Lattice results for exclusive D semileptonic decays

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• Introduction: D semi-leptonic decay

- D to K l v $D^0 \to K^- \, l \, \upsilon, \qquad D^+ \to \overline{K}^0 \, l \, \upsilon$
- D to π l v $D^0 \to \pi^- \, l \, \upsilon, \qquad D^+ \to \pi^0 \, l \, \upsilon$

$$\frac{d}{dq^2}\Gamma \propto |V_{cx}|^2 \, |f_+(q^2)|^2$$

- Testing Lattice formalism
  - $f_+(q^2=0)$ as well as the shape of $f_+(q^2)$
- Direct determination of CKM matrix elements
  - $|V_{cs}|$ and $|V_{cd}|$
- Unitarity tests
  - second row and column unitarity tests
- Testing the Standard Model!

- From experiment: $f_+^K(0)\,|V_{cs}|$ $f_+^\pi(0)\,|V_{cd}|$

	D to K	D to π
CLEO-c (2009)	1.2 %	3 %
BaBar (2007+update)	1.4 %	
Belle (2006)	3.3 %	5.8 %

- From Lattice: $f_+^K(0)$ $f_+^\pi(0)$

	D to K	D to π
HPQCD (2010)	2.5 %	
ETMC (2010, Preliminary)	5.2 % (stat.)	9 % (stat.)
Fermilab/MILC (2005)	10 %	10 %

- ongoing project: Fermilab/MILC and QCDSF

- Lattice calculations without using quenched approximations

	Configuration type	Light quark action	Heavy quark action	Lattice spacing (~fm)
HPQCD (2010, D to K)	MILC asqtad N_f=2+1	HISQ	HISQ	0.12, 0.09
ETMC (preliminary)	ETMC Twisted Mass N_f=2	Twisted Mass	Twisted Mass	0.102, 0.086, 0.068
Fermilab/MILC (preliminary)	MILC asqtad N_f=2+1	asqtad	Fermilab	0.12, 0.09
Fermilab/MILC (2005)	MILC asqtad N_f=2+1	asqtad	Fermilab	0.12

Outline

- Charm quarks on the lattice
- Fermilab/MILC (2005)
  - The first unquenched lattice calculation
- HPQCD (2010, D to K)
  - Most recent report with a factor of 4 smaller errors
  - New methods: form factor from a scalar current
    simultaneous modified z-expansion fit
- ETMC (2010, preliminary)
- Fermilab/MILC (2010, preliminary)
- Result overview
- Summary

• Charm quarks on the lattice

- It is difficult to satisfy

$$am_c = m_c / cutoff << 1$$

-> NRQCD, HQET, or Heavy clover with Fermilab interpretation

-> Operator matching may be required.

• Charm quarks on the lattice

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- Newly developed highly improved quark actions
  - **HISQ** (Highly Improved Staggered Quark) action
    - Leading error starts at $O(\alpha_s (am_c)^2 v^2/c^2)$ and $O((am_c)^4 v^2/c^2)$

      (If am_c=0.6, it is about 2%)
  - **Twisted Mass** or clover wilson action
    - set am_c<1/2 with $O((am_c)^2)$ errors

- Fermilab/MILC (2005)
  - First unquenched lattice calculation
  - Using MILC asqtad 2+1 configurations
    - asqtad light quarks and Fermilab charm quark
  - a~0.12 fm "coarse" ensembles
    - am_l down to $am_s/8$ -> easier chiral extrapolation
    - 400~500 confs
    - discretization effect considered by power counting
  - Becirevic and Kaidalov (BK) parametrization
  - Staggered ChPT

$f_+^K(0) = 0.73(3)(7)$

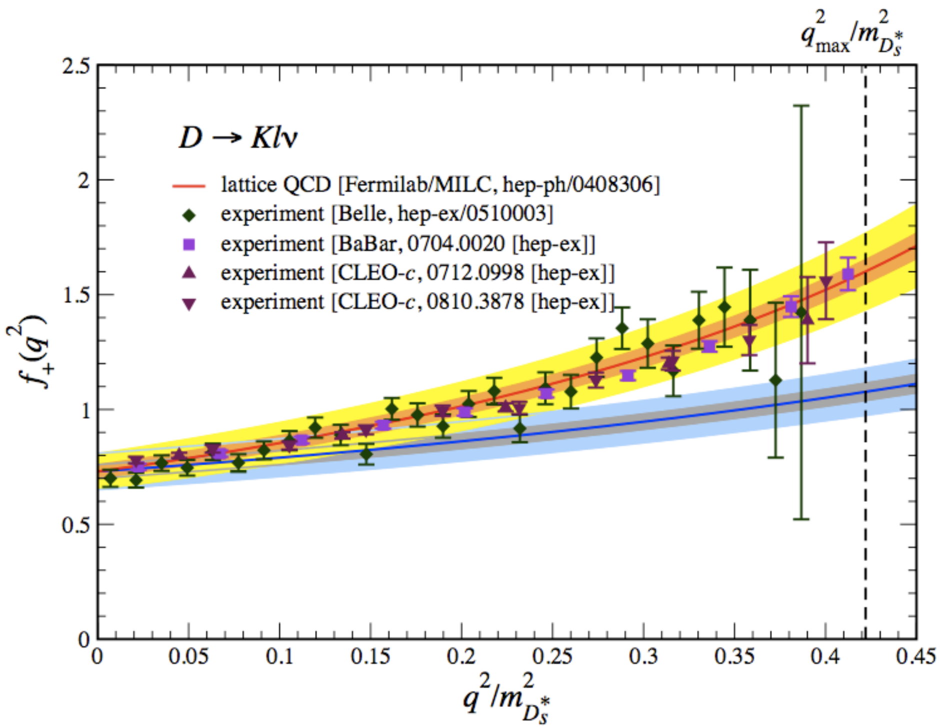
$f_+^\pi(0) = 0.64(3)(6)$

fit	3 %	scale	1 %
Chiral fit	3 (2) %	Light q	2 %
BK	2 %	Ep	5 %
ρVμ	1 %	Charm q	7 %

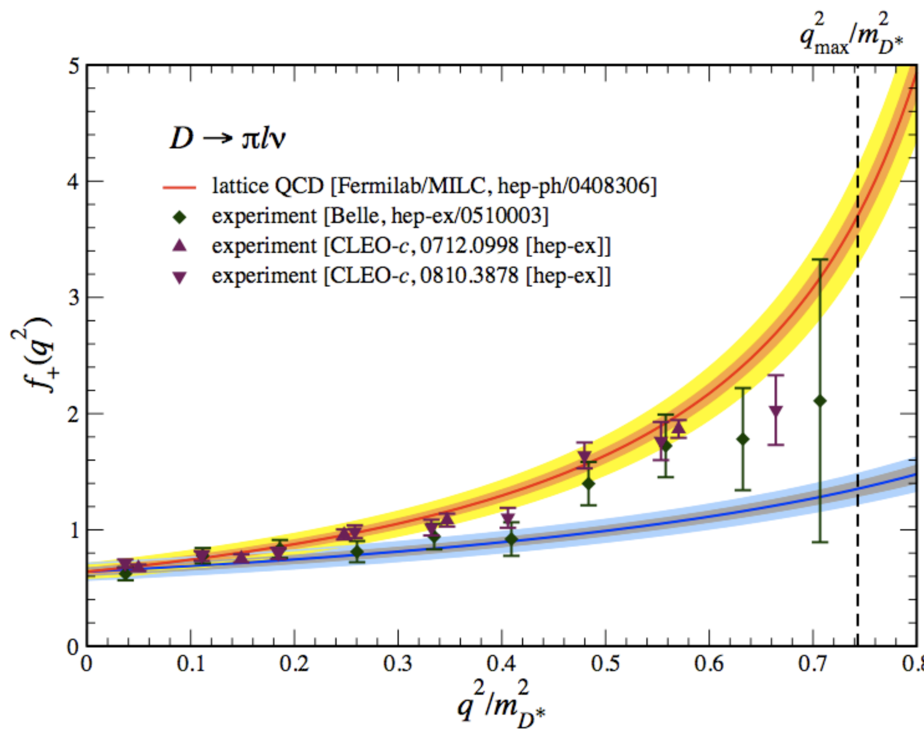
Total 10 % error

• Fermilab/MILC (2005)

