THE UNIVERSITY OF CHICAGO

Theory of Inclusive Radiative B Decays

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CKM 2010

Disclaimer

In this talk I will focus on

recent[a] developments

and mostly on $\bar{B} \to X_s \gamma$

[a] recent = after CKM 2008

Outline

- Introduction
- Recent developments in $\bar{B} \to X_s\gamma$: Perturbative
- Recent developments in $\bar{B} \to X_s\gamma$: Non-Perturbative
- Commnets on $\bar{B} \to X_d\gamma$
- Commnets on $\bar{B} \to X_s l^+ l^-$
- Summary and Outlook

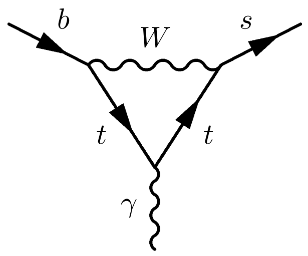
Introduction

$\bar{B} \to X_s\gamma$ in the SM

- $b \to s\gamma$ is a flavor changing neutral current (FCNC)

  In SM no FCNC at tree level

  Arises as a loop effect:



gives rise to the operator:

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} F^{\mu\nu}(1+\gamma_5)b$$

part of the effective Hamiltonian

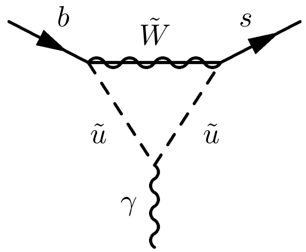
$$\mathcal{H}_{\text{eff}} \ni \frac{G_F}{\sqrt{2}} C_{7\gamma} Q_{7\gamma}$$

Constraints on New Physics: $\bar{B} \to X_s\gamma$

- **$\bar{B} \to X_s\gamma$ is an important probe of new physics**

  $b \to s\gamma$ can have contribution from new physics e.g. SUSY

  (only one diagram shown):



  leads to same operator, modifies $C_{7\gamma}$

- But $Q_{7\gamma}$ is not the whole story...

How To Make a Photon?

- Produce it directly...

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (1+\gamma_5) b$$

How To Make a Photon?

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$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} F^{\mu\nu}(1+\gamma_5)b$$

- Or make a gluon or a quark pair

$$\begin{aligned} Q_{8g} &= \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} G^{\mu\nu}(1+\gamma_5)b \\ Q_1^q &= (\bar{q}b)_{V-A}(\bar{s}q)_{V-A} \quad (p = u, c) \end{aligned}$$

and convert them to a photon

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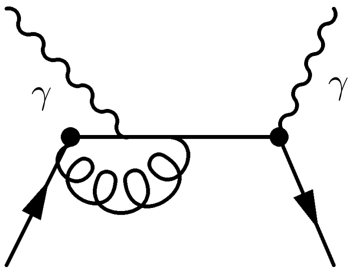
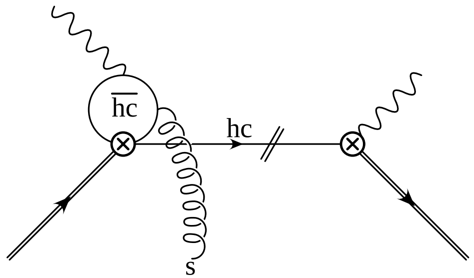
and convert them to a photon

- But it will cost you..

α_s or $1/m_b$





Effective Hamiltonian

- For $\bar{B} \to X_s \gamma$ need Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\ldots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

  - At leading power only $Q_{7\gamma} - Q_{7\gamma}$ contribute
  - At higher orders need other $Q_i - Q_j$ contributions
  - Most important: $Q_{7\gamma}, Q_{8g}$, and Q_1

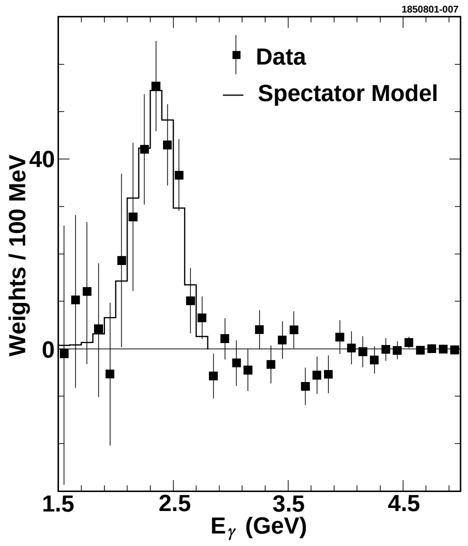
$$\begin{aligned} Q_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1+\gamma_5) F^{\mu\nu} b \\ Q_{8g} &= \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1+\gamma_5) G^{\mu\nu} b \\ Q_1^q &= (\bar{q} b)_{V-A} (\bar{s} q)_{V-A} \quad (q = u, c) \end{aligned}$$

- Ratios of Wilson coefficients:

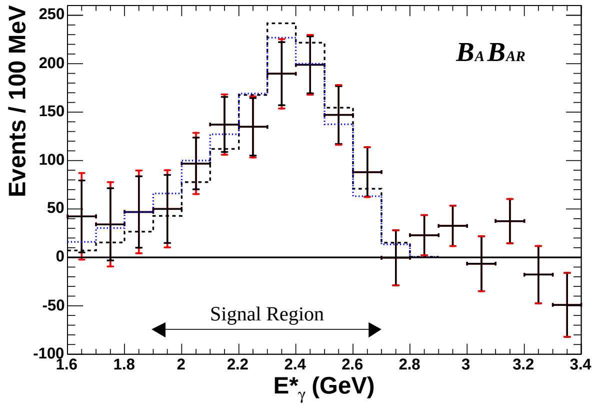
$$\begin{array}{ccccc} C_1 & : & C_{7\gamma} & : & C_{8g} \\ 3 & : & 1 & : & \frac{1}{2} \end{array}$$

