$|V_{ub}|$measurements with inclusive $B \rightarrow X_u \ell \nu$ decays

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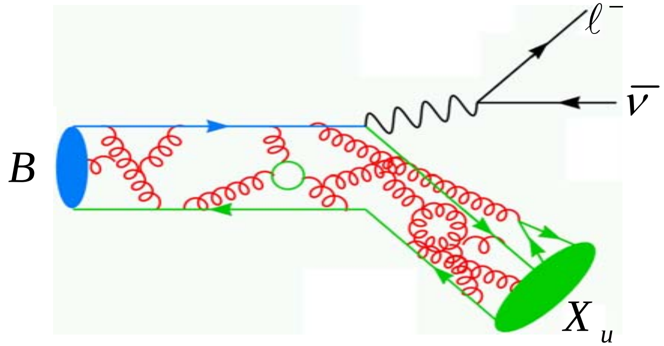
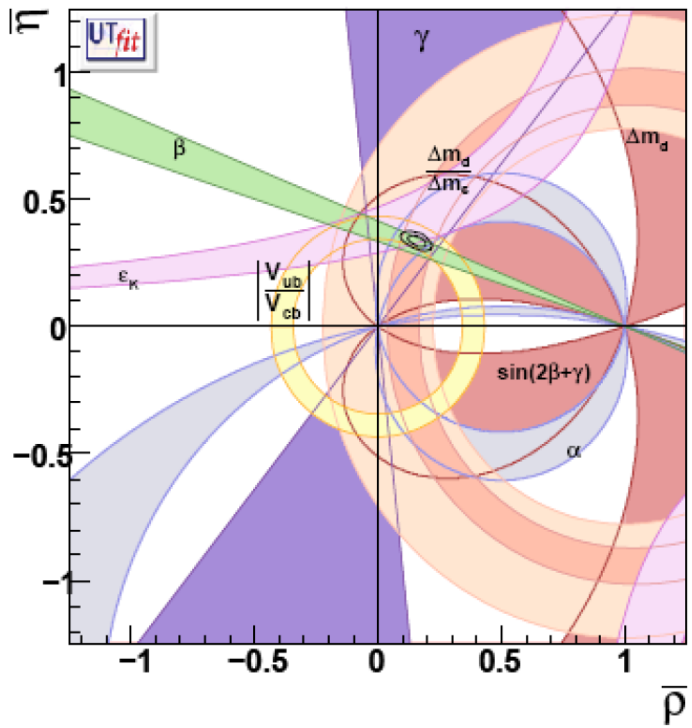
Outline:

- Introduction
- Endpoint analyses
- Recoil analyses: new results from Babar
  limits on weak annihilation
- HFAG averages
- Conclusion

V_{ub} and semileptonic decays

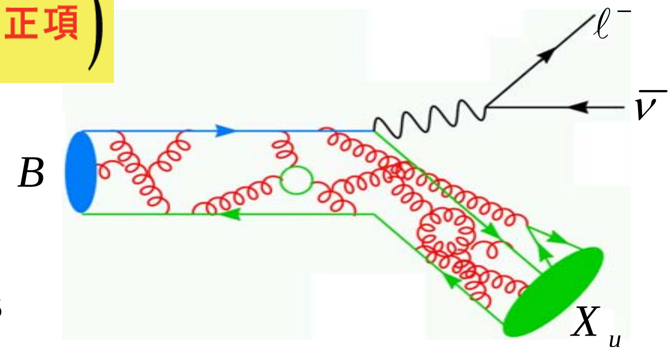
- Hadronic and leptonic currents factorize
- Mature theoretical description
  - QCD corrections to quark-level decay
  - Operator Product Expansion in α_s and Λ/m_b
- Uncertainty on the predicted total decay rate below 5%
- Nevertheless, $|V_{ub}|$ is a limiting factor in CKM precision tests
  - about 7% uncertainty, dominated by theory



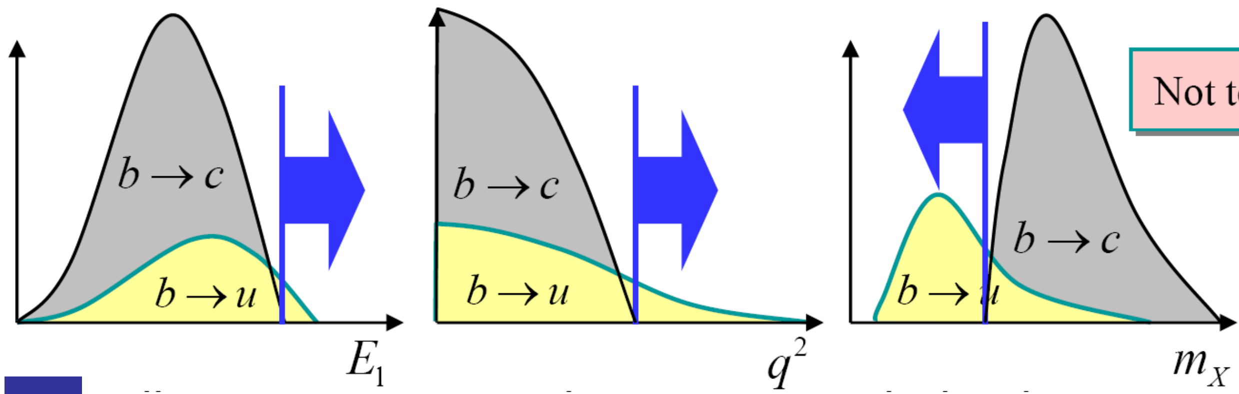


Inclusive charmless decays

- In principle: $\Gamma(b \to u \ell \bar{\nu}) = \frac{G_F^2}{192\pi^2} |V_{ub}|^2 m_b^5 \left(1 + 補正項\right)$
- Unfortunately: $\frac{\Gamma(b \to u \ell \bar{\nu})}{\Gamma(b \to c \ell \bar{\nu})} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}$



- Measurements in restricted kinematic regions



Not to scale!

E_1 = lepton energy

q^2 = dilepton mass squared

m_X = hadron system mass

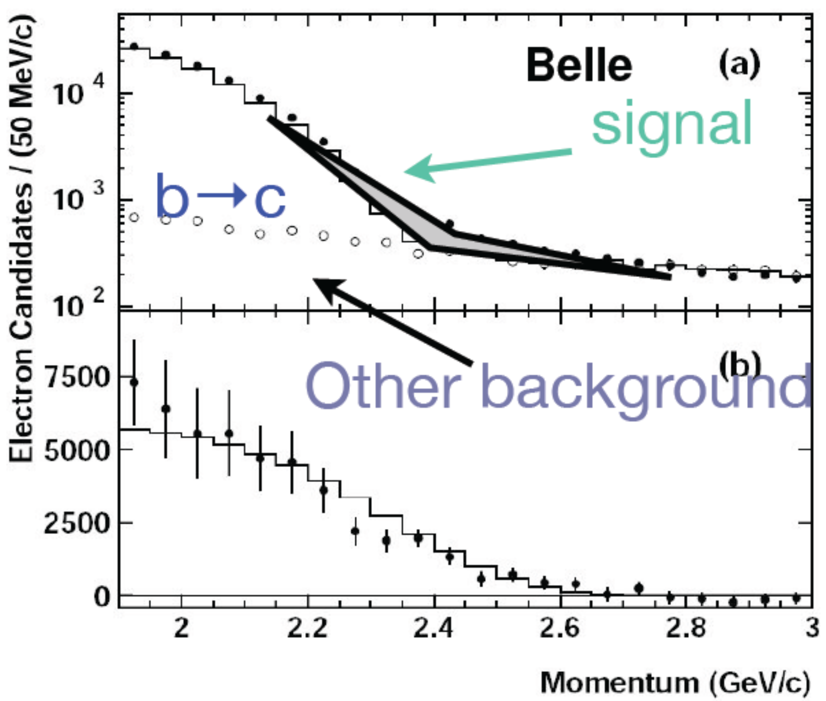
$P_+ = E_X - |\boldsymbol{p}_X|$

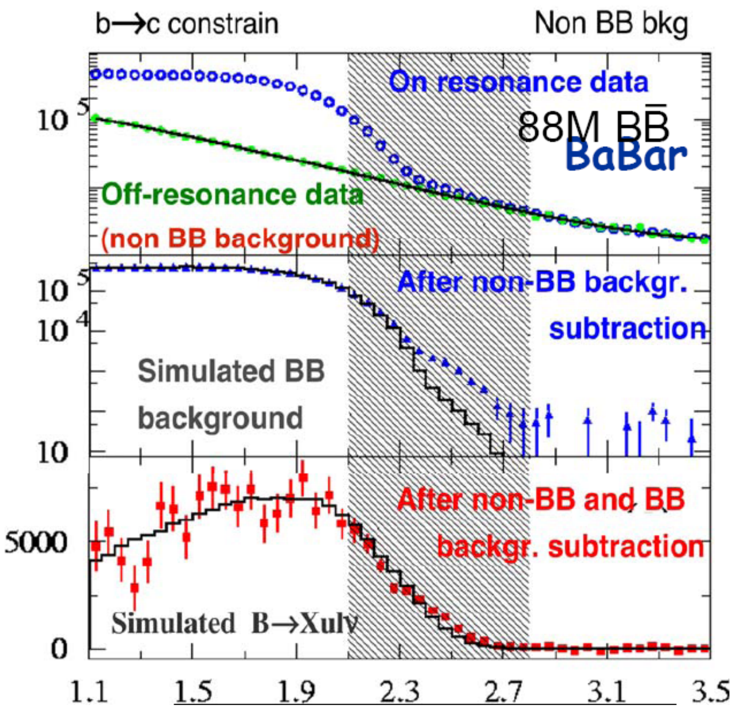
See Einan's talk

- Unfortunately in these regions
  - OPE breaks down
  - need a-priori unknown "shape function" to resum non-perturbative physics
  - Increased sensitivity to m_b
  - Possible weak annihilation effects
- Theory and background subtraction give conflicting requirements → trade-off must be found

“Classic” endpoint analyses

- Typical requirements: missing momentum, event shape
- S/B ~ 1/10, ε<~40%, measurements limited by background knowledge