D to K



D to π



The lattice calculation successfully predicted the shape!

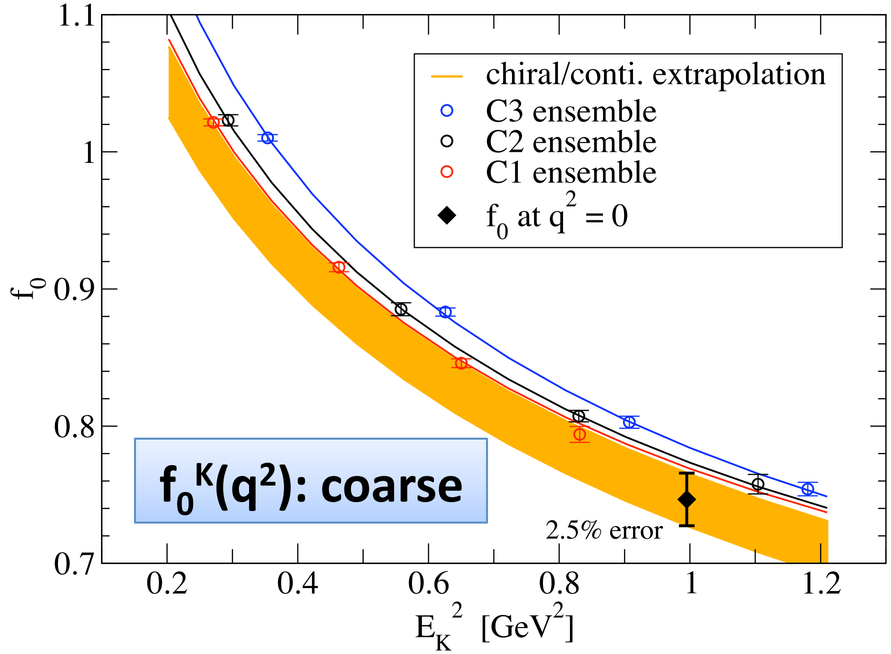
• HPQCD (2010) **D to K** l ν

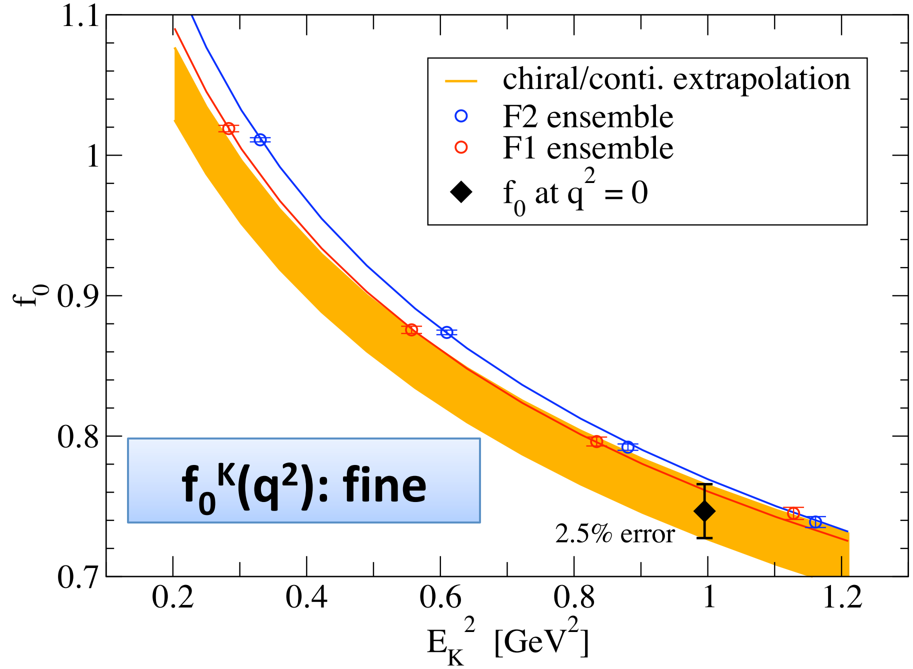
* HN et al. arXiv:1008.4562

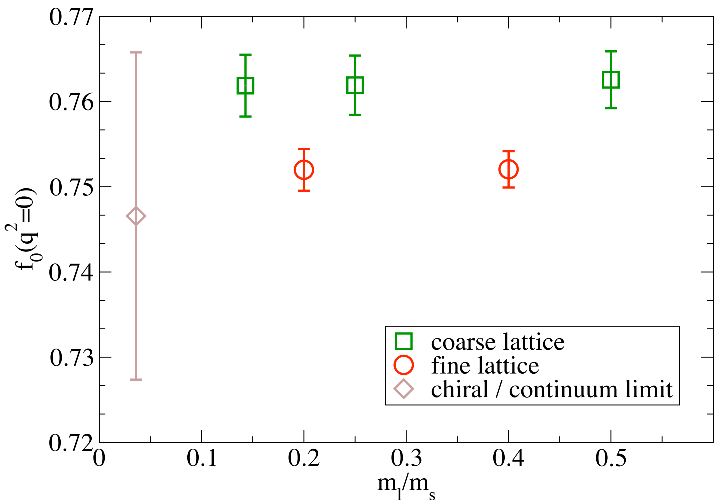
- Using MILC asqtad 2+1 configurations
    - HISQ valence light and charm quarks
- a~0.12 fm “coarse” and a~0.09 fm “fine” ensembles
    - 600 confs for each ensemble
    - random-wall source
- Total 2.5 % error on $f_+^K(0)$
    - Using a scalar current -> no need for operator matching
    - Simultaneous modified z-expansion fit for chiral / continuum extrapolations
    - sophisticated simultaneous correlator fits
- Only for $f_0(q^2)$, but $f_+(0) = f_0(0)$
- D to π l ν is under way

