$\overline{\text{MS}}$ Charm Quark Mass from Relativistic Sum Rules at $\mathcal{O}(\alpha_s^3)$

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Outline

- **General remarks on heavy quark masses**
  - Schemes. Renormalons.
  - Why a precise charm quark mass?
  - Recent results: are 13-15 MeV errors realistic ?
- **Treatment of experimental data**
  - Treatment of data in most recent sum rule analyses
  - Aim: reduce theory input for unmeasured cross sections
  - Combine different datasets: errors and correlations

NEW

- **Theoretical developments**
  - Known: results at $\mathcal{O}(\alpha_s^3)$
  - Aim: proper and conservative estimate of perturbative errors.

NEW

- **Preliminary results for m_c**
- **Open issues**

B. Dehnadi, V. Mateu, M.D. Zebarjad, AHH

To appear soon!

Remarks on Quark Masses

- Important QCD input parameters for SM predictions
- Confinement $\Longrightarrow$ quark masses not physical observables

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} G^2_{\mu\nu} + \sum_i \bar{\psi}_i \, (\not{D} - M_i) \, \psi$$

  - ▷ defined as formal parameters in QCD action → $\boxed{\delta m_Q < \Lambda_{\rm QCD} \text{ possible}}$
  - ▷ (renormalization) scheme dependent
  - ▷ to be well defined: $m_q^{\rm schemeA} = m_q^{\rm schemeB}(1 + \alpha_s + \alpha_s^2 + \ldots)$
  - ▷ some schemes more appropriate than others

> We only want to use short-distance mass scheme that do not suffer from the $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon inherent to the pole scheme!

Short-Distance Masses

$$m_{\text{pole}} = m_{\text{sd}} + \delta m$$

short-distance mass without $\mathcal{O}(\Lambda_{\text{QCD}})$ ambiguity

perturbation series that contains the pole mass ambiguity of $\mathcal{O}(\Lambda_{\text{QCD}})$

$$\delta m \sim \mu \sum_{n \gg 1} \alpha_s^{n+1}(\mu)\, 2^n\, \beta_0^n\, n!$$

What's the best way to define δm ?

- infinitely many possible schemes exist
- but only certain classes might be used for certain types of problems.

How relevant it is to find a good scheme depends on the size of the uncertainties one has anyway or is willing to accept.

Bottom and Charm Masses

→ m_b and m_c are not very large.

→ Only two distinct classes need to be defined in practice.

Threshold Schemes:

- B/D physics (inclusive decays)
- Quarkonia: $b\bar{b}$, $c\bar{c}$
- non-relativistic sum rules

Kinetic mass:

- o from B meson form factor sum rules
- o cut-off dependent

Bigi, Uraltsev

$\mathcal{O}(\alpha_s^2), \mathcal{O}(\alpha_s^3\beta_0)$

$$m^{\rm kin}(\mu_f) = m^{\rm pole} - \left[\bar{\Lambda}(\mu_f)\right]_{\rm pert} - \left[\frac{\mu_\pi^2(\mu_f)}{2m_{\rm pole}}\right]_{\rm pert} + \ldots$$

1S mass:

- o from pert. $\Upsilon(1S)$ mass
- o scale independent

Ligeti, Manohar, AHH

$$m^{\rm 1S} = \frac{1}{2}\left[M_{\Upsilon_{Q\bar{Q}}(1\,^3S_1)}\right]_{\rm pert}$$

$\mathcal{O}(\alpha_s^3)$

$$m^{\rm sd}(R) = m^{\rm pole} - R\left(a_1\frac{\alpha_s}{4\pi} + a_2\left(\frac{\alpha_s}{4\pi}\right)^2 + \ldots\right) \quad \text{with } R \ll m_Q$$

Bottom and Charm Masses

→ m_b and m_c are not very large.

→ Only two distinct classes need to be defined in practice.

MSbar Mass Scheme:

- high-energy, inclusive processes

  e.g. $\sigma(e^+e^- \to b\bar{b})$, $\sigma(pp \to b\bar{b})$

  e.g. $\Gamma(Z \to b\bar{b})$

- off-shell, highly virtual b and c quarks

  e.g. $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$

  e.g. $\mathcal{B}(B \to X_s \gamma)$

$$m^{\text{sd}}(R) = m^{\text{pole}} - R\left(a_1 \frac{\alpha_s}{4\pi} + a_2 \left(\frac{\alpha_s}{4\pi}\right)^2 + \ldots\right) \quad \text{with } R \approx m_Q$$

Bottom and Charm Masses

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MSbar Mass Scheme:

- high-energy, inclusive processes

  e.g. $\sigma(e^+e^- \to b\bar{b})$, $\sigma(pp \to b\bar{b})$

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- off-shell, highly virtual b and c quarks

  e.g. $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$

  e.g. $\mathcal{B}(B \to X_s \gamma)$

Distinction between MSbar and threshold schemes probably not really relevant for the charm quark: $R \ll m_Q$ not feasible

Impact of Precision

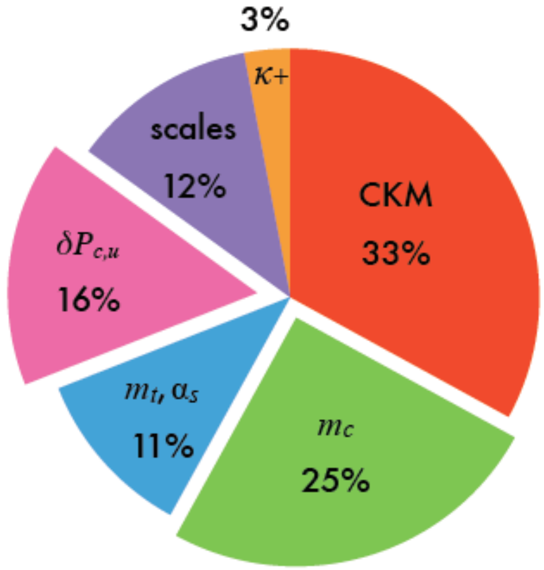
SM prediction(s) of $K^+ \to \pi^+ \nu\bar{\nu}$: error budget

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \{8.57 \pm 0.93, 8.51 \pm 0.73, 8.04 \pm 0.98\} \times 10^{-11}$$

$$m_c(m_c) = (1.30 \pm 0.05)\ \mathrm{GeV}$$

$$m_c(m_c) = (1.286 \pm 0.013)\ \mathrm{GeV}^*$$

$$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054)\ \mathrm{GeV}^\dagger$$



