$K\pi$ vector form factor constrained by $\tau \to K\pi\nu_\tau$ and K_{l3} decays

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Purpose: to present a model for the $K\pi$ vector form factor using a dispersive representation and incorporating constraints from K_{l3} decays suited to describe both $\tau\to K\pi\nu_\tau$ and K_{l3} decays simultaneously

Why? because a good knowledge of the $K\pi$ f.f.'s s is of fundamental importance for the determination of V_{us} from K_{l3} decays and m_s

Outline:

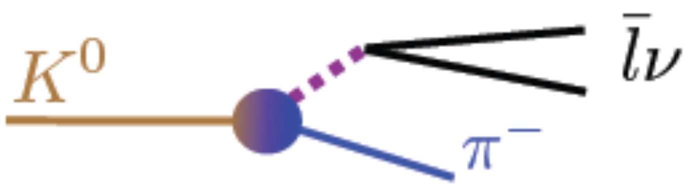
- *Introduction*
- $K\pi$ *form factors*
- *Fit to* $\tau\to K\pi\nu_\tau$
- *Fit to* $\tau\to K\pi\nu_\tau$ *with restrictions from* K_{l3}
- *Conclusions*

in collab. with D. R. Boito and M. Jamin, JHEP in press

• *Introduction*

- K_{l3} decays are the main route towards the determination of $|V_{us}|^2$

  H. Leutwyler and M. Roos, ZPC 25 (1984) 91



$$\Gamma_{K_{l3}} \propto |V_{us}|^2 |F_+(0)|^2 I_{K_{l3}}$$

with

$$I_{K_{l3}} = \frac{1}{m_K^8} \int dt\, (\text{p.s.}) \left[\tilde{F}_+(t)^2 + \eta(t, m_l) \tilde{F}_0(t)^2 \right]$$

and $\tilde{F}_{+,0}(q^2) \equiv \frac{F_{+,0}(q^2)}{F_+(0)}$

- $F_{+,0}(0)$ the normalization from ChPT, Lattice
- $\tilde{F}_{+,0}(q^2)$ the energy dependence from (R)ChPT, dispersion relations

• Kπ *form factors*

Definition

vector f.f. scalar f.f.

$$\langle \pi^-(p)|\bar{s}\,\gamma^\mu\,u|K^0(k)\rangle = \left[(k+p)^\mu - \frac{m_K^2 - m_\pi^2}{q^2}(k-p)^\mu\right] F_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2}(k-p)^\mu F_0(q^2)$$

with $F_+(0) = F_0(0)$

Kπ f.f. representation for K_{l3} decays

$$m_l^2 < q^2 < (m_K - m_\pi)^2$$

slope curvature

$$F_{+,0}(q^2) = F_{+,0}(0)\left[1 + \lambda'_{+,0}\frac{q^2}{m_{\pi^-}^2} + \frac{1}{2}\lambda''_{+,0}\left(\frac{q^2}{m_{\pi^-}^2}\right)^2 + \cdots\right]$$

In this kinematical region the f.f. are real

Kπ f.f. representation for $\tau \to K\pi\nu_\tau$ decays

$$(m_K + m_\pi)^2 < q^2 < m_\tau^2$$

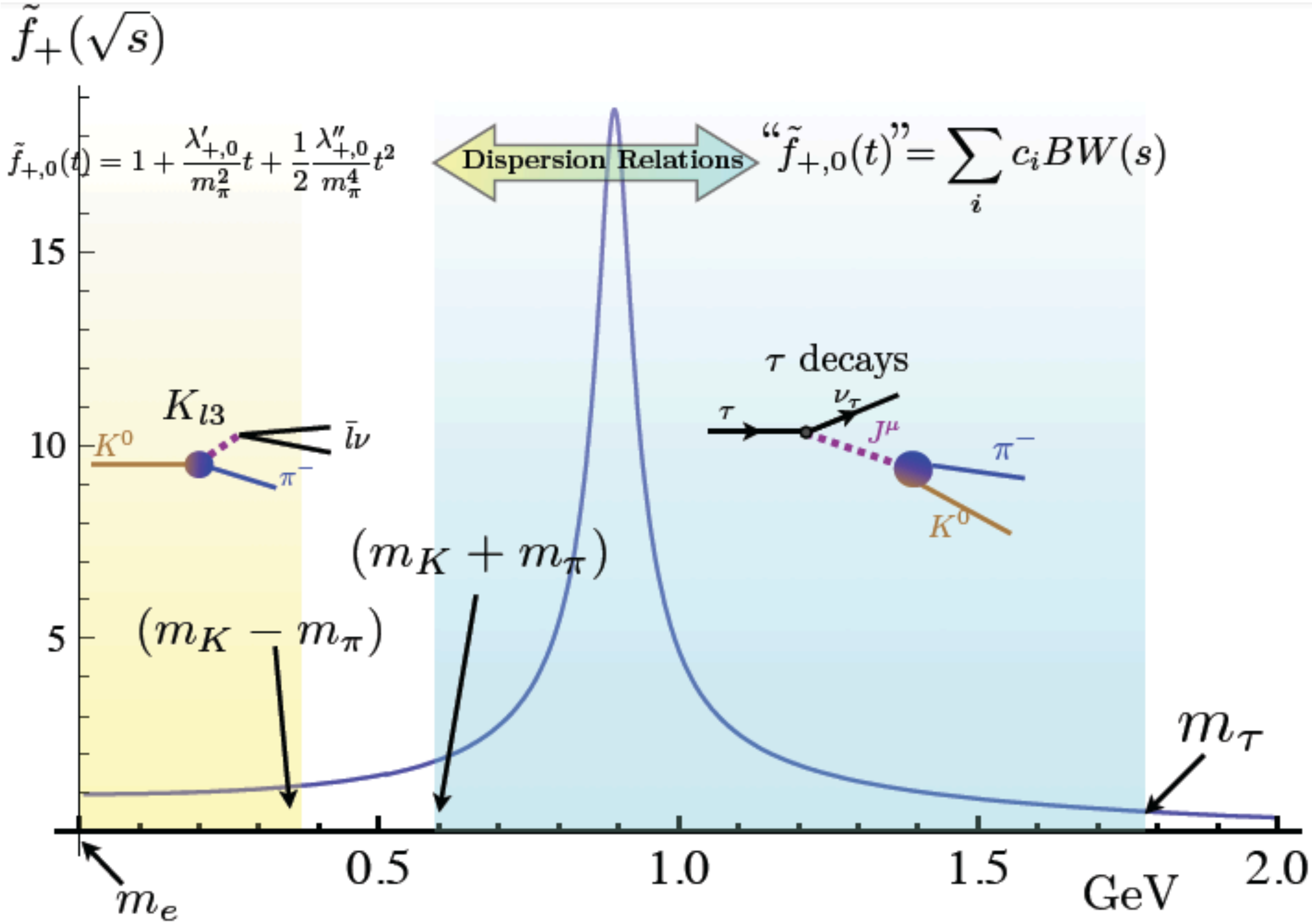
In this kinematical region the f.f. are complex