Photon Spectrum

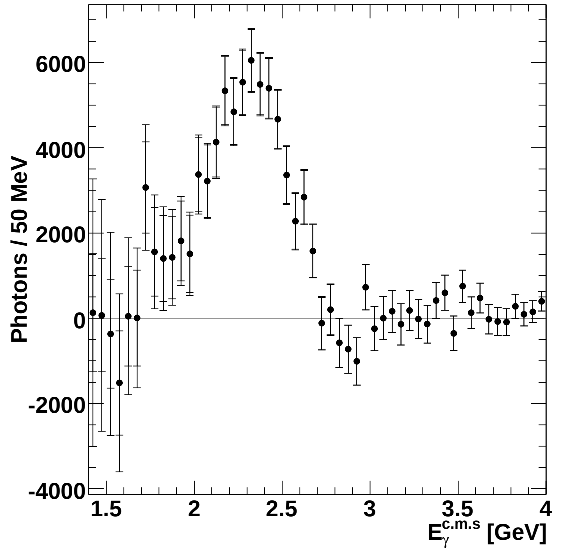
CLEO (2001)



BaBar (2006)



Belle (2008)



- Often seperate discussion to
  - Integrated rate:
  - Spectrum:

Photon Spectrum

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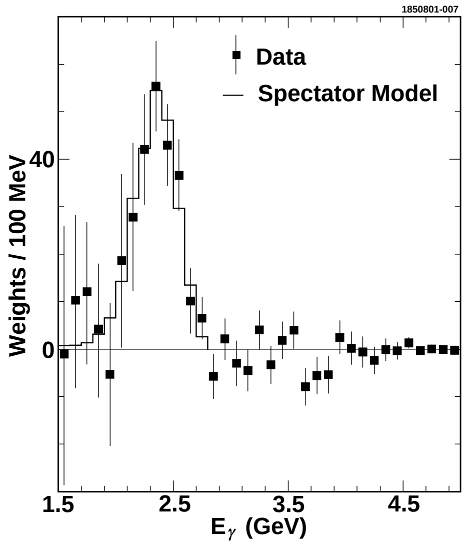

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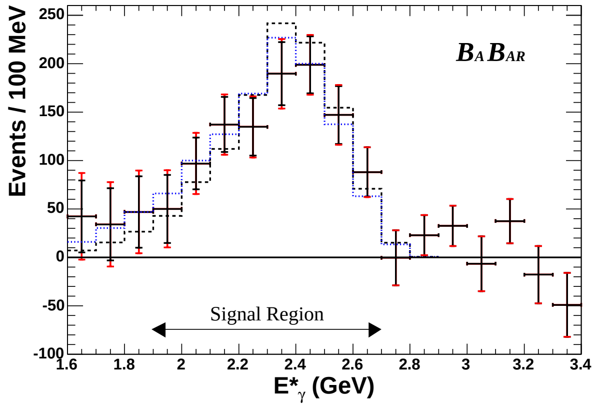

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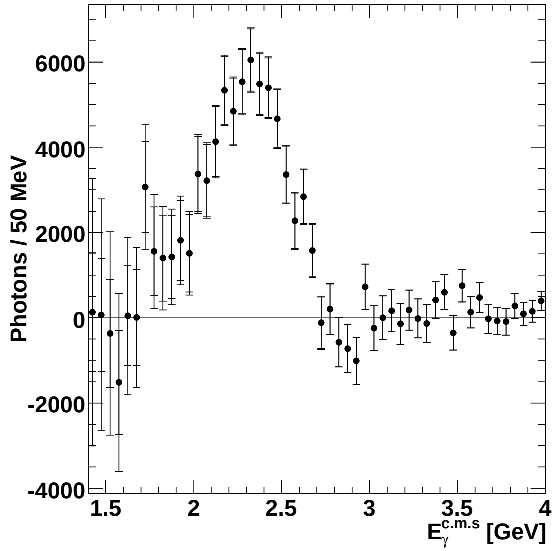

- Often seperate discussion to
  - Integrated rate: Expansion in terms of local operators
  - Spectrum: Expansion in terms of non-local operators

Photon Spectrum

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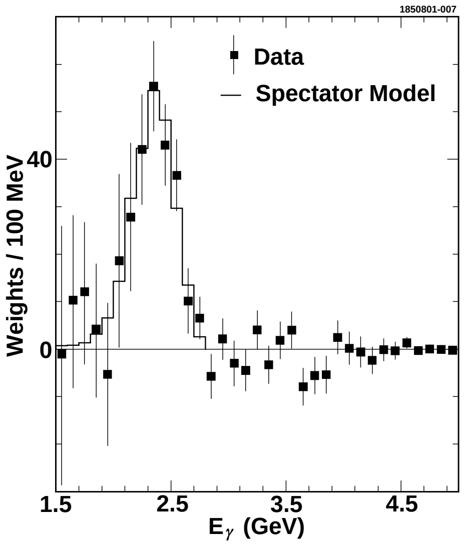

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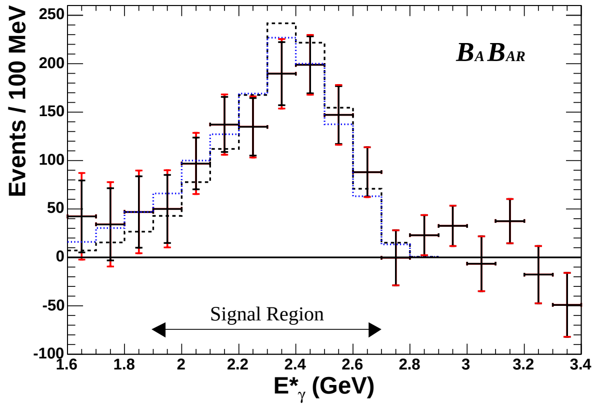

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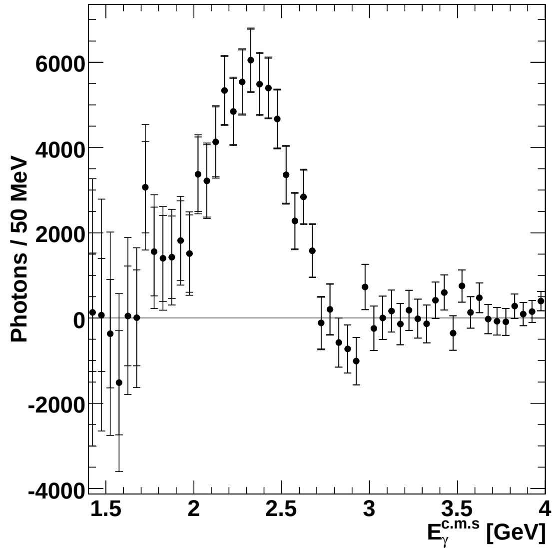

