Status of SIMBA

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Global Fit Approach to $B \to X_s\gamma$ and $B \to X_u\ell\nu$

Our aim: Provide global fit that combines all available information

- Simultaneously determine
  - Overall normalization: $|V_{ub}|$, $\mathcal{B}(B \to X_s\gamma)$
  - Required input parameters: m_b, shape function

  $\Rightarrow$ Same strategy as for $|V_{cb}|$ (but a bit more complicated now)
- Combine different decay modes and measurements
  - Different $B \to X_s\gamma$ spectra
  - Different $B \to X_u\ell\nu$ partial BFs (or even better spectra)
  - Eventually also $B \to X_s\ell^+\ell^-$
  - External constraints on m_b, μ_π^2 (λ_1) (from $B \to X_c\ell\nu$ or other)

What we gain from a global fit

- Minimize uncertainties by making maximal use of all available data
- Consistent treatment of correlated uncertainties (experimental, theoretical, input parameters)

Outline

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1. A Little Bit of Theory
2. Global Fit to $B \to X_s\gamma$
3. Outlook on $|V_{ub}|$

Requirements on Theory

Model-independent framework for shape function

- Small uncertainties have to be reliable uncertainties
- SF uncertainty should reflect the actual information we have
  - Perturbative constraints (RGE running and perturbative tail)
  - Constraints on moments from m_b, μ_π^2 (λ_1)
  - Shape information from $B \to X_s\gamma$ and $B \to X_u\ell\nu$ spectra

Different measurements probe different phase-space regions

- SF region: large E_γ, E_ℓ (near peak/endpoint)
- Local OPE region: small E_γ, E_ℓ, large q^2
- Something in between: $m_X \sim m_D$, moderately large E_γ, E_ℓ

$\Rightarrow$ Combination of optimal theory descriptions for each region

Factorized Shape Function

[Ligeti, Stewart, FT (2008)]

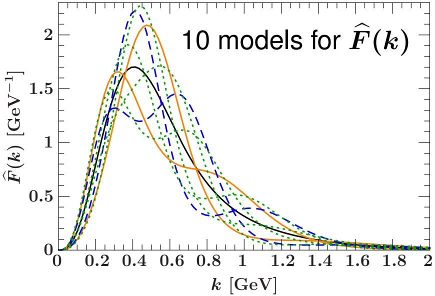
$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, \widehat{C}_0(\omega - k, \mu_\Lambda)\, \widehat{F}(k)$$

$\widehat{F}(k)$ nonperturbative part

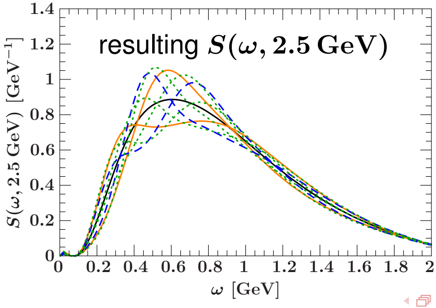
- Determines peak region
- Fit from data

$\widehat{C}_0(\omega, \mu_\Lambda)$ perturbative part

- Generates perturbative tail consistent with RGE



$\Rightarrow$



Factorized Shape Function

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- Determines peak region
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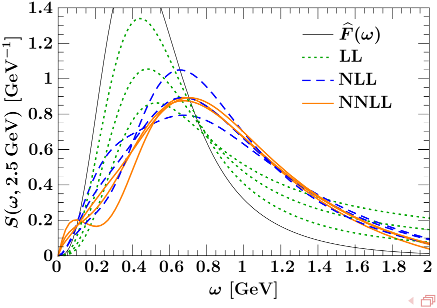
$\widehat{C}_0(\omega, \mu_\Lambda)$ perturbative part

- Generates perturbative tail consistent with RGE

$\Rightarrow$ If we know $\widehat{F}(k)$ we can *compute* $S(\omega, \mu_\Lambda)$ in perturbation theory