→ Taylor expansion inadmissible → more sophisticated treatments

- $K\pi$ *form factors*

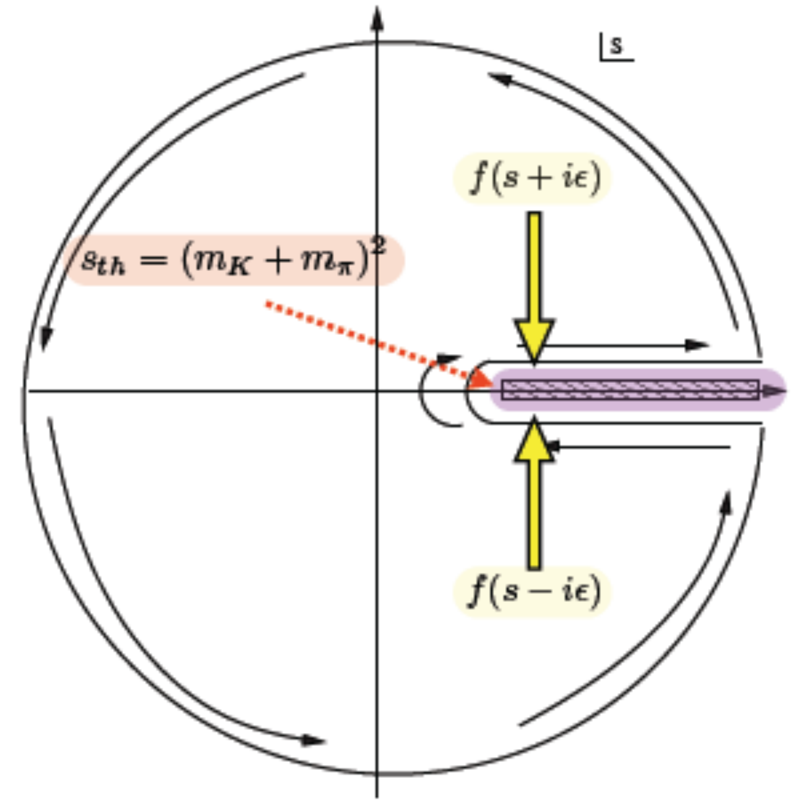
$K\pi$ f.f. dispersive representations

Suited to described both $\tau \to K\pi\nu_\tau$ and K_{l3} decays



• Kπ *form factors*

Kπ f.f. dispersive representations



Analyticity →

$$f(s)=\frac{1}{\pi}\int_{s_{th}}^{\infty}ds'\frac{\mathrm{Im}\,f(s')}{s'-s-i\epsilon}$$

Analyticity + Unitarity

→ Muskelishivili-Omnès equation

$$f(s)=\frac{1}{\pi}\int_{s_{th}}^{\infty}ds'\frac{\tan\delta(s')\mathrm{Re}\,f(s')}{s'-s-i\epsilon}$$

solution

$$f(s)=f(0)\exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s'-s-i\epsilon}\right]$$

generalized solution (n subtractions at s=0)

$$f(s)=\exp\left[\alpha_1+\alpha_2 s\cdots+\alpha_{n-1}s^{n-1}+\frac{s^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s')^n}\frac{\delta(s')}{s'-s-i\epsilon}\right]$$

Recent dispersive representations:

B. Moussallam, EPJC 53 (2008) 401
V. Bernard *et. al.*, PRD 80 (2009) 034034
D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821
V. Bernard *et. al.*, PLB 638 (2006) 480
M. Jamin, J.A. Oller and A. Pich, NPB 587 (2000) 331 & 622 (2002) 279, PRD 74 (2006) 074009

• *Fit to* $\tau \to K\pi\nu_\tau$

Differential decay distribution

$|V_{us}|F_+(0) = 0.2163(5)$ M. Antonelli *et. al.*, arXiv:1005.2323

$$\frac{\mathrm{d}\Gamma_{K\pi}}{\mathrm{d}\sqrt{s}} = \frac{G_F^2 |V_{us}F_+(0)|^2 m_\tau^3}{32\pi^3 s} S_{\mathrm{EW}} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times$$
$$\times \left[\left(1 + 2\frac{s}{m_\tau^2}\right) q_{K\pi}^3 |\tilde{F}_+(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s} q_{K\pi} |\tilde{F}_0(s)|^2\right]$$

with $\tilde{F}_{+,0}(q^2) \equiv \frac{F_{+,0}(q^2)}{F_+(0)}$

normalized vector f.f. normalized scalar f.f.

Ansatz to analyse the data:

$$N_i^{\mathrm{th}} = \mathcal{N}_T \, \frac{1}{2} \, \frac{2}{3} \, \Delta_{\mathrm{b}}^i \, \frac{1}{\Gamma_\tau \bar{B}_{K\pi}} \frac{\mathrm{d}\Gamma_{K\pi}}{\mathrm{d}\sqrt{s}}(s_{\mathrm{b}}^i)$$

with $\mathcal{N}_T = 53110$ and $\Delta_{\mathrm{b}} = 11.5\ \mathrm{MeV}$

D. Epifanov *et. al.* (Belle Collaboration), PLB 654 (2007) 65

Model for the scalar f.f.

M. Jamin, J.A. Oller and A. Pich, NPB 622 (2002) 279

• *Fit to* $\tau \to K\pi\nu_\tau$

Our model for the vector f.f.