- Often seperate discussion to
  - Integrated rate: Expansion in terms of local operators
  - Spectrum: Expansion in terms of non-local operators
  - **Only true if ignoring $\Lambda_{\rm QCD}/m_b$ corrections or only considering $Q_{7\gamma} - Q_{7\gamma}$**

Photon Spectrum

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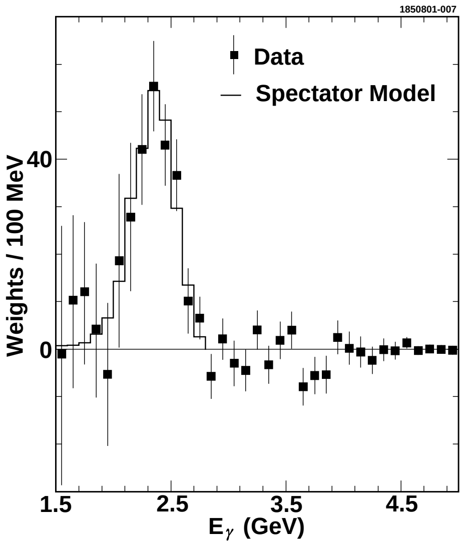

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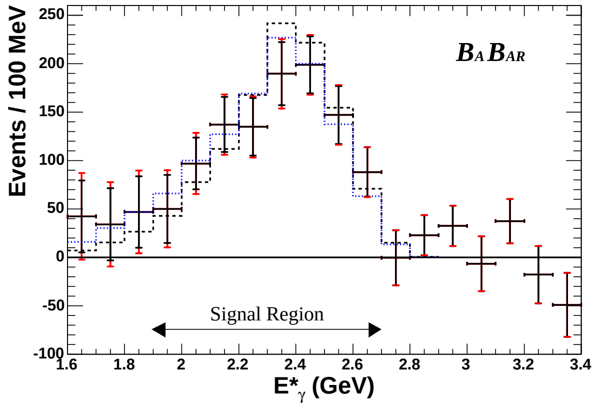

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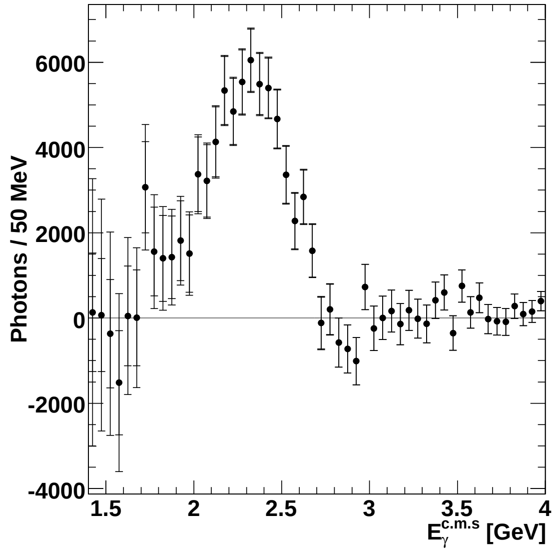

- Often seperate discussion to
  - Integrated rate: Expansion in terms of local operators
  - Spectrum: Expansion in terms of non-local operators
  - **Only true if ignoring $\Lambda_{\rm QCD}/m_b$ corrections or only considering $Q_{7\gamma} - Q_{7\gamma}$**
  - Otherwise, Integrated rate: Expansion in terms of non-local operators

Recent Developments

in

$\bar{B} \to X_s\gamma$: Perturbative

$\Gamma(b \to s\gamma)$

- Since

$$\Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to s\gamma) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

  Current effort: complete $\Gamma(b \to s\gamma)$ at NNLO

  For details see Greub CKM 2008

- Out of 3 ingrediants
  - Matching at $\mu \sim M_W$ ✓
  - Running from $\mu \sim M_W$ to $\mu \sim m_b$ ✓
  - Matrix elements at $\mathcal{O}(\alpha_s^2)$ almost done

- Current (2006) value Misiak et. al. '06:

  $\Gamma(b \to s\gamma) = (3.15 \pm 0.23) \times 10^{-4}, \quad E_\gamma > 1.6$ GeV

  Four types of uncertainties:
  - nonperturbative (5%) from $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$
  - parametric (3%)
  - higher-order (3%)
  - m_c-interpolation ambiguity (3%)

$\Gamma(b \to s\gamma)$: Recent Developments

- The $\mathcal{O}(\alpha_s^2)$ corrections from $Q_{7\gamma} - Q_{8g}$ calculated

  Asatrian, Ewerth, Ferroglia, Greub, Ossola, [ArXiv:1005.5587]

  "the correction... will not alter the central value... by more than **1%**"

- Details on evaluation of the NNLO QCD corrections in the heavy charm limit ($m_c \gg m_b/2$)

  Misiak, Steinhauser NPB 840, 271 (2010) [ArXiv:1005.1173]

- Complete $\mathcal{O}(\alpha_s^2)$ calculation of $Q_1 - Q_{7\gamma}$ and $Q_2 - Q_{7\gamma}$ is underway

  Goal: "make the perturbative uncertainties... negligible with respect to the non- perturbative... and experimental... ones"

$Q_{7\gamma} - Q_{7\gamma}$: Power Corrections

- Considering **only** $Q_{7\gamma} - Q_{7\gamma}$

  Power corrections are from $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right)$ local operators

$$\frac{\langle \bar{B}|\bar{h}(iD)^2 h|\bar{B}\rangle}{2M_B} \equiv \langle O_2^K\rangle \quad \frac{\langle \bar{B}|\bar{h}\sigma_{\mu\nu}\, g\, G^{\mu\nu} h|\bar{B}\rangle}{2M_B} \equiv \langle O_2^G\rangle$$

- The contribution to Γ_{77}

$$\Gamma_{77} = c_0\langle O_0\rangle + c_2^K \frac{\langle O_2^K\rangle}{m_b^2} + c_2^G \frac{\langle O_2^G\rangle}{m_b^2} + \ldots$$

- c_0 at $\mathcal{O}(\alpha_s^2)$, as for C_2^i
  - Falk, Luke, Savage '93

    C_2^i at $\mathcal{O}(\alpha_s^0)$
  - Ewerth, Gambino, Nandi NPB 830, 278 (2010) [arXiv:0911.2175]

