$B \rightarrow K^*(\rightarrow K\pi)l^+l^-$ Theory: A symmetry point of view

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Table of contents

Motivation

Few processes contain a richer phenomenology than the $b \longrightarrow s$ semileptonic exclusive decay $B \longrightarrow K^* l^+ l^-$:

Observables:

- Forward-Backward asymmetry A_{FB},
- Isospin asymmetry A_I,
- K^* spin/helicity amplitude observables of the 4-body decay: $A_T^{(i)}$ and more

Main goal: Identify signals of specific NP models in the flavor sector to complement direct research.

Condition: Construct the best (less QCD uncertainties) observables. How?

Description of the Method for $B \to K^*(\to K\pi)l^+l^-$

The steps of the present method are

- **Construction** of a quantity using spin or helicity amplitudes as the milestones.
  - **Maximize** sensitivity to certain type of New Physics
  - **Minimize** dependence on hadronic uncertainties (soft form factors).
- **Identification** of all symmetries of the distribution.
- Check that the quantity **fulfills** all the symmetries ⇒ Observables.

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- **Find hidden correlations** (dependencies) between the coefficients of the distribution.
  - Stability of fit and extra experimental checks.

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Differential decay distributions

The decay $\bar{\mathbf{B}}_\mathbf{d} \to \bar{\mathbf{K}}^{*\mathbf{0}}(\to \mathbf{K}^-\pi^+)\mathbf{l}^+\mathbf{l}^-$ with the K^{*0} on the mass shell is described by s and three angles $\theta_\mathbf{l}$, $\theta_\mathbf{K}$ and ϕ

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$

- $q^2 = s$ square of the lepton-pair invariant mass.
- θ_l angle between $\vec{p_{l^+}}$ in l^+l^- rest frame and dilepton's direction in rest frame of $\bar{B}_d$
- θ_K angle between $\vec{p_{K^-}}$ in the $\bar{K}^{*0}$ rest frame and direction of the $\bar{K}^{*0}$ in rest frame of $\bar{B}_d$
- ϕ angle between the planes defined by the two leptons and the $K - \pi$ planes.

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$$J(q^2, \theta_l, \theta_K, \phi) =$$
$$J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l + J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$
$$+J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi + (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l$$
$$+J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi + J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\,.$$

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R\,*} + A_\parallel^L A_\parallel^{R*}\right),$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R*})\right] + \beta_\ell^2|A_S|^2,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right], \quad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + (L\to R)\right],$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + (L\to R)\right], \quad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L\,*}) + (L\to R)\right],$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L\,*}) - (L\to R) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^R A_S^*)\right],$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L\,*}) - (L\to R)\right], \quad J_{6c} = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}\left[A_0^L A_S^* + (L\to R)\right],$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L*}) - (L\to R) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* + A_\perp^R A_S^*)\right],$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L\,*}) + (L\to R)\right], \quad J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L\,*} A_\perp^L) + (L\to R)\right]$$

SCALARS: We have 8 complex amplitudes ($A_{\perp,||,0,(L,R)S,t}$) and 12 experimental inputs

NO SCALARS: We have 7 complex amplitudes ($A_{\perp,||,0,(L,R)}, t$) and 11 experimental inputs

Symmetries of the distribution

Experimental (J_i) $\leftrightarrow$ theoretical (A_i) degrees of freedom

$$\mathbf{n_C - n_d = 2n_A - n_s}$$

- $\mathbf{n_C}$: # coefficients of differential distribution: J_i
- $\mathbf{n_d}$: # relations between J_i
- $\mathbf{n_A}$: # spin amplitudes
- $\mathbf{n_s}$: # symmetries of the distribution

Case: **Massless leptons with no scalars: ML-NS**

$\mathbf{n_C = 11}$, $\mathbf{n_d = 3}$ ($J_{1s} = 3J_{2s}$, $J_{1c} = -J_{2c}$ and a **third relation**),

$$\Updownarrow$$

$\mathbf{n_A = 6}$ (spin amplitudes), $\mathbf{n_s = 4}$ **symmetries**.

What is this third relation and which are those symmetries?

Counting symmetries

Infinitesimal symmetry transformation of the distribution

$$\mathbf{A}' = \mathbf{A} + \delta\mathbf{S} \, .$$

$$\begin{aligned} \vec{A} \;=\; & \Big(\mathrm{Re}(A_\perp^L), \mathrm{Im}(A_\perp^L), \mathrm{Re}(A_\parallel^L), \mathrm{Im}(A_\parallel^L), \mathrm{Re}(A_0^L), \mathrm{Im}(A_0^L), \\ & \mathrm{Re}(A_\perp^R), \mathrm{Im}(A_\perp^R), \mathrm{Re}(A_\parallel^R), \mathrm{Im}(A_\parallel^R), \mathrm{Re}(A_0^R), \mathrm{Im}(A_0^R)\Big) \end{aligned}$$

S represents a symmetry of the distribution if and only if

$$\forall i \in (J_{1s}...J_9) : \vec{\nabla}(J_i) \perp \mathbf{S} \, .$$

n independent infinitesimal symmetries $\leftrightarrow$
n linearly independent vectors $\mathbf{S}_j$ with $j = 1, ..n$.

$\Rightarrow$ In the masless case $\mathbf{n} = \mathbf{4}$

From infinitesimal to continuous symmetry.