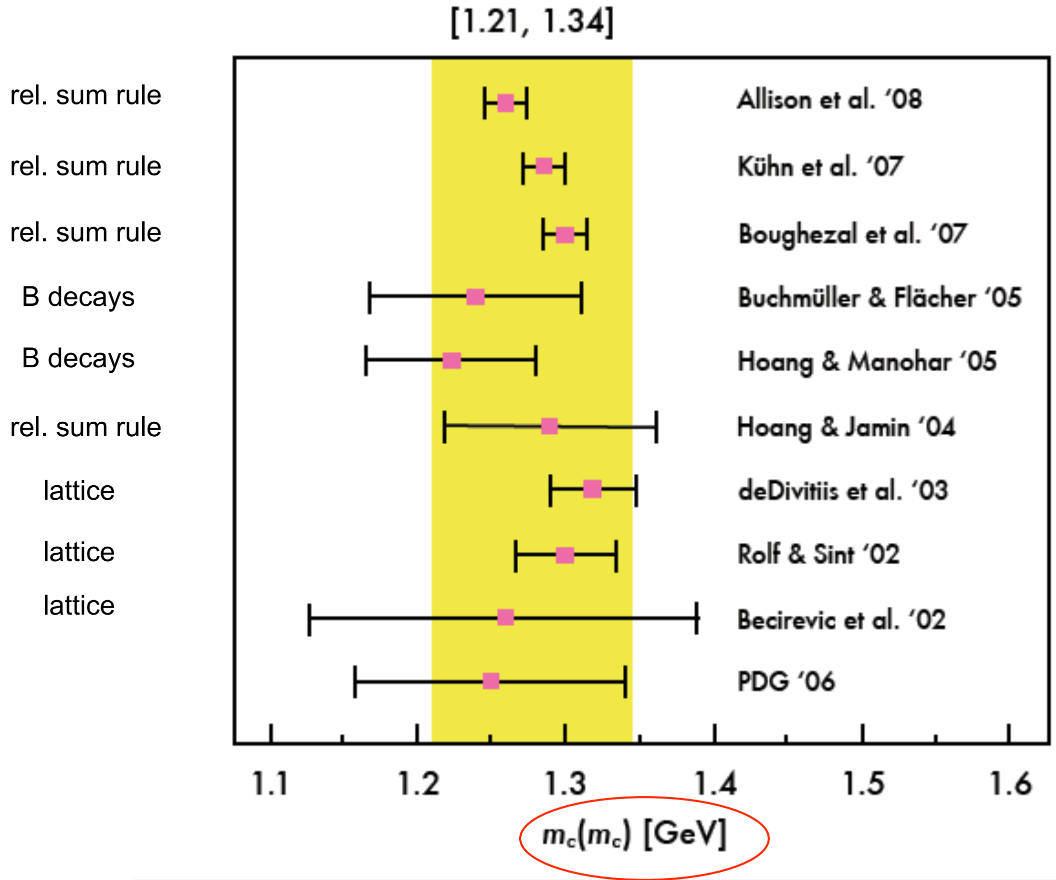
*Kühn et al. '07 †Hoang & Manohar '05

from U. Haisch

Charm Quark Mass Determinations

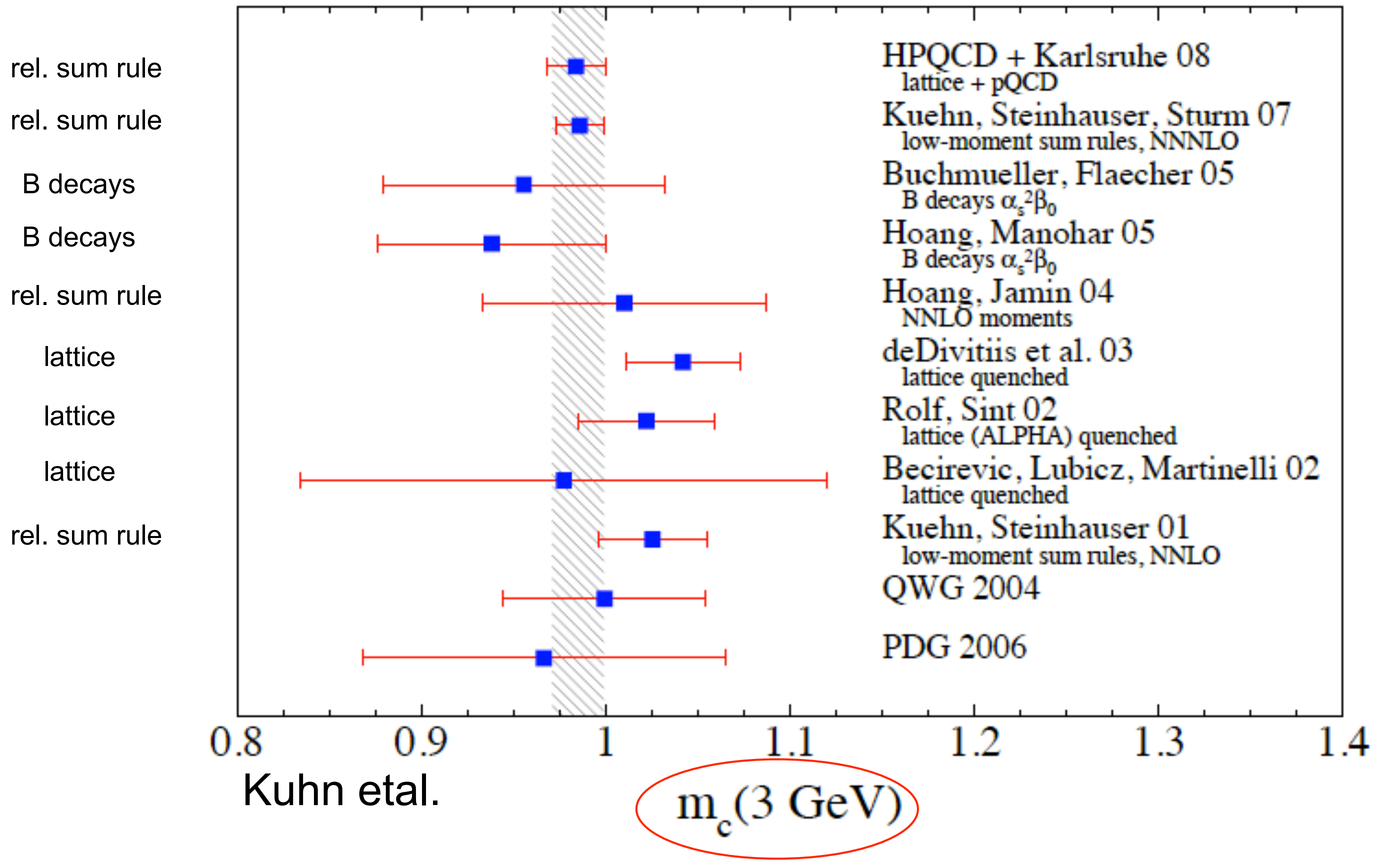
Recent determinations of charm mass

from U. Haisch

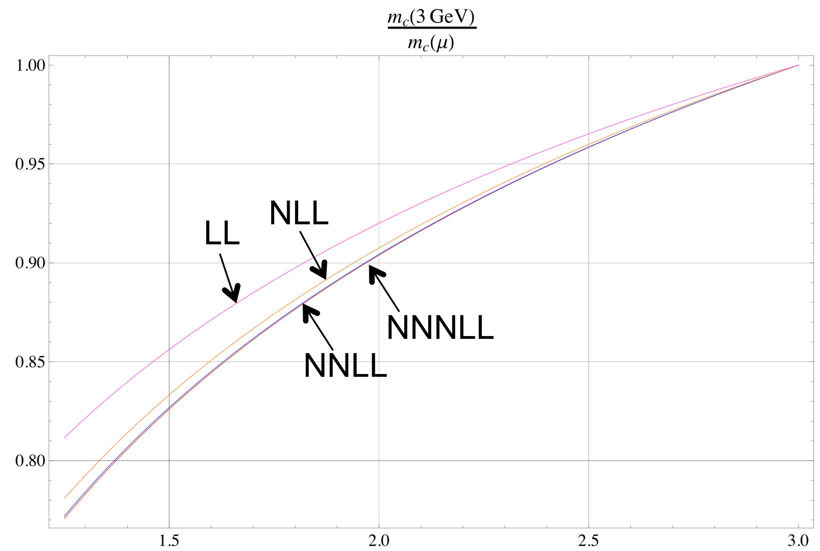


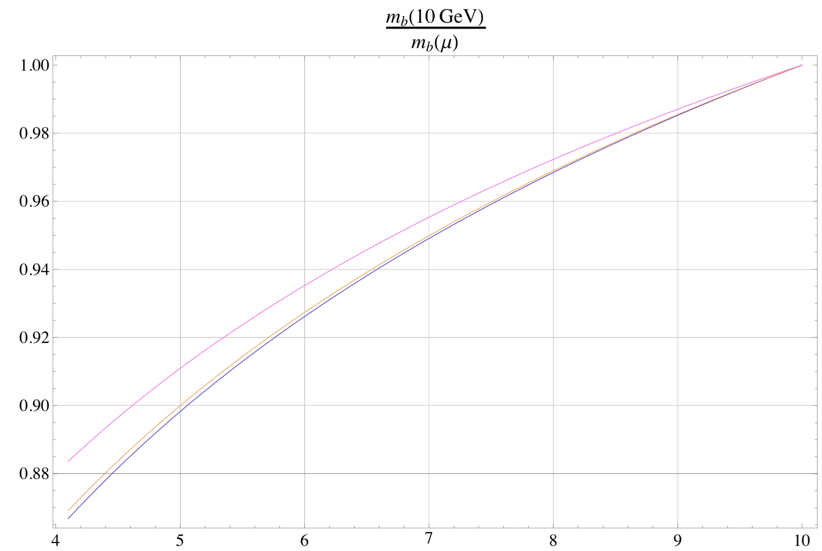
$m_c(m_c)$ [GeV]	method
1.266 ± 0.014	lattice, unquenched, staggered
1.286 ± 0.013	low-momentum sum rules, N^3LO
1.295 ± 0.015	low-momentum sum rules, N^3LO
1.24 ± 0.07	fit to B-decay distribution, $\alpha_s^2\beta_0$
1.224 ± 0.017 ± 0.054	fit to B-decay data, $\alpha_s^2\beta_0$
1.29 ± 0.07	NNLO moments
1.319 ± 0.028	lattice, quenched
1.301 ± 0.034	lattice, quenched
1.26 ± 0.04 ± 0.12	lattice, quenched
1.25 ± 0.09	PDG 2006

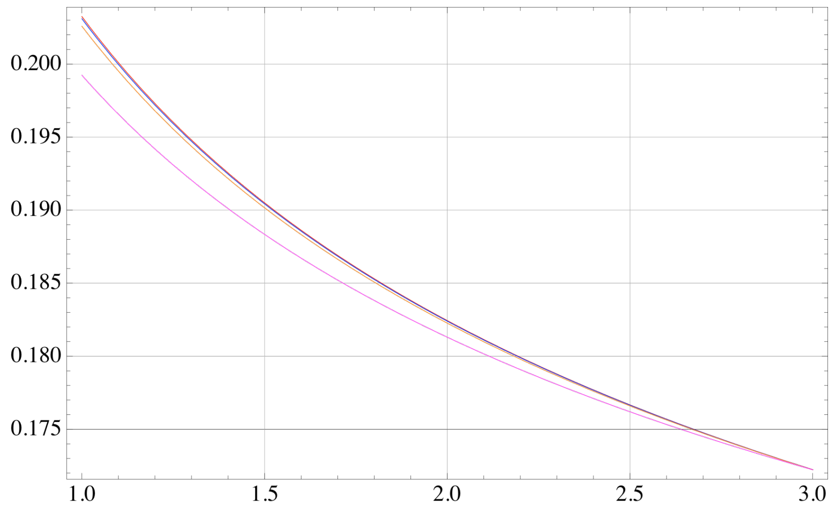
Charm Quark Mass Determinations



Mass and Coupling Running







- Excellent convergence of the running of quark masses and QCD coupling
- No failure of perturbative RG-evolution even down to 1 GeV

Use of $\overline{m}_c(\overline{m}_c)$ is fine !

Relativistic Sum Rules: Status

→ Method with the most advanced theoretical computations: (OPE based !)

$$\mathcal{M}_n^{\rm th} \equiv \frac{12\pi^2}{n!}\left(\frac{\rm d}{{\rm d}q^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n \quad + \ldots\text{(SVZ)}$$

