A Fresh Look at $B_{s,d} \to \pi\pi, \pi K, KK$ Decays

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- Two main targets: $\to$ U-spin related B decays
  - $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$
  - $B_d \to \pi^\mp K^\pm$, $B_s \to \pi^\pm K^\mp$
- Picture emerging from data: $\to$ γ determinations, predictions, ...

Update of R.F., *Eur. Phys. J.* **C 52** (2007) 267 $\oplus$ work with Rob Knegjens

NIKHEF

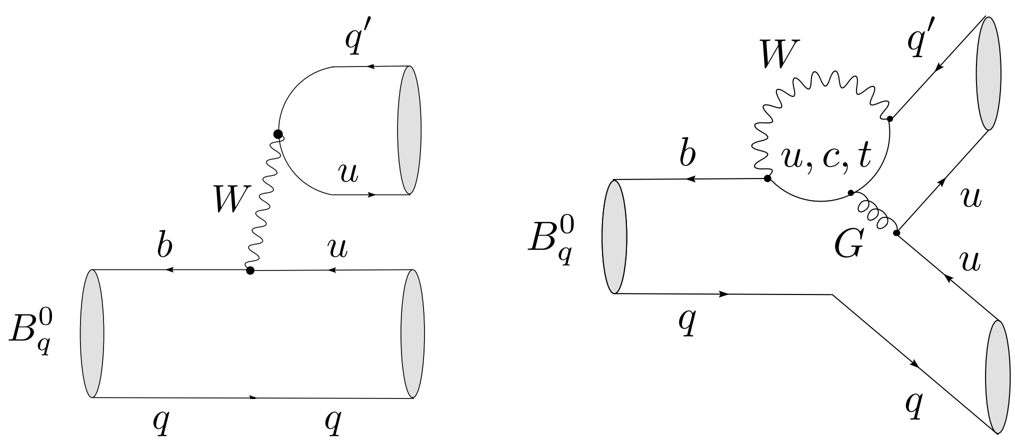
Preliminaries

- Key problem in phenomenological analysis of non-leptonic B decays:

Hadronic matrix elements!? $\rightarrow$ get them from data...

- Particularly interesting: [R.F., *Phys. Lett.* **B 459** (1999) 306]

U-spin-related decays: $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$



$\Rightarrow$ extraction of $\gamma \oplus$ hadronic parameters

- The advantage of this U-spin strategy with respect to the conventional $SU(3)$ flavour-symmetry strategies is twofold:
  - no additional dynamical assumptions have to be made, which could be spoiled by large rescattering effects;
  - EW penguins, which are not invariant under the isospin symmetry because of the different up- and down-quark charges, can be included.

- Observables:
  - CP-averaged branching ratios;
  - Direct and mixing-induced CP asymmetries:[1]

$$\mathcal{A}_{\rm CP}(t) \equiv \frac{\Gamma(B^0_q(t)\to f)-\Gamma(\bar{B}^0_q(t)\to f)}{\Gamma(B^0_q(t)\to f)+\Gamma(\bar{B}^0_q(t)\to f)}$$

$$= \left[\frac{\mathcal{A}_{\rm CP}^{\rm dir}(B_q\to f)\,\cos(\Delta M_q t)+\mathcal{A}_{\rm CP}^{\rm mix}(B_q\to f)\,\sin(\Delta M_q t)}{\cosh(\Delta\Gamma_q t/2)-\mathcal{A}_{\Delta\Gamma}(B_q\to f)\,\sinh(\Delta\Gamma_q t/2)}\right]$$

- Another U-spin-related pair: [Gronau & Rosner, PLB **482** (2000) 7]

$B_d\to\pi^\mp K^\pm$, $B_s\to\pi^\pm K^\mp$, but further input required: $B^\pm\to\pi^\pm K$.

[1] Similar sign convention also for direct CP asymmetries of flavour-specific decays.

Experimental Picture Autumn 2010 (HFAG)

- Results for $B \to \pi\pi, \pi K$ decays:

$$\begin{aligned} \text{BR}(B_d \to \pi^+\pi^-) &= (5.16 \pm 0.22) \times 10^{-6} \\ \text{BR}(B_d \to \pi^\mp K^\pm) &= (19.4 \pm 0.6) \times 10^{-6} \\ \text{BR}(B^\pm \to \pi^\pm K) &= (23.1 \pm 1.0) \times 10^{-6} \end{aligned}$$

$$\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-) = \begin{cases} 0.68 \pm 0.10 \pm 0.03 & \text{(BaBar)} \\ 0.61 \pm 0.10 \pm 0.04 & \text{(Belle)} \end{cases}$$

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to \pi^+\pi^-) = \begin{cases} -0.25 \pm 0.08 \pm 0.02 & \text{(BaBar)} \\ -0.55 \pm 0.08 \pm 0.05 & \text{(Belle)} \end{cases}$$

  - Nice agreement for $\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-) \to -0.65 \pm 0.07$.
  - $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to \pi^\mp K^\pm) = 0.098^{+0.011}_{-0.012}$ favours the BaBar measurement:

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to \pi^+\pi^-) \stackrel{SU(3)}{=} -\left(\frac{f_\pi}{f_K}\right)^2 \frac{\text{BR}(B_d \to \pi^\mp K^\pm)}{\text{BR}(B_d \to \pi^+\pi^-)} \mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to \pi^\mp K^\pm)$$

$$= -0.26 \pm 0.03 \quad \text{[see also R.F., Recksiegel \& Schwab ('07)]}$$

- Results for B_s decays [CDF & Belle@$\Upsilon(5S)$]:

$$\begin{aligned} \text{BR}(B_s \to \pi^\pm K^\mp) &= (5.0 \pm 0.7 \pm 0.8) \times 10^{-6} \\ \text{BR}(B_s \to K^+K^-) &= (26.5 \pm 4.4) \times 10^{-6} \end{aligned}$$

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \to \pi^\pm K^\mp) = -0.39 \pm 0.15 \pm 0.08 = -0.39 \pm 0.17$$

$$B_d \to \pi^+\pi^-,\ B_s \to K^+K^-$$

Some Technical Details

- Decay amplitudes: $[\epsilon = \lambda^2/(1-\lambda^2) = 0.053$, with Wolfenstein Parameter $\lambda]$

$$A(B_d^0 \to \pi^+\pi^-) = e^{i\gamma}\left(1-\frac{\lambda^2}{2}\right)\mathcal{C}\left[1 - d\,e^{i\theta}e^{-i\gamma}\right]$$

$$A(B_s^0 \to K^+K^-) = e^{i\gamma}\lambda\,\mathcal{C}'\left[1 + \frac{1}{\epsilon}d' e^{i\theta'}e^{-i\gamma}\right]$$

- Implications of the U-spin symmetry:

(i) $d' = d$, $\theta' = \theta$:

$*$ $de^{i\theta}$ and $d'e^{i\theta'}$ are actually ratios of certain hadronic amplitudes;

$*$ U-spin-breaking form factors and decay constants *cancel*:

$\to$ *no* factorizable U-spin-breaking corrections.