The differential distribution is invariant under $n = 4$ independent symmetry transformations of the amplitudes:

- 1. An independent phase transformation of L-amplitudes

$$A'_{\perp L} = e^{i\phi_L} A_{\perp L}, \qquad A'_{\| L} = e^{i\phi_L} A_{\| L}, \qquad A'_{0L} = e^{i\phi_L} A_{0L},$$

- 2. An independent phase transformation of the R-amplitudes,

$$A'_{\perp R} = e^{i\phi_R} A_{\perp R}, \qquad A'_{\| R} = e^{i\phi_R} A_{\| R}, \qquad A'_{0R} = e^{i\phi_R} A_{0R},$$

- 3. A first continuous $L \leftrightarrow R$ rotation (I)

$$\begin{aligned} A'_{\perp L} &= +\cos\theta A_{\perp L} + \sin\theta A^*_{\perp R} \quad & A'_{\perp R} &= -\sin\theta A^*_{\perp L} + \cos\theta A_{\perp R} \\ A'_{\| L} &= +\cos\theta A_{\| L} - \sin\theta A^*_{\| R} \quad & A'_{\| R} &= +\sin\theta A^*_{\| L} + \cos\theta A_{\| R} \\ A'_{0L} &= +\cos\theta A_{0L} - \sin\theta A^*_{0R} \quad & A'_{0R} &= +\sin\theta A^*_{0L} + \cos\theta A_{0R} \end{aligned}$$

- 4. A second continuous $L \leftrightarrow R$ transformation (II)

$$
\begin{aligned}
A'_{\perp L} &= +\cosh\bar{\theta} A_{\perp L} + \sinh\bar{\theta} A^*_{\perp R} \quad & A'_{\perp R} &= -\sinh\bar{\theta} A^*_{\perp L} + \cosh\bar{\theta} A_{\perp R} \\
A'_{\parallel L} &= +\cosh\bar{\theta} A_{\parallel L} - \sinh\bar{\theta} A^*_{\parallel R} \quad & A'_{\parallel R} &= +\sinh\bar{\theta} A^*_{\parallel L} + \cosh\bar{\theta} A_{\parallel R} \\
A'_{0L} &= +\cosh\bar{\theta} A_{0L} - \sinh\bar{\theta} A^*_{0R} \quad & A'_{0R} &= +\sinh\bar{\theta} A^*_{0L} + \cosh\bar{\theta} A_{0R}
\end{aligned}
$$

$$\bar{\theta} = \mathbf{i}\,\theta'$$

Any quantity constructed out of A has to fulfill all symmetries of the distribution

Consequence: The quantity $A_T^{(1)} = -2\frac{\mathrm{Re}(A_\parallel A_\perp^*)}{|A_\perp|^2+|A_\parallel|^2}$ is not invariant under 3 and 4 $\Rightarrow$ it cannot be extracted from angular distribution.

A bit more on symmetries

Define

$$n_1 = (A_\parallel^L, A_\parallel^{R*}) \qquad m_1 = (H_{+1}^L, H_{-1}^{R}{}^*)$$
$$n_2 = (A_\perp^L, -A_\perp^{R*}) \quad \text{or} \quad m_2 = (H_{-1}^L, H_{+1}^{R}{}^*)$$
$$n_3 = (A_0^L, A_0^{R*}) \qquad m_3 = (H_0^L, H_0^{R*})$$

Spin amplitudes — Helicity amplitudes

All physical information of the distribution encoded in 3 moduli + 3 relative angles (complex) - 1 constrain (**third relation)**.

$$|n_1|^2 = \frac{2}{3}J_{1s} - J_3\,, \qquad |n_2|^2 = \frac{2}{3}J_{1s} + J_3\,, \qquad |n_3|^2 = J_{1c}$$
$$n_1 \cdot n_2 = \frac{J_{6s}}{2} - iJ_9\,, \qquad n_1 \cdot n_3 = \sqrt{2}J_4 - i\frac{J_7}{\sqrt{2}}\,, \qquad n_2 \cdot n_3 = \frac{J_5}{\sqrt{2}} - i\sqrt{2}J_8$$

Interpretation of the symmetry: moduli and complex scalar products kept invariant.

What do we learn/gain out of those symmetries?

- **Identify** the conditions to construct observables out of spin amplitudes.
- **Solve** the system of A's in terms of J's.
- **Stability and convergence of the fit** by identifying all hidden correlations inside the distribution.
- Identify a **non-linear and non-trivial correlation** (third relation) between the coefficients of the angular distribution.
- Moreover, this is a more general view of angular distributions **reinterpreted** in terms of moduli and angle between certain complex vectors.

Explicit solution and New non-trivial relation

We can solve the system of A's in terms of J's:

- Global phase symmetry L (1) $\Rightarrow \phi_L$ such that $\mathrm{Im} A_\parallel^L = 0$
- Global phase symmetry R (2) $\Rightarrow \phi_R$ such that $\mathrm{Im} A_\parallel^R = 0$ (simplicity)
- Continuous $L \leftrightarrow R$ rotation (3) $\Rightarrow \theta$ such that $\mathrm{Re} A_\parallel^R = 0$

This implies $n_1 = \left(0, A_\parallel^R\right)$ with $\mathrm{Im} A_\parallel^R = 0$. The system is then easily solved

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Amp.	LEFT	RIGHT
$A_\perp$	$\left[\|n_2\|^2 - \frac{\|(n_1 . n_2)\|^2}{\|n_1\|^2}\right]^{\frac{1}{2}} e^{i\phi_\perp^L} = \left[\frac{\frac{4}{9}J_{1s}^2 - J_3^2 - \frac{1}{4}J_{6s}^2 - J_9^2}{\frac{2}{3}J_{1s} - J_3}\right]^{\frac{1}{2}} e^{i\phi_\perp^L}$	$-\frac{n_1 . n_2}{\sqrt{\|n_1\|^2}} = -\frac{(J_{6s} - 2iJ_9)}{2\sqrt{\frac{2}{3}J_{1s} - J_3}}$,
$A_\parallel$	0	$\sqrt{\|n_1\|^2} = \sqrt{\frac{2}{3}J_{1s} - J_3}$
A_0	$\left[\|n_3\|^2 - \frac{\|(n_1 . n_3)\|^2}{\|n_1\|^2}\right]^{\frac{1}{2}} e^{i\phi_0^L} = \left[\frac{J_{1c}\left(\frac{2}{3}J_{1s} - J_3\right) - 2J_4^2 - \frac{1}{2}J_7^2}{\frac{2}{3}J_{1s} - J_3}\right]^{\frac{1}{2}} e^{i\phi_0^L}$	$\frac{n_1 . n_3}{\sqrt{\|n_1\|^2}} = \frac{2J_4 - iJ_7}{\sqrt{\frac{4}{3}J_{1s} - 2J_3}}$