$\mathcal{O}(\alpha_s^2)$ moments — Chetyrkin, Kuhn, Steinhauser (1994-1998)

$\mathcal{O}(\alpha_s^3)$ moments $n=1$ — Boughezal, Czakon, Schutzmeier (2006)

$n = 1-4$ — Kuhn, Steinhauser, Sturm (2006)

$\Pi(q^2)$ function at $\mathcal{O}(\alpha_s^3)$ — Mateu, Zebarjad, Hoang (2008)

Kiyo, Meier, Meierhofer, Marquard (2009)

→ dominated by perturbation theory for

$$\Delta E \sim \frac{m_c}{n} > \Lambda_{\rm QCD}$$

$$\mathcal{M}_n = \int \frac{{\rm d}s}{s^{n+1}} R_c(s)$$

Perturbation theory works most reliably for n=1. → used in this analysis

(n=2 appears fine as well)

Relativistic Sum Rules: m_b & m_c

Analyses with smallest errors I:

Chetyrkin, Kuhn, Meier, Meierhofer, Marquard Steinhauser (2009)

n	$m_c(3\,\mathrm{GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

n	$m_b(10\,\mathrm{GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

- $m_c(3\,\mathrm{GeV}) = 986 \pm 13\,\mathrm{MeV}$
- $m_c(m_c) = 1279 \pm 13\,\mathrm{MeV}$

- $m_b(10\,\mathrm{GeV}) = 3610 \pm 16\,\mathrm{MeV}$
- $m_b(m_b) = 4163 \pm 16\,\mathrm{MeV}$

- theory predictions and errors taken for missing data
- $\alpha_s(\mu)$ and $\overline{m}_Q(\mu)$ taken as theory parameters, $\mu = 2 - 4\ \mathrm{GeV}$, fixed order

Analyses with smallest errors II:

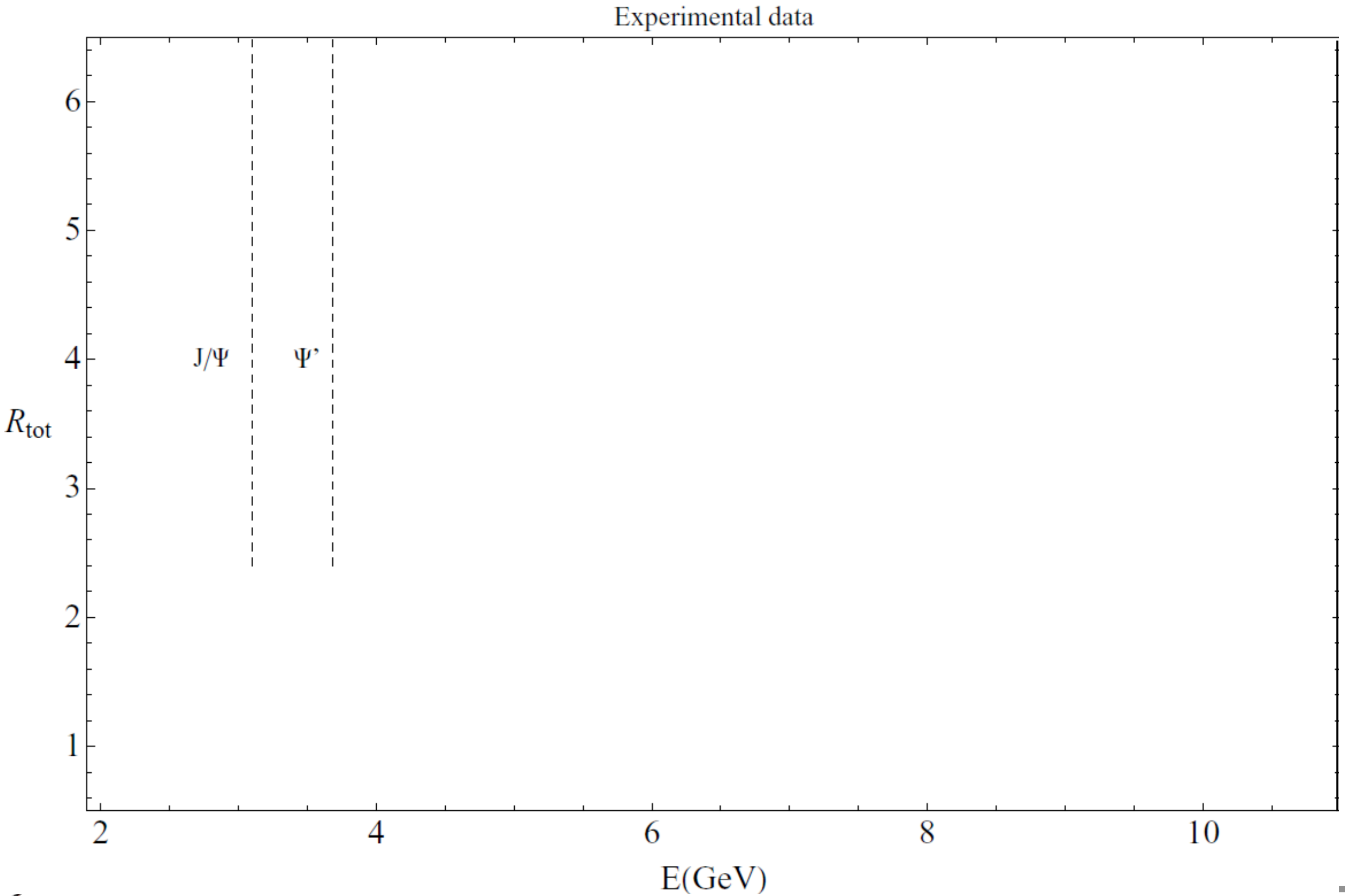
HPQCD, Chetyrkin, Kuhn, Steinhauser, Sturm (2008)

- Lattice data for moments instead of experimental data (lattice error: $\sim 2\ \mathrm{MeV}$)