(ii) $|\mathcal{C}'/\mathcal{C}| = 1$:

$*$ Here the decay constants and form factors do *not* cancel:

$$\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|_{\rm fact} = \frac{f_K}{f_\pi}\frac{F_{B_sK}(M_K^2;0^+)}{F_{B_d\pi}(M_\pi^2;0^+)}\left(\frac{M_{B_s}^2 - M_K^2}{M_{B_d}^2 - M_\pi^2}\right) \to \left|\frac{\mathcal{C}'}{\mathcal{C}}\right|_{\rm fact}^{\rm QCDSR} = 1.41^{+0.20}_{-0.11}$$

[Updated QCD light-cone sum rule calculation: Duplancic & Melic (2008)]

Observables

- CP-violating $B_d \to \pi^+\pi^-$ asymmetries:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-) = -\left[\frac{2\,d\sin\theta\sin\gamma}{1-2\,d\cos\theta\cos\gamma+d^2}\right]$$

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-) = +\left[\frac{\sin(\phi_d+2\gamma)-2\,d\,\cos\theta\,\sin(\phi_d+\gamma)+d^2\sin\phi_d}{1-2\,d\cos\theta\cos\gamma+d^2}\right]$$

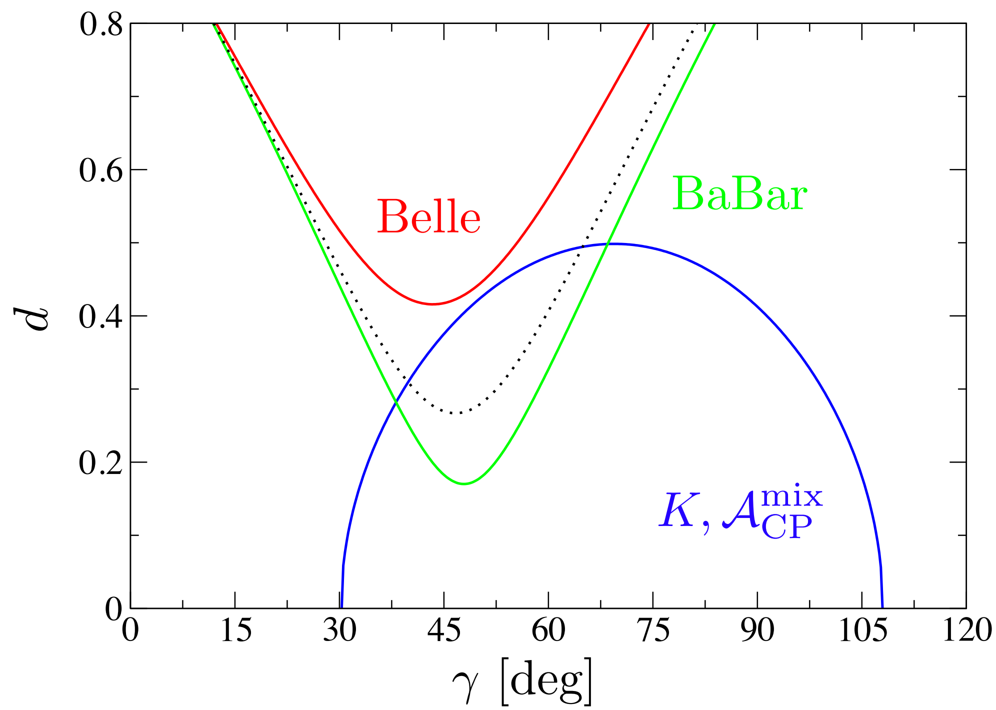
[$\phi_d = (42.2 \pm 1.8)^\circ$ is the B_d^0–$\bar{B}_d^0$ mixing phase; HFAG average of $B_d \to J/\psi K_{\rm S,L}$, etc.]

- CP-averaged branching ratios: $\to B_s \to K^+K^-$ measurement enters:

$$K = \frac{1}{\epsilon}\left|\frac{\mathcal{C}}{\mathcal{C}'}\right|^2\left[\frac{M_{B_s}}{M_{B_d}}\,\frac{\Phi(M_\pi/M_{B_d},\,M_\pi/M_{B_d})}{\Phi(M_K/M_{B_s},\,M_K/M_{B_s})}\,\frac{\tau_{B_d}}{\tau_{B_s}}\right]\left[\frac{\text{BR}(B_s\to K^+K^-)}{\text{BR}(B_d\to\pi^+\pi^-)}\right]$$