BUT, there is a last equation

$$e^{i(\phi_\perp^L - \phi_0^L)} = \frac{(n_2 \cdot n_3)|n_1|^2 - (n_2 \cdot n_1)(n_1 \cdot n_3)}{\left([|n_1|^2|n_2|^2 - |(n_2 \cdot n_1)|^2)\,(|n_1|^2|n_3|^2 - |(n_3 \cdot n_1)|^2)\right]^{1/2}}$$
$$= \frac{J_5\left(\frac{2}{3}J_{1s} - J_3\right) - J_4 J_{6s} - J_7 J_9 - i\left(\frac{4}{3}J_{1s}J_8 - 2J_3J_8 + 2J_4J_9 - \frac{1}{2}J_{6s}J_7\right)}{\left[2\left(\frac{4}{9}J_{1s}^2 - J_3^2 - \frac{1}{4}J_{6s}^2 - J_9^2\right)\left(J_{1c}\left(\frac{2}{3}J_{1s} - J_3\right) - 2J_4^2 - \frac{1}{2}J_7^2\right)\right]^{1/2}}.$$

Remarks:

a) Condition of the L.H.S. being a phase ⇒ the non-trivial new relation:

$$\mathbf{J_{1c} = -J_{2c} = 6\frac{(2J_{1s} + 3J_3)\left(4J_4^2 + J_7^2\right) + (2J_{1s} - 3J_3)\left(J_5^2 + 4J_8^2\right)}{16J_{1s}^2 - 9\left(4J_3^2 + J_{6s}^2 + 4J_9^2\right)}}$$
$$\mathbf{- 36\frac{J_{6s}(J_4J_5 + J_7J_8) + J_9(J_5J_7 - 4J_4J_8)}{16J_{1s}^2 - 9\left(4J_3^2 + J_{6s}^2 + 4J_9^2\right)} \equiv f}$$

True in massless leptons case without scalars. In massless leptons with scalars ($J_{1c} \neq -J_{2c}$) fulfilled for $-J_{2c}$ and not fulfilled for J_{1c}. Similar expression (but with β) for massive leptons with no scalars. **Not fulfilled** for massive leptons with scalars. **Large deviations in** $J_{1c} = f \Rightarrow$ **scalars. Large deviations in** $-J_{2c} = f \Rightarrow$ **experimental problem.**

b) 4th symmetry manifest in the freedom to chose $\phi_\perp^L$ or $\phi_0^L = 0$

More general cases

The discussion of the differential symmetries can be generalised to:

a) **Massless leptons with scalars**: $\mathbf{n_C = 11, n_d = 2, n_A = 7, n_s = 5}$

- Amplitudes ML-NS + scalar amplitude A_S: Seven amplitudes.
- Four explicit symmetries and

$$A_S^{'} = e^{i\phi_S} A_S$$

The phase of A_S cannot be determined.

b) **Massive leptons without scalars**: $\mathbf{n_C = 11, n_d = 1, n_A = 7, n_s = 4}$

- Amplitudes ML-NS + A_t: Seven amplitudes
- Symmetries:
  - One global phase transformation $\phi_L = \phi_R$.
  - Two continuous LR symmetries are broken.
  - A new symmetry concerning the phase of A_t given as:

$$A_t^{'} = e^{i\phi_t} A_t$$

Four symmetries of differential distribution required.

c) **Massive leptons with scalars**: $\mathbf{n_C = 12, n_d = 0, n_A = 8, n_s = 4}$

- Amplitudes: ML-NS $+A_s+A_t$: Eight amplitudes.
- Coefficients of the distribution 12: ML-NS $+J_{6c}$.
- Symmetries:
  - The global phase transformation, $\phi_L = \phi_R$.
  - The phase transformation of A_t in b) is valid.

In this case, there is NO dependency between J's, and four symmetries of the differential form required.

Case	Coefficients	Dependencies	Amplitudes	Symmetries
$m_l = 0,\ A_S = 0$	11	3	6	4 (4)
$m_l = 0,\ A_S <> 0$	11	2	7	5 (5)
$m_l > 0,\ A_S = 0$	11	1	7	4 (2)
$m_l > 0,\ A_S <> 0$	12	0	8	4 (2)

Remind: $\mathbf{n_C - n_d = 2n_A - n_s}$

Construction of Observables: $A_T^{(i)}$ i=2,3,4,5

Theory framework: NLO QCDF including Λ/m_b corrections.