    C_2^i at $\mathcal{O}(\alpha_s)$: Analytical expressions

- "The effect on the $\bar{B} \to X_s\gamma$ rate is below 1% for $E_\gamma < 1.8$ GeV"

$Q_{7\gamma} - Q_{7\gamma}$: Factorization

- Considering **only** $Q_{7\gamma} - Q_{7\gamma}$

$$d\Gamma \sim \overbrace{H \cdot J \otimes S + \frac{1}{m_b}\sum_i H \cdot J \otimes s_i}^{\text{known}} + \overbrace{\frac{1}{m_b}\sum_i H \cdot j_i \otimes S}^{\text{new}} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

  - Leading order term: $H \cdot J \otimes S$ (Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01)
  - Subleading shape functions: s_i (Bauer, Luke, Mannel '01, K. Lee, Stewart '04; Bosch, Neubert, GP '04; Beneke, Campanario, Mannel, Pecjak '04)
  - New term: Subleading jet functions: j_i (GP JHEP **0906** (2009) 083 [arXiv:0903.3377])

- j_i start at order $\mathcal{O}(\alpha_s) \Rightarrow$ SJF contribution $\mathcal{O}\left(\alpha_s \cdot \frac{\Lambda_{\text{QCD}}}{m_b}\right)$
- Relevant for "NNLO" extraction of $|V_{ub}|$
- SJF reduce to local operators for integrated rate

Perturbative corrections: Summary

- New Corrections are at NNLO level in α_s and/or $\dfrac{\Lambda_{\rm QCD}}{m_b}$

- Almost at the theoretical limit

- New perturbative corrections $\sim 1\%$ for $\Gamma(\bar{B} \to X_s \gamma)$

Recent Developments

in

$\bar{B} \to X_s\gamma$: Non Perturbative

M. Benzke, S.J. Lee, M. Neubert, GP

JHEP 1008:099 (2010) [arXiv:1003.5012]

Total Rate

- Previous studies of $Q_i - Q_j$ contributions focus on $\Gamma(\bar{B} \to X_s\,\gamma)$ and mostly on α_s suppressed effects
- Common lore:
  like $\Gamma(\bar{B} \to X_u\, l\, \bar{\nu})$ non perturbative effects arise at $1/m_b^2$

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- Hints that not all is well
  - $Q_{8g} - Q_{8g}$ (Ali, Greub '95; Kapustin, Ligeti, Politzer '95)
  - $Q_1 - Q_{7\gamma}$ (Voloshin '96; Ligeti, Randall, Wise '97; Grant, Morgan, Nussinov, Peccei '97; Buchalla, Isidori, Rey '97)
  - No local OPE for $\Gamma(\bar{B} \to X_s \gamma)$ (Ligeti, Randall, Wise '97)

  But effects were thought to be under control or small ...

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  But effects were thought to be under control or small ...

- **Never** a systematic study!

  In fact uncertainty from $Q_{7\gamma} - Q_{8g}$ was **missed**!

  (Lee, Neubert, GP '06)

  Non perturbative effects in $\Gamma(\bar{B} \to X_s\,\gamma)$ arise at $1/m_b$

- SCET allows for a systematic analysis

  What do we find from such an analysis?

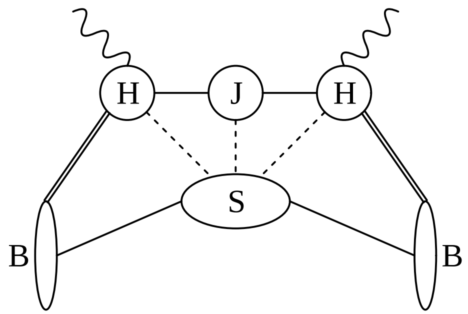
New Factorization Formula: Schematically

At the endpoint region: $m_b - 2E_\gamma \sim \Lambda_{\rm QCD}$

- Considering only $Q_{7\gamma} - Q_{7\gamma}$: factorization formula for $d\Gamma/dE_\gamma$

  (Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01)

$H \cdot J \otimes S$



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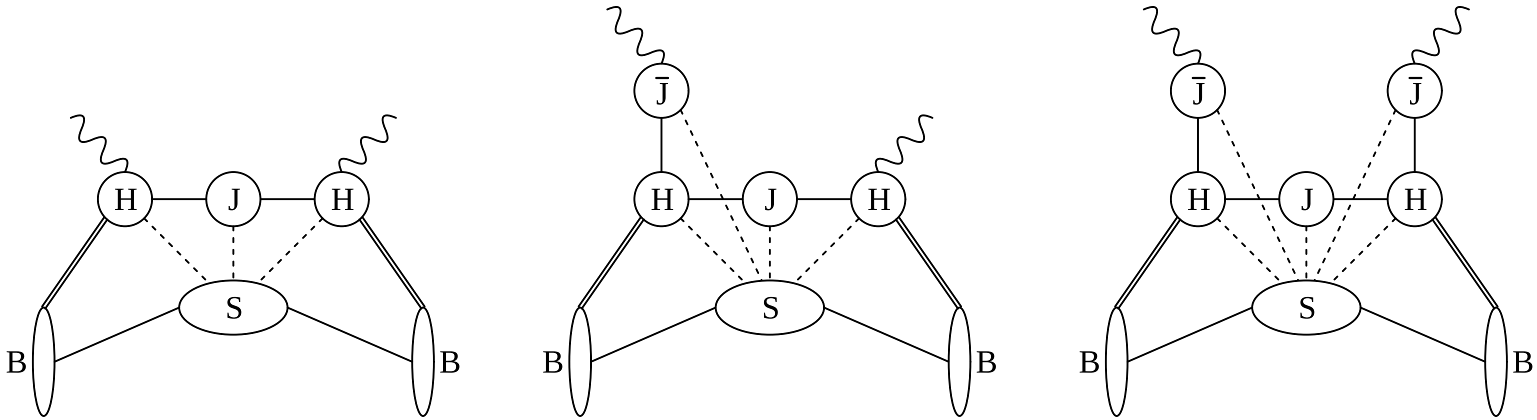
  (Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01)