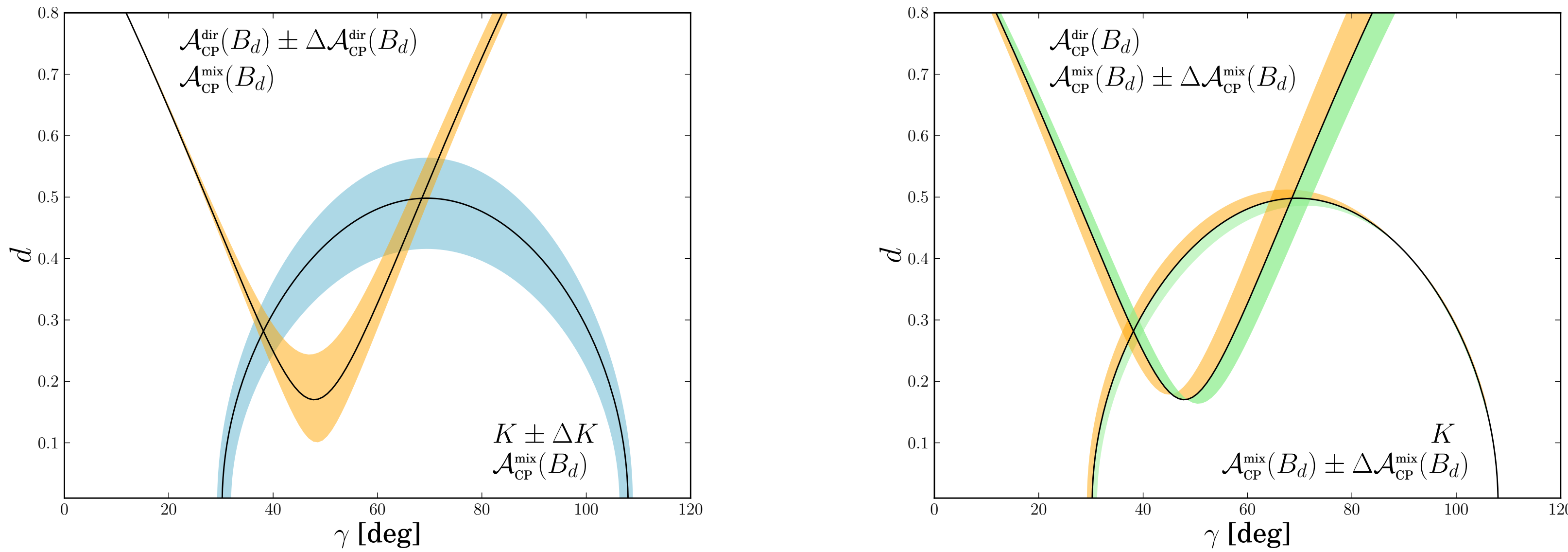
$$= \frac{1}{\epsilon^2}\left[\frac{\epsilon^2+2\epsilon d\cos\theta\cos\gamma+d^2}{1-2d\cos\theta\cos\gamma+d^2}\right] \overset{\rm exp}{=} 51.8^{+12.7}_{-14.9}$$

- Contours in the γ–d plane: $\rightarrow$ eliminate the strong phase θ ...
  - $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$: *theoretically clean*;
  - K and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$: U-spin symmetry enters:



$\Rightarrow$ BaBar measurement of $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ favoured;
will be used in the following numerical analysis...

Extraction of γ, d and θ



- We obtain the following numerical results:

$$\begin{aligned}
\gamma &= (38.1^{+2.0+0.9+2.0}_{-1.3-0.5-2.2})^\circ &&= (38.1^{+3.0}_{-2.6})^\circ \\
d &= 0.282^{+0.025+0.015+0.001}_{-0.035-0.009-0.001} &&= 0.282^{+0.029}_{-0.036} \qquad \text{(A)} \\
\theta &= (30.0^{+6.6+10.9+1.7}_{-3.5-10.0-1.3})^\circ &&= (30.0^{+12.9}_{-10.7})^\circ
\end{aligned}$$

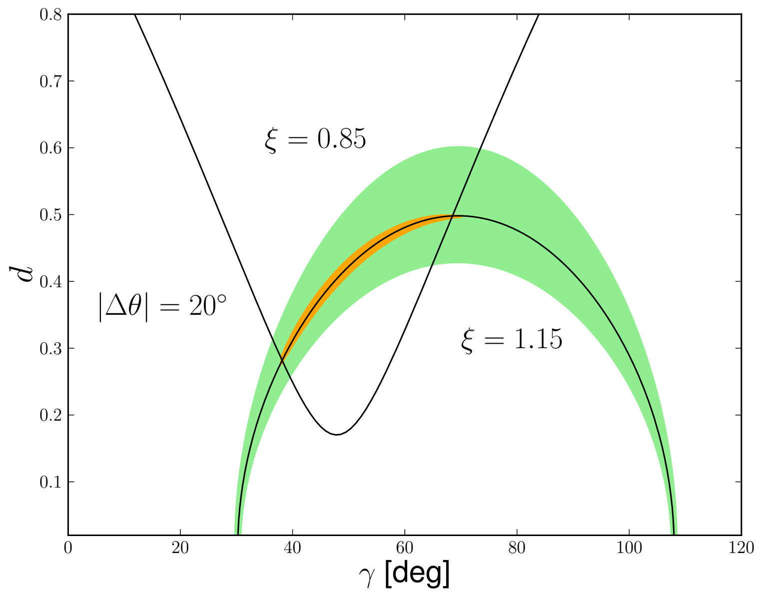
$$\begin{aligned}
\gamma &= (68.5^{+3.2+1.2+3.0}_{-4.2-1.9-3.5})^\circ &&= (68.5^{+4.5}_{-5.8})^\circ \\
d &= 0.498^{+0.065+0.000+0.013}_{-0.086-0.001-0.012} &&= 0.498^{+0.066}_{-0.087} \qquad \text{(B)} \\
\theta &= (154.8^{+2.6+8.5+0.9}_{-4.7-9.5-1.2})^\circ &&= (154.8^{+8.9}_{-10.7})^\circ
\end{aligned}$$

– Here we show the errors arising from K, $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$, and have finally added them in quadrature.

Impact of U-Spin-Breaking Effects

- Parametrized as follows: $\xi \equiv d'/d, \quad \Delta\theta \equiv \theta' - \theta$

$$\Rightarrow \; K = \frac{1}{\epsilon^2}\left[\frac{\epsilon^2 + 2\epsilon\xi d\cos(\theta+\Delta\theta)\cos\gamma + \xi^2 d^2}{1 - 2d\cos\theta\cos\gamma + d^2}\right]:$$



$$\underbrace{\begin{aligned}\gamma &= (38.1^{+3.0+1.4+0.2}_{-2.6-1.7-0.3})^\circ\\ d &= 0.282^{+0.029+0.032+0.006}_{-0.036-0.026-0.003}\\ \theta &= (30.0^{+12.9+4.6+0.5}_{-10.7-4.5-0.9})^\circ\end{aligned}}_{\text{(A)}} \qquad \underbrace{\begin{aligned}\gamma &= (68.5^{+4.5+5.0+0.1}_{-5.8-3.7-0.2})^\circ\\ d &= 0.498^{+0.066+0.101+0.002}_{-0.087-0.074-0.005}\\ \theta &= (154.8^{+8.9+3.8+0.1}_{-10.7-3.9-0.2})^\circ\end{aligned}}_{\text{(B)}}$$

[1st errors: input; 2nd errors: ξ, 3rd errors: $\Delta\theta$]

Discrete Ambiguities

- For each of the solutions given above we obtain an additional one through:

$$\gamma \to \gamma - 180^\circ, \quad d \to d, \quad \theta \to \theta - 180^\circ$$

  - The range of $-180^\circ \leq \gamma \leq 0^\circ$ is excluded by ε_K. But NP ...