After a detailed analysis in D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

Three-times-subtracted dispersion relation

cut-off to check stability

$$\tilde{F}_+(s) = \exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{\rm cut}} ds' \frac{\delta(s')}{(s')^3(s'-s-i0)}\right]$$

with $\lambda'_+ = \alpha_1$ and $\lambda''_+ = \alpha_2 + \alpha_1^2$

Our model for the phase

$$\delta(s) = \tan^{-1}\left[\frac{\mathrm{Im}\ \tilde{f}_+(s)}{\mathrm{Re}\ \tilde{f}_+(s)}\right]$$

where
$$\tilde{f}_+(s) = \frac{\tilde{m}_{K^*}^2 - \kappa_{K^*}\tilde{H}_{K\pi}(0) + \gamma\, s}{D(\tilde{m}_{K^*}, \gamma_{K^*})} - \frac{\gamma\, s}{D(\tilde{m}_{K^{*\prime}}, \gamma_{K^{*\prime}})}$$

2 vector resonances form inspired by RChPT

M. Jamin, A. Pich and J. Portolés, PLB 640 (2006) 176 & 664 (2008) 78

and

$$D(\tilde{m}_n, \gamma_n) \equiv \tilde{m}_n^2 - s - \kappa_n \,\mathrm{Re}\, \tilde{H}_{K\pi}(s) - i\,\tilde{m}_n\gamma_n(s)$$

$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma(\tilde{m}_n^2)^3}\frac{\gamma_n}{\tilde{m}_n} \qquad \gamma_n(s) = \gamma_n \frac{s}{\tilde{m}_n^2}\frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(\tilde{m}_n^2)}$$

$\tilde{H}_{K\pi}(s)$ is the one-loop $K\pi$ bubble integral

Physical masses and widths are obtained from

$$D(\tilde{m}_n, \gamma_n) = 0$$

for $s \to s_R$ with $\sqrt{s_R} = m_R - \frac{i}{2}\Gamma_R$

R. Escribano *et. al.*, EPJC 28 (2003) 107

• *Fit to* τ→Kπν$_\tau$

Results Update of D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

$B_{K\pi}^{\rm exp} = 0.418(11)\%$

$$\chi^2 = \sum_{i=1}^{90}{}' \left(\frac{N_i^{\rm th} - N_i^{\rm exp}}{\sigma_{N_i^{\rm exp}}}\right)^2 + \left(\frac{\bar{B}_{K\pi} - B_{K\pi}^{\rm exp}}{\sigma_{B_{K\pi}^{\rm exp}}}\right)^2$$

$1.8\,\mathrm{GeV} < \sqrt{s_{\rm cut}} < \infty$

	$s_{\rm cut} = 3.24\ \mathrm{GeV}^2$	$s_{\rm cut} = 4\ \mathrm{GeV}^2$	$s_{\rm cut} = 9\ \mathrm{GeV}^2$	$s_{\rm cut} \to \infty$
$\bar{B}_{K\pi}$	$0.416 \pm 0.011\%$	$0.417 \pm 0.011\%$	$0.418 \pm 0.011\%$	$0.418 \pm 0.011\%$
$(B_{K\pi}^{\rm th})$	(0.414%)	(0.414%)	(0.415%)	(0.415%)
m_{K^*} [MeV]	892.00 ± 0.19	892.02 ± 0.19	892.03 ± 0.19	892.03 ± 0.19
Γ_{K^*} [MeV]	46.14 ± 0.44	46.20 ± 0.43	46.25 ± 0.42	46.25 ± 0.42
$m_{K^{*\prime}}$ [MeV]	1281^{+25}_{-33}	1280^{+25}_{-28}	1278^{+26}_{-27}	1278^{+26}_{-27}
$\Gamma_{K^{*\prime}}$ [MeV]	243^{+92}_{-70}	193^{+72}_{-56}	177^{+66}_{-52}	177^{+66}_{-52}
$\gamma \times 10^2$	$-5.1^{+1.7}_{-2.6}$	$-3.9^{+1.3}_{-1.8}$	$-3.4^{+1.1}_{-1.6}$	$-3.4^{+1.1}_{-1.6}$
$\lambda'_+ \times 10^3$	24.15 ± 0.72	24.55 ± 0.68	24.86 ± 0.66	24.88 ± 0.66
$\lambda''_+ \times 10^4$	11.99 ± 0.19	11.95 ± 0.19	11.93 ± 0.19	11.93 ± 0.19
$\chi^2/\mathrm{n.d.f.}$	$74.1/79$	$75.7/79$	$77.2/79$	$77.3/79$

• *Fit to τ→Kπν$_τ$ with restrictions from K$_{l3}$*

M. Antonelli *et. al.*, arXiv:1005.2323

$\lambda_+^{\prime\,\mathrm{exp}} = (24.9 \pm 1.1) \times 10^{-3}$
$\lambda_+^{\prime\prime\,\mathrm{exp}} = (16 \pm 5) \times 10^{-4}$
$\rho_{\lambda'_+,\lambda''_+} = -0.95$

Results

$B_{K\pi}^{\mathrm{exp}} = 0.418(11)\%$

$$\chi^2 = \sum_{i=1}^{90}{}' \left(\frac{N_i^{\mathrm{th}} - N_i^{\mathrm{exp}}}{\sigma_{N_i^{\mathrm{exp}}}}\right)^2 + \left(\frac{\bar{B}_{K\pi} - B_{K\pi}^{\mathrm{exp}}}{\sigma_{B_{K\pi}^{\mathrm{exp}}}}\right)^2 + (\boldsymbol{\lambda}_+^{\mathrm{th}} - \boldsymbol{\lambda}_+^{\mathrm{exp}})^{\mathrm{T}} V^{-1} (\boldsymbol{\lambda}_+^{\mathrm{th}} - \boldsymbol{\lambda}_+^{\mathrm{exp}})$$