- Considering also other operators $\Rightarrow$ **new** factorization formula

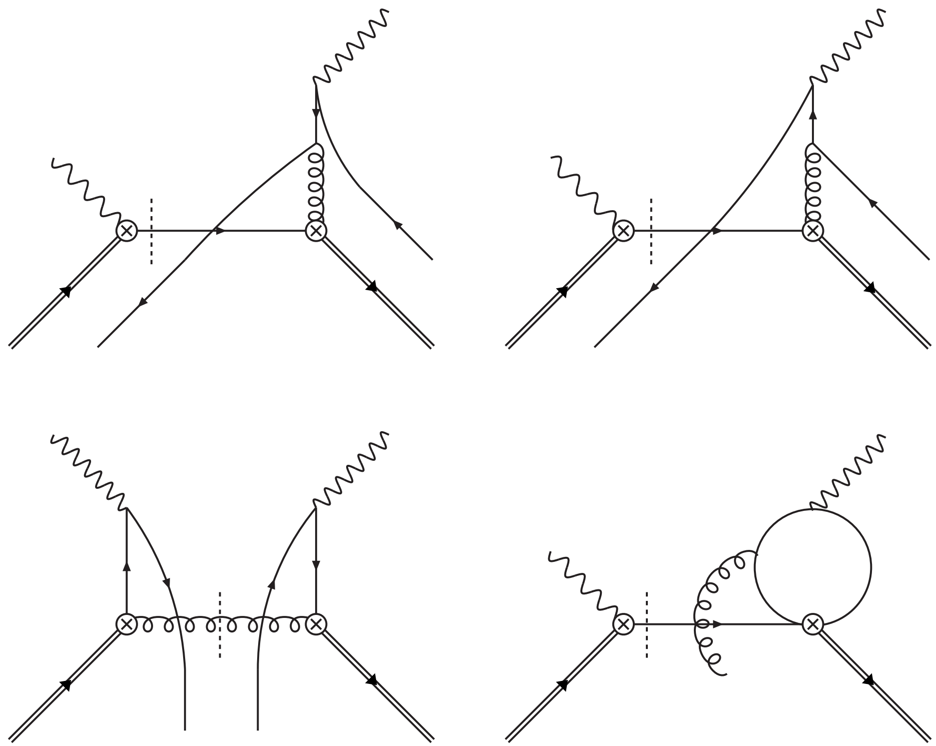
  for $d\Gamma/dE_\gamma$ (Benzke, Lee, Neubert, GP '10)

$H \cdot J \otimes S$ $\qquad$ $H \cdot J \otimes S \otimes \bar{J}$ $\qquad$ $H \cdot J \otimes S \otimes \bar{J} \otimes \bar{J}$



- No analog for semileptonic decays
- New "resolved photon" contributions $\Rightarrow$ $1/m_b$ corrections to $\Gamma(\bar{B} \to X_s\,\gamma)$
- What are they?

Resolved Photon Contributions



Top line: $Q_{7\gamma} - Q_{8g}$

Bottom left: $Q_{8g} - Q_{8g}$

Bottom right: $Q_1 - Q_{7\gamma}$

- $Q_1 - Q_{8g}$ and $Q_1 - Q_1$ give a $1/m_b^2$ effect

Resolved Photon Contributions

- Non perturbative effects arise from "Resolved Photon Contributions"

  They have the form

$$\Delta\Gamma \sim \underset{\text{Calc. in PT}}{\underset{\uparrow}{\bar{J}}} \otimes \underset{\text{Non pert.}}{\underset{\uparrow}{h}}$$

- The non perturbative functions h_{ij} are

$$h_{88}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B}|\bar{b}(0)\cdots s(un)\bar{s}(r\bar{n})\cdots b(0)|\bar{B}\rangle$$

$$h_{17}(\omega_1) \quad \text{F.T. of} \quad \langle \bar{B}|\bar{b}(0)\cdots G(s\bar{n})\cdots b(0)|\bar{B}\rangle$$

$$h_{78}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B}|\bar{b}(0)\cdots b(0)\sum_q e_q\, \bar{q}(r\bar{n})\cdots q(s\bar{n})|\bar{B}\rangle$$

The Integrated Rate

- For a photon energy cut $E_\gamma > E_0$ define

$$\mathcal{F}_E(\Delta) = \frac{\Gamma(E_0) - \Gamma(E_0)|_{\rm OPE}}{\Gamma(E_0)|_{\rm OPE}},$$

where $\Delta = m_b - 2E_0$ and $\Gamma(E_0)|_{\rm OPE}$ is the older calculation

- Assuming $\Delta \gg \Lambda_{\rm QCD}$

$$\mathcal{F}_E(\Delta) = \frac{C_1(\mu)}{C_{7\gamma}(\mu)} \frac{\Lambda_{17}(m_c^2/m_b, \mu)}{m_b} + \frac{C_{8g}(\mu)}{C_{7\gamma}(\mu)} 4\pi\alpha_s(\mu) \frac{\Lambda_{78}^{\rm spec}(\mu)}{m_b} + \left(\frac{C_{8g}(\mu)}{C_{7\gamma}(\mu)}\right)^2 \left[4\pi\alpha_s(\mu) \frac{\Lambda_{88}(\Delta, \mu)}{m_b} - \frac{C_F\alpha_s(\mu)}{9\pi} \frac{\Delta}{m_b} \ln\frac{\Delta}{m_s}\right] + \ldots,$$

where **model independently**,

$$\Lambda_{17}\left(\frac{m_c^2}{m_b}, \mu\right) = e_c \, {\rm Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b\,\omega_1}\right) + \frac{m_b\,\omega_1}{12m_c^2}\right] h_{17}(\omega_1, \mu),$$

$$\Lambda_{78}^{\rm spec}(\mu) = {\rm Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1 + i\varepsilon} \int_{-\infty}^{\infty} \frac{d\omega_2}{\omega_2 - i\varepsilon} h_{78}{}^{(5)}(\omega_1, \omega_2, \mu),$$

$$\Lambda_{88}(\Delta, \mu) = e_s^2 \left[\int_{-\infty}^{\Lambda_{\rm UV}} \frac{d\omega_1}{\omega_1 + i\varepsilon} \int_{-\infty}^{\Lambda_{\rm UV}} \frac{d\omega_2}{\omega_2 - i\varepsilon} 2h_{88}{}^{\rm cut}(\Delta, \omega_1, \omega_2, \mu) - \frac{C_F}{8\pi^2} \Delta \left(\ln\frac{\Lambda_{\rm UV}}{\Delta} - 1\right)\right].$$