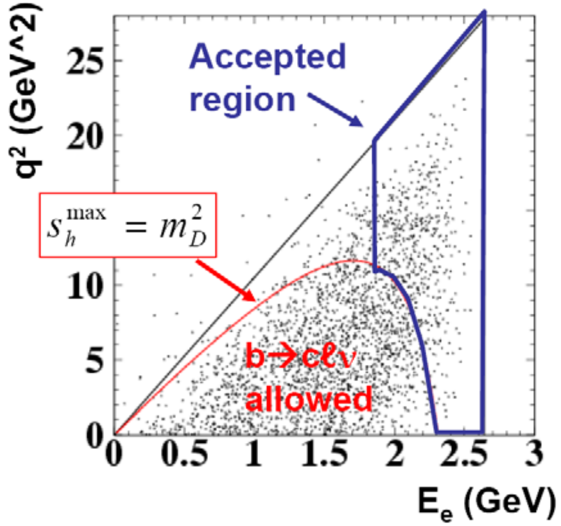
	$\mathcal{L}(fb^{-1})$	E_ℓ(GeV)	$\Delta\mathcal{B}(10^{-4})$	
BaBar	81.4	2.0–2.6	5.72±0.41±0.65	PRD 73, 012006 (2006)
Belle	27.0	1.9–2.6	8.5±0.4±1.5	PL B 621 (2005) 28
CLEO	9.13	2.2–2.6	2.30±0.15±0.35	PRL 88, 231803 (2002)

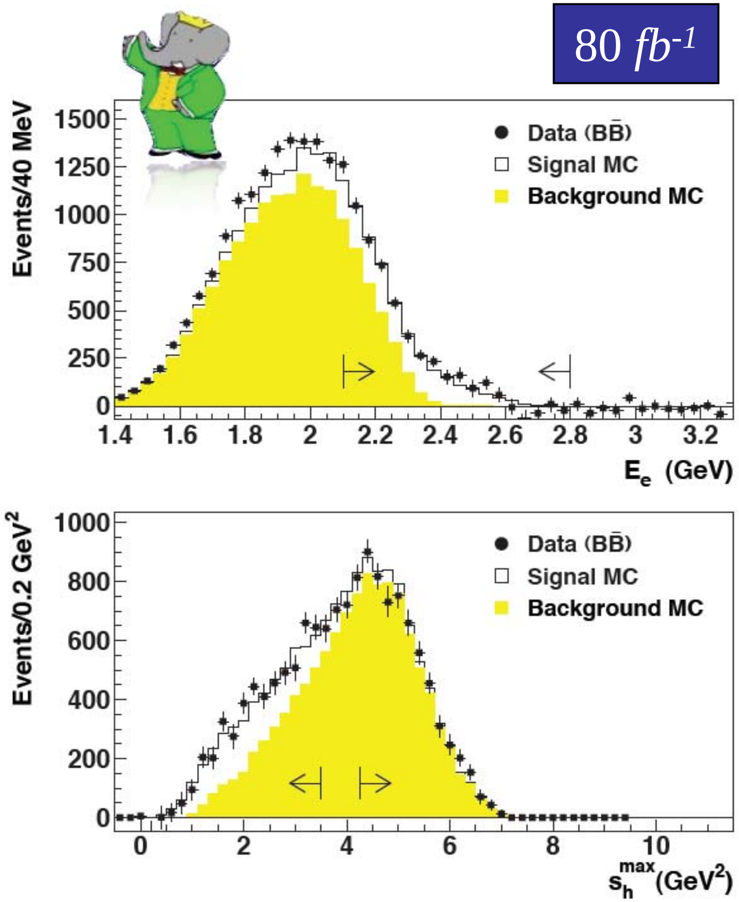
"improved" endpoint analysis

- Separate $b \to c l \nu$ background by using

$$s_h^{\max} = m_B^2 + q^2 - 2m_B\left(E_e + \frac{q^2}{4E_e}\right)$$

- S/B~1/2, ε~25%





BaBar (PRL 95, 111801, 2005
PRL 97, 019903 (2006) Err.)

$$\Delta\mathcal{B}(2.0, 3.5) = (4.41 \pm 0.42 \pm 0.42) \times 10^{-4}$$

Systematics dominated by K_L and neutral particle ID, charm SL decays

Analyses on recoil samples

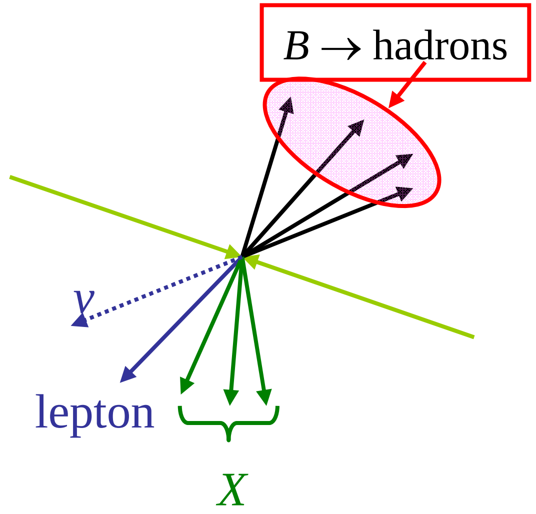
- Y(4S) decay products overlap
- Reconstruct a full decay chain of one B (B_{reco})

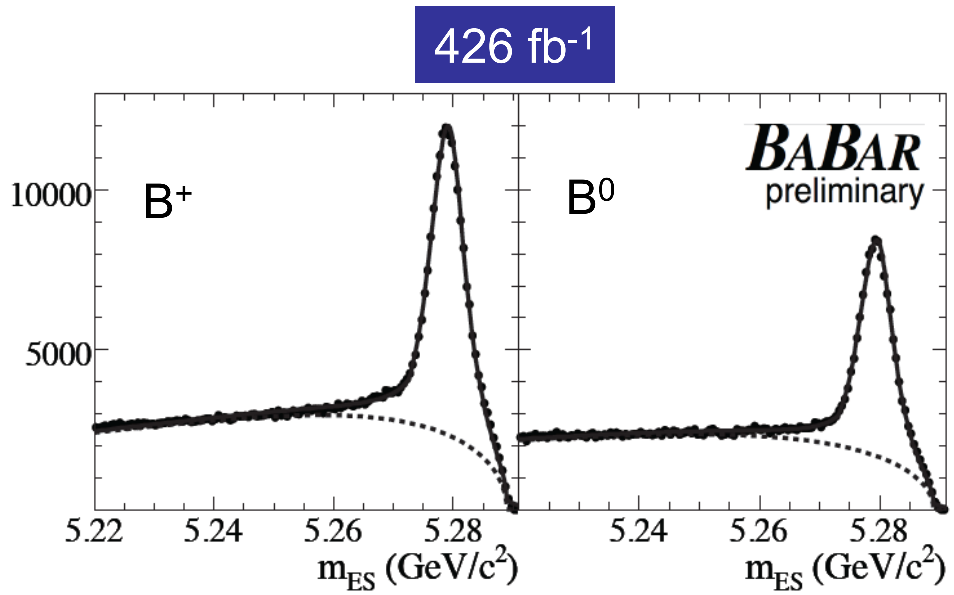
  $B \to D(*)\pi, D(*)\pi\,\pi^0, D(*)3\pi,$ *etc...(~1000 modes)*
- Study the recoiling B
  - Decay products are properly assigned
  - Require an high-momentum lepton (p*>1GeV/c) and missing mass consistent with neutrino
  - Kinematics completely determined → access to m_X, q^2, P_+
  - Low statistics

    (0.3%-0.5% efficiency)
- Subtract non-SL backgrounds by fitting the m_{ES} distribution

$$m_{\mathrm{ES}} = \sqrt{s/4 - \vec{p}_B^{\,2}}$$

Yields ~ 4000 B/fb^{-1}





New: Babar recoil analysis

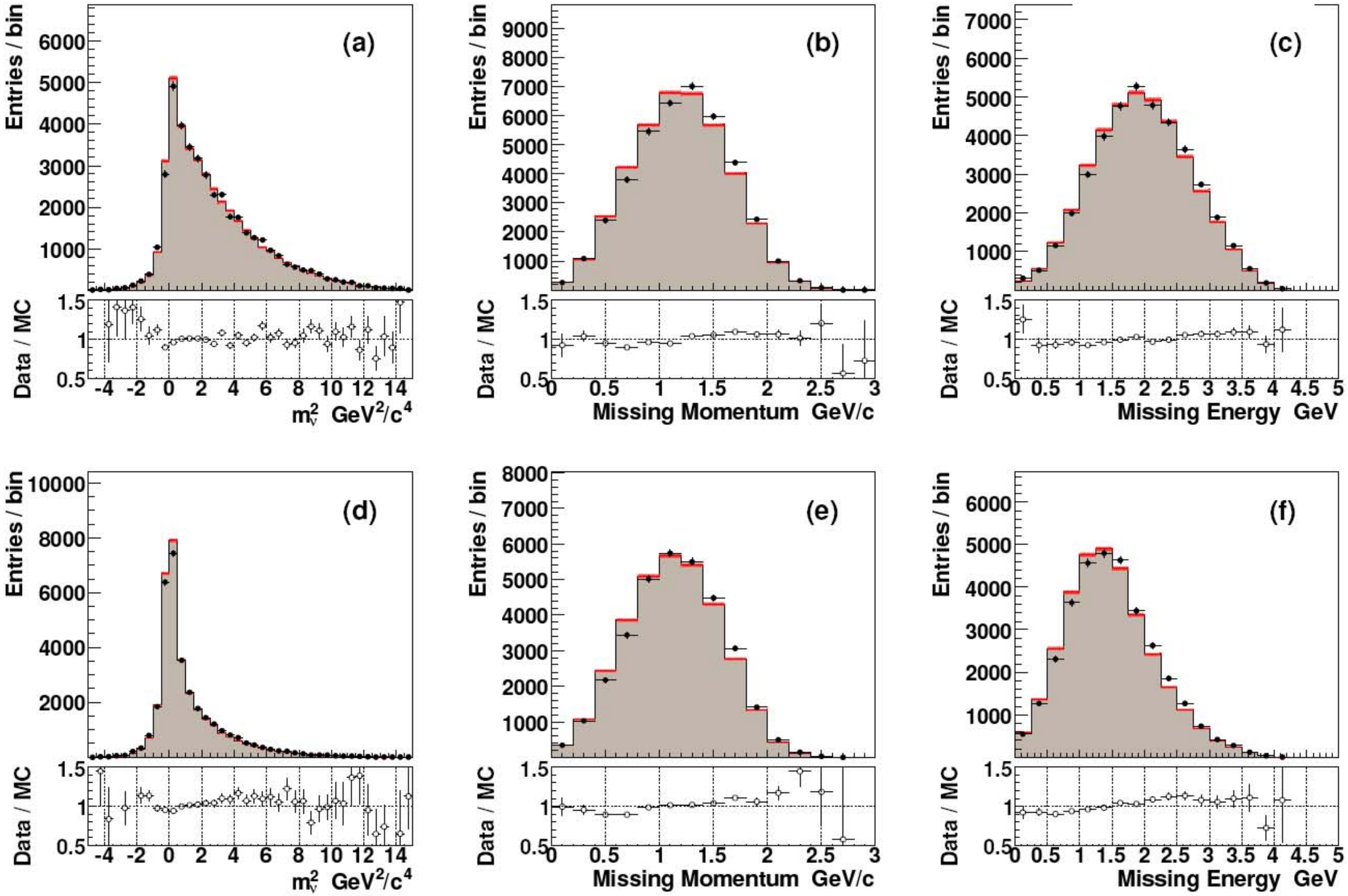
- Update of Phys. Rev. Lett. 100 (2008) 171802 on the full Babar dataset (426 fb^{-1})
- (incremental) improvements on B_{reco} selection and better treatment of systematic uncertainties
- More regions of phase space analysed
  - Full correlation matrix available (see backup slides)
- Results also for charged and neutral B separately
- Select three samples on the recoil side:
  1. Semileptonic (for normalization): at least one lepton with p*>1GeV
  2. $B \to X_u \ell \nu$ signal-enhanced: vetoes on kaons & soft pions from $D^* \ell \nu$, requirements on missing mass, event charge and charge correlations
  3. $B \to X_u \ell \nu$ signal-depleted: reverse kaon and $D^* \ell \nu$ vetoes and check data-MC agreement