- Look at the cosines of θ:

$$\cos\theta = +0.866^{+0.079}_{-0.128} \text{ (A)}, \quad \cos\theta = -0.905^{+0.091}_{-0.056} \text{ (B)}$$

  - Although non-factorizable effects have a significant impact on θ, we do *not* expect a change the sign of $\cos\theta$, which is *negative*.
  - We may therefore exclude solution (A), which can also be done through $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ (see below), and the "mirror" solution of (B).

- Current data for $B_d \to \pi^\mp K^\pm$, $B^\pm \to \pi^\pm K$ allow us also to exclude (A) and its "mirror" solution (see below). Therefore only (B) remains:

$$\Rightarrow \quad \gamma = (68.5^{+4.5}_{-5.8}|_{\rm input}{}^{+5.0}_{-3.7}|_\xi{}^{+0.1}_{-0.2}|_{\Delta\theta})^\circ$$

[UTfit: $\gamma = (69.6 \pm 3.1)^\circ$; CKMfitter: $\gamma = (67.2 \pm 3.9)^\circ \Rightarrow$ excellent agreement!]

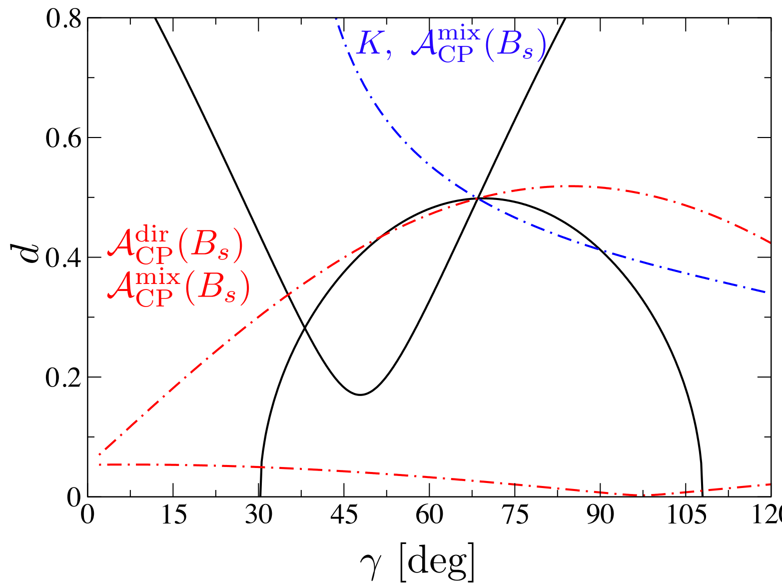
CP Violation in $B_s \to K^+K^-$

- We obtain the following SM predictions ($\phi_s = -2^\circ$):

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-) = +0.090^{+0.046}_{-0.034}\Big|_{\rm input}{}^{+0.014}_{-0.014}\Big|_{\xi}{}^{+0.057}_{-0.071}\Big|_{\Delta\theta}$$
$$\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-) = -0.218^{+0.027}_{-0.027}\Big|_{\rm input}{}^{+0.007}_{-0.006}\Big|_{\xi}{}^{+0.045}_{-0.020}\Big|_{\Delta\theta}$$
$$\mathcal{A}_{\Delta\Gamma}(B_s \to K^+K^-) = -0.972^{+0.009}_{-0.006}\Big|_{\rm input}{}^{+0.000}_{-0.000}\Big|_{\xi}{}^{+0.001}_{-0.002}\Big|_{\Delta\theta}$$

  – 1st errors: input; 2nd errors: $\xi = 1 \pm 0.15$, 3rd errors: $\Delta\theta = \pm 20^\circ$;

- Impact on the situation in the γ–d space (SM case):



[Note: the red $\mathcal{A}_{\rm CP}^{\rm dir}(B_s)$–$\mathcal{A}_{\rm CP}^{\rm mix}(B_s)$ contour is *theoretically clean*!]

Impact of New Physics

- Agreement between $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ result for γ and UT fits:

$$\Rightarrow \boxed{\textit{dramatic}\ \text{NP effects @ amplitude level are}\ \textit{excluded}\ \ldots}$$

  – But the experimental picture has still to be improved considerably!

- NP can enter via B_s^0–$\bar{B}_s^0$ mixing:

$$\Rightarrow \boxed{\text{most recent Tevatron results from CPV in } B_s \to J/\psi\phi\text{:}}$$

  – CDF finds the following ranges (68% C.L.):

$$\phi_s \in [-59.6^\circ, -2.29^\circ] \sim -30^\circ \vee [-177.6^\circ, -123.8^\circ] \sim -150^\circ$$

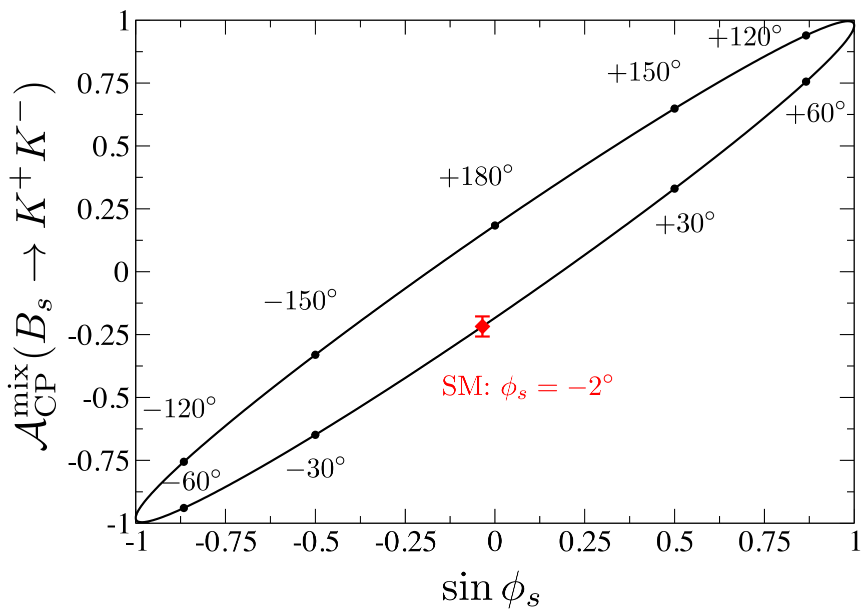
  – DØ takes also the dimuon charge asymmetry and data for BR($B_s \to D_s^{(*)+}D_s^{(*)-}$) into account, yielding the best fit value $\phi_s \sim -45^\circ$.