The Integrated Rate

- We need to estimate three parameters:

$$\Lambda_{17}\left(\frac{m_c^2}{m_b},\mu\right) = e_c\,\mathrm{Re}\int_{-\infty}^{\infty}\frac{d\omega_1}{\omega_1}\left[1-F\left(\frac{m_c^2-i\varepsilon}{m_b\,\omega_1}\right)+\frac{m_b\,\omega_1}{12m_c^2}\right]h_{17}(\omega_1,\mu)\,,$$

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$$\Lambda_{88}(\Delta,\mu) = e_s^2\left[\int_{-\infty}^{\Lambda_{\mathrm{UV}}}\frac{d\omega_1}{\omega_1+i\varepsilon}\int_{-\infty}^{\Lambda_{\mathrm{UV}}}\frac{d\omega_2}{\omega_2-i\varepsilon}\,2h_{88}{}^{\mathrm{cut}}(\Delta,\omega_1,\omega_2,\mu)-\frac{C_F}{8\pi^2}\,\Delta\left(\ln\frac{\Lambda_{\mathrm{UV}}}{\Delta}-1\right)\right].$$

- Naively, if $\Lambda_{ij}\sim\Lambda_{\mathrm{QCD}}\sim 0.5$ GeV and since

$$C_1 \quad : \quad C_{7\gamma} \quad : \quad C_{8g}$$

$$3 \quad : \quad 1 \quad : \quad \frac{1}{2}$$

  Effect on the rate can be up to 30%

- Fortunately, it is possible to constrain Λ_{17} and $\Lambda_{78}^{\mathrm{spec}}$

  We now discuss each of the three parameters

Λ_{17}

- We need to estimate

$$\Lambda_{17}\Big(\frac{m_c^2}{m_b},\mu\Big) = e_c\,\mathrm{Re}\int_{-\infty}^{\infty}\frac{d\omega_1}{\omega_1}\left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b\,\omega_1}\right) + \frac{m_b\,\omega_1}{12m_c^2}\right]h_{17}(\omega_1,\mu)$$

- Recall

$$h_{17}(\omega_1) \quad \text{F.T. of} \quad \langle\bar{B}|\bar{b}(0)\cdots G(s\bar{n})\cdots b(0)|\bar{B}\rangle$$

  where intuitively ω_1 is the soft gluon momentum

- $h_{17}(\omega_1)$
  - An even function of ω_1
  - Its normalization is $2\lambda_2 \approx 0.24\,\mathrm{GeV}^2$

- Using an exponential or a Gaussian as a model for $h_{17}(\omega_1)$

$$h_{17}(\omega_1,\mu) = \frac{\lambda_2}{\sigma}\,e^{-\frac{|\omega_1|}{\sigma}} \quad \text{or} \quad h_{17}(\omega_1,\mu) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma}\,e^{-\frac{\omega_1^2}{2\sigma^2}}$$

- Varying σ
  - $(\Lambda_{17}^{\mathrm{exp}})_{\max} = -4.6\,\mathrm{MeV}$ (for $\sigma = 0.51\,\mathrm{GeV}$)
  - $(\Lambda_{17}^{\mathrm{Gauss}})_{\max} = -8.1\,\mathrm{MeV}$ (for $\sigma = 0.77\,\mathrm{GeV}$)

- Not a conservative bound!

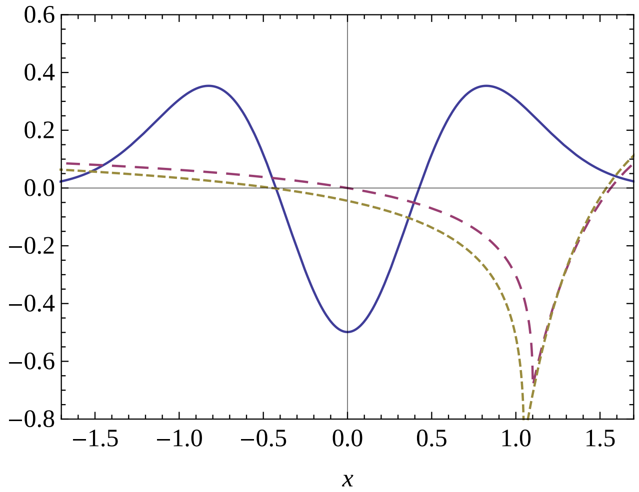
Λ_{17}

- We need to estimate

$$\Lambda_{17}\Big(\frac{m_c^2}{m_b},\mu\Big) = e_c \,\mathrm{Re}\int_{-\infty}^{\infty}\frac{d\omega_1}{\omega_1}\left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b\,\omega_1}\right) + \frac{m_b\,\omega_1}{12m_c^2}\right] h_{17}(\omega_1,\mu)$$

- $h_{17}(\omega_1)$ doesn't have to be positive, e.g.

$$h_{17}(\omega_1,\mu) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma}\,\frac{\omega_1^2 - \Lambda^2}{\sigma^2 - \Lambda^2}\, e^{-\frac{\omega_1^2}{2\sigma^2}}$$



- Blue line: $\sigma = 0.5\,\mathrm{GeV}$ and $\Lambda = 0.425\,\mathrm{GeV} \Rightarrow \Lambda_{17} = -42\,\mathrm{MeV}$
- Another choice: $\sigma = 0.5\,\mathrm{GeV}$ and $\Lambda = 0.575\,\mathrm{GeV} \Rightarrow \Lambda_{17} = 27\,\mathrm{MeV}$.
- Final range: $-60\,\mathrm{MeV} < \Lambda_{17} < 25\,\mathrm{MeV}$

Λ_{78}

- We need to estimate

$$\Lambda_{78}^{\rm spec}(\mu) = {\rm Re}\int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1 + i\varepsilon} \int_{-\infty}^{\infty} \frac{d\omega_2}{\omega_2 - i\varepsilon}\, {h_{78}}^{(5)}(\omega_1, \omega_2, \mu)$$

- Recall

$$h_{78}(\omega_1, \omega_2) \qquad \text{F.T. of} \quad \langle \bar{B}|\bar{b}(0)\cdots b(0)\sum_q e_q\, \bar{q}(r\bar{n})\cdots q(s\bar{n})|\bar{B}\rangle$$