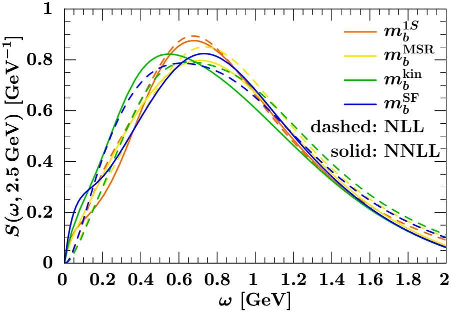
- Vary μ_Λ to estimate perturbative uncertainty in SF
  (Here: $\mu_\Lambda = (1.0, 1.3, 1.8)\,\mathrm{GeV}$
  + RGE up to $\mu = 2.5\,\mathrm{GeV}$)



Different Short Distance Schemes

$\widehat{C}$ and $\widehat{F}$ defined in certain short distance scheme (can use any m_b scheme)

$$
\begin{aligned}
S(\omega) &= \int \mathrm{d}k\, C_0^{\mathrm{pole}}(\omega - k)\, F^{\mathrm{pole}}(k) \\
&= \int \mathrm{d}k\, C_0^{1S}(\omega - k)\, F^{1S}(k) \\
&= \int \mathrm{d}k\, C_0^{\mathrm{kin}}(\omega - k)\, F^{\mathrm{kin}}(k) \\
&= \int \mathrm{d}k\, C_0^{\mathrm{SF}}(\omega - k)\, F^{\mathrm{SF}}(k)
\end{aligned}
$$



Moments of $\widehat{F}(k)$ given by respective SD parameters $\widehat{m}_b$, $\widehat{\lambda}_1$, etc. to all orders in α_s, e.g.

$$
\int \mathrm{d}k\, k^n\, F^{1S\mathrm{i}}(k) = M_n = \begin{cases} 1 & (n = 0) \\ m_B - m_b^{1S} & (n = 1) \\ -\lambda_1^{\mathrm{i}}/3 + (m_B - m_b^{1S})^2 & (n = 2) \end{cases}
$$

Basis Expansion for $\widehat{F}(k)$

Expand $\widehat{F}(k)$ into suitable orthonormal basis

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\Big[\sum_{n=0}^{\infty} c_n f_n(x)\Big]^2$$

$$\int \mathrm{d}k\, \widehat{F}(k) = \sum_{n=0}^{\infty} c_n^2 = 1$$

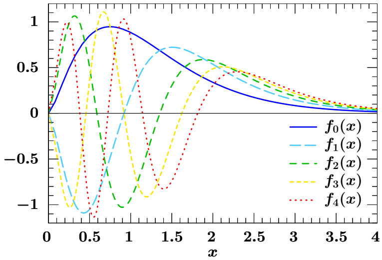
- Provides model-independent description

Fit for $\widehat{F}(k)$ by fitting basis coefficients c_n

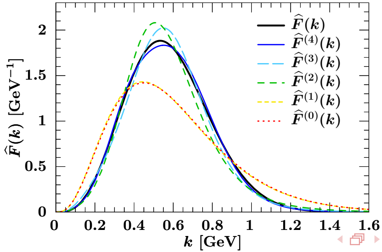
- Experimental uncertainties and correlations are captured in covariance matrix of fitted coefficients c_n

⇒ Allows for *data driven*, reliable estimation of SF uncertainties

Basis functions



Expansion of Gaussian $\widehat{F}(k)$



Residual Basis Dependence from Series Truncation

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\Bigl[\sum_{n=0}^{N} c_n f_n(x)\Bigr]^2$$

In practice, series must be truncated

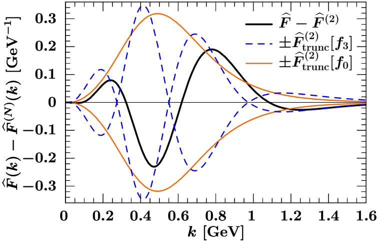
- Induces residual basis (model) dependence
- Truncation error scales as $1 - \sum_{n=0}^{N} c_n^2$

Optimal N and λ are determined from data

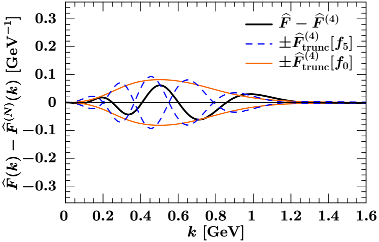
- Choose λ so series converges quickly
- Choose N so truncation error is small compared to exp. uncertainties
- Add more terms with more precise data