$\Rightarrow$ situation is far from being conclusive :-(

Such NP would also have footprints in $B_s \to K^+K^-$ ...

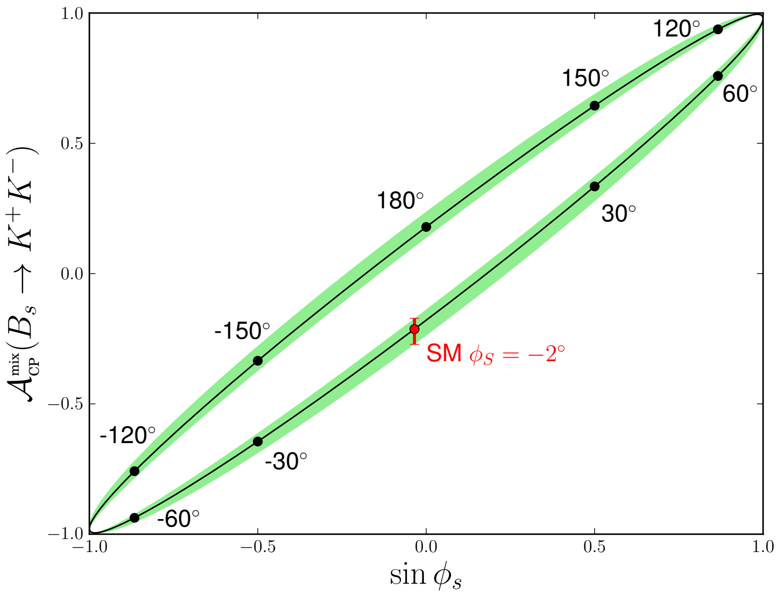
Target Space for $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ Measurement

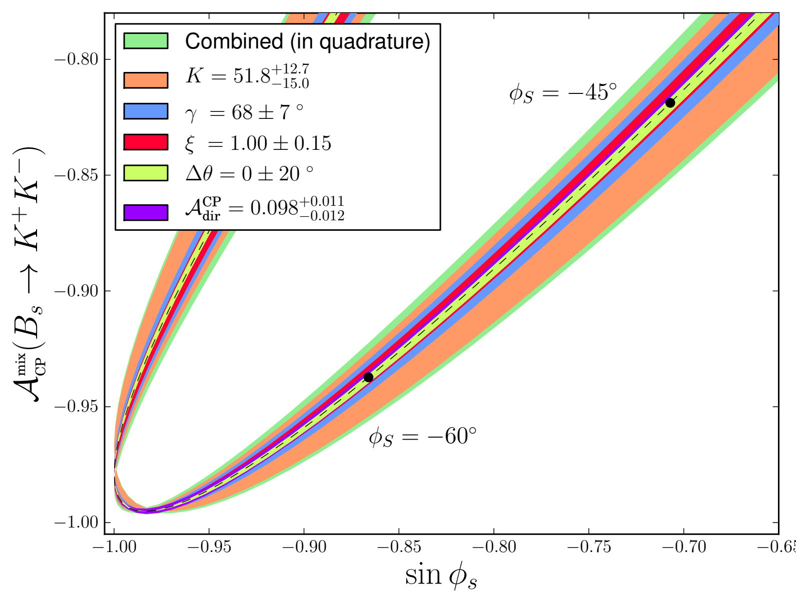
- Hadronic Parameters & γ as determined above: $\Rightarrow$



$\Rightarrow$ current picture for ϕ_s would correspond to $\mathcal{A}_{\rm CP}^{\rm mix} \sim -0.8$!

- This correlation can also be *calculated directly* from K: ($\to$ *new study:*)
  - Use γ as an input parameter (we assume $\gamma = 68 \pm 7°$);
  - Use $\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-) \approx \mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm) = 0.098^{+0.011}_{-0.012}$ (see below) to fix the direct CP violation in $B_s \to K^+K^-$ $\Rightarrow$





- Corresponding SM prediction:

$$\mathcal{A}_{CP}^{dir}(B_s \to K^+K^-)|_{SM} = -0.213^{+0.031}_{-0.053}|_K {}^{+0.022}_{-0.020}|_\gamma {}^{+0.005}_{-0.005}|_{\mathcal{A}_{CP}^{dir}} {}^{+0.014}_{-0.010}|_\xi {}^{+0.004}_{-0.007}|_{\Delta\theta}$$

$$= -0.213^{+0.041}_{-0.058}$$

R.F. & Rob Knegjens (in progress)

$$B_d \to \pi^{\mp} K^{\pm},\ B_s \to \pi^{\pm} K^{\mp}$$

First Insights into U-Spin-Breaking Effects

- Parametrization of the decay amplitudes:

$$\begin{aligned} A(B_d^0 \to \pi^- K^+) &= -P\left[1 - re^{i\delta}e^{i\gamma}\right] \\ A(B_s^0 \to \pi^+ K^-) &= P_s\sqrt{\epsilon}\left[1 + \frac{1}{\epsilon} r_s e^{i\delta_s} e^{i\gamma}\right] \end{aligned}$$

- U-spin symmetry: $\Rightarrow$ relations between strong parameters:

$$r_s = r, \quad \delta_s = \delta$$

$$\left|\frac{P_s}{P}\right|_{\rm fact} = \frac{f_\pi}{f_K}\frac{F_{B_sK}(M_\pi^2;0^+)}{F_{B_d\pi}(M_K^2;0^+)}\left(\frac{M_{B_s}^2 - M_K^2}{M_{B_d}^2 - M_\pi^2}\right) \to \left|\frac{P_s}{P}\right|_{\rm fact}^{\rm QCDSR} = 0.99^{+0.17}_{-0.06}$$