- Method I: Fierz and use VIA

$$h_{78}(\omega_1, \omega_2)\big|_{\rm VIA} = -e_{\rm spec}\, \frac{f_B^2\, M_B}{8} \left(1 - \frac{1}{N_c^2}\right) \phi_+^B(-\omega_1)\, \phi_+^B(-\omega_2)$$

where $\phi_+^B(-\omega_1)$ is B-meson LCDA (“wave function”)

- Λ_{78} depends on LCDA’s inverse moment λ_B

$$\Lambda_{78}^{\rm spec}\big|_{\rm VIA} = -e_{\rm spec}\left(1 - \frac{1}{N_c^2}\right) \frac{f_B^2\, M_B}{8\lambda_B^2(\mu)} \in e_{\rm spec}\, [-386\,{\rm MeV}, -35\,{\rm MeV}]$$

$e_{\rm spec}= \frac{1}{3}$ for $\bar{B}^0$, $-\frac{2}{3}$ for B^-

Λ_{78}

- We need to estimate

$$\Lambda_{78}^{\text{spec}}(\mu) = \text{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1 + i\varepsilon} \int_{-\infty}^{\infty} \frac{d\omega_2}{\omega_2 - i\varepsilon} h_{78}{}^{(5)}(\omega_1, \omega_2, \mu)$$

- Recall

$$h_{78}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \, \bar{q}(r\bar{n}) \cdots q(s\bar{n}) | \bar{B} \rangle$$

- Method II: **Assuming** $SU(3)$ flavor symmetry

$\Rightarrow$ $\Lambda_{78}^{\text{spec}}$ is determined by isospin asymmetry

Misiak Acta Phys. Polon. B 40, 2987 (2009) [arXiv:0911.1651]

$$\Delta_{0-} = \frac{\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)}{\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^- \to X_s\gamma)}$$

measured by BaBar to be $\Delta_{0-} = (-1.3 \pm 5.9)\%$

- Including 30% $SU(3)$ flavor breaking gives

$$\Lambda_{78}^{\text{spec}} \approx -4.5\,\text{GeV}\,(e_{\text{spec}} \pm 0.05)\,\Delta_{0-}$$

Λ_{88}

- We need to estimate

$$\Lambda_{88}(\Delta,\mu) \quad = \quad e_s^2 \left[\int_{-\infty}^{\Lambda_{\rm UV}} \frac{d\omega_1}{\omega_1+i\varepsilon} \int_{-\infty}^{\Lambda_{\rm UV}} \frac{d\omega_2}{\omega_2-i\varepsilon}\, 2{h_{88}}^{\rm cut}(\Delta,\omega_1,\omega_2,\mu) - \frac{C_F}{8\pi^2}\,\Delta\left(\ln\frac{\Lambda_{\rm UV}}{\Delta}-1\right)\right]$$

- Recall

$$h_{88}(\omega_1,\omega_2) \qquad \text{F.T. of} \quad \langle\bar B|\bar b(0)\cdots s(un)\bar s(r\bar n)\cdots b(0)|\bar B\rangle$$

- We model $\Lambda_{88}(\Delta,\mu)$ by

$$\Lambda_{88}(\Delta,\mu)\approx e_s^2\,\Lambda(\mu)\,, \qquad \Lambda(\mu)>0\,,$$

with $0<\Lambda(\mu)<1\,\text{GeV}$

Total Uncertainty

$$\mathcal{F}_E(\Delta) = \frac{C_1(\mu)}{C_{7\gamma}(\mu)} \frac{\Lambda_{17}(m_c^2/m_b, \mu)}{m_b} + \frac{C_{8g}(\mu)}{C_{7\gamma}(\mu)} 4\pi\alpha_s(\mu) \frac{\Lambda_{78}^{\text{spec}}(\mu)}{m_b}$$
$$+ \left(\frac{C_{8g}(\mu)}{C_{7\gamma}(\mu)}\right)^2 \left[4\pi\alpha_s(\mu) \frac{\Lambda_{88}(\Delta, \mu)}{m_b} - \frac{C_F \alpha_s(\mu)}{9\pi} \frac{\Delta}{m_b} \ln \frac{\Delta}{m_s}\right] + \dots ,$$

- Using the above values for Λ_{17} and Λ_{88}

$$\mathcal{F}_E\big|_{17} \in [-1.7, +4.0]\,\%\,,$$
$$\mathcal{F}_E\big|_{88} \in [-0.3, +1.9]\,\%\,.$$

- While for Λ_{78} we have

$$\mathcal{F}_E\big|_{78}^{\text{VIA}} \in [-2.8, -0.3]\,\% \quad \text{or} \quad \mathcal{F}_E\big|_{78}^{\text{exp}} \in [-4.4, +5.6]\,\% \quad (95\%\ \text{CL})$$

- "Scanning" over the ranges we have

$$-4.8\% < \mathcal{F}_E(\Delta) < +5.6\% \quad (\text{VIA for } \Lambda_{78}^{\text{spec}})$$

or

$$-6.4\% < \mathcal{F}_E(\Delta) < +11.5\% \quad (\Lambda_{78}^{\text{spec}} \text{ from } \Delta_{0-})$$

- Even if the error on Δ_{0-} was zero

$$-4.0\% < \mathcal{F}_E(\Delta) < +4.8\% \quad (\text{ideal case})\,.$$

$\Gamma(\bar{B} \to X_s\gamma)$ in SM

- Experiment
  - Experimental value of $\text{Br}(\bar{B} \to X_s\gamma)$:
    Extrapolated from measured $E_\gamma \sim 1.9$ GeV to $E_\gamma > 1.6$ GeV
    (HFAG ICHEP 2010 Update)
    $\text{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV}) = (3.55 \pm 0.24 \pm 0.09) \cdot 10^{-4}$ (error 7%)
- Theory NNLO:
  - OPE: Assume 1.6 GeV is in the OPE region (Misiak et. al. '06)
    $\text{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV}) = (3.15 \pm 0.23) \cdot 10^{-4}$ (error 7%)
  - MSOPE: 1.6 GeV is still in MSOPE region (Becher, Neubert '06)
    $\text{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV}) = (2.98 \pm 0.26) \cdot 10^{-4}$ (error 9%)
  - DGE: My extraction from (Andersen, Gardi '06)
    $\text{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV}) = (3.47 \pm 0.48 \pm 0.17) \cdot 10^{-4}$
    (error $14\%_{\text{Andersen,Gardi}} + 5\%_{\text{new non.pert.}}$)
- Largest error "non perturbative": estimated **5%**

  based on $Q_{7\gamma} - Q_{8g}$ (Lee, Neubert, GP '06) times 1.5
- Improved numerical estimate based on all $1/m_b$ contributions

  (Benzke, Lee, Neubert, GP '10): **5%**

Future

- Still to be done (Benzke, Lee, Neubert, GP, in progress)
  - New contributions to CP asymmetry in $\bar{B} \to X_s\gamma$
  - Effect on the spectrum and the extraction of the HQET parameters

Other ways for improvement

- Two other issues

1) Resummation
   - To resum or not to resum this is the question!