Spin amplitudes $A_{\perp L,R}, A_{\parallel L,R}, A_{0L,R}$ are functions:

- $B \to K^*$ Form factors: $A_{0,1,2}(s)$, $V(s)$, $T_{1,2,3}(s)$.
- Wilson Coefficients: $\mathcal{C}_7^{(\text{eff})}, \mathcal{C}_7^{'(\text{eff})}, \mathcal{C}_9^{(\text{eff})}, \mathcal{C}_{10}$

$$\mathbf{A}_{\perp \mathbf{L},\mathbf{R}} = N\sqrt{2}\lambda^{1/2}\left[(\mathcal{C}_9^{(\text{eff})} \mp \mathcal{C}_{10})\frac{V(q^2)}{m_B + m_K^*} + \frac{2m_b}{q^2}(\mathcal{C}_7^{(\text{eff})} + \mathcal{C}_7^{'(\text{eff})})T_1(q^2)\right]$$

$$\mathbf{A}_{\parallel \mathbf{L},\mathbf{R}} = -N\sqrt{2}(m_B^2 - m_{K^*}^2)\left[(\mathcal{C}_9^{(\text{eff})} \mp \mathcal{C}_{10})\frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2}(\mathcal{C}_7^{(\text{eff})} - \mathcal{C}_7^{'(\text{eff})})T_2(q^2)\right],$$

$$\mathbf{A}_{0\mathbf{L},\mathbf{R}} = -\frac{N}{2m_{K^*}\sqrt{q^2}} \times \left[(\mathcal{C}_9^{(\text{eff})} \mp \mathcal{C}_{10})\left\{(m_B^2 - m_{K^*}^2 - q^2)(m_B + m_{K^*})A_1(q^2) - \right.\right.$$
$$\left. -\lambda\frac{A_2(q^2)}{m_B + m_{K^*}}\right\} + 2m_b(\mathcal{C}_7^{(\text{eff})} - \mathcal{C}_7^{'(\text{eff})})\left\{(m_B^2 + 3m_{K^*}^2 - q^2)T_2(q^2) - \right.$$
$$\left.\left. -\frac{\lambda}{m_B^2 - m_{K^*}^2}T_3(q^2)\right\}\right],$$

HOW to deal with the form factors? Two alternatives:

- Framework of QCDF at LO+α_s-NLO+Λ/m_b corrections.

  Egede et al '08 and '10

- Mix QCD LCSR FF (LO) + α_s-QCDF NLO (neglect Λ/m_b)

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All FF (V, A_i, T_i) in the limit $m_B \to \infty$ and $E_K^* \to \infty \Rightarrow \xi_\perp(\mathbf{E_K^*}), \xi_\parallel(\mathbf{E_K^*})$

$$A_1(s) = \frac{2E_{K^*}}{m_B + m_{K^*}}\xi_\perp(\mathbf{E_{K^*}}), \qquad A_2(s) = \frac{m_B}{m_B - m_{K^*}}\left[\xi_\perp(\mathbf{E_{K^*}}) - \xi_\parallel(\mathbf{E_{K^*}})\right],$$

$$A_0(s) = \frac{E_{K^*}}{m_{K^*}}\xi_\parallel(\mathbf{E_{K^*}}), \qquad V(s) = \frac{m_B + m_{K^*}}{m_B}\xi_\perp(\mathbf{E_{K^*}}),$$

$$T_1(s) = \xi_\perp(\mathbf{E_{K^*}}), \quad T_2(s) = \frac{2E_{K^*}}{m_B}\xi_\perp(\mathbf{E_{K^*}}), \quad T_3(s) = \xi_\perp(\mathbf{E_{K^*}}) - \xi_\parallel(\mathbf{E_{K^*}}).$$

In this limit spin amplitudes reduce to a very simple form:

$$\mathbf{A}_{\perp \mathbf{L},\mathbf{R}} = \sqrt{2} N m_B (1-\hat{s}) \left[(\mathcal{C}_9^{(\mathrm{eff})} \mp \mathcal{C}_{10}) + \frac{2\hat{m}_b}{\hat{s}} (\mathcal{C}_7^{(\mathrm{eff})} + \mathcal{C}_7^{\prime(\mathrm{eff})}) \right] \xi_\perp(E_{K^*}),$$

$$\mathbf{A}_{\parallel \mathbf{L},\mathbf{R}} = -\sqrt{2} N m_B (1-\hat{s}) \left[(\mathcal{C}_9^{(\mathrm{eff})} \mp \mathcal{C}_{10}) + \frac{2\hat{m}_b}{\hat{s}} (\mathcal{C}_7^{(\mathrm{eff})} - \mathcal{C}_7^{\prime(\mathrm{eff})}) \right] \xi_\perp(E_{K^*}),$$

$$\mathbf{A}_{\mathbf{0L},\mathbf{R}} = -\frac{N m_B}{2\hat{m}_{K^*}\sqrt{\hat{s}}} (1-\hat{s})^2 \left[(\mathcal{C}_9^{(\mathrm{eff})} \mp \mathcal{C}_{10}) + 2\hat{m}_b (\mathcal{C}_7^{(\mathrm{eff})} - \mathcal{C}_7^{\prime(\mathrm{eff})}) \right] \xi_\parallel(E_{K^*}),$$

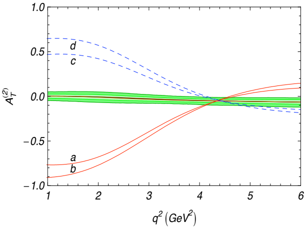
- Corrections to FF relations:
    - order α_s in QCDF at NLO (factor. and non-factor.)
    - $\mathbf{\Lambda/m_b}$ breaking contributions: order 5 and 10%.

EXAMPLE Transverse Asymmetries: A_T^2

Definition Kruger,J.M. '05

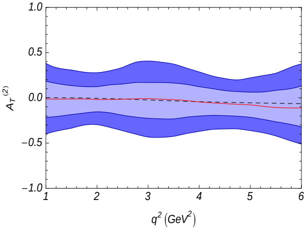
$$A_T^2 = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2} = -2\frac{\mathrm{Re} H_+^* H_-}{|H_+|^2 + |H_-|^2}$$

- Physics Sensitivity: Deviation from SM left-handed structure: $A_T^2\Big|_{SM} \sim 0.$
- Cleanliness: Soft form factor ($\xi_\perp(0)$) dependence cancel exactly at LO and very mild dependence at NLO.
- Domain: Low-Region $1 \leq q^2 \leq 6\ \mathrm{GeV}^2$ (High region, see G. Hiller et al.)