• HPQCD (2010) **D to K** l ν

- Chiral / continuum extrapolations using modified z-expansion







$f_0^K(q^2=0)$ at the physical limit and each ensemble

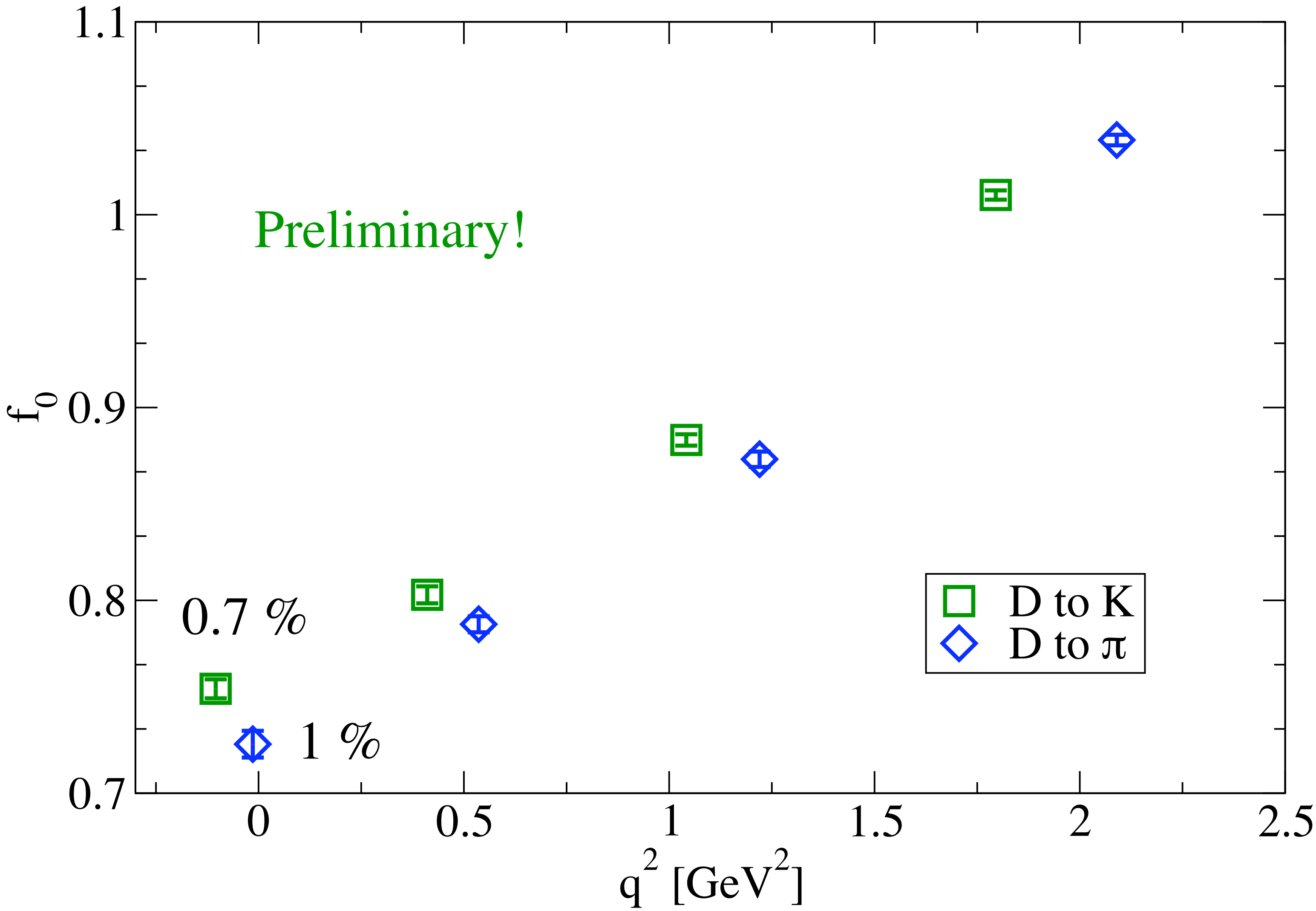
• HPQCD (2010) **Error budget and results on $f_+^K(0)$.**

statistical	1.5 %
Lattice scale	0.2 %
Input meson mass	0.1 %
Light quark dependence	0.6 %
Strange quark dependence	0.7 %
Sea quark dependence	0.4 %
am_c extrapolation	1.4 %
aE_K extrapolation	1.0 %
Finite volume	0.01 %
Charm quark tuning	0.05 %
Total	2.5 %

$f_+^K(0) = 0.747\,(11)(15)$

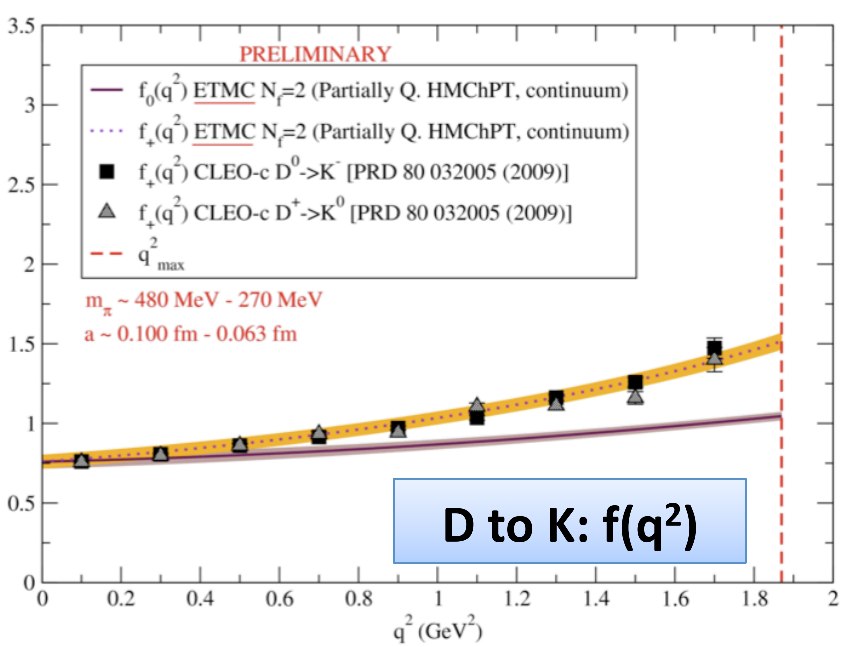
$|V_{cs}| = 0.961\,(11)(24)$

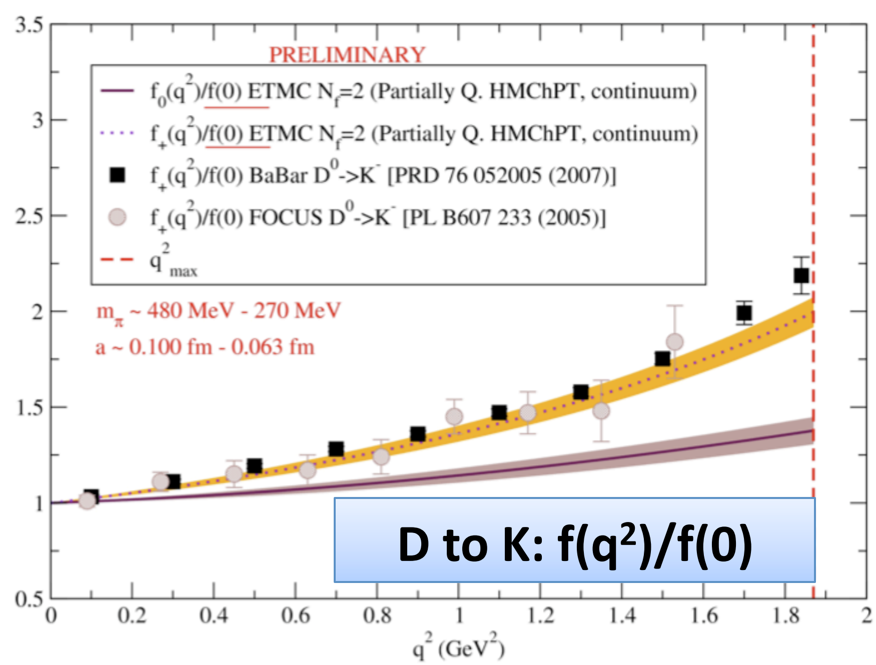
• HPQCD   **D to π l ν**



• ETMC (preliminary)