     M. Misiak [arXiv:0808.3134]
   - See also Gardi CKM 2008
   - Discussion only via confrences talks and proceedings

2) Extrapolation
   - Experimental value of $\mathrm{Br}(\bar{B} \to X_s\gamma)$:
     Extrapolated from measured $E_\gamma \sim 1.9$ GeV to $E_\gamma > 1.6$ GeV
   - HFAG ICHEP 2010 Update:

     "In this average... we still use the extrapolation factors [arXiv:hep-ph/0507253] obtained by O. Buchmüller and H. Flächer..."
   - Considering new knowledge of non-perturbative effects

     and new Belle measurment with $E_\gamma > 1.7$ GeV
   - It's time to revisit the issue!

Comments on

$$\bar{B} \to X_d\gamma$$

Comments on $\bar{B} \to X_d\gamma$

- $\bar{B} \to X_d\gamma$ analogous to $\bar{B} \to X_s\gamma$:

  replace $V_{qb}V_{qs}^*$ by $V_{qb}V_{qd}^*$

- But for $\bar{B} \to X_d\gamma$, Q_1^u is not CKM suppressed

  $\Rightarrow Q_1^u - Q_{7\gamma}$ is not suppressed

  Buchalla, Isidori, Rey '97: $Q_1^u - Q_{7\gamma}$ scales as Λ_{QCD}/m_b

- Effect not calculated in [Buchalla, Isidori, Rey '97]

  [Benzke, Lee, Neubert, GP '10]: For CP averaged rate $Q_1^u - Q_{7\gamma}$ vanishes!

$$\int P \frac{d\omega_1}{\omega_1} \quad \times \quad h_{17}(\omega_1) \quad = 0$$

$$\underset{\text{Odd}}{\uparrow} \qquad\qquad \underset{\text{Even}}{\uparrow}$$

- Removes largest source of uncertainty

  $\bar{B} \to X_d\gamma$ as theoretically clean as $\bar{B} \to X_s\gamma$

  Hurth, Nakao [ArXiv:1005.1224]

Comments on

$$\bar{B} \to X_s\, l^+ l^-$$

$\bar{B} \to X_s\, l^+ l^-$

- Region of low $q^2 \in [1...6]\,\text{GeV}^2$ and $m_X \leq m_X^{\text{cut}}$

  $d\Gamma_i$ factorizes similarly to $d\Gamma_{77}$ of $\bar{B} \to X_s\gamma$

$$d\Gamma_i \sim H_i \cdot J \otimes S + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right), \quad i = T, A, L$$

- Recent progress

  - K. S. M. Lee, F. J. Tackmann

    Calculation of $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$ "primary" SSF

    PRD 79, 114021 (2009) [arXiv:0812.0001]

  - G. Bell, M. Beneke, T. Huber and X. Q. Li

    Two loop calculation of H_i

    [arXiv:1007.3758]

$\bar{B} \to X_s\, l^+ l^-$: Power Corrections

- K. S. M. Lee, F. J. Tackmann, PRD 79, 114021 (2009) [arXiv:0812.0001]:

  Calculate contribution of SSF that appear also in $\bar{B} \to X_u\, l\bar{\nu}$ ("primary")

- Subleading power corrections give sizeable corrections of order 5% to 10%

- Cause a shift of $\sim -0.05\,\mathrm{GeV}^2$ to $-0.1\,\mathrm{GeV}^2$

  in the zero of the forward-backward asymmetry

$\bar{B} \to X_s\, l^+ l^-$: Perturbative Corrections

- G. Bell, M. Beneke, T. Huber and X. Q. Li [arXiv:1007.3758]:

  Two loop calculation of H_i

- Shift in zero of the forward-backward asymmetry:

  NLO: -2.2% NNLO: -3%

- Final result, **including** the "primary" $1/m_b$ corrections

$$q_0^2 = (3.34 \dots 3.40)^{+0.22}_{-0.25}\,\text{GeV}^2 \quad \text{for} \quad m_X^{\text{cut}} = (2.0 \dots 1.8)\,\text{GeV}$$

$\bar{B} \to X_s l^+ l^-$: Future Directions

- Following the completed analysis for $\Gamma(\bar{B} \to X_s \gamma)$

  What is the effect from "non-primary" SSF?

- For example, soft gluon attachments to the charm-loop diagrams:

$$\langle \bar{B} | \bar{b}(0) \cdots G(s\bar{n}) \cdots b(0) | \bar{B} \rangle$$

- Point also stressed in

  G. Bell, M. Beneke, T. Huber and X. Q. Li [arXiv:1007.3758]

Summary and Outlook

Summary

- Inclusive Radiative B decays is a mature field
- New factorization formula for photon spectrum in endpoint region

$$d\Gamma(\bar{B} \to X_s\gamma) = \overbrace{\sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)}}^{\text{known}}$$

$$+ \underbrace{\sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]}_{\text{new}}$$

- New “resolved photon” contributions $\Rightarrow$ $1/m_b$ corrections to $\Gamma(\bar{B} \to X_s\,\gamma)$
- From a systematic study:

  An irreducible error of $\sim 5\%$ on $\Gamma(\bar{B} \to X_s\gamma)$

  This non perturbative error is the largest

$\Rightarrow$ no prospect for reducing **total** theoretical error below $\sim 5\%$

Future Directions

- Perturbative error is expected to be reduced

  below non-perturbative and experimental error

- New non-perturbative contributions to CP asymmetry in $\bar{B} \to X_s \gamma$

- New non-perturbative effects on the spectrum

  and extraction of HQET parameters

- The issues of Resummation and Extrapolation should be revisited

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  - Perturbative: beyond NNLO seems impossible
  - Non Perturbative: irreducible hadronic uncertainty

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Definitive theoretical predictions for inclusive radiative B decays!