$1.8\,\mathrm{GeV} < \sqrt{s_{\mathrm{cut}}} < \infty$

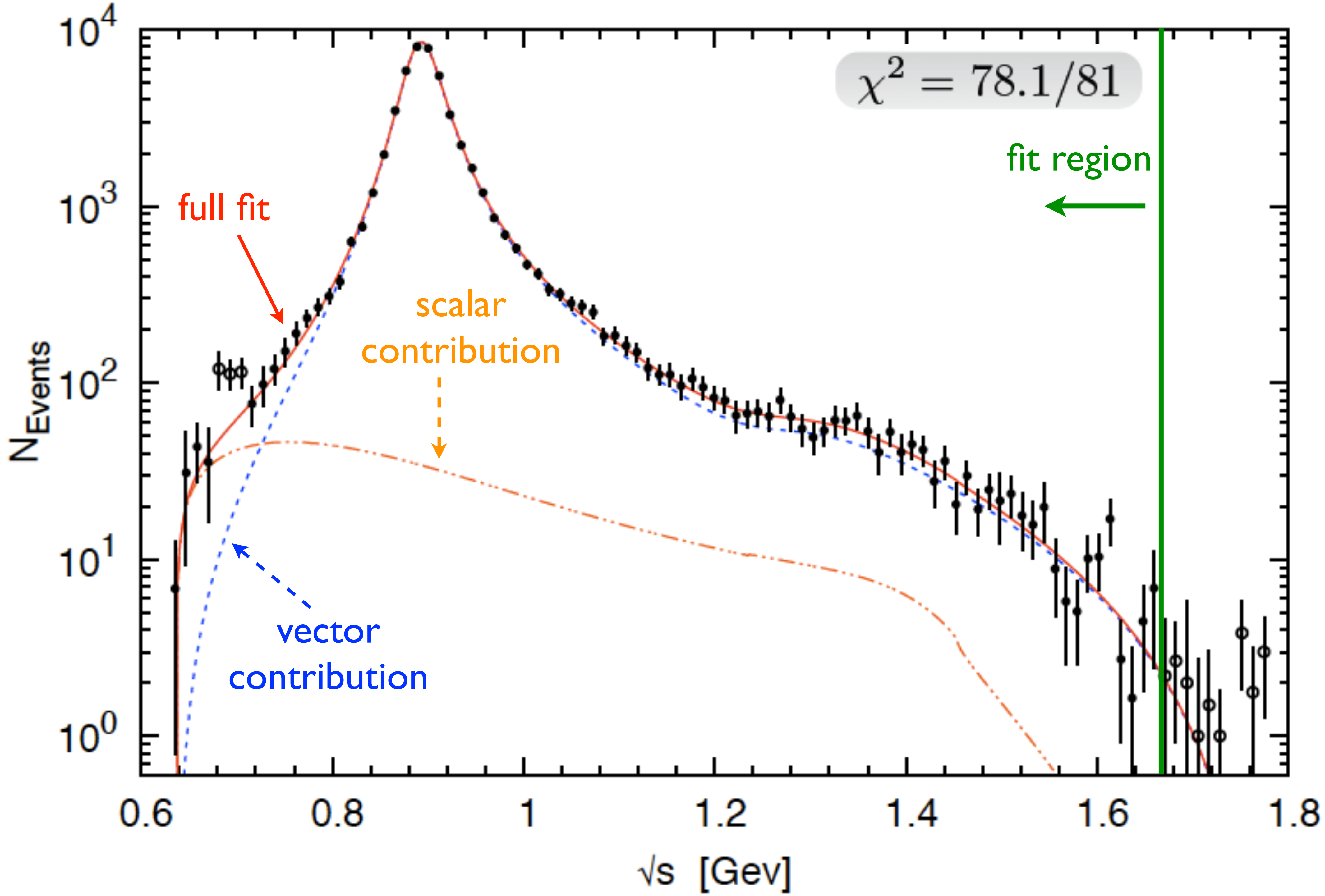
	$s_{\mathrm{cut}} = 3.24\ \mathrm{GeV}^2$	$s_{\mathrm{cut}} = 4\ \mathrm{GeV}^2$	$s_{\mathrm{cut}} = 9\ \mathrm{GeV}^2$	$s_{\mathrm{cut}} \to \infty$
$B_{K\pi}$	0.429 ± 0.009	$0.427 \pm 0.008\%$	$0.426 \pm 0.008\%$	$0.426 \pm 0.008\%$
$(B_{K\pi}^{\mathrm{th}})$	(0.426%)	(0.425%)	(0.423%)	(0.423%)
m_{K^*} [MeV]	892.04 ± 0.20	892.02 ± 0.20	892.03 ± 0.19	892.03 ± 0.19
Γ_{K^*} [MeV]	46.58 ± 0.38	46.52 ± 0.38	46.48 ± 0.38	46.48 ± 0.38
$m_{K^{*\prime}}$ [MeV]	1257^{+30}_{-45}	1268^{+25}_{-32}	1270^{+24}_{-29}	1271^{+24}_{-29}
$\Gamma_{K^{*\prime}}$ [MeV]	321^{+95}_{-76}	238^{+75}_{-57}	206^{+67}_{-50}	205^{+67}_{-50}
$\gamma \times 10^2$	$-8.2^{+2.2}_{-3.5}$	$-5.4^{+1.4}_{-2.0}$	$-4.4^{+1.2}_{-1.6}$	$-4.4^{+1.2}_{-1.6}$
$\lambda'_+ \times 10^3$	25.43 ± 0.30	25.49 ± 0.30	25.55 ± 0.30	25.55 ± 0.30
$\lambda''_+ \times 10^4$	12.31 ± 0.10	12.20 ± 0.10	12.12 ± 0.10	12.12 ± 0.10
χ^2/n.d.f.	$77.9/81$	$78.1\ /81$	$79.0\ /81$	$79.1/81$