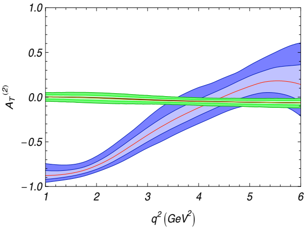
Theoretical sensitivity

Λ/m_b: light(dark) green $\pm 5\%$ ($\pm 10\%$)



Exper. sensitivity SM ($10\mathrm{fb}^{-1}$)

light(dark) blue 1σ (2σ)



Exper. SUSY sens.

(Egede et al. 08)

Understanding A_T^2

In the large E_K^* and m_B limit (only C_7')

$$A_T^2 \sim 4\mathcal{C}_7^{\prime(\text{eff})}\frac{m_b M_B}{s}\frac{\Delta_- + \Delta_+^*}{2C_{10}^2 + |\Delta_-|^2 + |\Delta_+|^2}$$

$$\Delta_\pm = \mathcal{C}_9^{\text{eff}} + 2\frac{m_b M_B}{s}(\mathcal{C}_7^{(\text{eff})} \pm \mathcal{C}_7^{\prime(\text{eff})})$$

BUT

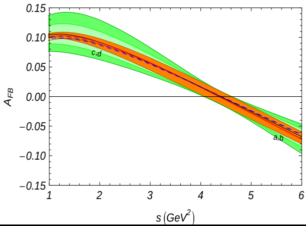
$$\Delta_+ + \Delta_-^* = 2\mathcal{C}_9^{\text{eff}} + 4\frac{m_b M_B}{s}(\mathcal{C}_7^{(\text{eff})})$$

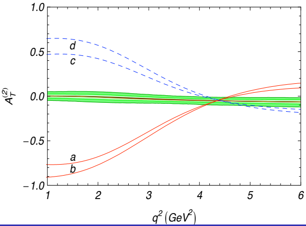
- Enhance sensitivity to $\mathcal{C}_7^{\prime(\text{eff})}$ (modulus+sign) at low s ($1 < s < 2\,\text{GeV}^2$) and 1/s-slope:

$$\underline{\mathbf{A_{FB}}} = \frac{3}{2}\frac{\text{Re}(A_{\parallel L}A_{\perp L}^*) - \text{Re}(A_{\parallel R}A_{\perp R}^*)}{|A_0|^2+|A_\parallel|^2+|A_\perp|^2} \quad \textbf{versus} \quad \underline{\mathbf{A_T^2}} = \frac{|A_\perp|^2-|A_\parallel|^2}{|A_\perp|^2+|A_\parallel|^2}$$

Only FF protection at q_0^2
q_0^2 at LO (and NLO)

FF protection from $1 < q^2 < 6\,\text{GeV}^2$
SAME q_0^2 at LO (and NLO) ($C_7' \neq 0$)





Understanding A_T^2

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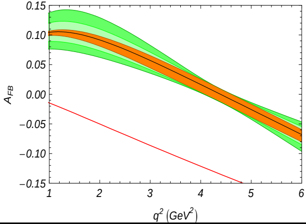
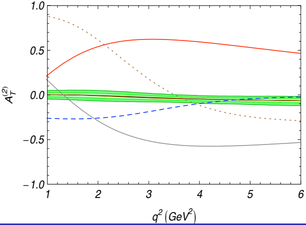
Only FF protection at q_0^2
q_0^2 at LO + **Absence of zero**

FF protection from $1 < q^2 < 6\,{\rm GeV}^2$
SAME q_0^2 at LO+ **Absence of zero**





Understanding A_T^2

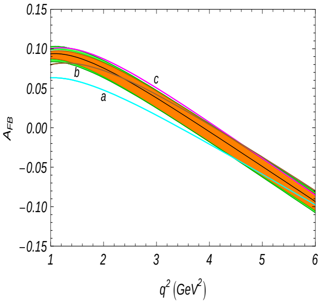
- A_T^2: CP violating phase (O_7') sensitivity BETTER than CP violating observables

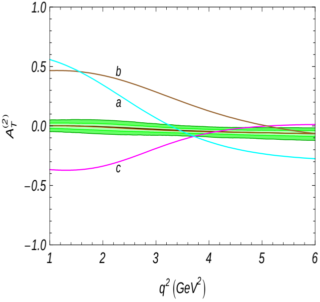
$$\mathbf{A_{FB}} = \frac{3}{2}\frac{\mathrm{Re}(A_{\parallel L}A^*_{\perp L}) - \mathrm{Re}(A_{\parallel R}A^*_{\perp R})}{|A_0|^2+|A_\parallel|^2+|A_\perp|^2} \quad \textbf{versus} \quad \mathbf{A_T^2} = \frac{|A_\perp|^2-|A_\parallel|^2}{|A_\perp|^2+|A_\parallel|^2}$$

A_{FB}: **Mild** sensitivity to C_7' mod+phase $\qquad$ A_T^2: **Strong** sensitivity to C_7' mod+phase

$$\mathrm{Num}(A_{FB}) \sim \frac{2m_b M_B}{q^2} C_7^{eff} + C_9 + \frac{2m_b M_B}{q^2}|C_7^{NP}|\cos\phi_7^{NP}$$

$$\mathrm{Num}(A_T^2) = \frac{4m_b M_B}{q^2}\left[\left(\frac{2m_b M_B}{q^2} C_7^{eff} + C_9\right)|C_7'|\cos\phi_7' + \frac{2m_b M_B}{q^2}|C_7'||C_7^{NP}|\cos(\phi_7' - \phi_7^{NP})\right]$$



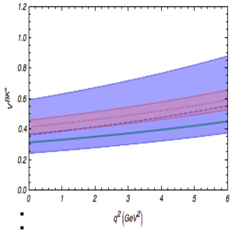


$C_7^{NP} e^{i\phi_7^{NP}}$	$C_7' e^{i\phi_7'}$
$0.26e^{-i\frac{7\pi}{16}}$	$0.2e^{i\pi}$ (a)
$0.07e^{i\frac{3\pi}{5}}$	$0.3e^{i\frac{3\pi}{5}}$ (b)
$0.03e^{i\pi}$	0.07 (c)

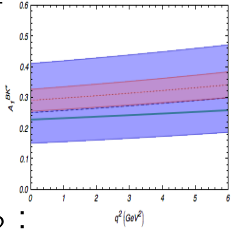
How solid are these predictions under FF changes?