- Using ETMC $N_f = 2$ configurations
- a~0.102, 0.086, and 0.068 fm
- Twisted boundary conditions
- Using ratios and double ratios
  - reduce errors and cancel the renormalization factors
- HMChPT extrapolations
  - including $O(a^2)$ terms





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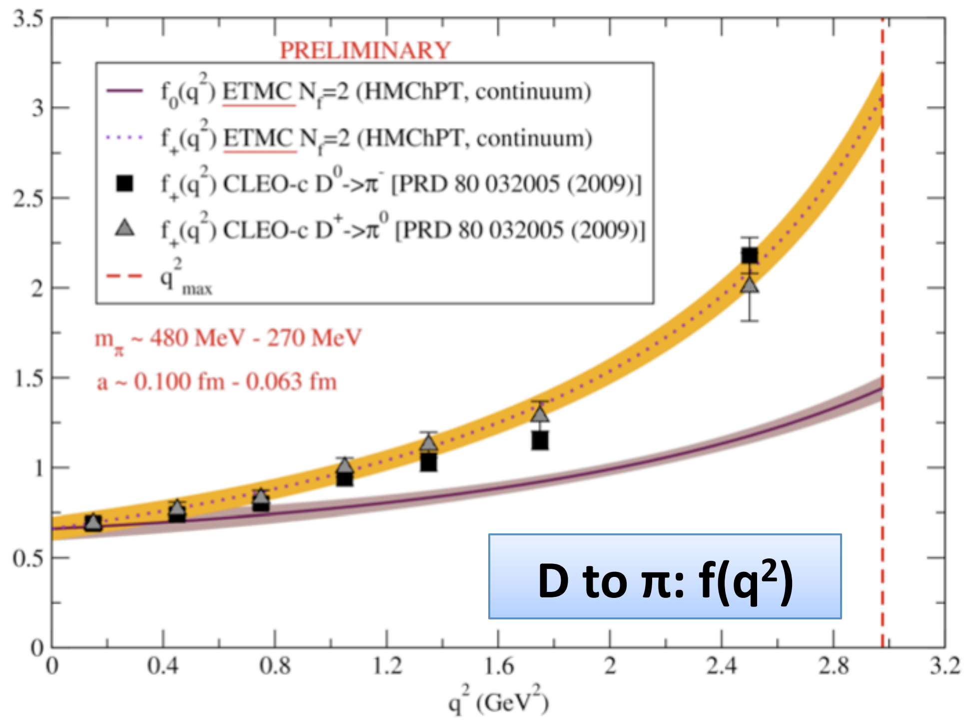
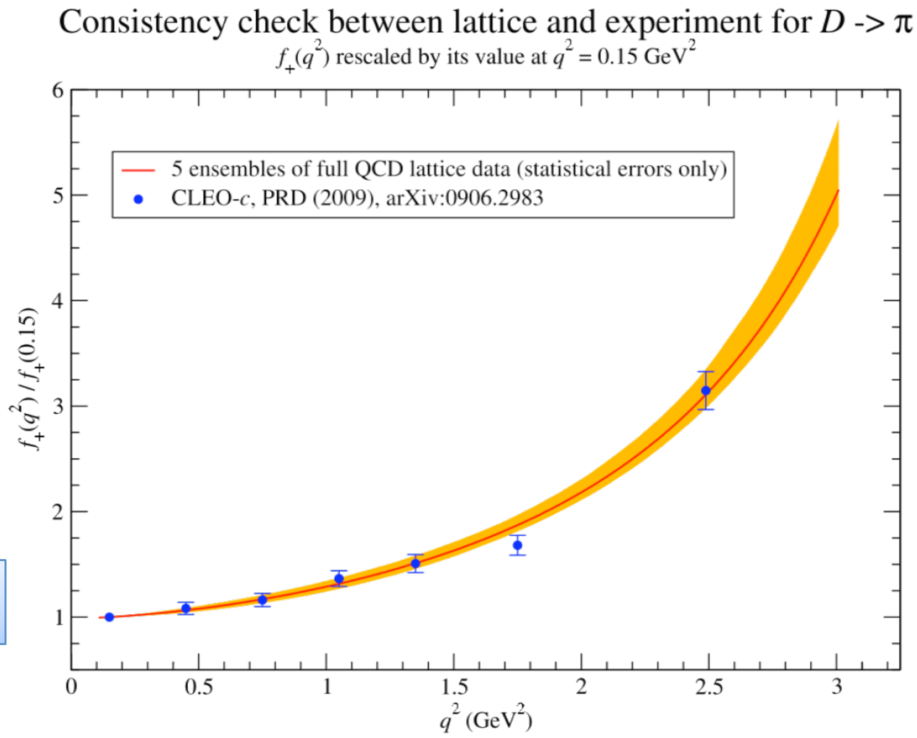
$f_+^K(0) = 0.76\ (4)_{stat.}$
$f_+^\pi(0) = 0.66\ (6)_{stat.}$