• *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

Fit to Belle spectrum

D. Epifanov *et. al.* (Belle Collaboration), PLB 654 (2007) 65

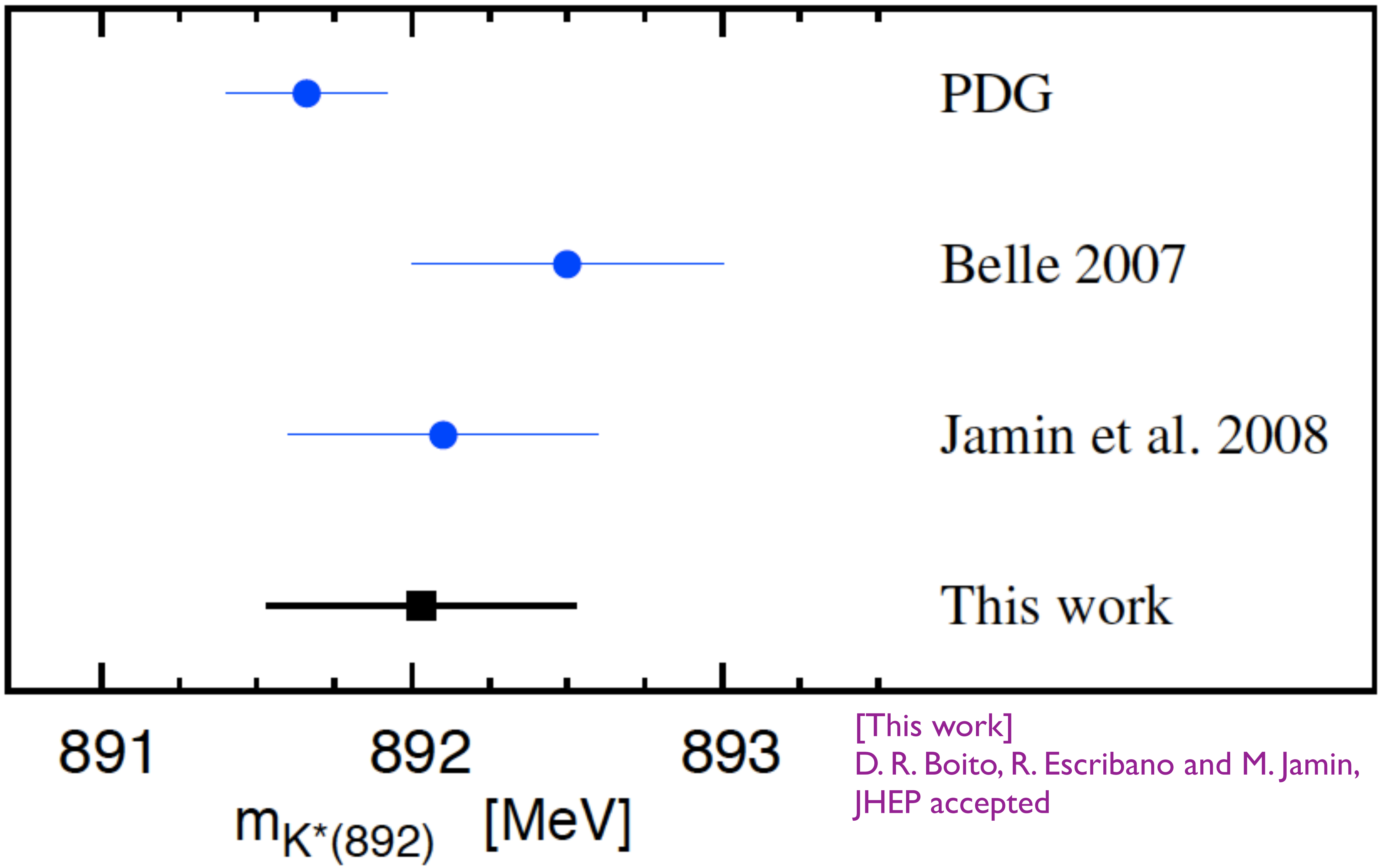
$s_{\rm cut} = 4\ {\rm GeV}^2$



- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

K*(892)± pole mass

$$m_{K^*(892)^\pm} = 892.03 \pm (0.19)_{\rm stat} \pm (0.44)_{\rm sys}\ \mathrm{MeV}$$