- Another U-spin symmetry implication: [$\to$ further info needed for γ]

$$\frac{\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to \pi^\pm K^\mp)}{\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm)} \sim -\left|\frac{P_s}{P}\right|^2\left[\frac{\mathrm{BR}(B_d \to \pi^\mp K^\pm)}{\mathrm{BR}(B_s \to \pi^\pm K^\mp)}\right]$$

$$\Rightarrow \left|\frac{P_s}{P}\right|_{\rm exp} = \left|\frac{P_s}{P}\right|\sqrt{\left[\frac{r_s}{r}\right]\left[\frac{\sin\delta_s}{\sin\delta}\right]} = 1.04 \pm 0.26$$

Further Information: $B^+ \to \pi^+ K^0$ and $B^+ \to K^+ \bar{K}^0$

- For the extraction of γ, the overall normalization P has to be fixed:
  - Neglect colour-suppressed EWPs and use the $SU(2)$ isospin symmetry:

$$A(B^+ \to \pi^+ K^0) = P\left[1 + \epsilon \rho_{\pi K} e^{i\theta_{\pi K}} e^{i\gamma}\right]$$

- Hadronic parameter $\rho_{\pi K} e^{i\theta_{\pi K}}$ is expected to play a minor rôle because of the ϵ suppression, but could be enhanced through FSI effects(!?):

$$\mathcal{A}_{\rm CP}^{\rm dir}(B^\pm \to \pi^\pm K) = -\left[\frac{2\epsilon\rho_{\pi K}\sin\theta_{\pi K}\sin\gamma}{1 + 2\epsilon\rho_{\pi K}\cos\theta_{\pi K}\cos\gamma + \epsilon^2\rho_{\pi K}^2}\right] = -0.009 \pm 0.025$$

$\Rightarrow$ no anomalous behaviour indicated!

- U-spin-related $b \to d$ penguin mode $B^\pm \to K^\pm K$ (already observed):

$$A(B^+ \to K^+ \bar{K}^0) = \sqrt{\epsilon} P_{KK}\left[1 - \rho_{KK} e^{i\theta_{KK}} e^{i\gamma}\right]$$

$$\rho_{KK} = \rho_{\pi K}, \quad \theta_{KK} = \theta_{\pi K}$$

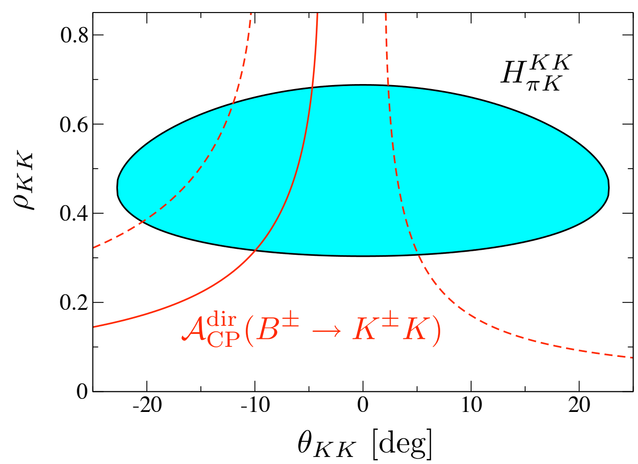
- Allows us to determine ρ_{KK} and θ_{KK} for a given value of γ:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B^\pm \to K^\pm K) = \frac{2\rho_{KK}\sin\theta_{KK}\sin\gamma}{1-2\rho_{KK}\cos\theta_{KK}\cos\gamma+\rho_{KK}^2} \stackrel{\rm exp}{=} -0.12^{+0.18}_{-0.17}$$

$$H_{\pi K}^{KK} \sim \frac{1}{\epsilon}\left|\frac{P}{P_{KK}}\right|^2 \left[\frac{\mathrm{BR}(B^\pm \to K^\pm K)}{\mathrm{BR}(B^\pm \to \pi^\pm K)}\right]$$

$$= \frac{1-2\rho_{KK}\cos\theta_{KK}\cos\gamma+\rho_{KK}^2}{1+2\epsilon\rho_{\pi K}\cos\theta_{\pi K}\cos\gamma+\epsilon^2\rho_{\pi K}^2} \stackrel{\rm exp}{=} 0.64 \pm 0.15$$

- We arrive at a pretty resticted region in parameter space:



$$\Rightarrow \quad \begin{array}{l} \rho_{KK} \approx \rho_{\pi K} \sim 0.5 \\ \theta_{KK} \approx \theta_{\pi K} \sim 0^\circ \end{array}$$

- Consequently, we find $\epsilon\rho_{\pi K}|_{\rm exp} \sim 0.025$:
  - We *do not* have to worry about the effects of this parameter;
  - Toy models of large FSI effects are ruled out by the B-factory data!

Extracting the UT Angle γ

- Let's first have a look at the $B_d \to \pi^\mp K^\pm$, $B^\pm \to \pi^\pm K$ system:

$$R \sim \frac{\tau_{B^+}}{\tau_{B_d}} \left[\frac{\mathrm{BR}(B_d \to \pi^\mp K^\pm)}{\mathrm{BR}(B^\pm \to \pi^\pm K)} \right] \overset{\mathrm{exp}}{=} 0.902 \pm 0.049$$

$$\Rightarrow \boxed{w^2 R = 1 - 2r\cos\delta\cos\gamma + r^2}$$

$$w = \sqrt{1 + 2\epsilon\rho_{\pi K}\cos\theta_{\pi K} + \epsilon^2\rho_{\pi K}^2} \overset{\mathrm{exp}}{\sim} 1.02 \;\to\; \text{neglegt } \rho_{\pi K} \text{ effect!}$$

- R can be converted into a bound on γ: [R.F. & Mannel (1997)]

$$\boxed{\sin^2\gamma \leq R \Rightarrow \gamma \leq \left(71.8^{+5.4}_{-4.3}\right)^\circ}$$

$\to$ effectively constrains γ in a phenomenologically interesting region!

- Further information from direct CP violation: $\to$ γ–r contours:[2]

$$\boxed{A_0 \equiv \mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^\mp K^\pm) R = 2r\sin\delta\sin\gamma}$$

[2] Detailed analysis: R.F., *Eur. Phys. J.* **C6** (1999) 647.