A_T^2 is function of form factors: V, A_1, T_1 and $T_2 \Rightarrow \xi_\perp$.

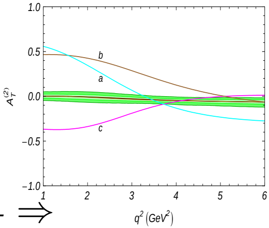
V:



A_1 :

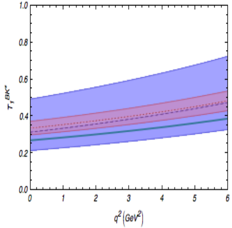


A_T^2 with Ball '98

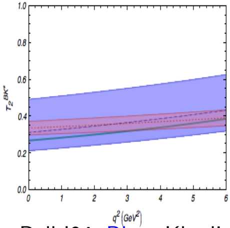


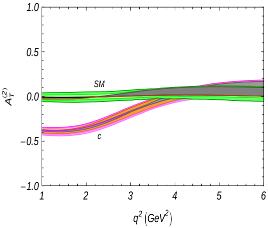
$A_T^2 \Rightarrow$

T_1 :



T_2 :





Green: Egede'10, Magenta: Ball '04, Blue: Khodjamirian '10

A_T^2 with Khodjamirian '10

Other sensitivities of A_T^2 : O_{10}'

A_T^2 may serve also as an excellent test of O_{10}' if ONLY switched on.
In the limit $m_b \to \infty$, $E_K^* \to \infty$

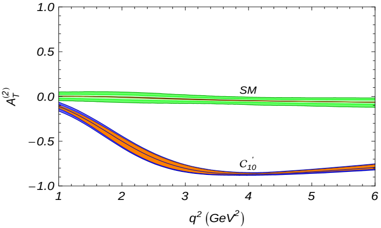
$$A_T^2 = \frac{2\,C_{10}\,C_{10}'\cos\phi_{10}'}{C_{10}^2 + |C_{10}'|^2 + (2m_b M_B C_7/q^2 + C_9)^2}$$

- A_T^2 shows a linear dependence on C_{10}' like for $C_7^{eff\,\prime}$
- But the q^2-dependence is different:
  - It does not show up a ZERO
  - Maximal around the standard minima of the A_{FB}.

A combined analysis with $A_T^{(3,4)}$ and A_T^5 allow to disentangle different WC.

Green $\to$ SM
Orange/Blue $\to$ $C_{10}' = 3e^{i\pi/8}$

All uncertainties included



A new example: A_T^5

Definition:

$$A_T^{(5)} = \frac{|A_\parallel^{R*} A_\perp^L + A_\parallel^L A_\perp^{R*}|}{|A_\parallel|^2 + |A_\perp|^2}$$

a) It probes spin amplitudes $A_\perp$ and $A_\parallel$ differently from A_T^2.

b) No angular coefficient mixes L/R with $\perp/\parallel$ simultaneously.

c) In the large recoil limit

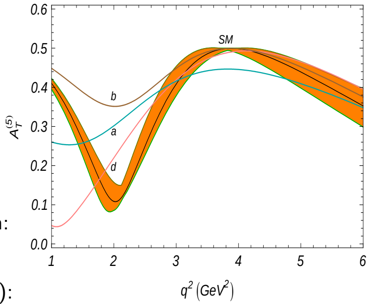
$$A_T^{(5)}\Big|_{SM} = \frac{\left|-C_{10}^2 + (2m_b M_B C_7^{eff}/q^2 + C_9^{eff})^2\right|}{2\left[C_{10}^2 + (2m_b M_B C_7^{eff}/q^2 + C_9^{eff})^2\right]},$$

Minimum at LO of $A_T^5 \Rightarrow$ **NEW** relation:

$$C_{10}^2 = (2m_b M_B C_7^{eff}/q_1^2 + C_9^{eff})^2$$

Maximum at LO of $A_T^5 \Rightarrow$ by **OLD** (A_{FB}-zero) relation:

$$-C_9^{eff} = 2m_b M_B C_7^{eff}/q_0^2$$



d) Expresion in terms of J's (using explicit solution):

$$A_T^{(5)}\Big|_{m_\ell=0} = \frac{\sqrt{16 J_1^{s\,2} - 9 J_6^{s\,2} - 36(J_3^2 + J_9^2)}}{8 J_1^s}.$$

A_T^5 sensitivities

C_7' sensitivity weaker, BUT dependence on C_9, C_{10} and C_{10}' is transparent.
At large recoil in presence of O_{10}', O_7, O_9, O_{10}

$$A_T^{(5)}\Big|_{10'} = \frac{\left|-C_{10}^2 + |C_{10}'|^2 + \left(2m_b M_B C_7^{eff}/q^2 + C_9^{eff}\right)^2\right|}{2\left[C_{10}^2 + |C_{10}'|^2 + \left(2m_b M_B C_7^{eff}/q^2 + C_9^{eff}\right)^2\right]}$$

a) **Maximum (LO)** in SM when $2m_b M_B C_7^{eff}/q_0^2 + C_9^{eff} = 0$ and $C_{10}' = 0$ then $A_T^5\Big|_{max} = \frac{1}{2}$ (True for NLO also).

b) **Maximum** moves also by NP contributions from C_7^{eff} or C_9^{eff} like A_{FB}.

c) **Minimum (LO)** moves by NP contribution from C_{10}'.