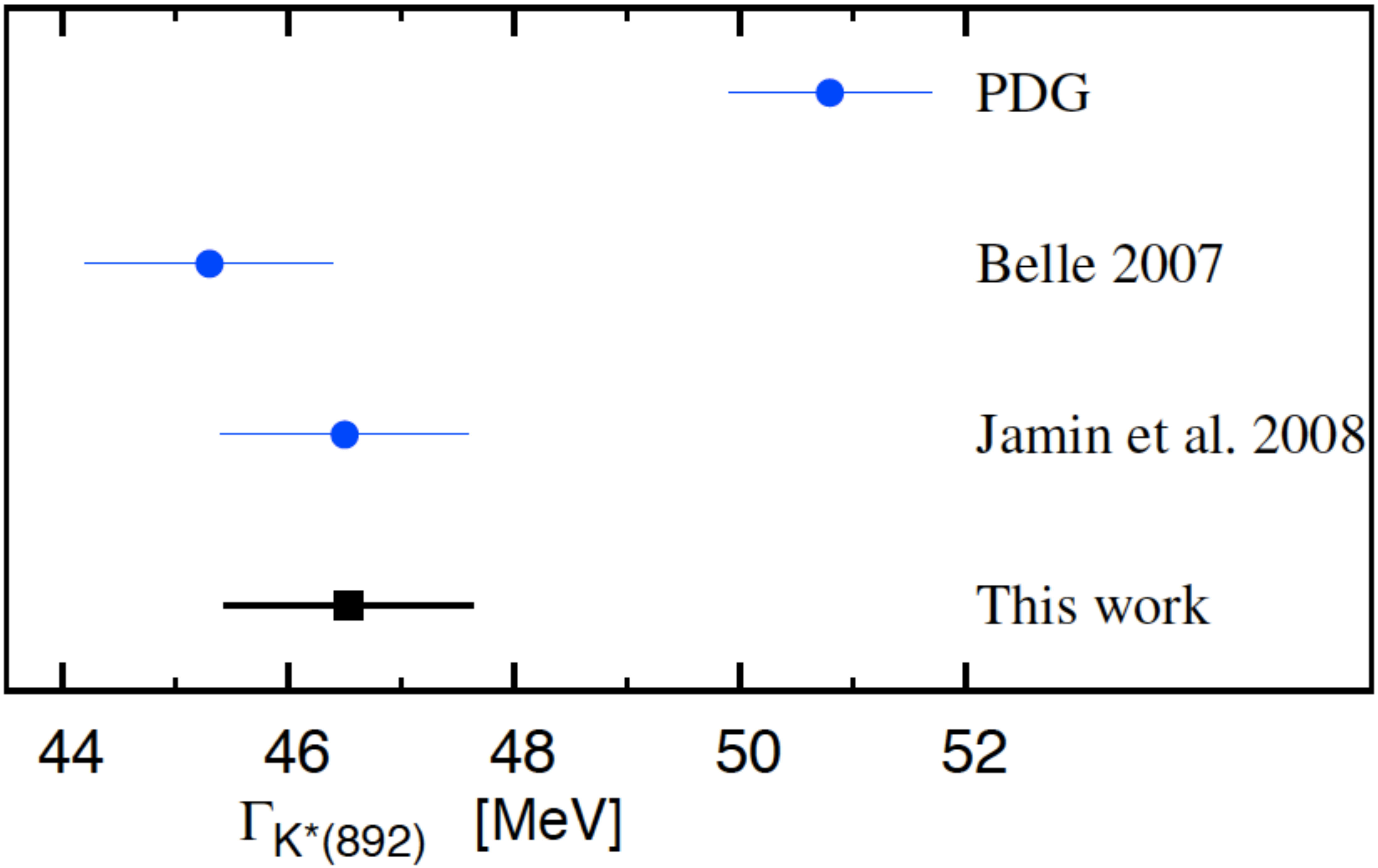


[This work]
D. R. Boito, R. Escribano and M. Jamin,
JHEP accepted

- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

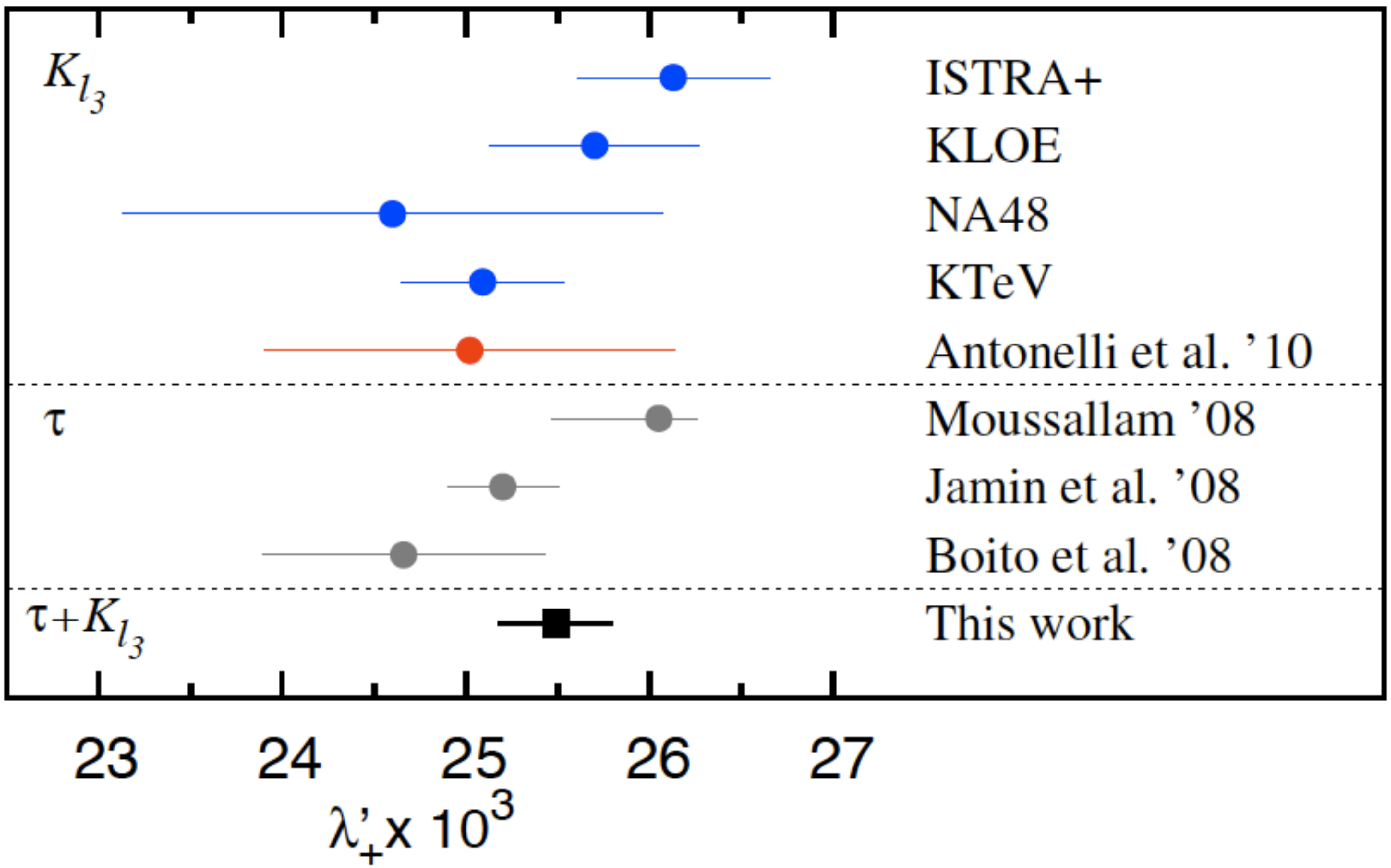
$K^*(892)^\pm$ pole width

$$\Gamma_{K^*(892)^\pm} = 46.53 \pm (0.38)_{\rm stat} \pm (1.0)_{\rm sys}\ \mathrm{MeV}$$