$$m_c(3\mathrm{GeV}) = 986(10)\ \mathrm{MeV}$$

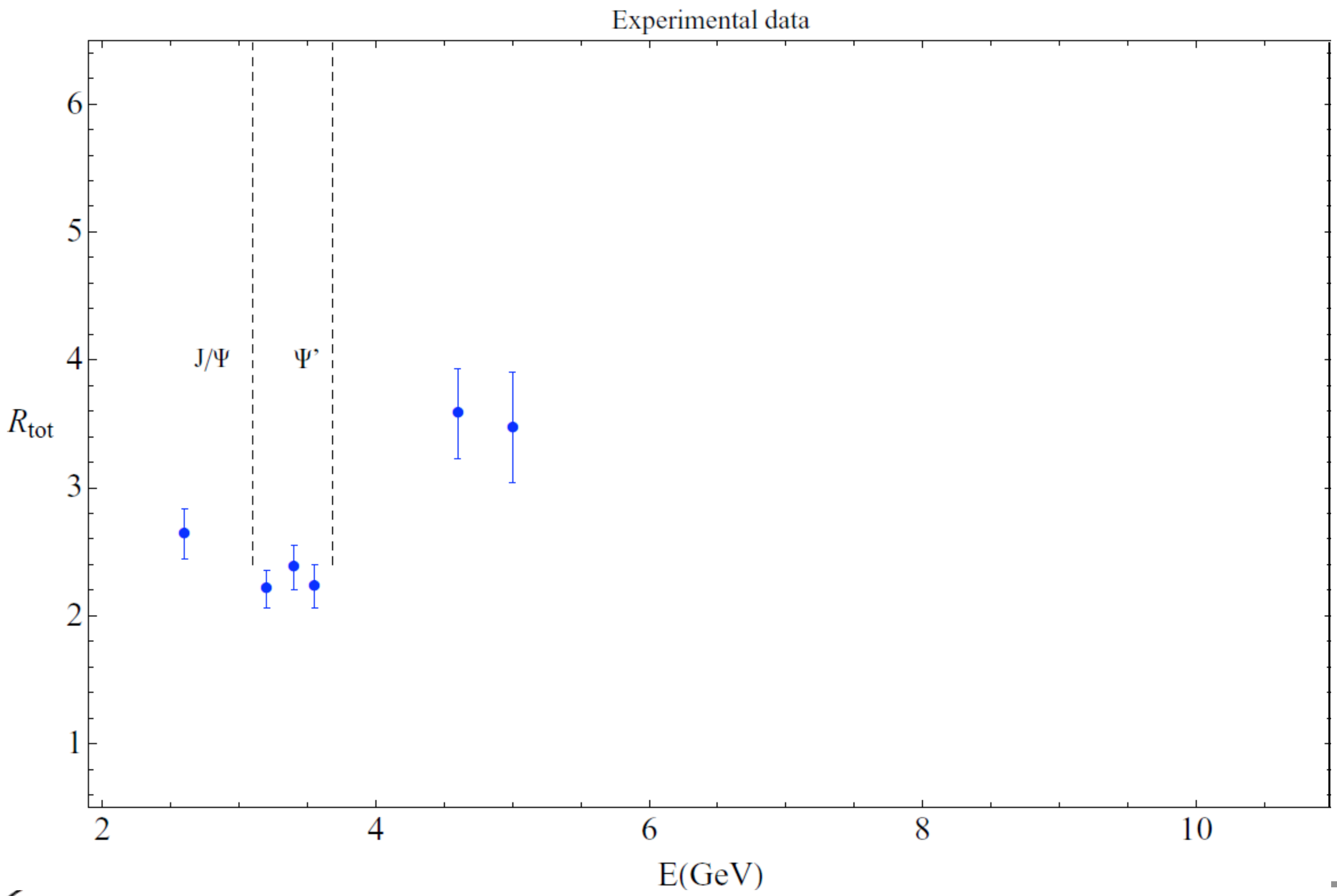
Experimental Data

Narrow resonances



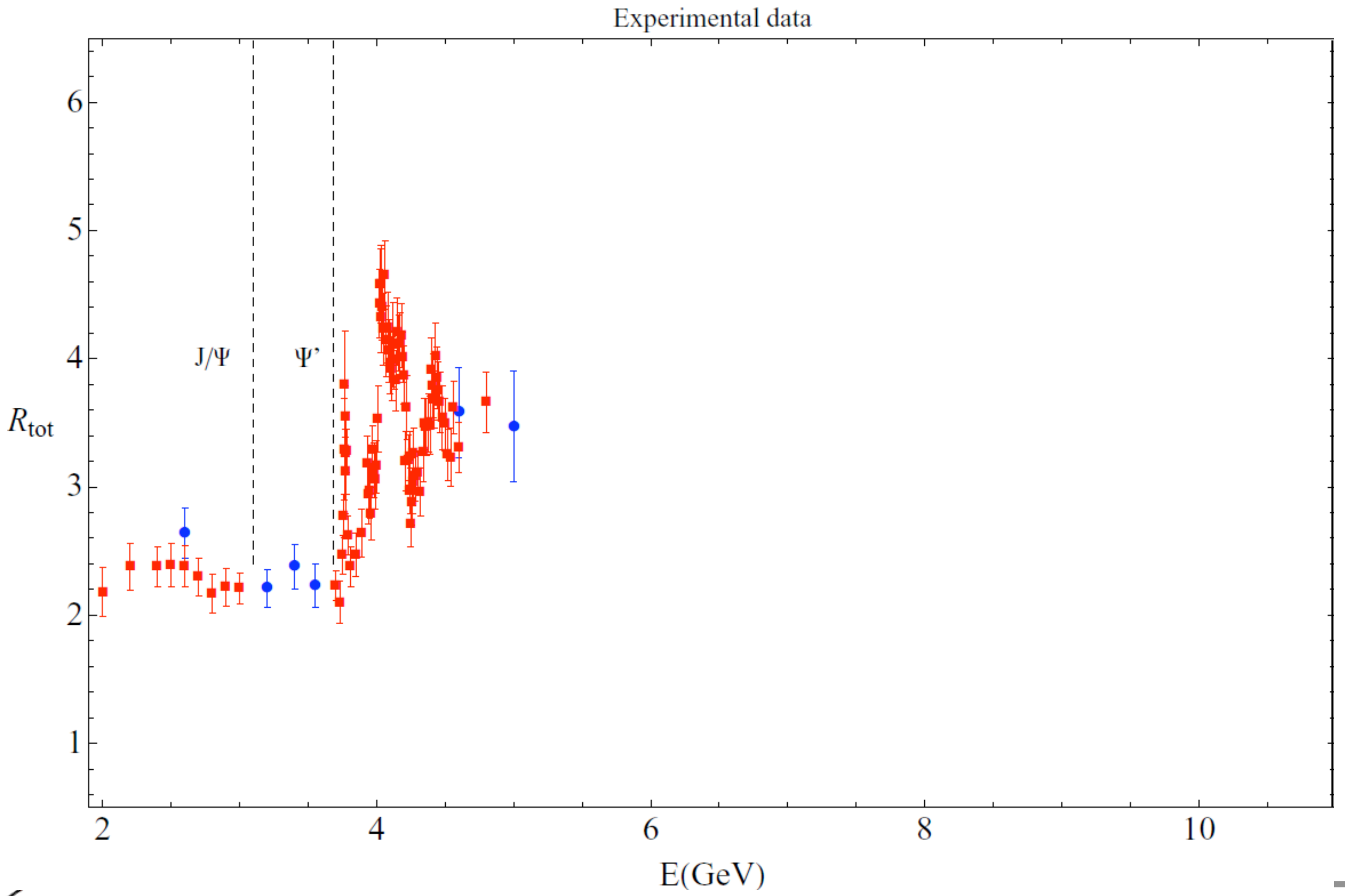
Experimental Data

Sub-threshold and threshold   BES 1999