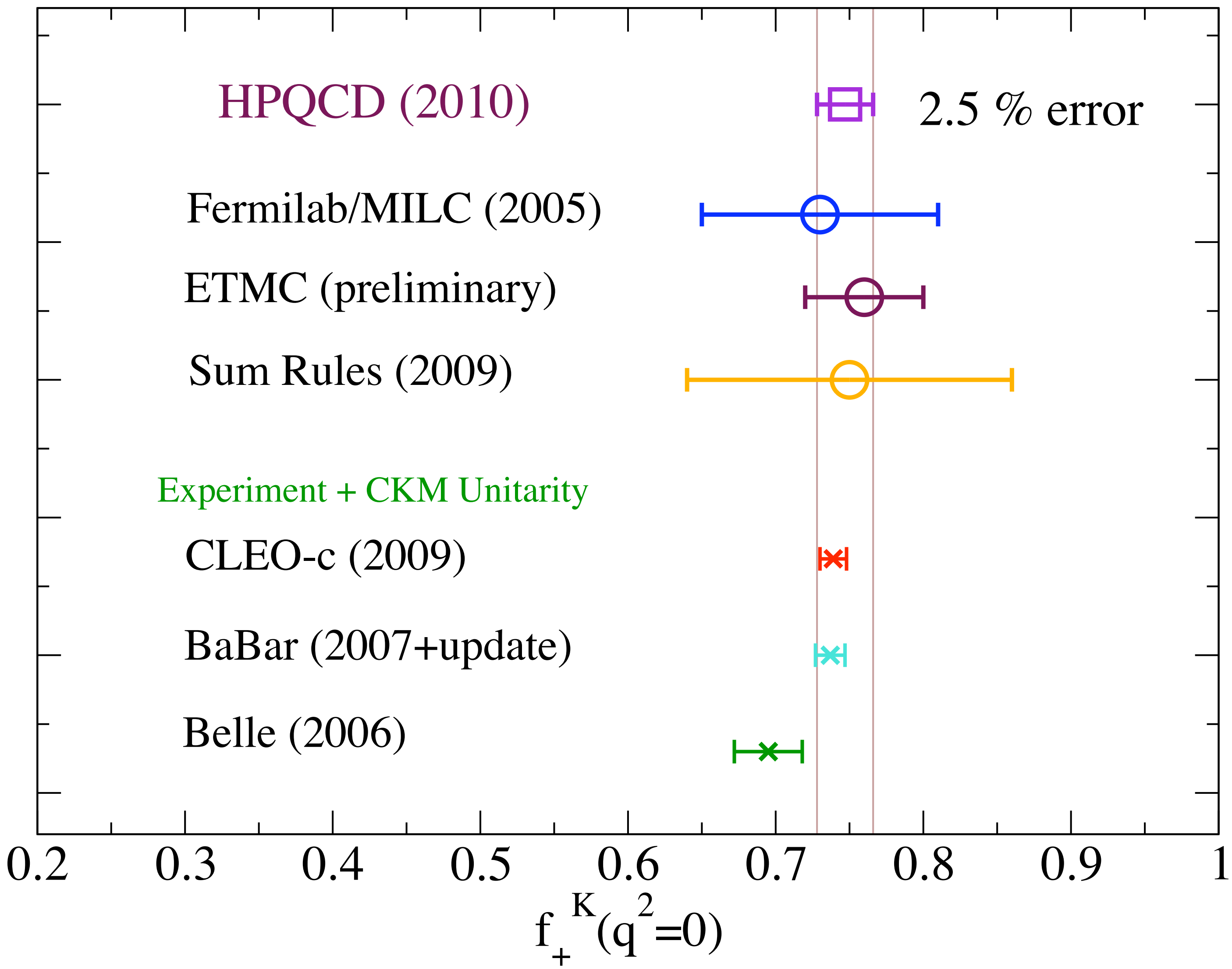
• Fermilab/MILC (preliminary)

- Using MILC asqtad 2+1 configurations
  - asqtad light quark and Fermilab charm quark
- a~0.12, and 0.09 fm
  - ~2000 confs. high statistics.
- PQSHMChPT extrapolations
  - include NLO and analytic NNLO terms
  - Use only up to p = (1,1,0)