- Introduce similar quantities for the $B_s \to \pi^\pm K^\mp$, $B^\pm \to \pi^\pm K$ system:

$$R_s \sim \left|\frac{P}{P_s}\right|^2 \left[\frac{\text{BR}(B_s \to \pi^\pm K^\mp)}{\text{BR}(B^\pm \to \pi^\pm K)}\right] = \epsilon + 2r_s \cos\delta_s \cos\gamma + \frac{r_s^2}{\epsilon}$$

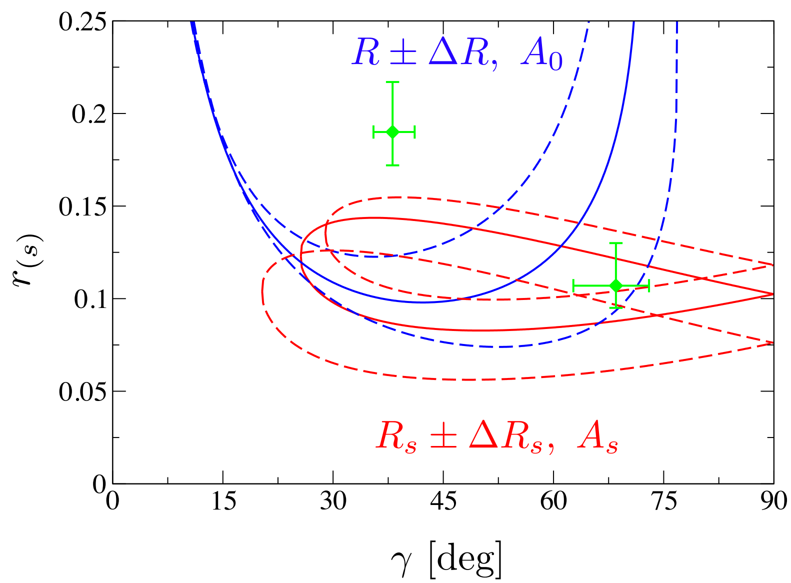
$$A_s \equiv \mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \to \pi^\pm K^\mp) R_s = -2r_s \sin\delta_s \sin\gamma$$

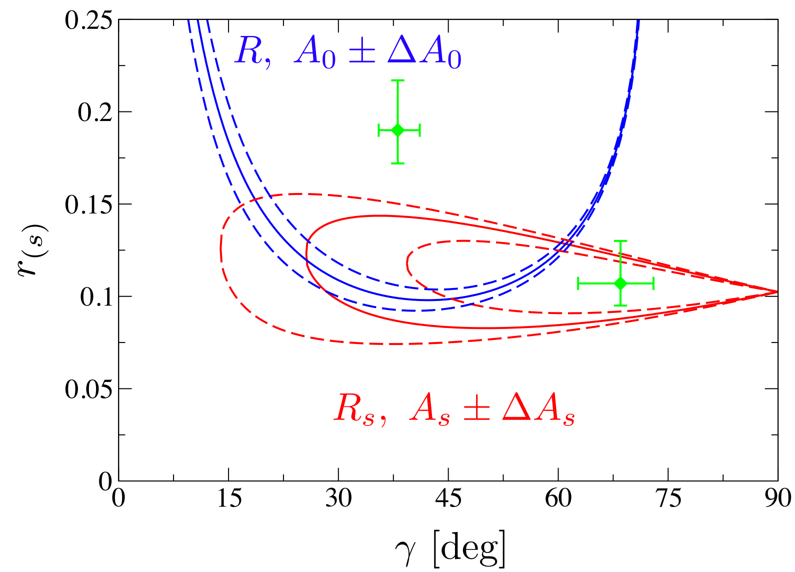
$\to$ γ–r_s contours (in analogy to the γ–r contours)

- U-spin symmetry: $\boxed{r = r_s,\ \delta = \delta_s}$
  - Intersection of the γ–r and γ–r_s contours: $\Rightarrow \gamma$, $r = r_s$.
  - Moreover, the strong phases δ and δ_s can be extracted $\Rightarrow$ test!

- A closer look shows the following additional features:
  - $\cos\delta$ positive for $-90^\circ \leq \gamma \leq +90^\circ \Rightarrow 0^\circ \leq \gamma \leq +90^\circ$ (see above).
  - The requirement of $\cos\delta_s > 0$ imposes further constraints ...

- Situation not as fortunate as in the case of $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$:





  – The FM bound is nicely visible for the blue γ–r contours;

  – Because of the $\text{sgn}(\cos\delta_s) = \text{sgn}(\cos\delta) = 1$ constraint, only the lower branches of the red γ–r_s contours are effective:

$$\Rightarrow \quad \boxed{24^\circ \leq \gamma \leq 71^\circ, \quad 0.07 \leq r \leq 0.13}$$

- Consider the upper $1\,\sigma$ values of $R_s = 0.315$ and $R = 0.951$:

$$\Rightarrow \quad \gamma = 71.1^\circ, \quad r = 0.105, \quad \delta = 27.9^\circ, \quad \delta_s = 38.3^\circ,$$

which would look quite reasonable.

Interplay with the $B_s \to K^+K^-$, $B_d \to \pi^+\pi^-$ Strategy

- $B_s^0 \to K^+K^-$ and $B_d^0 \to \pi^- K^+$ differ only in their spectator quarks:

  – Difference only through exchange and penguin annihilation topologies, which contribute to $B_s^0 \to K^+K^-$ but not to $B_d^0 \to \pi^- K^+$:

$$\sqrt{\frac{1}{2}\left[\frac{\mathrm{BR}(B_d \to K^+K^-)}{\mathrm{BR}(B^\pm \to \pi^\pm\pi^0)}\right]\frac{\tau_{B^+}}{\tau_{B_d}}}$$

$$\approx \left|\frac{\mathcal{E} - (\mathcal{PA})_{tu}}{\mathcal{T} + \mathcal{C}}\right| \sqrt{1 + 2\varrho_{\mathcal{PA}}\cos\vartheta_{\mathcal{PA}}\cos\gamma + \varrho_{\mathcal{PA}}^2} = 0.12^{+0.04}_{-0.06}$$

$$\sqrt{\frac{\epsilon}{2}\left[\frac{\mathrm{BR}(B_s \to \pi^+\pi^-)}{\mathrm{BR}(B^\pm \to \pi^\pm\pi^0)}\right]\frac{\tau_{B^+}}{\tau_{B_s}}} \approx \frac{1}{R_b}\left|\frac{(\mathcal{PA})_{tc}}{\mathcal{T} + \mathcal{C}}\right| = 0.05^{+0.03}_{-0.04}$$

$\Rightarrow$ **data do not indicate any anomalous behaviour** $\Rightarrow$ neglect!