$\Rightarrow$ Must be careful not to "overtune"

Truncation error at $N = 2$

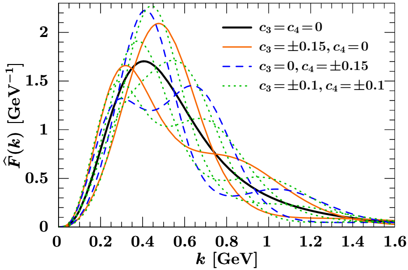


Truncation error at $N = 4$



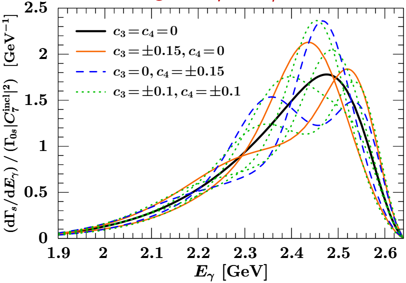
Master Formula for Differential Spectra

$\widehat{F}(k)$ with different c_n



$\Rightarrow$

Resulting $\mathrm{d}\Gamma_s/\mathrm{d}E_\gamma$ (leading power)



$$\mathrm{d}\Gamma_s = |V_{tb}V_{ts}^*|^2 m_b^2\, |C_7^{\mathrm{incl}}|^2 \int \mathrm{d}k\, \widehat{W}_{77}(E_\gamma; k)\, \widehat{F}(m_B - 2E_\gamma - k) + \cdots$$

$$\mathrm{d}\Gamma_u = |V_{ub}|^2 \int \mathrm{d}k\, \widehat{W}_u(p_X^-, p_X^+, E_\ell; k)\, \widehat{F}(p_X^+ - k) + \cdots$$

Insert expansion for $\widehat{F}(k)$

$$\mathrm{d}\Gamma_s = |V_{tb}V_{ts}^*|^2 m_b^2\, |C_7^{\mathrm{incl}}|^2 \sum_{m,n} c_m c_n \mathrm{d}\Gamma_{mn}^{77} \qquad \mathrm{d}\Gamma_u = |V_{ub}|^2 \sum_{m,n} c_m c_n \mathrm{d}\Gamma_{mn}^{u}$$

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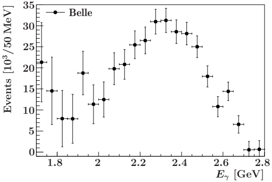
Global Fit to $B \to X_s\gamma$



Current status of experiment to theory comparison

- HFAG extrapolation down to $E_\gamma^{\rm cut} = 1.6\,\mathrm{GeV}$ (adds model dependence)

$$\mathcal{B}(E_\gamma > 1.6\,\mathrm{GeV}) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}$$

- Fixed-order NNLO estimate by Misiak et al. (2006)

$$\mathcal{B}(E_\gamma > 1.6\,\mathrm{GeV}) = (3.15 \pm 0.23) \times 10^{-4}$$

Sensitivity to new physics lies in normalization, parametrized by $|V_{tb}V_{ts}^* C_7^{\rm incl}|$

- Most sensitivity in data comes from large E_γ
- Fit determines both $|V_{tb}V_{ts}^* C_7^{\rm incl}|$ (normalization) and $\widehat{F}(k)$ (shape)
  - Can directly compare $|V_{tb}V_{ts}^* C_7^{\rm incl}|$ to its SM prediction
  - Avoids any extrapolation

Theory Inputs

$$d\Gamma_s \propto |V_{tb}V_{ts}^*|^2 m_b^2 \Big\{ \big|C_7^{\rm incl}\big|^2 \Big[\big(\widehat{W}_{77}^{\rm sing} + \widehat{W}_{77}^{\rm nons}\big) \otimes \widehat{F} + \sum_n W_{77,n} F_n^{\rm subl} \Big]$$
$$+ \sum_{i,j\neq 7} \Big[{\rm Re}(C_7^{\rm incl}) 2 C_i \widehat{W}_{7i}^{\rm nons} + C_i C_j \widehat{W}_{ij}^{\rm nons} \Big] \otimes \widehat{F} + \cdots \Big\}$$