Data-MC agreement

BABAR
preliminary

Signal-depleted

Signal-enhanced



Extraction of signal yields

- Fit the distributions of kinematic variables in several regions of phase space:
  - $M_X < 1.55\,\mathrm{GeV}/c^2$ and $M_X < 1.70\,\mathrm{GeV}/c^2$,
  - $M_X < 1.70\,\mathrm{GeV}/c^2,\ q^2 > 8.0\mathrm{GeV}^2/c^4$
  - $P_+ < 0.66\,\mathrm{GeV}/c$.
  - $p^*_\ell > 1.3\,\mathrm{GeV}/c$,
  - (M_X, q^2) fit by requiring p*>1GeV/c only
  - p* fits performed also from p*>1.0 to 2.3 GeV/c
- Subtract combinatorial background by fitting m_{ES} in each bin
- Reweight SL decays into P-wave D mesons by using the signal-depleted sample
  - Better fit χ^2, negligible impact on signal yields
  - $N_{D^{**}}/(N_D+N_{D^*}+N_{D^{**}})$ smaller in data than MC
- Normalize to semileptonic sample in order to minimize experimental systematic uncertainties

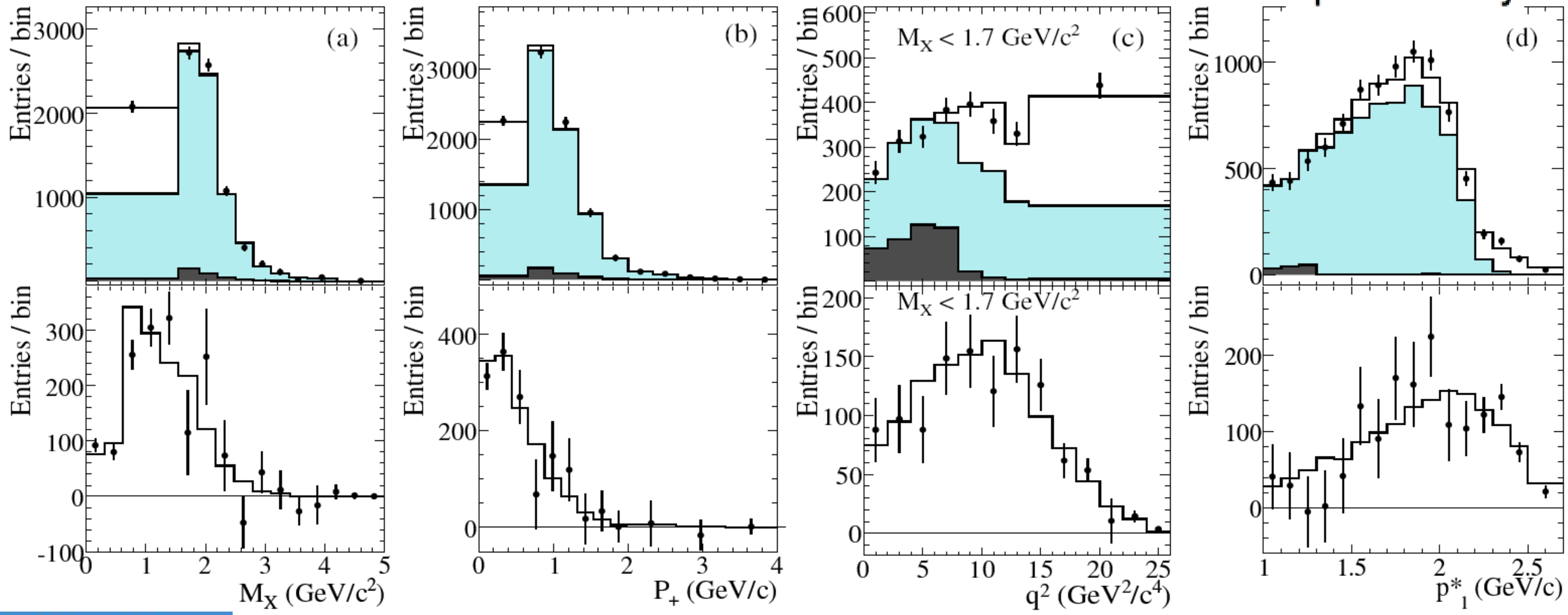
$$\Delta R_{\mathrm{u/sl}} = \frac{\Delta\mathcal{B}(B \to X_u \ell\nu)}{\mathcal{B}(B \to X\ell\nu)} = \frac{(N_u^{\mathrm{fit}})/(\epsilon^u_{\mathrm{sel}}\epsilon^u_{\mathrm{kin}})}{(N_{\mathrm{sl}}^{\mathrm{meas}} - BG_{\mathrm{sl}})} \times \frac{\epsilon_l^{\mathrm{sl}}\epsilon_t^{\mathrm{sl}}}{\epsilon_l^u \epsilon_t^u}$$

Multiply $R_{\mathrm{u/sl}}$ by $(10.66 \pm 0.15)\%$ to obtain $\Delta\mathcal{B}(\overline{B} \to X_u \ell\overline{\nu})$

Results

426 fb^{-1}

BABAR preliminary



(Lower row: background-subtracted distributions)

- $B \to X_u l n$
- $B \to X_c l n$ (bkgd)
- $B \to X_u l n$ (outside selected region)

Regions of phase space	N_u	$\epsilon^u_{\rm sel}\epsilon^u_{\rm kin}$	$\Delta\mathcal{B}(B \to X_u \ell\nu)$ (10^{-3})
(a) $M_X < 1.55\,\mathrm{GeV}/c^2$	1033 ± 73	0.365 ± 0.002	$1.08 \pm 0.08 \pm 0.06$
$M_X < 1.70\,\mathrm{GeV}/c^2$	1089 ± 82	0.370 ± 0.002	$1.15 \pm 0.10 \pm 0.08$
(b) $P_+ < 0.66\,\mathrm{GeV}$	902 ± 80	0.375 ± 0.003	$0.98 \pm 0.09 \pm 0.08$
(c) $M_X < 1.70\,\mathrm{GeV}/c^2, q^2 > 8\mathrm{GeV}^2/c^4$	665 ± 53	0.386 ± 0.003	$0.68 \pm 0.06 \pm 0.04$
$(M_X, q^2),\ p^*_\ell > 1\,\mathrm{GeV}/c$	1441 ± 102	0.338 ± 0.002	$1.80 \pm 0.13 \pm 0.15$
$p^*_\ell > 1.0\,\mathrm{GeV}/c$	1462 ± 137	0.339 ± 0.002	$1.76 \pm 0.16 \pm 0.18$
(d) $p^*_\ell > 1.3\,\mathrm{GeV}/c$	1326 ± 118	0.359 ± 0.002	$1.50 \pm 0.13 \pm 0.14$

Error budgets

	Babar preliminary						Belle
Source $\sigma(\Delta\mathcal{B}(B \to X_u \ell\nu))$	$M_X < 1.55$ GeV/c^2	$M_X < 1.70$ GeV/c^2	$P_+ < 0.66$ GeV	$M_X < 1.70$ GeV/c, $q^2 > 8$GeV^2/c^4	(M_X, q^2) $p^*_\ell > 1.0$ GeV/c	$p^*_\ell > 1.3$ GeV/c	$p^*_\ell > 1.0$ GeV/c
Statistical	**7.1**	**8.9**	**8.9**	**8.0**	**7.1**	**8.9**	**8.8**
MC statistics	**1.3**	**1.3**	**1.3**	**1.6**	**1.1**	**1.2**	
Detector-related	2.8	3.7	5.5	4.1	3.2	2.7	3.3
Fit-related	2.7	4.9	3.2	3.2	2.1	2.5	3.6
Signal model	2.7	3.0	3.5	1.9	**6.6**	**7.9**	**6.3**
Background model	2.0	2.6	3.4	2.8	2.8	2.2	1.7
Total syst	**5.2**	**6.3**	**8.1**	**6.2**	**8.1**	**9.0**	**8.1**
Total error	**8.9**	**11.0**	**12.1**	**10.3**	**10.8**	**12.7**	**12.0**

- Statistical accuracies: 7-9%
- Systematic uncertainties dominated by signal model in most inclusive analyses
- total uncertainties: 9-13% → ~4-6% on $|V_{ub}|$