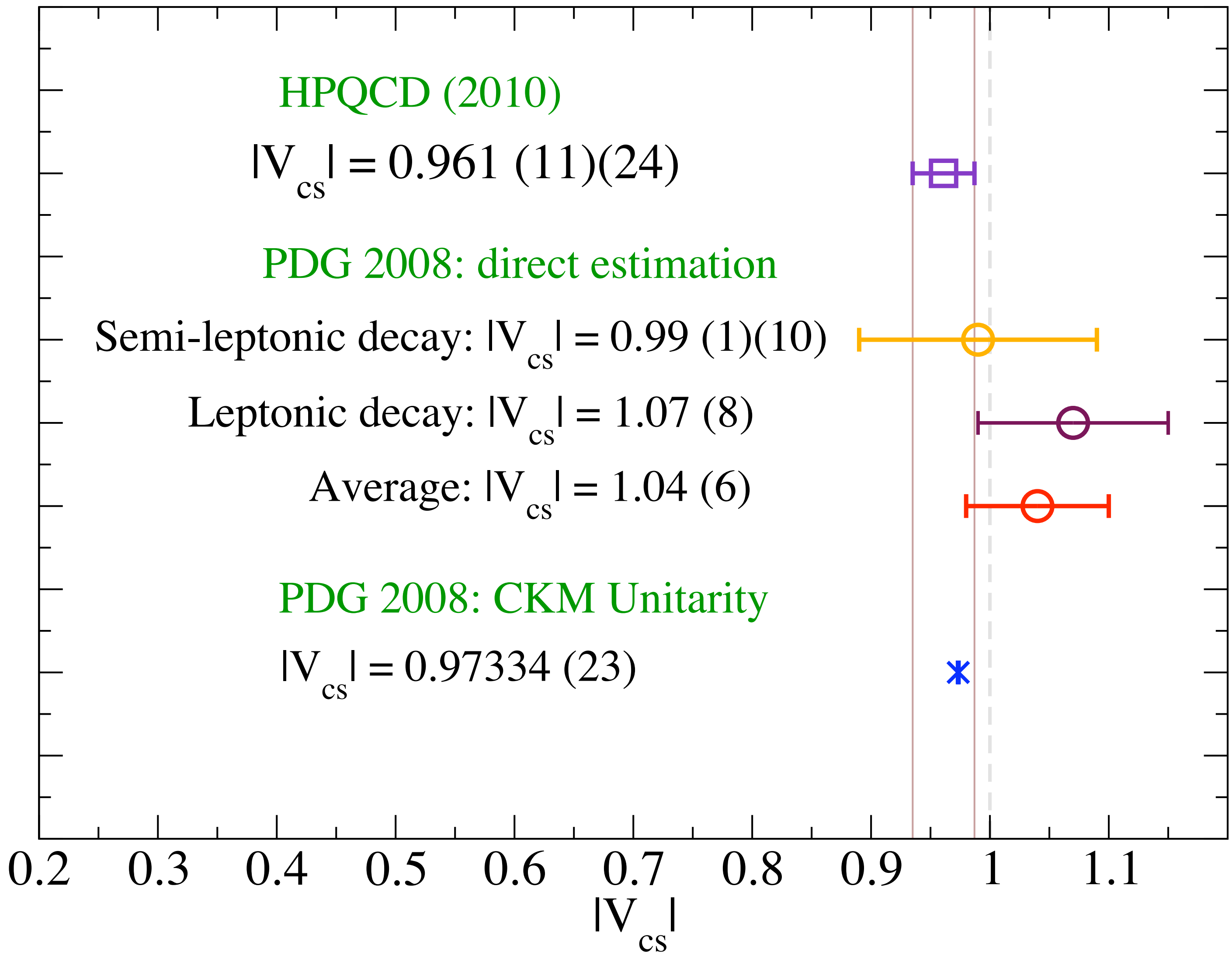


D to π: $f_+(q^2)/f_+(0.15\ \mathrm{GeV}^2)$

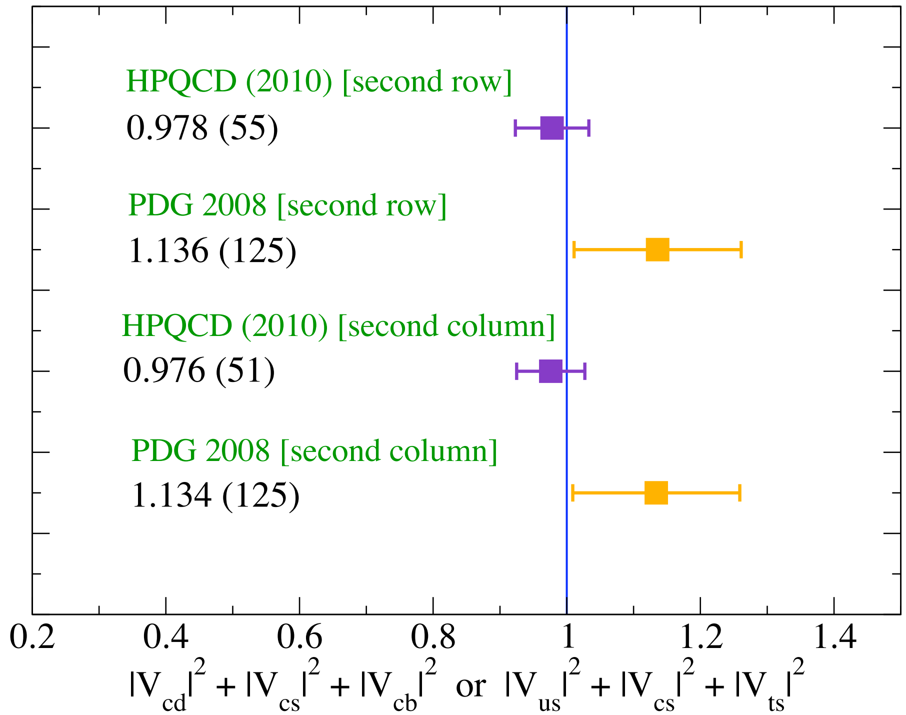
• Result overview **D to K** l ν $f_+(0)$

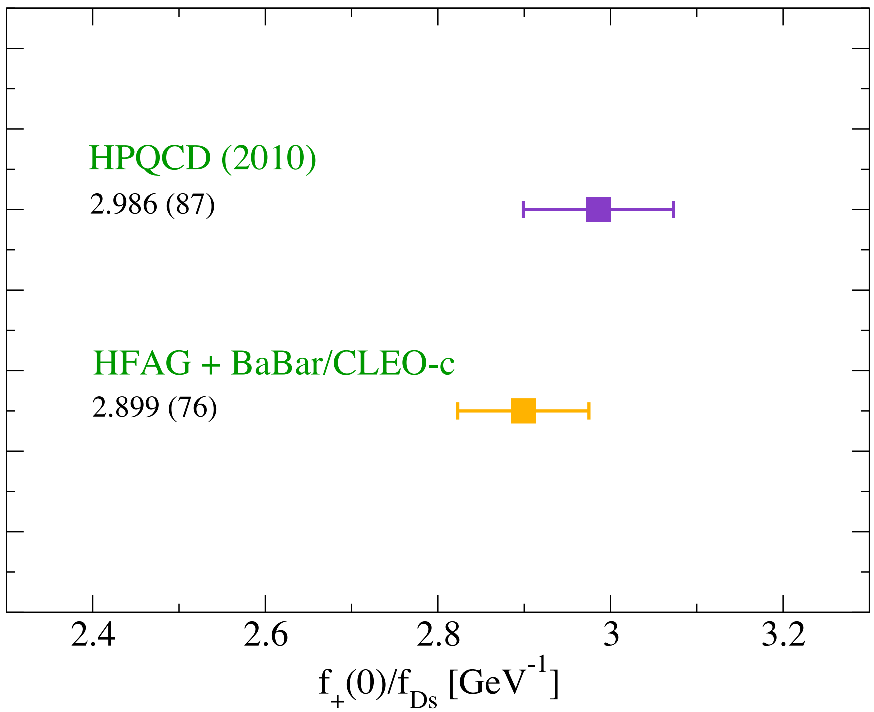


• Result overview **D to K l ν** **Direct determination of $|V_{cs}|$**

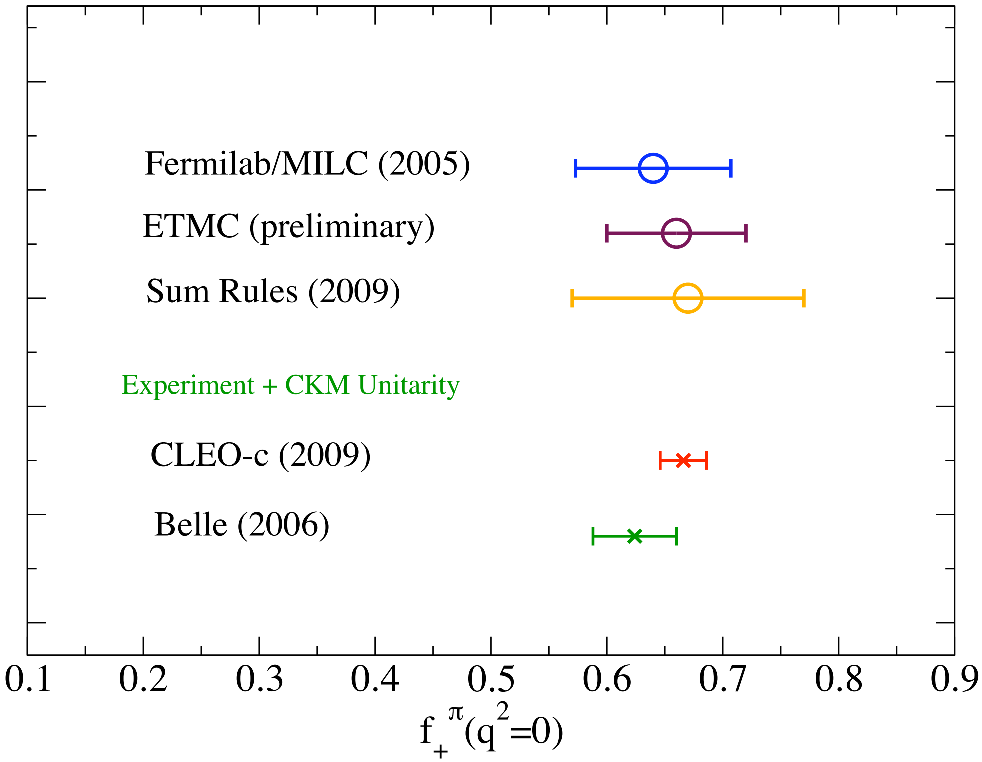


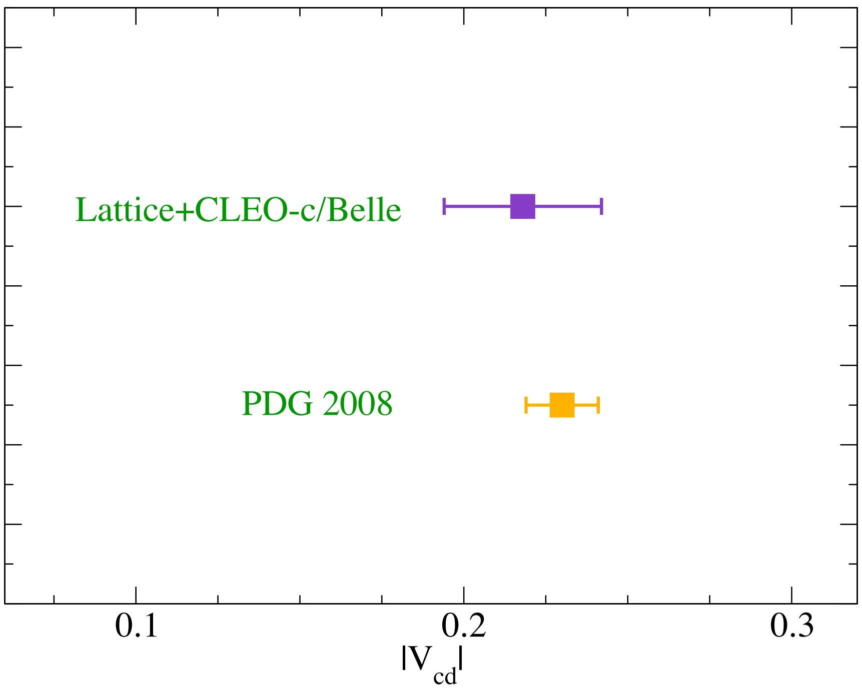
• Result overview **D to K l ν** **Unitarity check and $f_+^K(0)/f_{Ds}$**