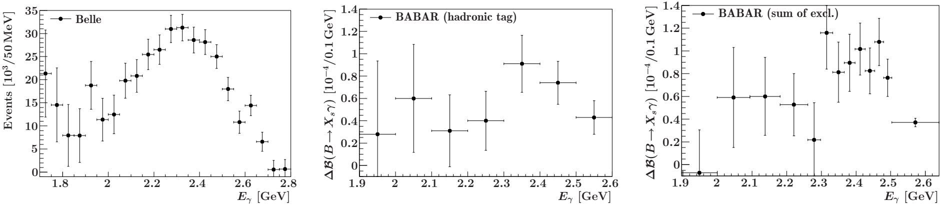
Leading C_7^2 contribution

- Included at full NNLL+NNLO (in $1S$ short-distance scheme)
- $1/m_b$ subleading shape functions absorbed into leading one
  - Have large impact on extracted value of m_b

Contributions from other operators $\sim C_i C_7, C_i C_j$

- Largest effects come from virtual corrections, are absorbed into $C_7^{\rm incl}$
  - Important charm-mass effects only enter SM prediction for $C_7^{\rm incl}$
- Remaining perturbative contributions included at NLO
  - Some NNLO are known, but NLO already have very small effect

Current Inputs for $B \to X_s\gamma$ Fit



Belle $605\,\mathrm{fb}^{-1}$

- Thanks to Belle, especially Antonio Limosani, for providing the covariance matrix, experimental efficiency and resolution!
- Efficiency and resolution effects folded into theory predictions

BaBar sum-over-exclusive-modes ($80\,\mathrm{fb}^{-1}$), hadronic tag ($210\,\mathrm{fb}^{-1}$)

- Correlations are available
- Spectra efficiency corrected, resolution not an issue
- Thanks to Francesca Di Lodovico and Henning Flächer

Fit Setup

χ^2 Fit

- Includes all experimental correlations
- Extensively validated with pseudo experiments
  - Just having a good χ^2/ndf is not enough

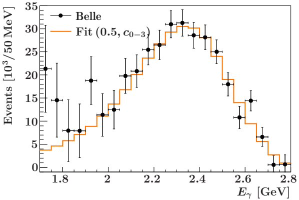
Shape function basis $\widehat{F}(\lambda x) = \frac{1}{\lambda}\left[\sum_n c_n f_n(x)\right]^2$

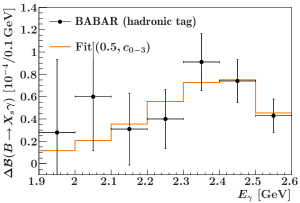
- Default basis parameter: $\lambda = 0.5\,\mathrm{GeV}$
- Include up to 5 basis coefficients (c_0 to c_4)
- Fix $\sum_n c_n^2 = 1$ to enforce correct normalization $\int \mathrm{d}k \widehat{F}(k) = 1$

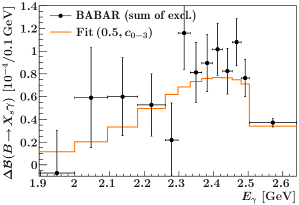
Disclaimer: What I am showing is active work in progress

- Numbers still subject to change
- Theoretical uncertainties not yet included in the fit