Phys.Rev.Lett.104:021801,2010

604 fb^{-1}

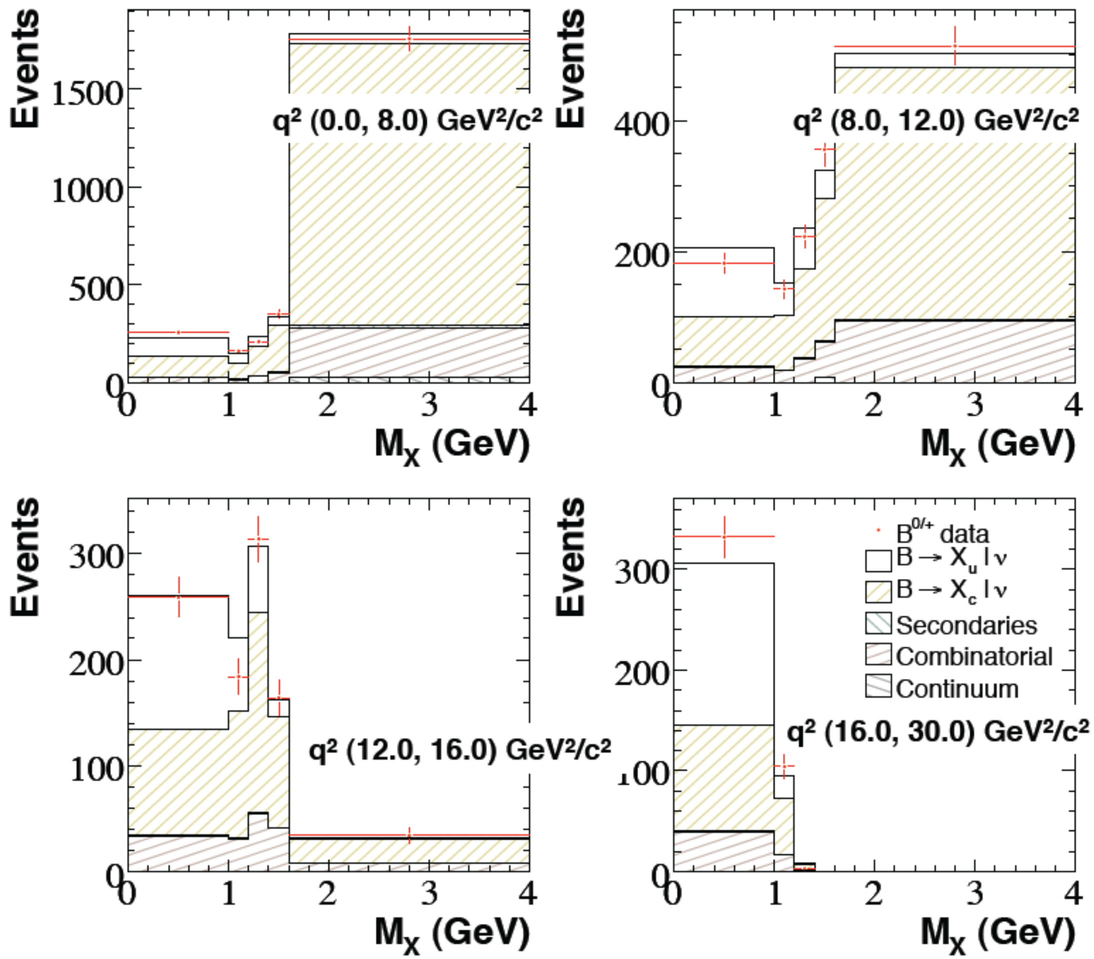
Belle recoil analysis

- Instead of using cuts, exploit non-linear correlations between kinematic and event variables available in recoil sample to separate b→u and b→c.
- Boosted Decision Tree (BDT) based selection, use many event parameters from the full reconstruction sample.
- ε~22%
- Require a lepton with p*>1GeV/c
- Fit (M_x,q^2) distribution with no cuts other than p* and BDT
- Do not fit m_{ES} distributions, estimate combinatorial from MC and normalization from sideband region
- Normalize to number of tags instead of semileptonic sample → measure absolute BR

$$\Delta\mathcal{B} = \frac{N^{\Delta}_{b\to u}}{(2\epsilon^{\Delta}_{b\to u} N_{\rm tag})}(1-\delta_{\rm rad})$$

Belle recoil analysis: results

~1032 ± 91 (stat) Events



Systematics

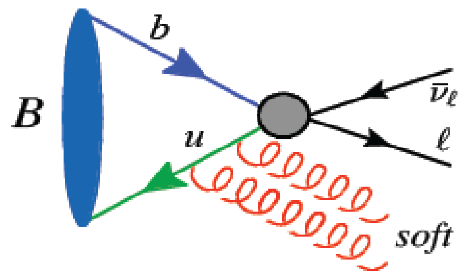
$p^{*B}_{\ell} > 1.0$ GeV	ΔBR/BR (%)
BR($D^{(*)}\ \ell\ \nu$)	1.2
FF($D^{(*)}\ \ell\ \nu$)	1.2
BR & FF ($D^{(**)}\ \ell\ \nu$)	0.2
SF ($X_u\ \ell\ \nu$)	3.6
X_u ($g \rightarrow ss$)	1.5
BR($\pi/\rho/\omega\ \ell\ \nu$)	2.3
BR($\eta/\eta'\ \ell\ \nu$)	3.2
BR($X_{unmeasured}\ \ell\ \nu$)	2.9
Continuum/Combinatorial	1.8
Secondaries/Fakes/Fit	1.0
Particle ID/Reconstruction	3.1
BDT	3.1
Systematics	**8.1**
Statistics	**8.8**

Phys.Rev.Lett.104:021801,2010

$BR(B \rightarrow X_u l\nu) \times 10^{-3}$ = 1.963 x (1 ± 0.088 (stat) ± 0.081 (sys))

604 fb^{-1}

Limits on WA

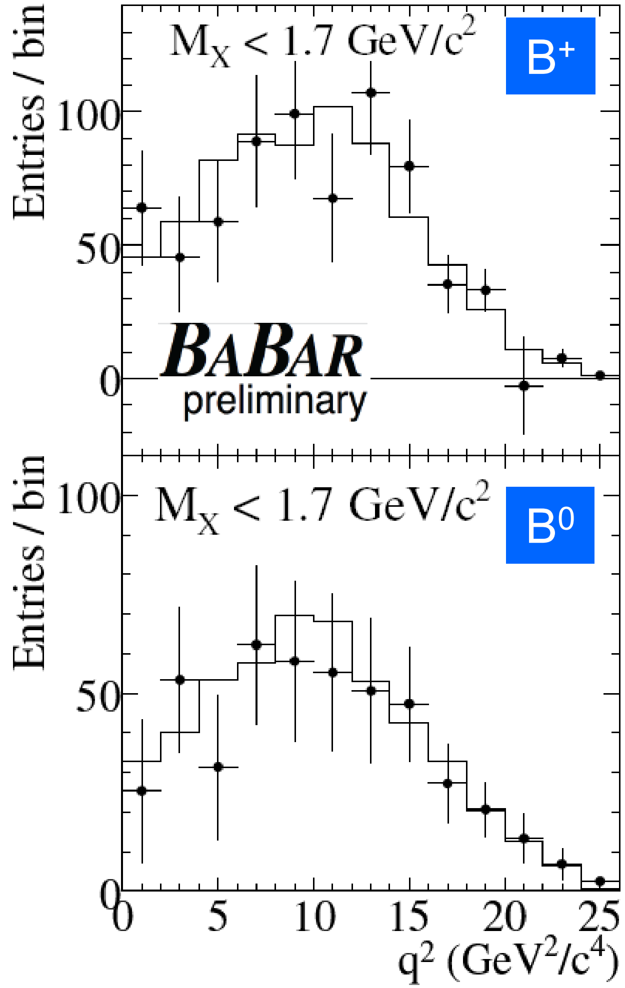


- PBF for charged and neutral B meson decays have also been measured
- They can be used to set a limit on weak annihilation (WA) in B+ decays

$$\frac{\gamma_{WA}}{\Gamma} = \frac{f_u}{f_{WA}} \cdot R^{+/0},$$

$$R^{+/0} = \frac{\Delta\Gamma^+}{\Delta\Gamma^0} = \frac{\tau^0}{\tau^+} \cdot \frac{\Delta\mathcal{B}(B^+ \to X_u \ell\nu)}{\Delta\mathcal{B}(B^0 \to X_u \ell\nu)}$$

Phase Space Region	$R^{+/0}-1$	f_u	C.L. (90%)
$M_X \le 1.70$ GeV/c^2, $q^2 \ge 8$GeV^2	0.042±0.066±0.009	0.31	$-0.07 \le \gamma_{WA}/\Gamma \le 0.15$
$M_X \le 1.55$ GeV/c^2	-0.020±0.066±0.003	0.47	$-0.13 \le \gamma_{WA}/\Gamma \le 0.09$
$M_X \le 1.70$ GeV/c^2	0.071±0.117±0.011	0.56	$-0.12 \le \gamma_{WA}/\Gamma \le 0.26$
(M_X, q^2) $p^*_\ell > 1.0$ GeV/c	0.109±0.157±0.019	0.87	$-0.15 \le \gamma_{WA}/\Gamma \le 0.37$