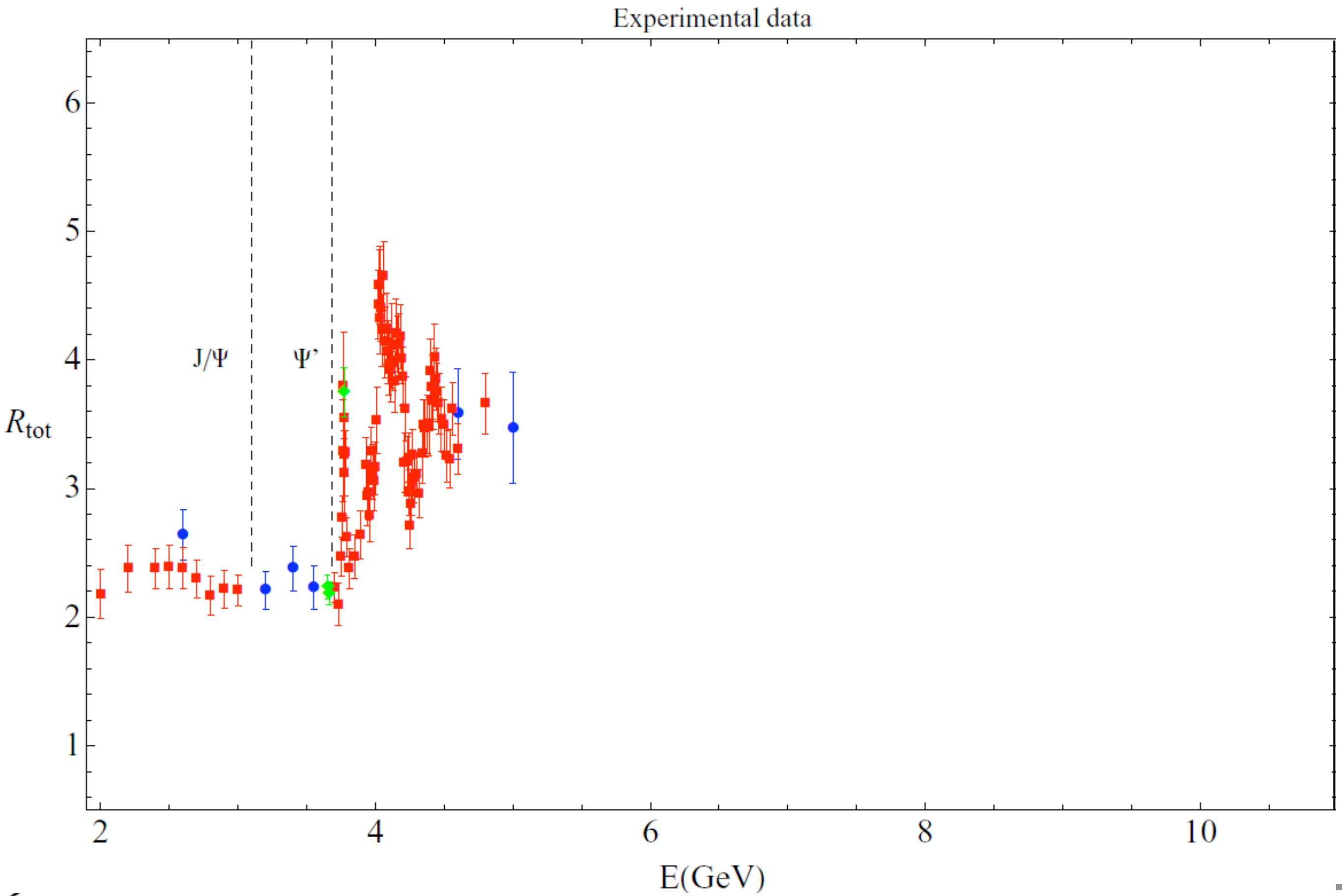
Experimental Data

Sub-threshold and threshold BES 2001



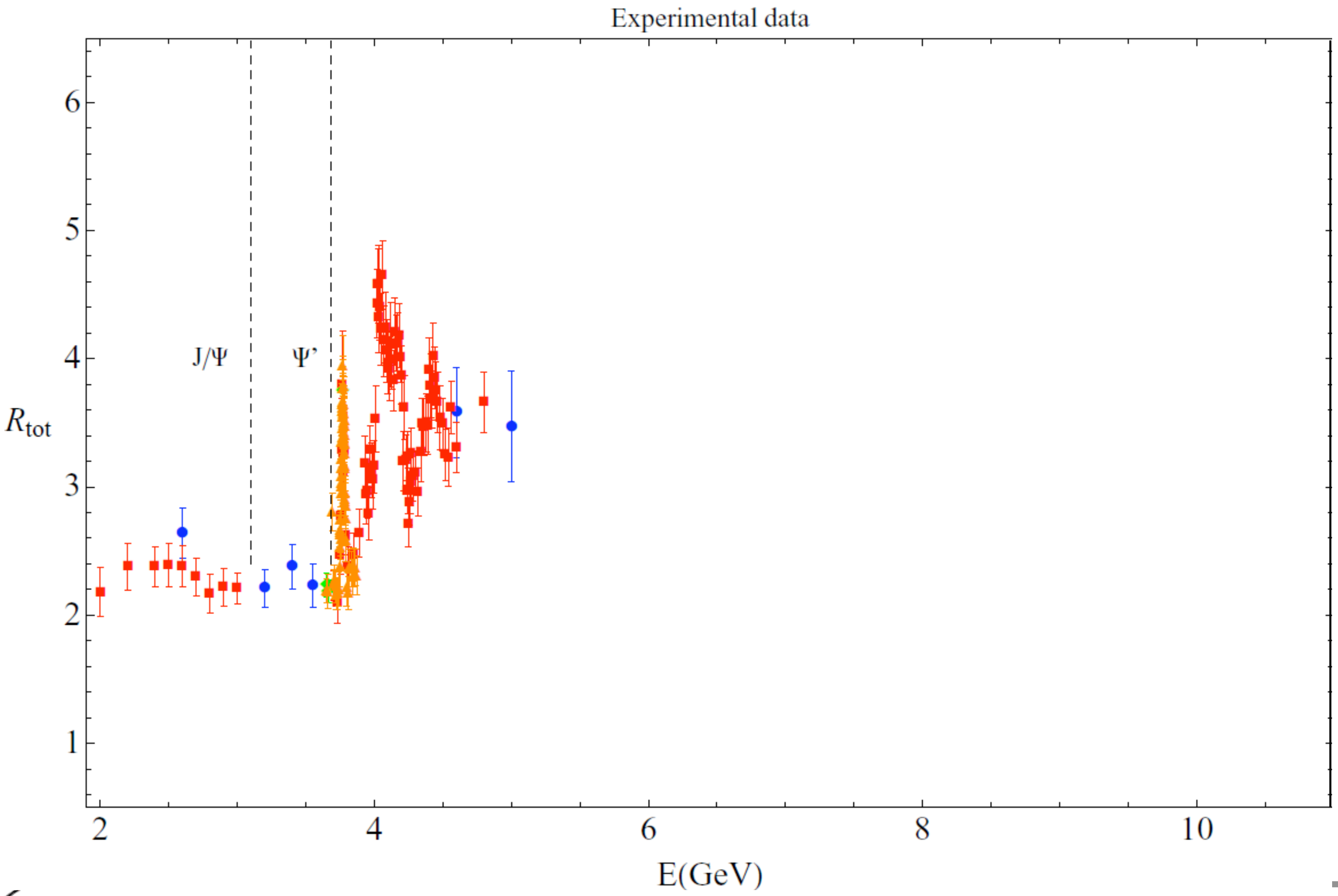
Experimental Data

Sub-threshold and threshold BES 2006 (I)