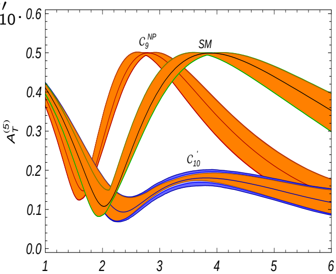
$$|C_{10}'|^2 = C_{10}^2 - (2m_b M_B C_7^{eff}/q^2 + C_9^{eff})^2$$

d) If only O_{10}' turn on and $C_{10}' < C_{10}$

<u>**Distance** between</u>

SM maximum and NP (O_{10}') maximum:

$$|C_{10}'^{NP}|^2/(C_{10}^2 + |C_{10}'^{NP}|^2)$$

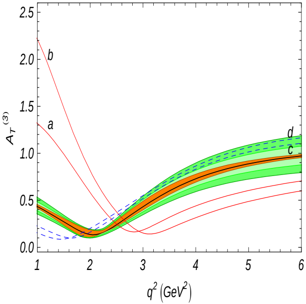


Transverse/Longitudinal Asymmetries: A_T^3 and A_T^4.

Open **longitudinal spin amplitude** A_0 sensitivity in a protected way.

$$A_T^3 = \frac{|A_{0L}A^*_{\parallel L} + A^*_{0R}A_{\parallel R}|}{\sqrt{|A_0|^2|A_\perp|^2}} \quad \text{and} \quad A_T^4 = \frac{|A_{0L}A^*_{\perp L} - A^*_{0R}A_{\perp R}|}{|A_{0L}A^*_{\parallel L} + A^*_{0R}A_{\parallel R}|}$$

- Invariant under massless symmetries and high experimental resolution.
- Constructed to cancel both $\xi_\perp(0)$ and $\xi_\parallel(0)$ dependence at LO.
- Offer different sensitivity to $\mathcal{C}_7^{\prime(\mathrm{eff})}$, C_9^{eff} and C_{10}:



- $\mathbf{A_T^3}$ at LO its **minimum** determines a relation:

$$C_{10}^2 = -(C_9^{eff} + 2\frac{m_b}{M_B}(C_7^{eff} - C_7^{eff\prime}))(C_9^{eff} + 2\frac{m_b M_B}{q^2}(C_7^{eff} - C_7^{eff\prime}))$$

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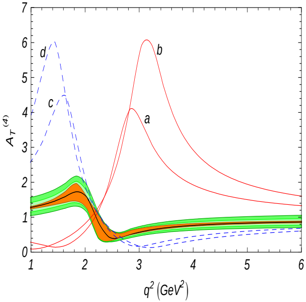


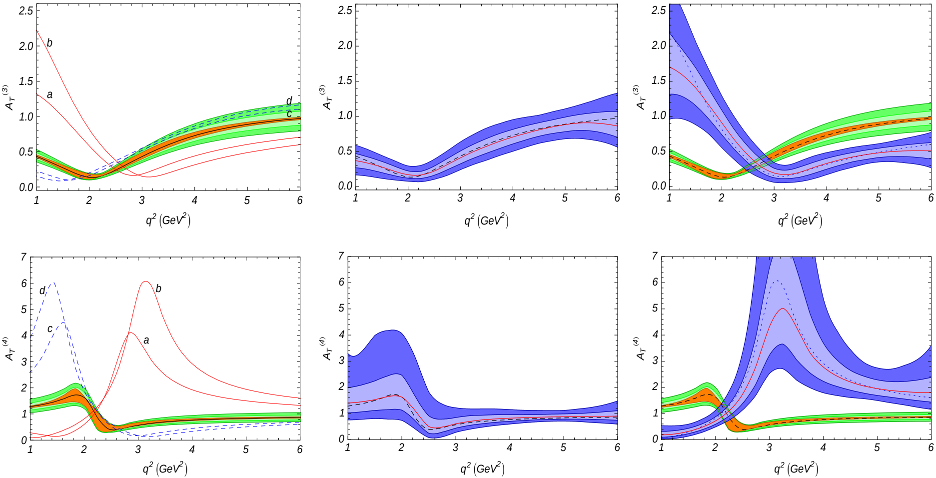
- A_T^3 at LO its **minimum** determines a relation:

$$C_{10}^2 = -(C_9^{eff} + 2\frac{m_b}{M_B}(C_7^{eff} - C_7^{eff\prime}))(C_9^{eff} + 2\frac{m_b M_B}{q^2}(C_7^{eff} - C_7^{eff\prime}))$$

- $\mathbf{A_T^4}$ at LO its **minimum** determines a relation:

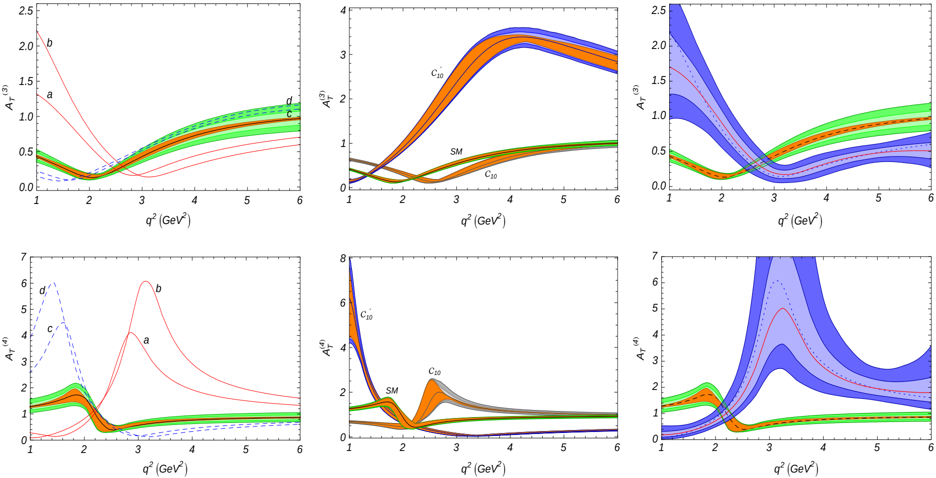
$$C_9^{eff} = -\frac{m_b}{M_B}(C_7^{eff} - C_7^{eff\prime}) - \frac{m_b M_B}{q^2}(C_7^{eff} + C_7^{eff\prime})$$