Fit Results





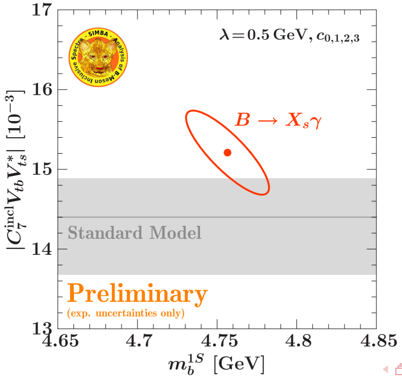


- $\chi^2/\mathrm{ndf} = 27.9/38$
- Standard Model prediction

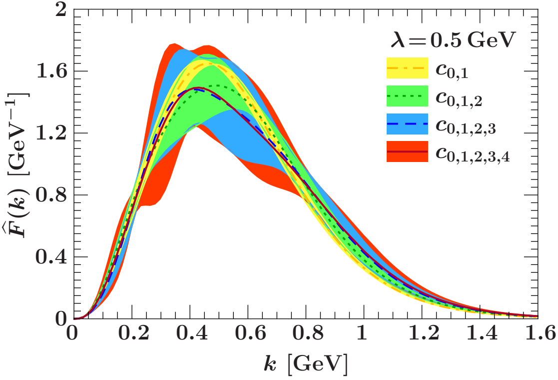
$$|C_7^{\mathrm{incl}}|^{\mathrm{SM}} = 0.354^{+0.011}_{-0.012}$$

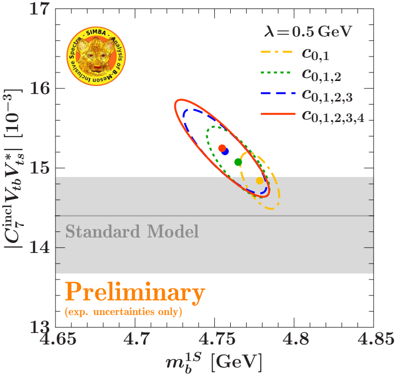
$$|V_{tb}V_{ts}^*| = (40.7^{+0.4}_{-1.5}) \times 10^{-3}$$

- Data slightly above SM prediction (same pattern as Misiak vs. HFAG)



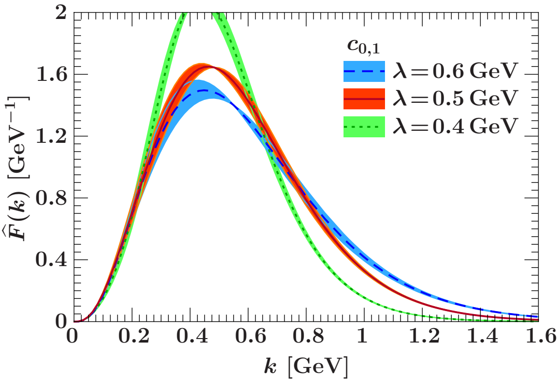
Convergence of Basis Expansion

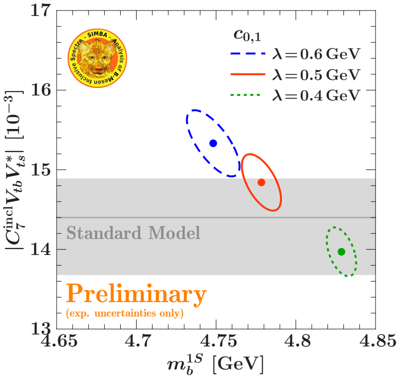




- Uncertainties underestimated with too few coefficients ($c_{0,1}$)
  - Would need to include additional uncertainty due to truncation
- Very little change from including 5th coefficient (c_4)
  - Truncation uncertainty negligible compared to other uncertainties

Basis Independence

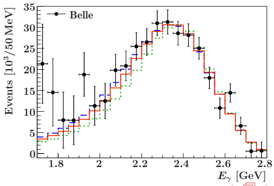




Fit with only two basis functions

- Equivalent to fixed model with fitted 1st moment
- All with good χ^2/ndf: $37.5/40$, $28.8/40$, $27.8/40$