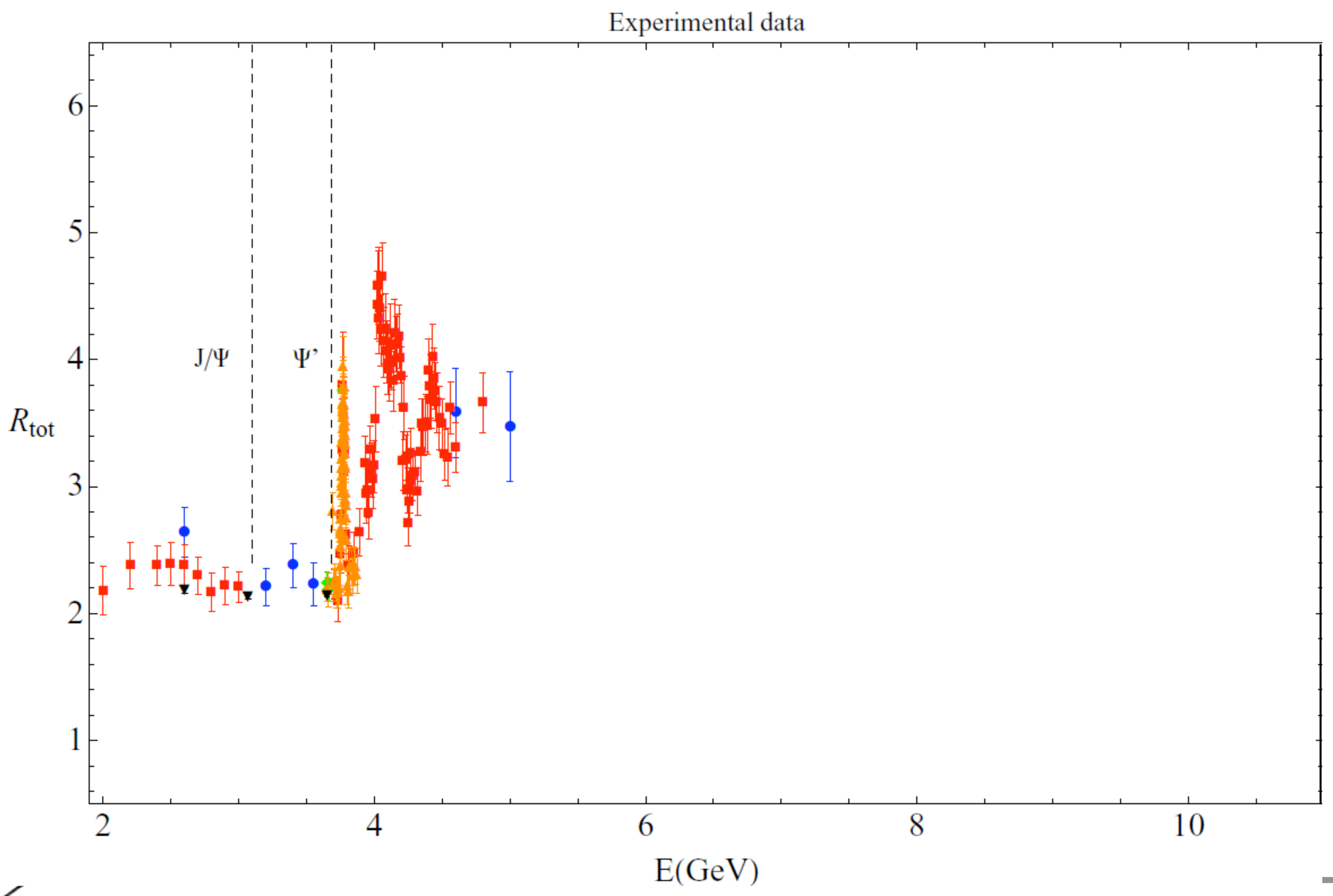
Experimental Data

Sub-threshold and threshold BES 2006 (II)



