M_{\alpha i}$: double Dalitz, N_{α} : $B^0 \rightarrow D_{fl} K^+ \pi^-$, K_i : $D^0 \rightarrow K_S \pi \pi$):

$$M_{\alpha i} = h \{ \bar{N}_{\alpha} K_i + N_{\alpha} K_{-i} + 2 \sqrt{N_{\alpha} K_i \bar{N}_{\alpha} K_{-i}} \times \\ [(\varkappa_{\alpha} c_i - \sigma_{\alpha} s_i) \cos \gamma - (\varkappa_{\alpha} s_i + \sigma_{\alpha} c_i) \sin \gamma] \}.$$

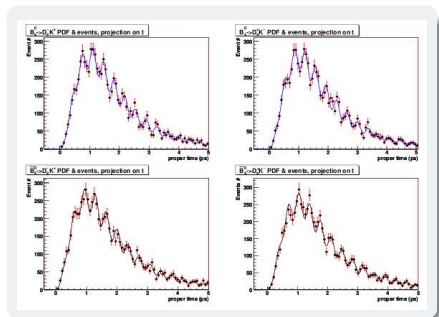
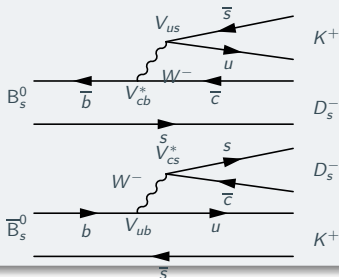
Phase parameters: c_i, s_i from charm data, \varkappa_{α} and σ_{α} — free parameters.

Very naive estimates for the sample $\sim 50 \text{ fb}^{-1}$ at LHCb (and without background)

Amplitude is unknown, sensitivity may vary significantly.



Time-dependent analyses: $B_s \rightarrow D_s K$



Measurement unique to LHCb: sensitive to $\phi_s + \gamma$.

Looking at the time-dependent distributions for flavour-tagged

$B_s \rightarrow D_s^\pm K^\mp$.

No external inputs needed except for ϕ_s (to be measured). Δm_s , mistag rate enter in the free parameters.

Sensitivity of 10° with 2 fb^{-1} at LHCb is expected \Rightarrow scale to $\sim 1.5 - 2^\circ$ with 50 fb^{-1} at SuperLHCb, but limiting systematic uncertainties still need to be understood.

In the modes with neutral D (time-integrated measurement), ignoring D mixing with $(x_D, y_D) \sim 1\%$ can affect a precision measurement.

- If the charm parameters (r_D, K_i , etc.) are extracted taking mixing into account, the mixing correction is linear and significant ($O(1^\circ)$).
[Silva, Soffer, PRD 61, 112001 \(2000\)](#)
- If mixing is ignored in both charm and B parameters, mixing correction is of 2nd order in $(x_D, y_D) \Rightarrow$ can safely ignore. True for ADS, model-dependent Dalitz.
[Grossman, Soffer, Zupan, PRD 72, 031501 \(2005\)](#)
- In model-independent Dalitz, c_i and s_i are extracted from correlated $D\bar{D}$ data with $\mathcal{C} = -1 \Rightarrow$ appear to be unaffected by mixing after integration over time. In that case, mixing corrections are linear, but suppressed by additional factors (incl. r_B). $\Delta\gamma \simeq 0.2^\circ$. Can be corrected. [Bondar, AP, Vorobiev, PRD 82, 034033 \(2010\)](#)
- Interestingly, in correlated $D\bar{D}$ decays produced with $\mathcal{C} = -1$, mixing effects are enhanced \Rightarrow allows to measure x_D, y_D in time-integrated analysis at DD^* threshold.

- Various independent methods should allow for $\sim 1 - 3^\circ$ measurement with SuperB and upgraded LHCb:
 - ADS + GLW modes
 - $B \rightarrow D(K_S \pi \pi)K$ Dalitz analysis
 - Self-tagging $B^0 \rightarrow DK\pi$ modes
 - $B_s \rightarrow D_s K$ (LHCb only)
- Overall, LHCb sensitivity with 50 fb^{-1} potentially looks better than that of SuperB, but higher backgrounds can reduce it. SuperB is more stable against "unlucky" parameter combinations when the sensitivity can be significantly reduced.
- Having a large ($\sim 10\text{-}20 \text{ fb}^{-1}$) sample at charm threshold is desirable for an efficient use of B data:
 - Significant fraction of BES-III sample.
 - Dedicated charm-tau factory.
 - SuperB operated at low energy.