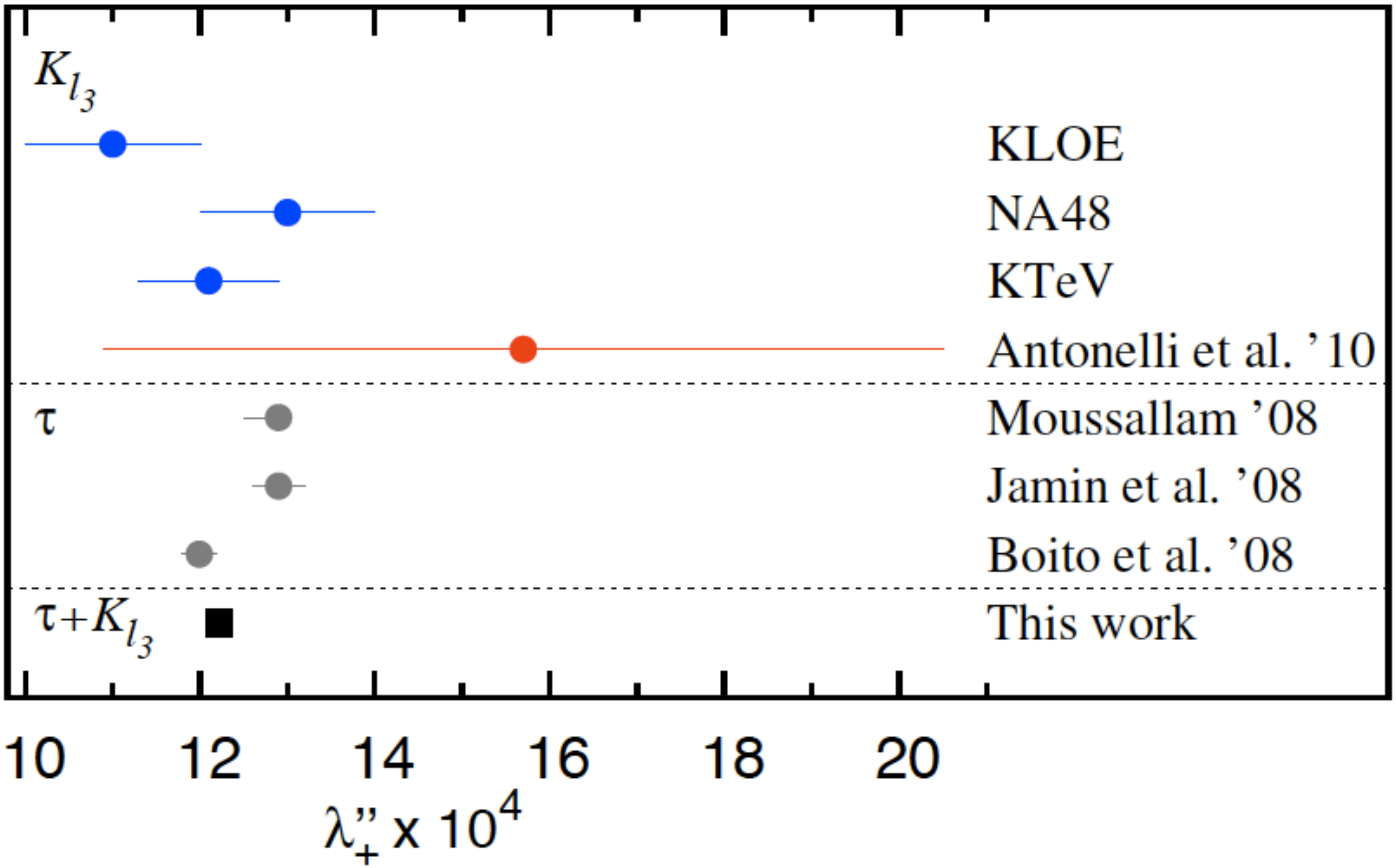
- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

$$\lambda'_+ \times 10^3 = 25.49 \pm (0.30)_{\rm stat} \pm (0.06)_{s_{\rm cut}}$$



- *Fit to* $\tau \to K\pi\nu_\tau$ *with restrictions from* K_{l3}

$$\lambda''_{+} \times 10^4 = 12.22 \pm (0.10)_{\rm stat} \pm (0.10)_{s_{\rm cut}}$$



- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

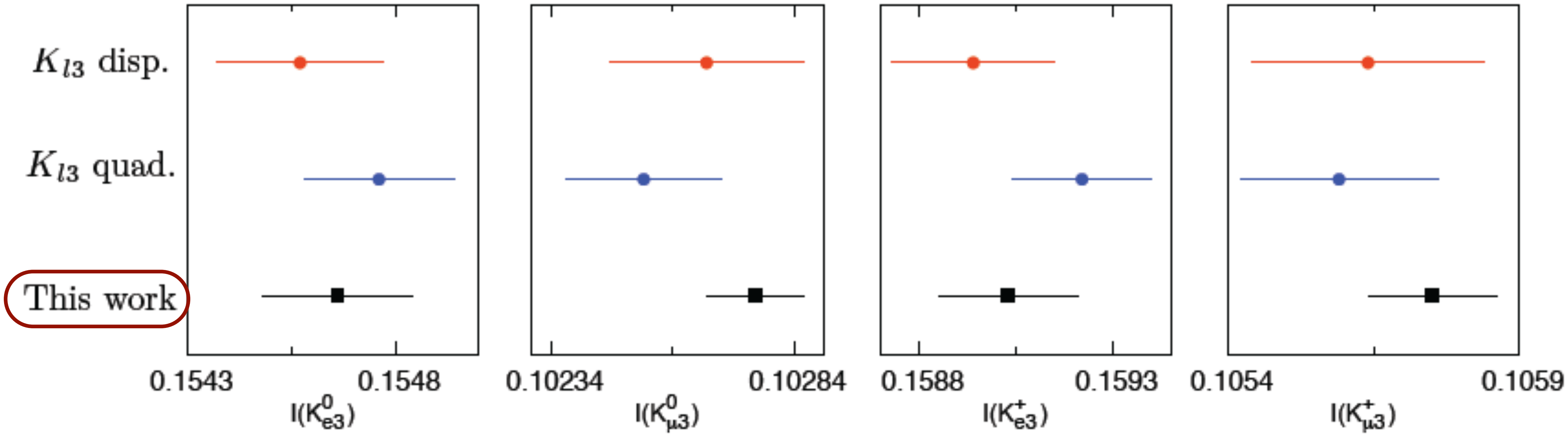
K_{l3} phase-space integrals

$$I_{K_{l_3}} = \frac{1}{m_K^2} \int_{m_l^2}^{(m_K - m_\pi)^2} \mathrm{d}t\, \lambda(t)^{3/2} \left(1 + \frac{m_l^2}{2t}\right) \left(1 - \frac{m_l^2}{t}\right)^2 \left(|\tilde{f}_+(t)|^2 + \frac{3\, m_l^2 (m_K^2 - m_\pi^2)^2}{(2t + m_l^2)\, m_K^4\, \lambda(t)} |\tilde{f}_0(t)|^2 \right)$$