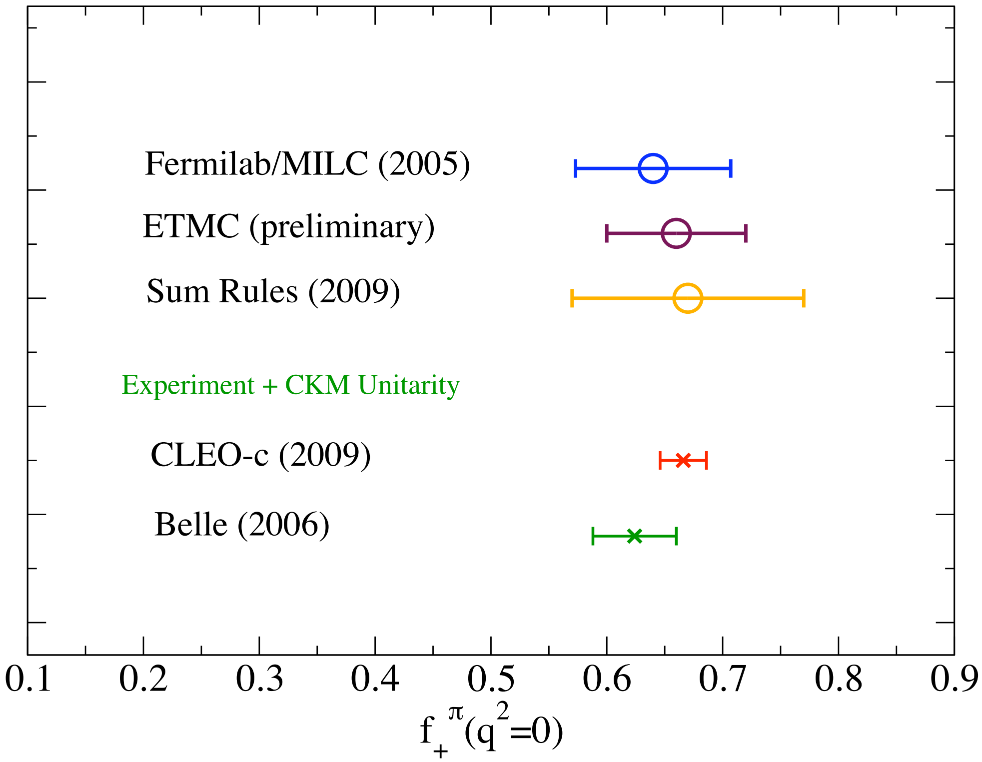
• Result overview D to π l ν $f_+^\pi(0)$ and $|V_{cd}|$

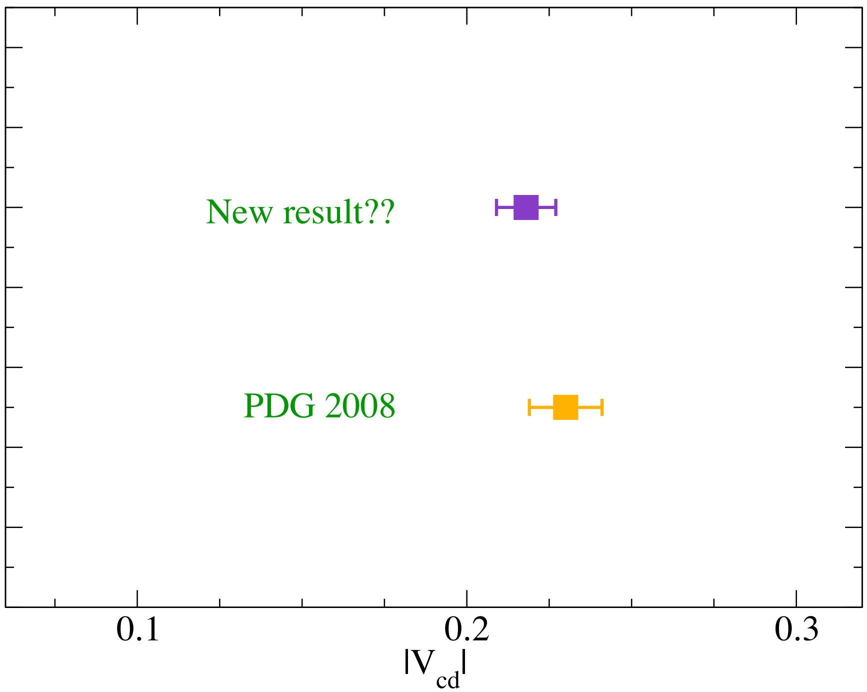




• **waiting for more results!**

• Result overview **D to π l ν** $f_+^\pi(0)$ and $|V_{cd}|$





• **waiting for more results!**

• Summary

- Studies for the exclusive D semileptonic decays have been very successful!
  - D to K: experiment 1% and lattice 3 % errors
  - D to π: experiment 3 % and lattice 10 % errors
- Many lattice projects on D semileptonic decays are ongoing
  - ETMC, Fermilab/MILC, HPQCD, and QCDSF
- Recently, HPQCD reports a new lattice calculation on D to K semileptonic decays with 3% errors.
  - The most precise direct determination of $|V_{cs}|$ to date
  - Applying new methods
    - scalar matrix element (no need for operator matching)
    - modified z-expansion fit
  - Need for D to π analysis

Backup slides

• HPQCD (2010) **New Method 1.**

- Using HISQ action for both charm and light quarks
- Form factors

$$< V^{\mu} > = [(p_D^{\mu} + p_K^{\mu}) - \frac{m_D^2 - m_K^2}{q^2} q^{\mu}] f_+(q^2) + \frac{m_D^2 - m_K^2}{q^2} q^{\mu} f_0(q^2)$$

- PCVC and "fully nonperturbative matching"

$$q^{\mu} < V_{\mu}^{conti} > = (m_c - m_q) < S^{conti} >$$

$$(m_D - E_{\pi}) < V_0^{latt.} > Z_t + \vec{p}_{\pi} \cdot < \vec{V}^{latt.} > Z_S = (m_c - m_q) < S^{latt.} >$$

- Furthermore, $f_0(q^2)$ and $f_+(0)$ can be determined just from <S> with **no need for operator matching**!

$$f_0(q^2) = \frac{m_c - m_q}{m_D^2 - m_K^2} < S >, \qquad f_+(q^2 = 0) \equiv f_0(q^2 = 0)$$