Other results:

$$\frac{|\Gamma_{WA}|}{\Gamma_u} < 7.4\%\ at\ 90\%C.L.$$

CLEO, studing the q^2 spectra PRL96,121801 (2006)

$$\frac{|\Gamma_{WA}|}{\Gamma_u} < \frac{3.8\%}{f_{WA}(2.3-2.6)}\ at\ 90\%C.L.$$

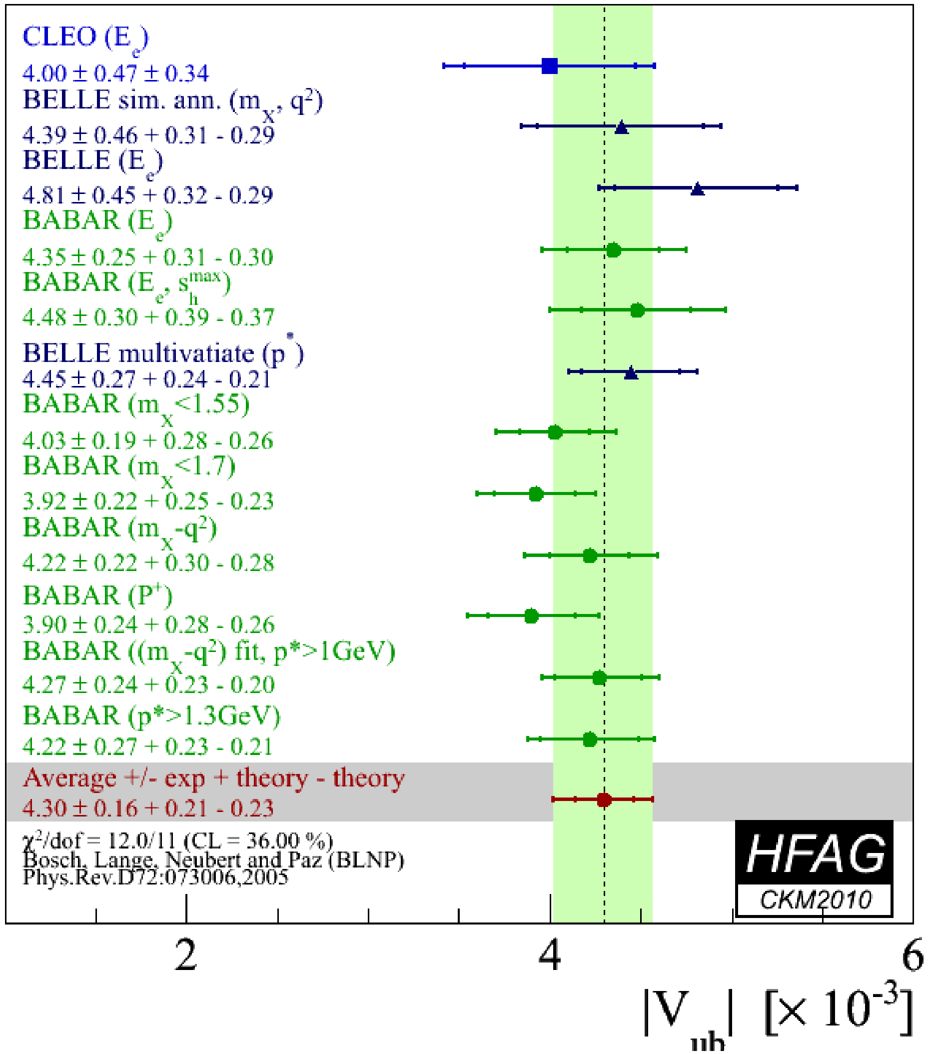
Babar
arXiv: 0708.1753 383 M BB

Preliminary HFAG averages

BLNP

Average of the six Babar determinations is

4.17 ± 0.18 +0.21 -0.23



Error budget:

$$+2.2_{stat}\ +1.7_{exp}\ +1.2_{b2c\ model}\ +1.9_{b2u\ model}\ +2.9_{HQE\ param}\ +0.4_{SF\ func}\ +0.6_{sub\ SF}\ +1.2_{WA}\ +3.7_{matching} = +6.1_{tot}$$

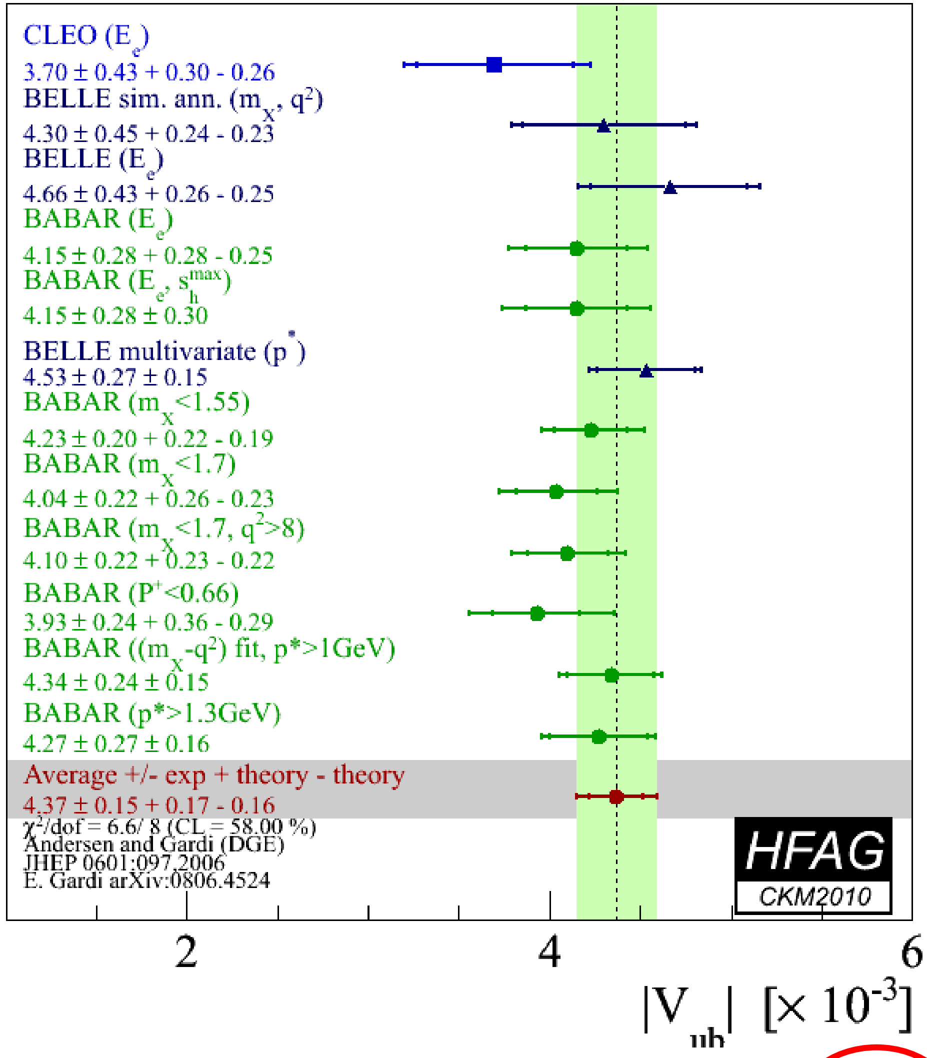
$$-2.3_{stat}\ -1.7_{exp}\ -1.2_{b2c\ model}\ -1.9_{b2u\ model}\ -3.4_{HQE\ param}\ -0.5_{SF\ func}\ -0.7_{sub\ SF}\ -1.2_{WA}\ -3.7_{matching} = -6.4_{tot}$$

Preliminary HFAG averages

DGE

Average of the six Babar determinations is

4.27 ± 0.17 +0.18 -0.17



Error budget:

$+2.0_{stat}$ $+1.7_{exp}$ $+1.2_{b2c\ model}$ $+2.0_{b2u\ model}$ $+0.4_{alpha\_s\ R\_CUT}$ $+3.5_{mb}$ $+1.3_{WA}$ $+0.4_{DGE\ theory}$ = $+5.2_{tot}$

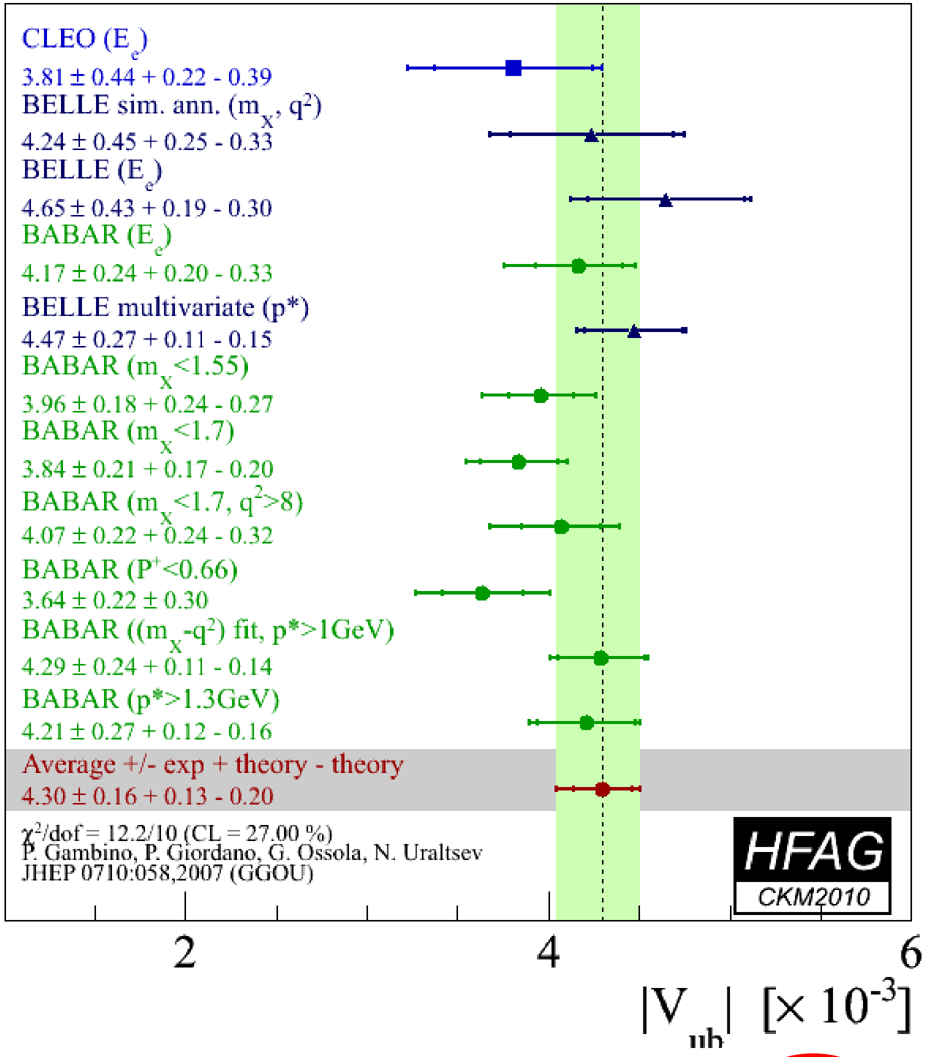
-2.0_{stat} -1.6_{exp} $-1.2_{b2c\ model}$ $-1.8_{b2u\ model}$ $-0.4_{alpha\_s\ R\_CUT}$ -3.5_{mb} -1.3_{WA} $-0.5_{DGE\ theory}$ = -5.0_{tot}

Preliminary HFAG averages

GGOU

Average of the six Babar determinations is

4.20 ± 0.19 +0.13 -0.18



Error budget:

$+2.3_{stat}$ $+1.9_{exp}$ $+1.2_{b2c\ model}$ $+1.6_{b2u\ model}$ $+2.5_{par.}$ $+1.5_{pert.}$ $+1.7_{q2*}$ $+0._{WA}$ $+0.5_{ff}$ = $+4.9_{tot}$

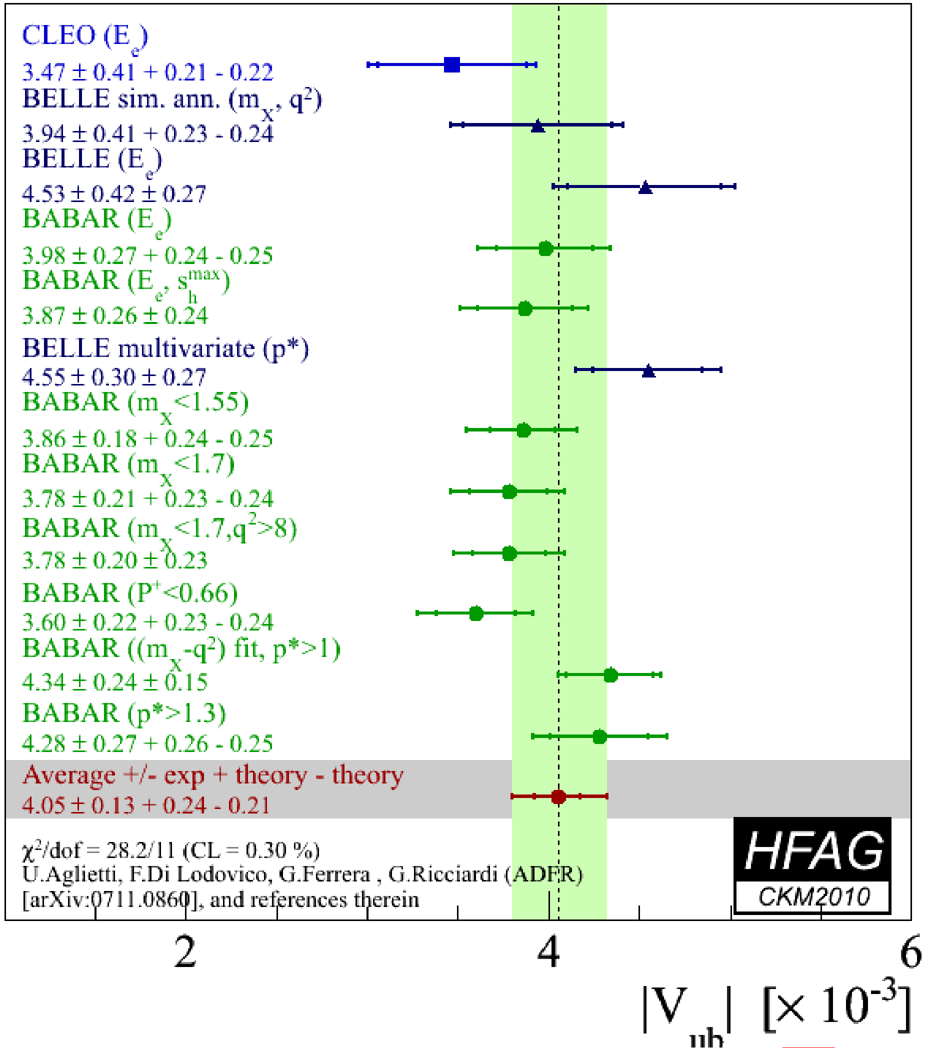
-2.3_{stat} -1.9_{exp} $-1.2_{b2c\ model}$ $-1.6_{b2u\ model}$ $-2.5_{par.}$ $-1.5_{pert.}$ -1.7_{q2*} -3.9_{WA} -0.2_{ff} = -6.3_{tot}

Preliminary HFAG averages

ADFR

Average of the six
Babar determinations is

3.96 ± 0.16 +0.23 -0.21



Error budget:

$+1.9_{stat}$ $+1.8_{exp}$ $+1.3_{b2c\ model}$ $+1.2_{b2u\ model}$ $+0.7_{alpha\_s}$ $+1.7_{Vcb}$ $+0.7_{mb}$ $+4.4_{mc}$ $+1.0_{BF}$ $+3.2_{model}$ $= +6.7_{tot}$

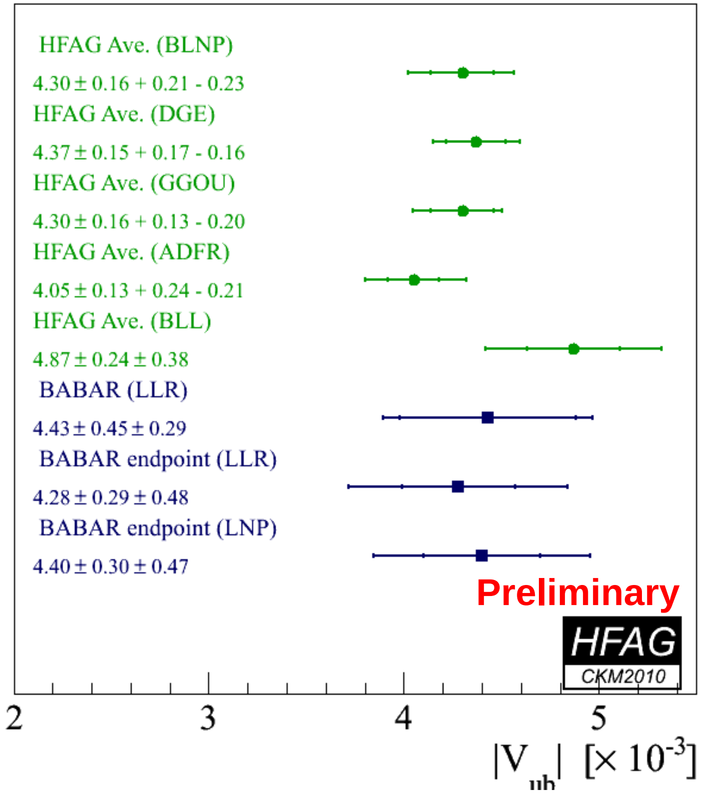
-1.9_{stat} -1.8_{exp} $-1.4_{b2c\ model}$ $-1.3_{b2u\ model}$ -1.2_{alpha_s} -1.7_{Vcb} -0.8_{mb} -4.4_{mc} -0.9_{BF} -3.2_{model} $= -6.9_{tot}$

Conclusions

- Partial branching fraction measurements performed in several phase space regions
  - Important for testing theoretical predictions
- Comparable statistical and systematic uncertainties (~8% each)
  - Signal modeling dominates (~6%) the most inclusive recoil analyses
  - Detector (K_L, PID) and m_{ES} fit modeling (~4-5%) follow
  - Background modeling dominates endpoint analyses
- are the current limits on WA useful at all?

- Inclusive $|V_{ub}|$ determinations for different calculations give similar theory uncertainties; the spread among calculations is comparable to the theory errors
- Total uncertainty on inclusive $|V_{ub}|$ determinations at the 6% level, dominated by parametric errors (e.g. ~4% from m_b)
  - BUT: NNLO calculation not included: sizeable change for BLNP!

(Pecjak et al; see Einan's talk)



Systematic uncertainties

BABAR preliminary

Source $\sigma(\Delta\mathcal{B}(B \to X_u \ell\nu))$	$M_X < 1.55$ GeV/c^2	$M_X < 1.70$ GeV/c^2	$P_+ < 0.66$ GeV	$M_X < 1.70$ GeV/c, $q^2 > 8$GeV^2/c^4	(M_X, q^2) $p^*_\ell > 1.0$ GeV/c	$p^*_\ell > 1.0$ GeV/c	$p^*_\ell > 1.3$ GeV/c
Statistical error	7.1	8.9	8.9	8.0	7.1	9.4	8.9
MC statistics	1.3	1.3	1.3	1.6	1.1	1.1	1.2
Detector-related:							
Tracking efficiency	0.4	1.0	1.1	1.7	0.7	1.2	0.1
Neutral efficiency	1.3	2.1	4.0	0.7	1.0	0.9	0.9
π^0 efficiency	1.2	0.9	1.1	0.9	0.9	2.9	1.1
PID eff. & misID	1.9	2.4	3.3	2.9	2.3	2.9	2.2
K_L	0.9	1.3	1.1	2.1	1.6	1.3	0.6
Fit related: (tbu)							
m_{ES} fit parameters	2.0	2.7	1.9	2.6	1.9	2.0	2.5
combinatorial backg.	1.8	1.8	2.6	1.8	1.0	2.1	0.5
Signal knowledge:							
SF parameters	$^{2.4}_{-1.6}$	$^{1.8}_{-0.9}$	$^{0.6}_{-1.8}$	$^{0.6}_{-0.9}$	$^{6.0}_{-4.9}$	$^{5.8}_{-7.1}$	$^{7.1}_{-6.1}$
SF form	1.2	1.6	2.6	1.2	1.5	1.1	1.1
Exclusive $B \to X_u \ell\nu$	0.6	1.3	1.6	0.7	1.9	5.3	3.4
Gluon splitting	1.2	1.6	1.1	1.0	2.7	3.1	2.4
Background knowledge:							
K_S veto	0.8	1.4	1.7	2.1	1.2	1.3	0.3
B SL branching ratio	0.9	1.4	1.5	1.4	1.0	0.8	0.7
D decays	1.1	0.6	1.1	0.6	1.1	1.6	1.5
$B \to D\ell\nu$ form factor	0.5	0.5	1.3	0.4	0.4	0.1	0.2
$B \to D^*\ell\nu$ form factor	0.7	0.7	0.9	0.7	0.7	0.7	0.7
$B \to D^{**}\ell\nu$ form factor	0.8	0.9	1.3	0.4	0.9	1.0	0.3
$B \to D^{**}$ reweight	0.4	1.0	1.1	0.7	1.6	0.1	1.2
Total systematics:	$^{5.3}_{-5.0}$	$^{6.4}_{-6.2}$	$^{8.0}_{-8.1}$	$^{6.2}_{-6.2}$	$^{8.5}_{-7.7}$	$^{10.5}_{-11.2}$	$^{9.4}_{-8.7}$
Total error:	$^{9.0}_{-8.8}$	$^{11.0}_{-10.9}$	$^{12.0}_{-12.1}$	$^{10.2}_{-10.3}$	$^{11.1}_{-10.5}$	$^{14.1}_{-14.6}$	$^{12.9}_{-12.4}$