$\Rightarrow$ Uncertainties underestimate model dependence



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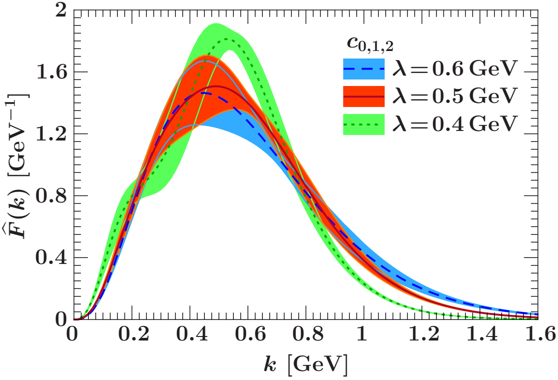
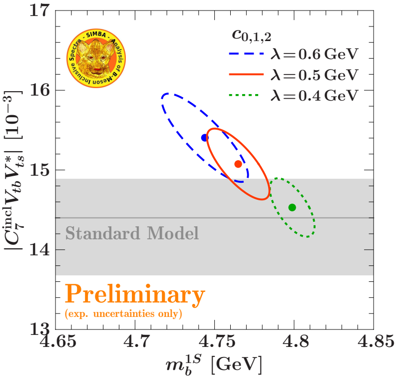
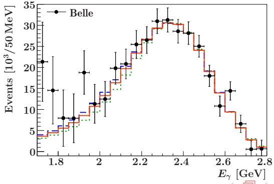
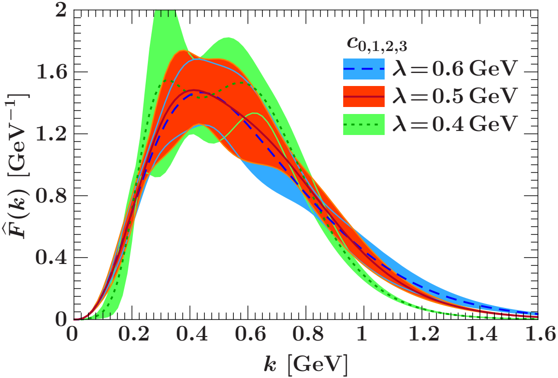
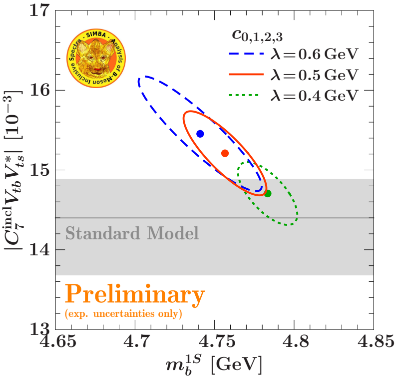


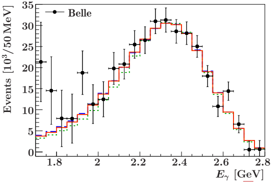




Basis Independence







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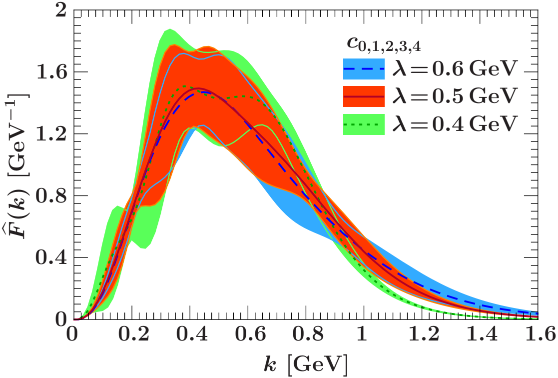
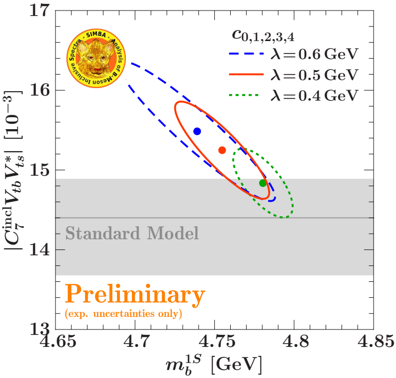
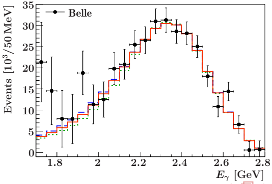




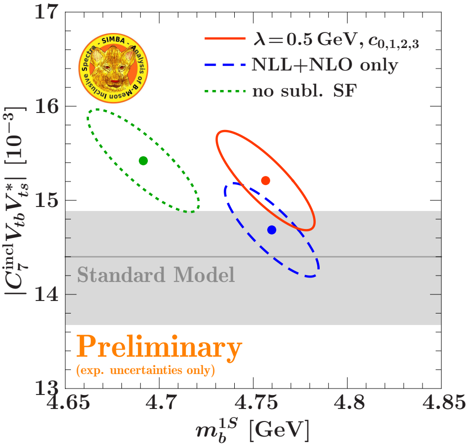


⇒ With enough coefficients results agree within uncertainties and become basis (model) independent