- $\mathbf{A_T^4}$ its **maximum** is related to the **minimum** of A_T^3



- Large-gluino scenario (a,b) with $\delta > 0$ clear at low-s region for A_T^3 and large-s for A_T^4
- Low-gluino scenario (c,d) with $\delta < 0$ clear at low-s region for A_T^4

Egede, Reece, Hurth, J.M, Ramon '08 update+Experimental sensitivity with 10 fb^{-1}



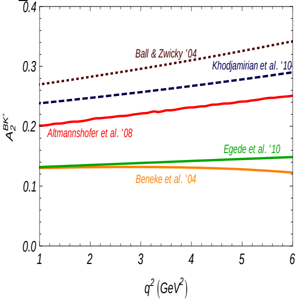
- A_T^3 large sensitivity to $\mathcal{O}_{10}'$ like in A_T^5
- A_T^4 stronger sensitivity to $\mathcal{O}_{10}$

Egede, Reece, Hurth, J.M, Ramon '10 update

How solid are these predictions under FF changes?

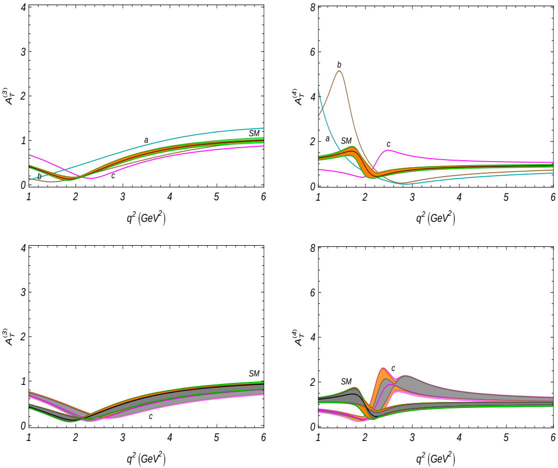
$A_T^{3,4}$ are function of form factors: A_1, A_2, T_2 and T_3

A_2:



On the contrary an example of unprotected observable is the $BR(B \to K^* l^+ l^-)$:
$BR(1-6\,\mathrm{GeV}^2) = 2.99x10^{-7}$ (Egede'10 with Ball '98)
$BR(1-6\,\mathrm{GeV}^2) = 1.91x10^{-7}$ (Khodjamirian '10)

A_T^3 Egede'10 with Ball '98 | A_T^4 Egede'10 with Ball '98



A_T^3 with Khodjamirian '10 | A_T^4 with Khodjamirian '10

Conclusions

- We have completed the method to construct QCD-protected observables A_T^i based on the exclusive 4-body B-meson decay $\bar{B}_d \to \bar{K}^{*0} l^+ l^-$ in the low dilepton mass region.
- We have identify the symmetries of the angular distribution and explore its consequences:
  - The possibility to fully solve the system
  - The construction of non-trivial observables
  - The emergence of non-trivial relations between the coefficients of the distribution.

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  - Their combined analysis together with A_{FB} may help in disentangling NP contributions to each Wilson coefficient.
  - A_T^2 emerges as an improved version of A_{FB}.

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BACK-UP slides

Ensemble method to compute Λ/m_b

- Parametrize each K^* spin amplitude:

$$A_i' = A_i(1 + C_i e^{i\theta_i}),$$

where C_i is the relative amplitude and θ_i the relative strong phase.

- The effective Hamiltonian which controls the decay has three terms,

$$\mathcal{H}_{\rm eff} = \mathcal{H}_{\rm eff}^{(u)\rm SM} + \mathcal{H}_{\rm eff}^{(t)\rm SM} + \mathcal{H}_{\rm eff}^{(t)\rm NP}.$$

SM CP violating / SM decay / NP contributions. Consequently:

$$A' = \Big[(A_{\rm SM}(\lambda_u \neq 0) - A_{\rm SM}(\lambda_u = 0)) \times (1 + C_1 e^{i\theta_1})\Big] + \Big[A_{\rm SM}(\lambda_u = 0) \times (1 + C_2 e^{i\theta_2})\Big] + \Big[(A_{\rm Full}(\lambda_u \neq 0) - A_{\rm SM}(\lambda_u \neq 0)) \times (1 + C_3 e^{i\theta_3})\Big].$$

An estimate of the theoretical uncertainty arising from the unknown Λ/m_b corrections is made using a randomly selected ensemble.

- For each member of the ensemble take values in the ranges $C_i \in [-0.1, 0.1]$ and $\theta_i \in [-\pi, \pi]$ using a random uniform distribution. Conditions
  - Assume each theory parameter C_i is uncorrelated.
  - True value of these parameters can be anywhere in a $\pm 10\%$ band with equal probability.
- To estimate the contribution to the theoretical uncertainties from Λ/m_b corrections for a particular observable, each element in the ensemble was used to calculate the value of that observable at a fixed value of q^2.
- Question: What the typical (66% of the time true value of the observable falls in the band) uncertainty coming from Λ_{QCD}/m_b is?
- This answer the question "how often a theory correction on the SM looks like NP" once a measurement is done.