• HPQCD (2010) **New Method 2.**

- z-expansion: model independent parameterization

$$f_0(q^2) = \frac{1}{P(q^2)\Phi_0(q^2,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(q^2,t_0)^k$$

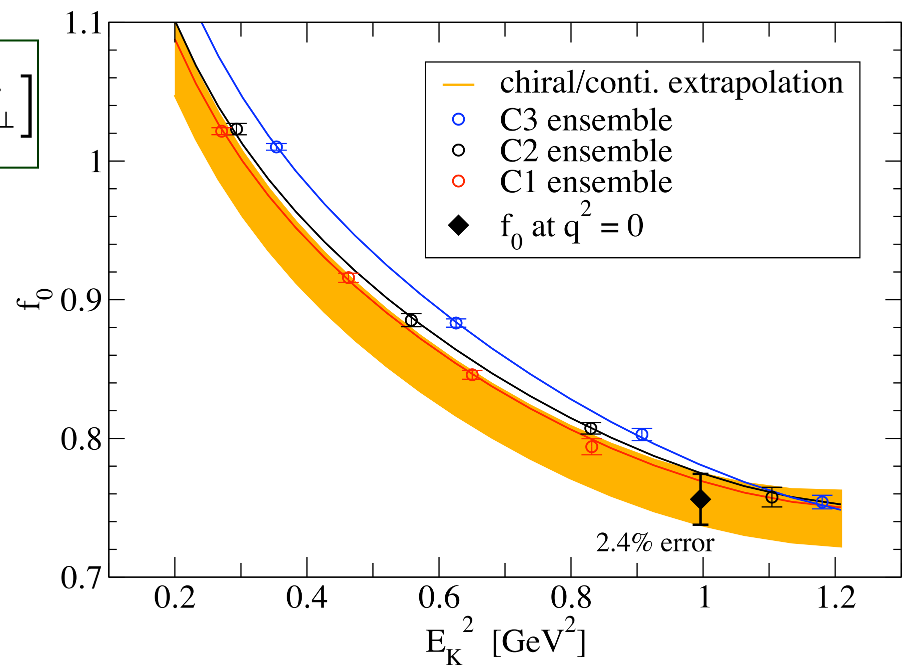
All kinematics are absorbed by P, Φ, and z

Applicable to individual ensembles

- HMPQChPT

$$f_0(q^2) = \frac{\sqrt{2M_D}}{M_D^2 - M_K^2}\left[(M_D - M_K)f_\parallel + (E_K^2 - M_K^2)f_\perp\right]$$

Questionable at large E_K^2



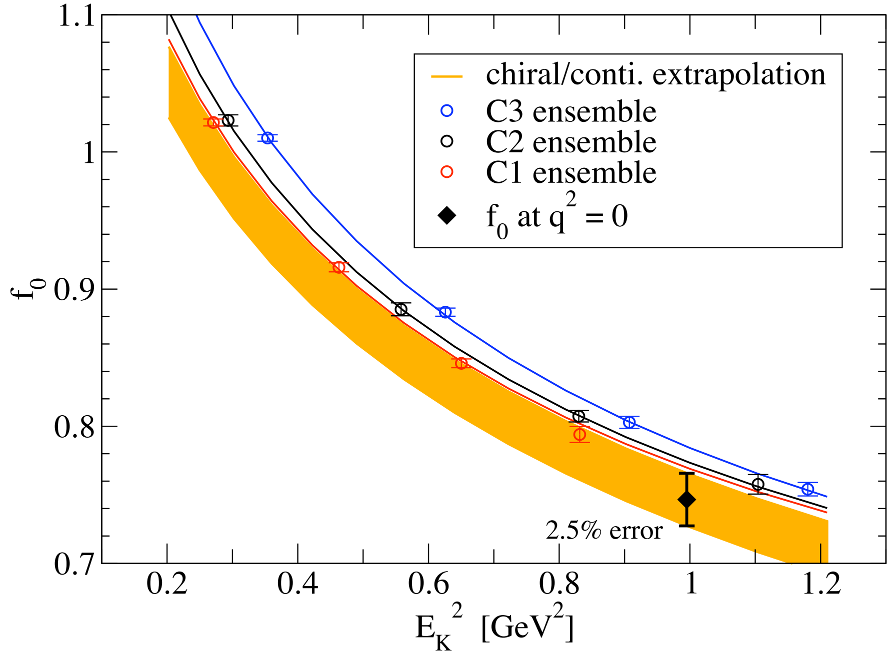
• HPQCD (2010) **New Method 2.**

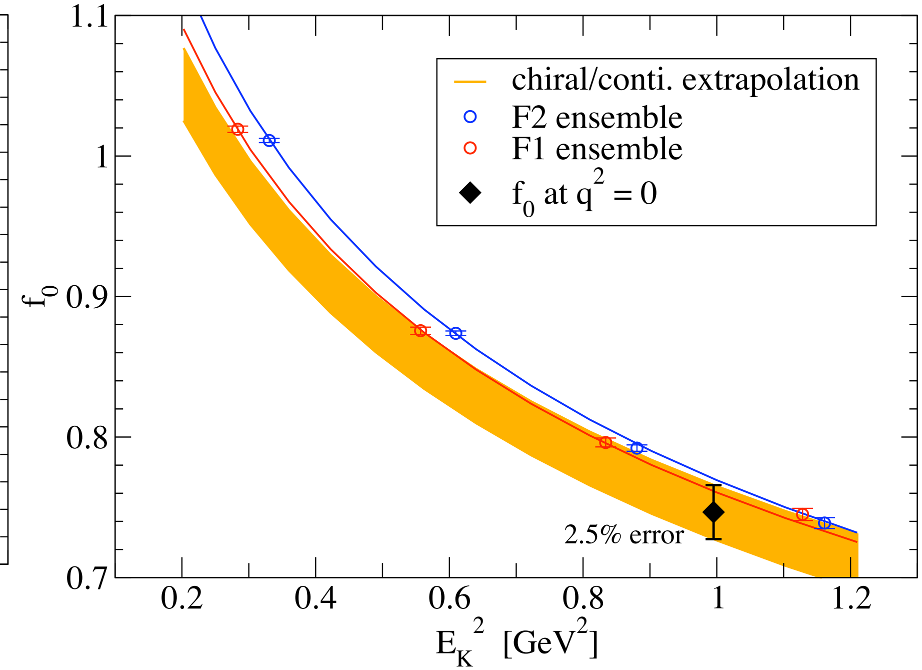
- Simultaneous modified z-expansion fit

$$f_0(q^2) = \frac{1}{P\,\Phi_0}\left(a_0 D_0 + a_1 D_1\, z + a_2 D_2\, z^2\right)\left(1 + b_1 (aE_K)^2 + b_2 (aE_K)^4\right)$$

$$D_i = 1 + c_1^i x_l + c_2^i \delta x_s + c_3^i x_l \log(x_l) + f_i (1/2\, \delta M_\pi^2 + \delta M_K^2) + d_i (am_c)^2 + e_i (am_c)^4$$

$$x_l = \frac{M_\pi^2}{(4\pi f_\pi)^2}, \qquad \delta x_s = \frac{M_{\eta_s}^2 - M_{\eta_s^{phys}}^2}{(4\pi f_\pi)^2}$$





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