Effect of Perturbative and $1/m_b$ Corrections

- NNLL+NNLO corrections move result up
- Subleading shape functions cause substantial shift in m_b given by their 1st moment

$$\frac{-\lambda_1 + 3\lambda_2}{2m_b} \sim 70\,\text{MeV}$$



Outline

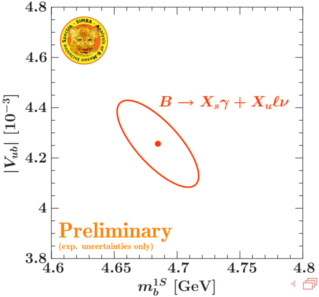
Additional Complications for $B \to X_u \ell \nu$

$$\mathrm{d}\Gamma_u \propto |V_{ub}|^2 \Big\{ \big(\widehat{W}_u^{\mathrm{sing}} + \widehat{W}_u^{\mathrm{nons}}\big) \otimes \widehat{F} + \sum_n W_{u,n} \widehat{F}_n^{\mathrm{subl}} + \cdots \Big\}$$

- Combining different phase-space regions for triple differential spectrum
  - E.g. $\widehat{W}_u^{\mathrm{sing}}$ known to $\mathcal{O}(\alpha_s^2)$ but $\widehat{W}_u^{\mathrm{nons}}$ only to $\mathcal{O}(\alpha_s, \alpha_s^2 \beta_0)$
- Subleading SFs are more tricky, cannot be absorbed anymore
  - Different $B \to X_u \ell \nu$ spectra would help

Proof-of-concept fit

- BaBar m_X, $m_X - q^2$, p_X^+, $E_\ell^\Upsilon \geq 2.2\,\mathrm{GeV}$
- Belle m_X, $E_\ell^\Upsilon \geq 2.3\,\mathrm{GeV}$
- $B \to X_s \gamma$ spectra
- Theory: NLL+NLO, no $1/m_b$



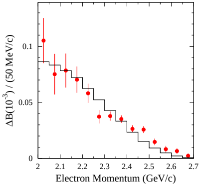
Getting More Out of Existing Data

Wishlist for experiments (or: if you want your measurement to be used ...)

- Correlations for spectra (or between partial branching fractions)
- Correction matrices if spectra are significantly affected by efficiency and resolution

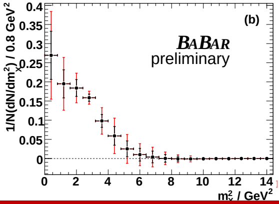
$\Rightarrow$ Right now this unfortunately excludes a lot of valuable inputs

- BaBar leptonic-tag $B \to X_s\gamma$ spectrum (updated analysis soon)
- BaBar E_ℓ^Υ partial BFs
- Belle hadronic-tag partial BFs



There is much more information we can gain from $\sim 1\,\mathrm{ab}^{-1}$ of Belle and BaBar data

- $B \to X_u\ell\nu$ *spectra* will help further constrain m_b and leading (subleading) SF
- Precise E_ℓ spectrum (maybe with cut on m_X) would be very useful



Summary

Global fit to $B \to X_s\gamma$ and $B \to X_u\ell\nu$
with a model-independent treatment of the shape function

- Minimizes uncertainties by making maximal use of available data
- SF and its uncertainties are determined by the data
  - No hidden or underestimated uncertainties from model dependence
- Eliminate extrapolation for comparison to the $B \to X_s\gamma$ rate

$\Rightarrow$ Will be key to precision $\mathcal{B}(B \to X_s\gamma)$ and $|V_{ub}|$ from super B factory

To reduce theory/parameter uncertainties with improved measurements now

- Measure (almost) the total rate, also has drawbacks:
  - Have to pay with (much) larger systematic uncertainties
  - Theory uncertainty creeps back in via signal MC model
- Measure $B \to X_u\ell\nu$ spectra with correlations (no drawbacks)

$\Rightarrow$ Ideally, should do both. With limited manpower the second option keeps potential for future improvements open.