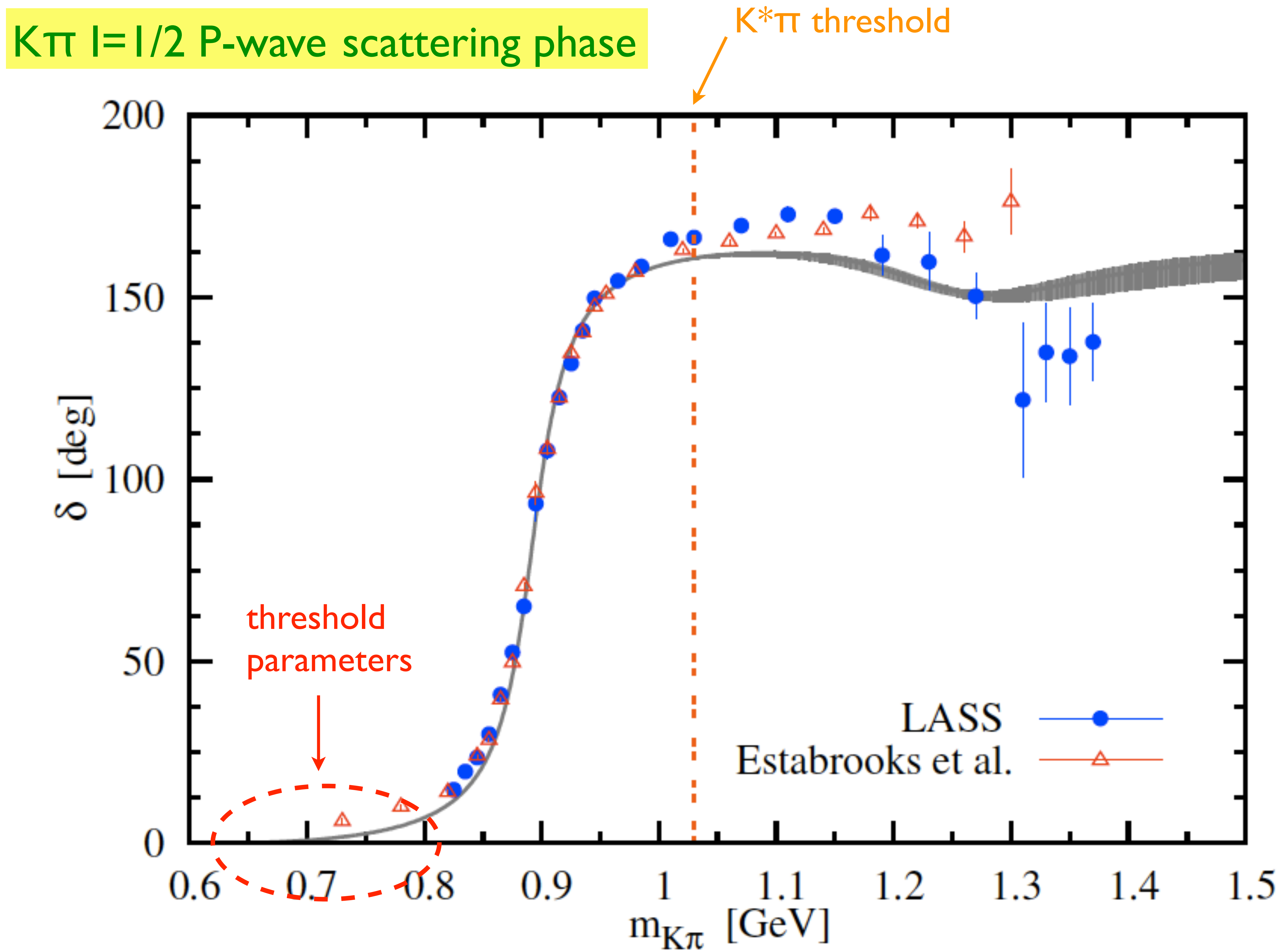
$$\lambda(t) = 1 + t^2/m_K^4 + r_\pi^4 - 2\, r_\pi^2 - 2\, r_\pi^2\, t/m_K^2 - 2\, t/m_K^2$$

	This Work	K_{l_3} disp. [9]	K_{l_3} quad. [9]
$I_{K^0_{e3}}$	0.15466(17)	0.15476(18)	0.15457(20)
$I_{K^0_{\mu 3}}$	0.10276(10)	0.10253(16)	0.10266(20)
$I_{K^+_{e3}}$	0.15903(17)	0.15922(18)	0.15894(21)
$I_{K^+_{\mu 3}}$	0.10575(11)	0.10559(17)	0.10564(20)

[9] M. Antonelli *et. al.*, arXiv:1005.2323



- *Fit to* $\tau \to K\pi\nu_\tau$ *with restrictions from* K_{l3}



- *Fit to τ→Kπν$_\tau$ with restrictions from K$_{l3}$*

Kπ I=1/2 P-wave threshold parameters

$$\frac{2}{\sqrt{s}}\mathrm{Re}\, t_l^I(s) = \frac{1}{2q}\sin 2\delta_l^I(q) = q^{2l}\left[a_l^I + b_l^I\, q^2 + c_l^I\, q^4 + \mathcal{O}(q^6)\right]$$

	This work	[60]	[61]	[62]	[48]
$m_{\pi^-}^3\, a_1^{1/2} \times 10$	0.166(4)	0.16(3)	0.18	0.18(3)	0.19(1)
$m_{\pi^-}^5\, b_1^{1/2} \times 10^2$	0.258(9)	-	-	-	0.18(2)
$m_{\pi^-}^7\, c_1^{1/2} \times 10^3$	0.90(3)	-	-	-	0.71(11)

[48] P. Büttiker, S. Descotes-Genon and B. Moussallam, EPJC 33 (2004) 209

[60] V. Bernard, N. Kaiser and U. G. Meißner, NPB 357 (1991) 129

[61] J. Bijnens, P. Dhonte and P. Talavera, JHEP 05 (2004) 036

[62] V. Bernard, N. Kaiser and U. G. Meißner, NPB 364 (1991) 283

• *Summary and Conclusions*

We have presented a model aimed at describing the $K\pi$ vector form factor using a dispersive representation and incorporating constraints from K_{l3} decays suited to describe both $\tau\to K\pi\nu_\tau$ and K_{l3} decays simultaneously

A good detemination of the $K\pi$ vector f.f. and resonance parameters is obtained from a fit of the $\tau\to K\pi\nu_\tau$ spectrum

Competitive results for the $K^*(892)^\pm$ pole mass and width, slope and curvature parameters, K_{l3} phase-space integrals, $K\pi$ I=1/2 P-wave scattering phase and threshold parameters are obtained

A combined fit of the $\tau\to K\pi\nu_\tau$ and K_{l3} spectra should be done in the future