This is a backup slide

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell \nu)}{\tau_B \cdot \Delta\Gamma_{theory}}}$$

BABAR preliminary

Calculation	Phase Space Region	$\Delta\Gamma_{theory}(B \to X_u \ell \nu)$ (ps^{-1})	$\|V_{ub}\|(10^{-3})$
BLNP	$M_X \leq 1.55\,\mathrm{GeV}/c^2$	42.0 ± 5.6	$4.03 \pm 0.15 \pm 0.11 \pm 0.28$
	$M_X \leq 1.70\,\mathrm{GeV}/c^2$	47.3 ± 5.9	$3.91 \pm 0.17 \pm 0.12 \pm 0.25$
	$P_+ \leq 0.66\,\mathrm{GeV}$	40.9 ± 5.6	$3.90 \pm 0.18 \pm 0.16 \pm 0.29$
	$M_X \leq 1.70\,\mathrm{GeV}/c^2,\ q^2 \geq 8\mathrm{GeV}^2$	24.3 ± 3.4	$4.22 \pm 0.19 \pm 0.12 \pm 0.30$
	$(M_X,\ q^2)\ p^*_\ell > 1.0\,\mathrm{GeV}/c$	62.7 ± 6.3	$4.27 \pm 0.15 \pm 0.18 \pm 0.26$
	$p^*_\ell > 1.0\,\mathrm{GeV}/c$	62.7 ± 6.3	$4.21 \pm 0.20 \pm 0.22 \pm 0.26$
	$p^*_\ell > 1.3\,\mathrm{GeV}/c$	53.4 ± 5.6	$4.22 \pm 0.19 \pm 0.20 \pm 0.27$
DGE	$M_X \leq 1.55\,\mathrm{GeV}/c^2$	38.3 ± 9.2	$4.23 \pm 0.16 \pm 0.12 \pm 0.23$
	$M_X \leq 1.70\,\mathrm{GeV}/c^2$	44.8 ± 6.2	$4.02 \pm 0.18 \pm 0.12 \pm 0.26$
	$P_+ \leq 0.66\,\mathrm{GeV}$	40.2 ± 10.2	$3.93 \pm 0.18 \pm 0.16 \pm 0.36$
	$M_X \leq 1.70\,\mathrm{GeV}/c^2,\ q^2 \geq 8\mathrm{GeV}^2$	25.6 ± 4.0	$4.10 \pm 0.18 \pm 0.12 \pm 0.23$
	$(M_X,\ q^2) p^*_\ell > 1.0\,\mathrm{GeV}/c$	60.7 ± 1.1	$4.34 \pm 0.16 \pm 0.18 \pm 0.28$
	$p^*_\ell > 1.0\,\mathrm{GeV}/c$	60.7 ± 1.1	$4.28 \pm 0.20 \pm 0.23 \pm 0.21$
	$p^*_\ell > 1.3\,\mathrm{GeV}/c$	53.7 ± 3.3	$4.27 \pm 0.19 \pm 0.19 \pm 0.21$
GGOU	$M_X \leq 1.55\,\mathrm{GeV}$	43.7 ± 6.8	$3.96 \pm 0.15 \pm 0.11 \pm 0.25$
	$M_X \leq 1.70\,\mathrm{GeV}$	49.4 ± 5.2	$3.97 \pm 0.16 \pm 0.11 \pm 0.19$
	$P_+ \leq 0.66\,\mathrm{GeV}$	47.0 ± 14.3	$3.64 \pm 0.17 \pm 0.15 \pm 0.52$
	$M_X \leq 1.70\,\mathrm{GeV},\ q^2 \geq 8\mathrm{GeV}^2$	26.0 ± 7.8	$4.07 \pm 0.18 \pm 0.12 \pm 0.28$
	$(M_X,\ q^2) p^*_\ell > 1.0\,\mathrm{GeV}/c$	62.1 ± 3.4	$4.29 \pm 0.15 \pm 0.18 \pm 0.18$
	$p^*_\ell > 1.0\,\mathrm{GeV}/c$	62.1 ± 3.4	$4.24 \pm 0.20 \pm 0.23 \pm 0.22$
	$p^*_\ell > 1.3\,\mathrm{GeV}/c$	53.6 ± 4.1	$4.23 \pm 0.19 \pm 0.19 \pm 0.23$
ADFR	$M_X \leq 1.55\,\mathrm{GeV}$	–	$3.84 \pm 0.14 \pm 0.11 \pm 0.25$
	$M_X \leq 1.70\,\mathrm{GeV}$	–	$3.96 \pm 0.17 \pm 0.14 \pm 0.25$
	$P_+ \leq 0.66\,\mathrm{GeV}$	–	$3.59 \pm 0.17 \pm 0.15 \pm 0.25$
	$M_X \leq 1.70\,\mathrm{GeV},\ q^2 \geq 8\mathrm{GeV}^2$	–	$3.77 \pm 0.17 \pm 0.12 \pm 0.23$
	$(M_X,\ q^2) p^*_\ell > 1.0\,\mathrm{GeV}/c$	–	$4.35 \pm 0.19 \pm 0.20 \pm 0.29$
	$p^*_\ell > 1.0\,\mathrm{GeV}/c$	–	$4.28 \pm 0.20 \pm 0.23 \pm 0.29$
	$p^*_\ell > 1.3\,\mathrm{GeV}/c$	–	$4.28 \pm 0.19 \pm 0.20 \pm 0.29$

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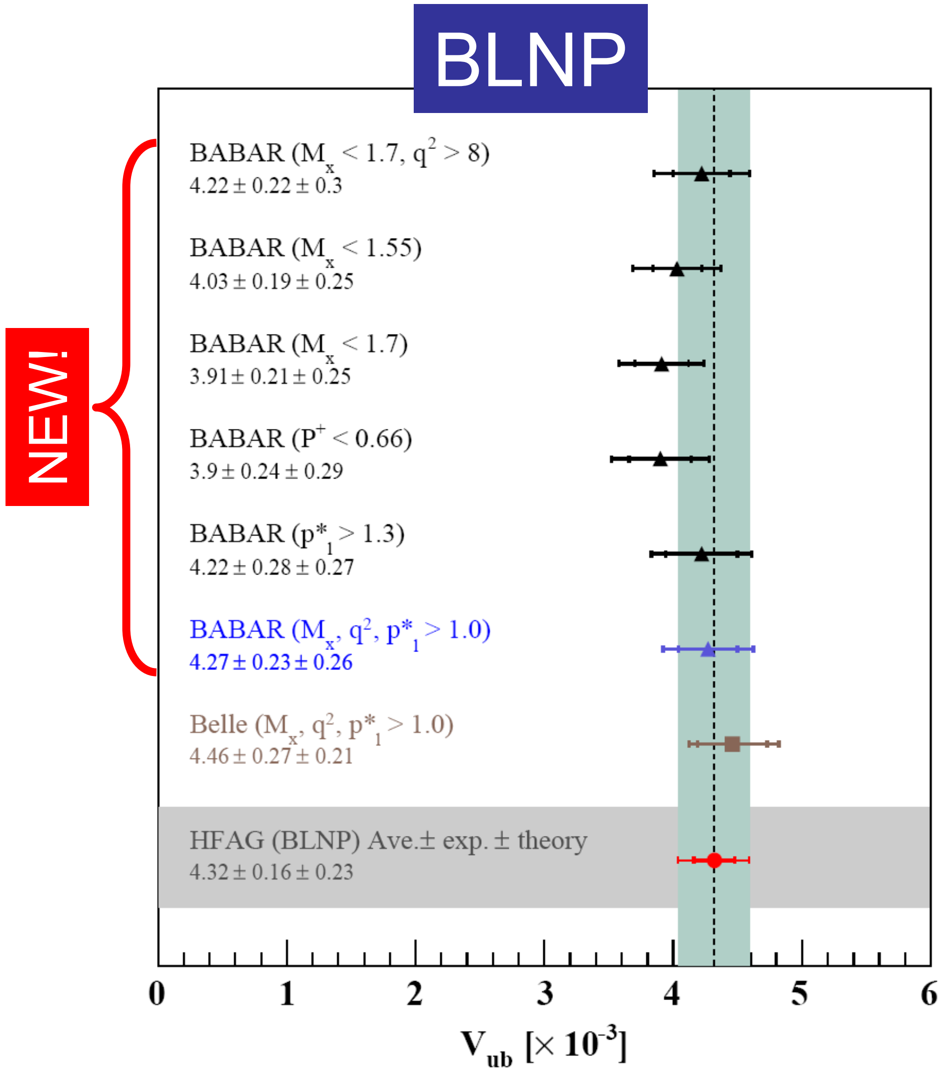
Correlation matrix for Babar analysis