- We obtain then the following "dictionary":

$$\boxed{re^{i\delta} = e^{i(\pi-\theta)}\epsilon/d}$$

- Translation of our $B_s \to K^+K^-$, $B_d \to \pi^+\pi^-$ solutions:

$$\underbrace{\begin{array}{rcl} \gamma & = & (38.1^{+3.0}_{-2.6})^\circ \\ r & = & 0.190^{+0.027}_{-0.018} \\ \delta & = & (150.0^{+10.7}_{-12.9})^\circ \end{array}}_{\text{(A)}} \qquad \underbrace{\begin{array}{rcl} \gamma & = & (68.5^{+4.5}_{-5.8})^\circ \\ r & = & 0.107^{+0.023}_{-0.012} \\ \delta & = & (25.2^{+10.7}_{-8.9})^\circ \end{array}}_{\text{(B)}}$$

  - Represented by green data points with error bars in the previous plot.
  - The γ–r contours exclude (A), as noted above, leaving us with (B).

- Calculation of the $B_d \to \pi^\mp K^\pm$, $B_s \to \pi^\pm K^\mp$, $B^\pm \to \pi^\pm K$ observables:

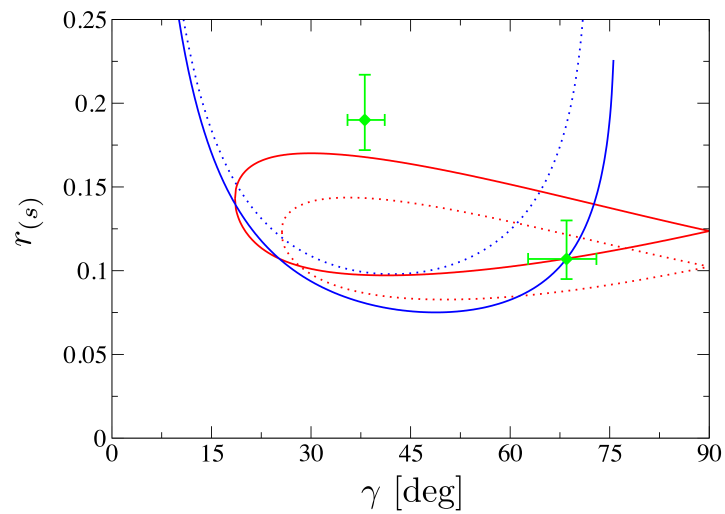
$$R = 0.940^{+0.016}_{-0.023} \stackrel{\text{exp}}{=} 0.902 \pm 0.049$$

$$R_s = 0.340^{+0.126}_{-0.063} \stackrel{\text{exp}}{=} 0.250^{+0.065}_{-0.088}$$

$$\to \quad \text{BR}(B_s \to \pi^\pm K^\mp) = (6.8^{+3.5}_{-1.6}) \times 10^{-6} \; (1\,\sigma \text{ larger than CDF})$$

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to \pi^\mp K^\pm) = +0.090^{+0.046}_{-0.034} \stackrel{\text{exp}}{=} 0.098^{+0.011}_{-0.012} \, [\to \mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \to K^+K^-)]$$

- Corresponding situation in the γ–$r_{(s)}$ plane: $\rightarrow$ serves as future scenario:



- Moreover:

$$\frac{\mathrm{BR}(B_s\to K^+K^-)}{\mathrm{BR}(B_d\to\pi^\mp K^\pm)} \sim \left(\frac{f_\pi}{f_K}\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|_{\mathrm{fact}}\right)^2 \Rightarrow \underbrace{\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|^{\mathrm{exp}}_{\mathrm{fact}} = 1.44\pm 0.12}_{\text{QCDSR: } 1.41^{+0.20}_{-0.11}}$$

$$\frac{\mathrm{BR}(B_s\to\pi^\pm K^\pm)}{\mathrm{BR}(B_d\to\pi^+\pi^-)} \sim \left(\frac{f_K}{f_\pi}\left|\frac{P_s}{P}\right|_{\mathrm{fact}}\right)^2 \Rightarrow \underbrace{\mathrm{BR}(B_s\to\pi^\pm K^\pm) = \left(6.8^{+2.5}_{-0.9}\right)\times 10^{-6}}_{\to\ \mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_s\to\pi^\pm K^\mp)\ \sim\ -0.29}$$

$$\mathrm{BR}(B_s\to\pi^\pm K^\pm) = \left[\frac{\mathrm{BR}(B_s\to K^+K^-)}{\mathrm{BR}(B_d\to\pi^\mp K^\pm)}\right]\mathrm{BR}(B_d\to\pi^+\pi^-) = (7.0\pm1.2)\times10^{-6}$$

$$\Delta^{\mathrm{NF}}_{SU(3)} \equiv 1 - \left[\frac{\mathrm{BR}(B_s\to K^+K^-)}{\mathrm{BR}(B_s\to\pi^\pm K^\pm)}\right]\left[\frac{\mathrm{BR}(B_d\to\pi^+\pi^-)}{\mathrm{BR}(B_d\to\pi^\mp K^\pm)}\right] = -0.4\pm 0.4$$

Final Remarks

- Detailed analysis of the $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ system:
  - The BaBar measurement of $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ is favoured.
  - A fortunate situation arises:

$$\boxed{\gamma = \left(68.5^{+4.5}_{-5.8}\big|_{\rm input}{}^{+5.0}_{-3.7}\big|_{\xi}{}^{+0.1}_{-0.2}\big|_{\Delta\theta}\right)^\circ} \to \text{very competitive!}$$

  - Measurement of $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ is the next important step:

    $\to$ interesting *correlations* with $(\sin\phi_s)_{B_s\to\psi\phi}$ $\Rightarrow$ probe of NP!

- Detailed analysis of the $B_d \to \pi^\mp K^\pm$, $B_s \to \pi^\pm K^\mp$ system:
  - FM bound $\gamma \leq \left(71.8^{+5.4}_{-4.3}\right)^\circ$ is effective in an interesting region!
  - Current $B_d \to \pi^\mp K^\pm$, $B_s \to \pi^\pm K^\mp$ data: $\Rightarrow 24^\circ \leq \gamma \leq 71^\circ$ ...

- Synergy between the two U-spin-related systems:
  - Resolves ambiguities for γ, thereby leaving us with a single solution.
  - Impressive consistency checks (U-spin-breaking effects, etc.).
  - Increase of BR$(B_s \to \pi^\pm K^\mp)$ is favoured...