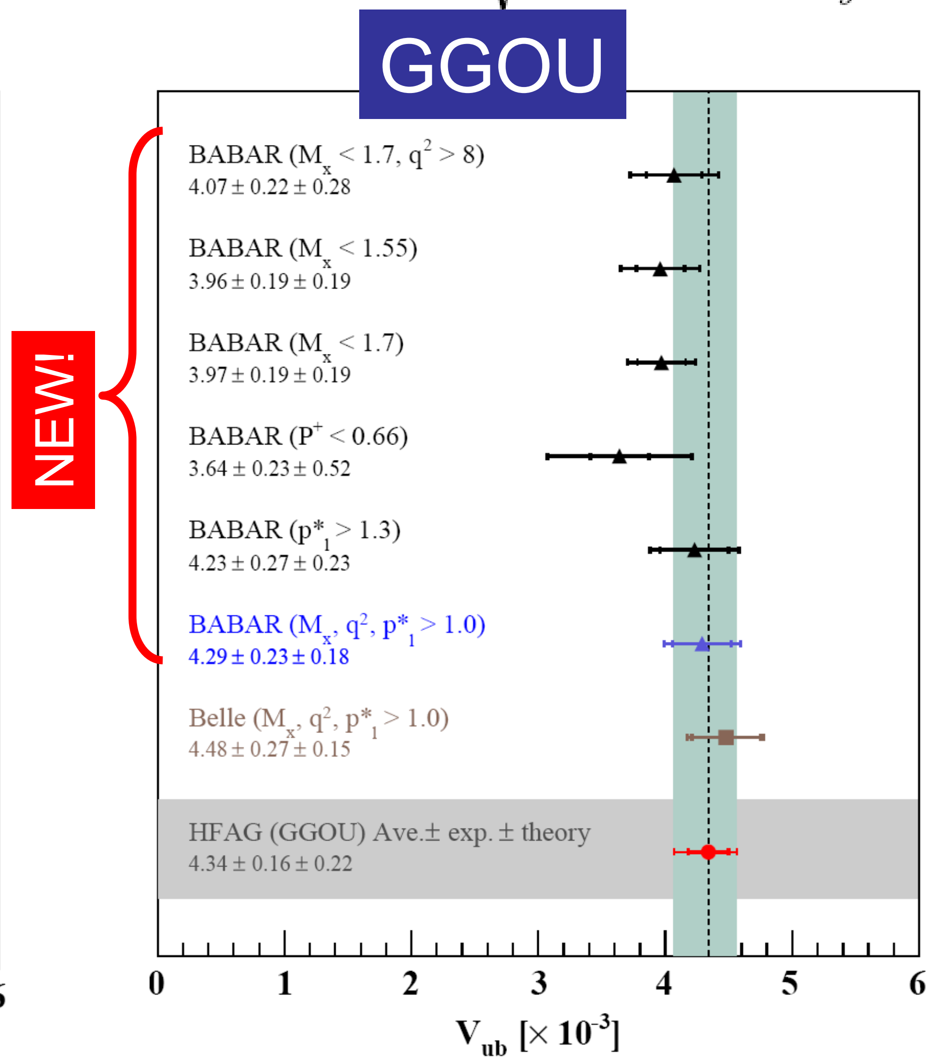
TABLE V: Matrix of statistical correlations between different analyses. The $p_\ell^* > 1\,\mathrm{GeV}/c$ requirement is implicitly assumed in the definitions of phase space regions, unless otherwise noted. The entries above the main diagonal refer to correlations between measurements of partial branching fractions; the entries below the main diagonal (in boldface) refer to correlations on $|V_{ub}|$ measurements. In the latter case, theoretical correlations have been included, as described in the text.

Analysis	$M_X < 1.55$ GeV/c^2	$M_X < 1.70$ GeV/c^2	$P_+ < 0.66$ GeV	$M_X < 1.70\,\mathrm{GeV}/c^2$, $q^2 > 8\mathrm{GeV}^2/\mathrm{c}^4$	(M_X, q^2) $p_\ell^* > 1.0\,\mathrm{GeV}/c$	$p_\ell^* > 1.3$ GeV/c
$M_X < 1.55\,\mathrm{GeV}/c^2$	1	0.77	0.74	0.50	0.72	0.57
$M_X < 1.70\,\mathrm{GeV}/c^2$	**0.81**	1	0.86	0.55	0.94	0.73
$P_+ < 0.66\,\mathrm{GeV}$	**0.69**	**0.81**	1	0.46	0.78	0.61
$M_X < 1.70\,\mathrm{GeV}/c^2, q^2 > 8\mathrm{GeV}^2/\mathrm{c}^4$	**0.40**	**0.46**	**0.38**	1	0.52	0.46
(M_X, q^2), $p_\ell^* > 1\,\mathrm{GeV}/c$	**0.58**	**0.88**	**0.67**	**0.34**	1	0.74
$p_\ell^* > 1.3\,\mathrm{GeV}/c$	**0.53**	**0.72**	**0.58**	**0.40**	**0.72**	1

V_{ub} from recoil analyses

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell\nu)}{\tau_B \cdot \Delta\Gamma_{theory}}}$$



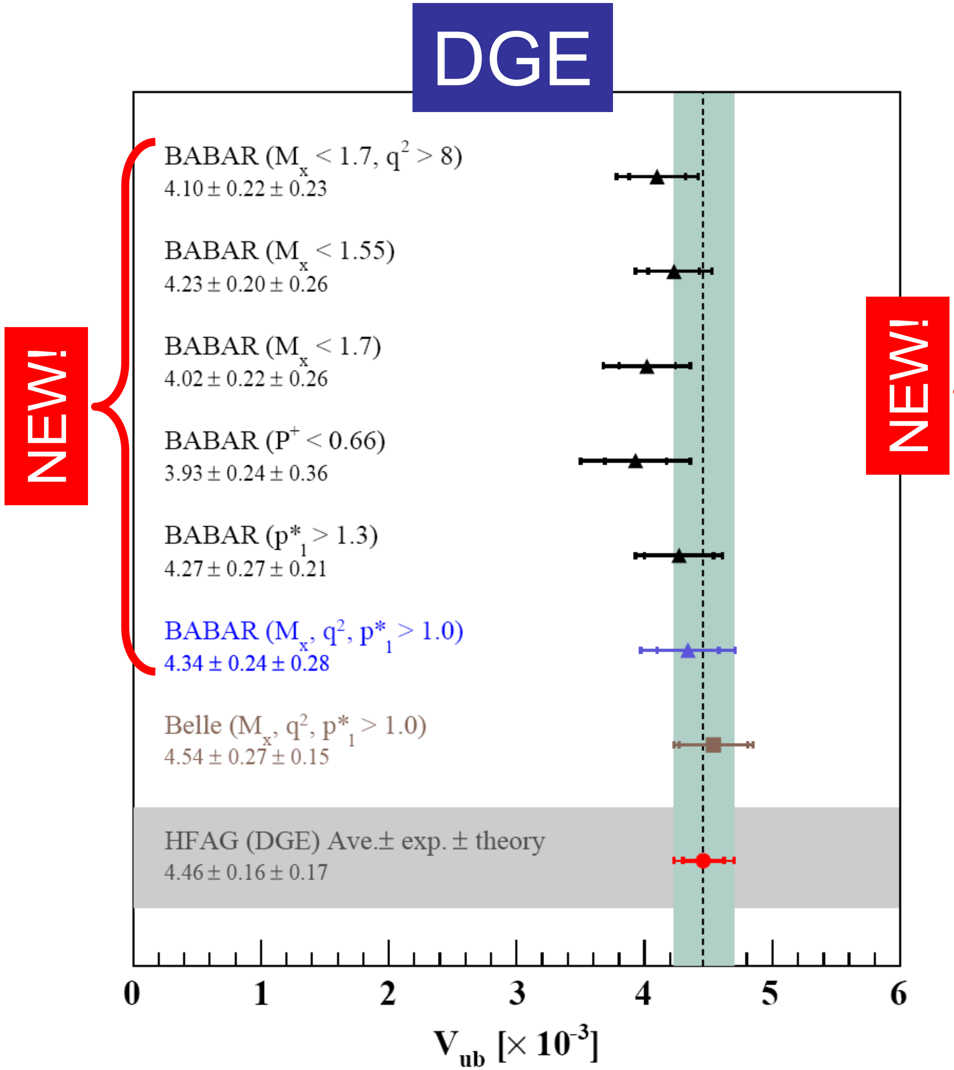


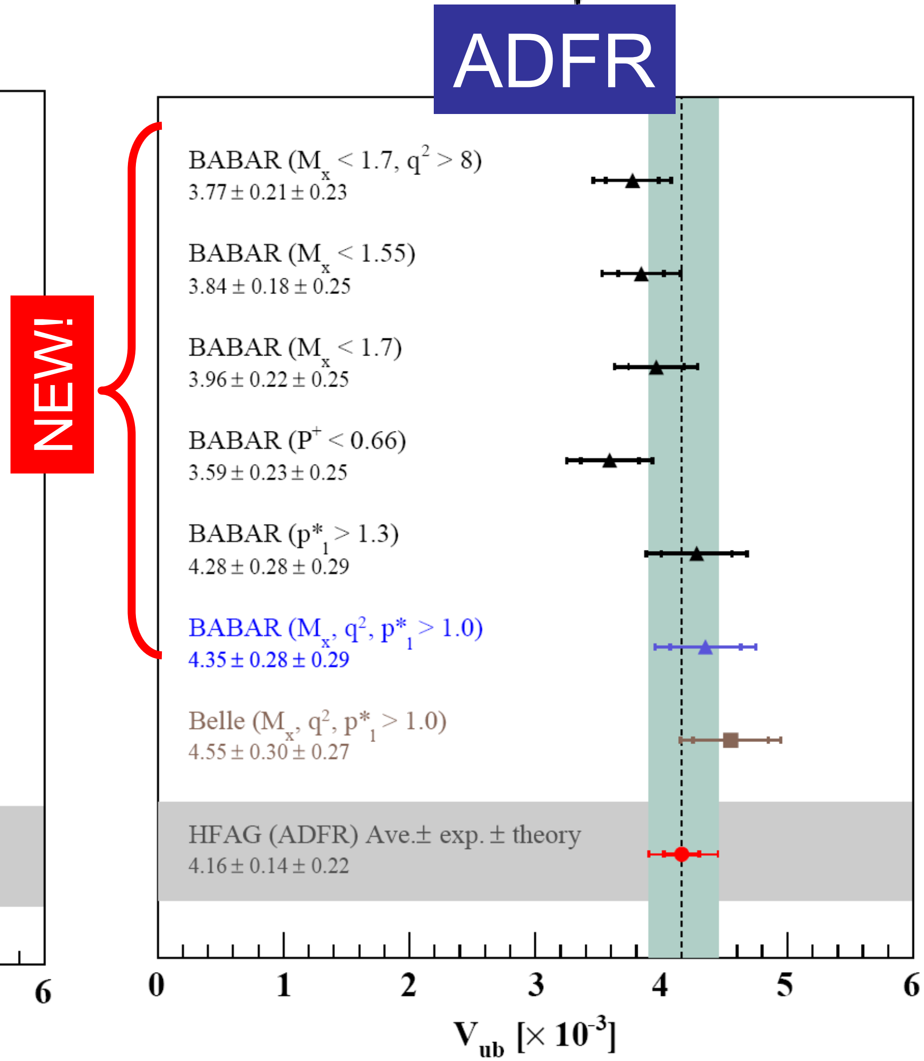
HFAG averages from http://www.slac.stanford.edu/xorg/hfag/semi/EndOfYear09/home.shtml

Endpoint measurements included, new Babar result not included

V_{ub} from recoil analyses

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell \nu)}{\tau_B \cdot \Delta\Gamma_{theory}}}$$





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Endpoint measurements included, new Babar result not included