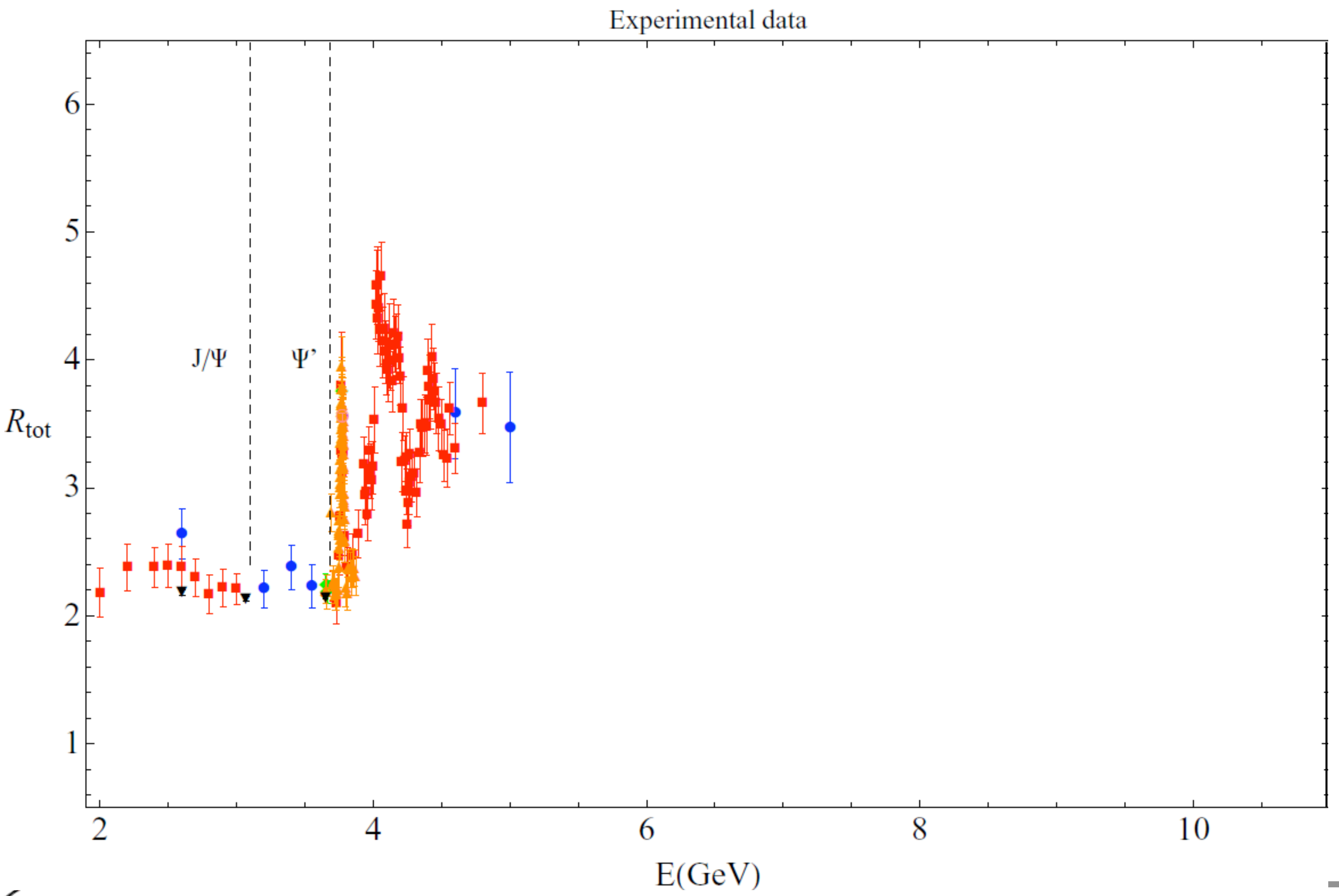
Experimental Data

Sub-threshold and threshold BES 2009



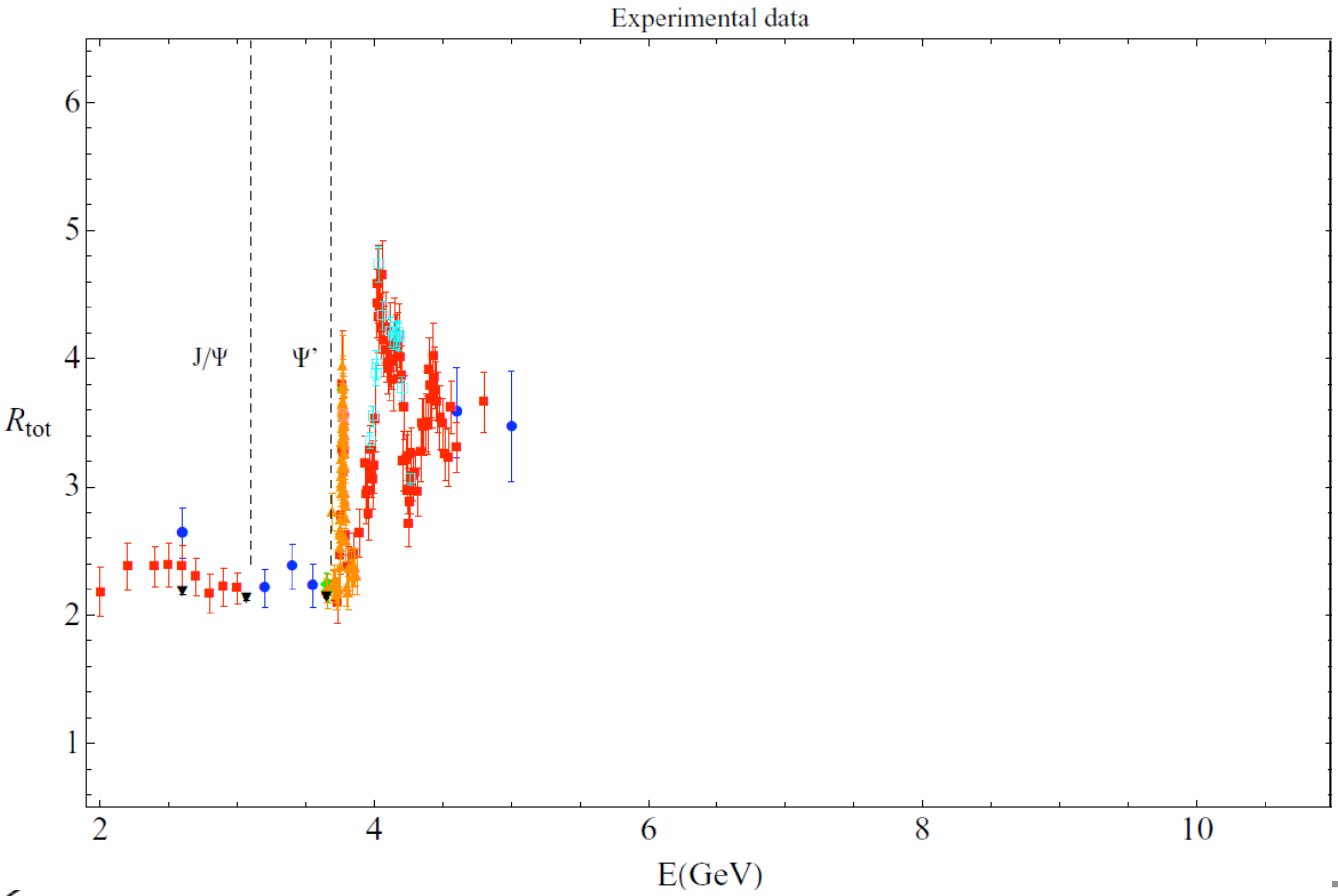
Experimental Data

Sub-threshold and threshold BES 2004



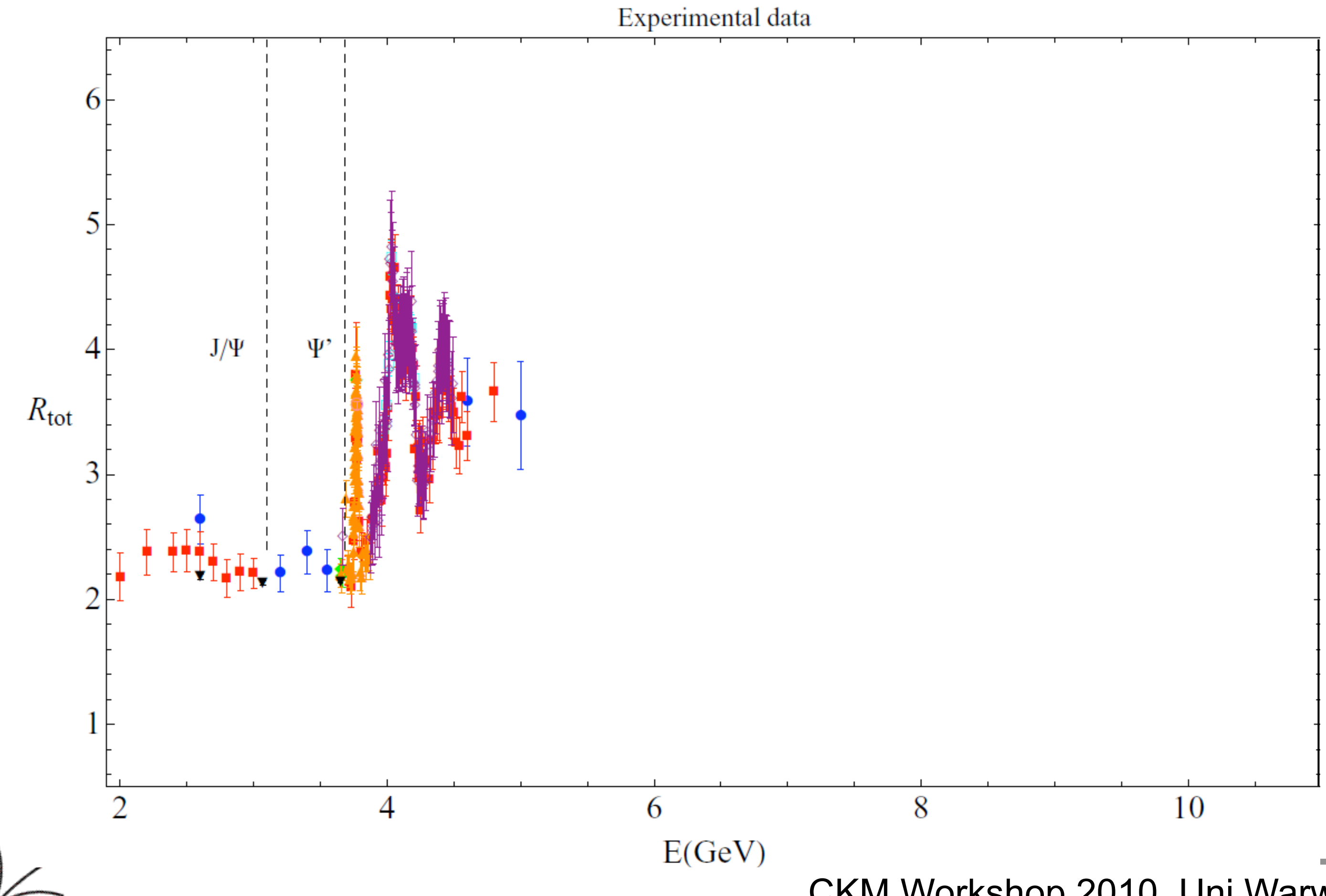
Experimental Data

Sub-threshold and threshold   CLEO 2009



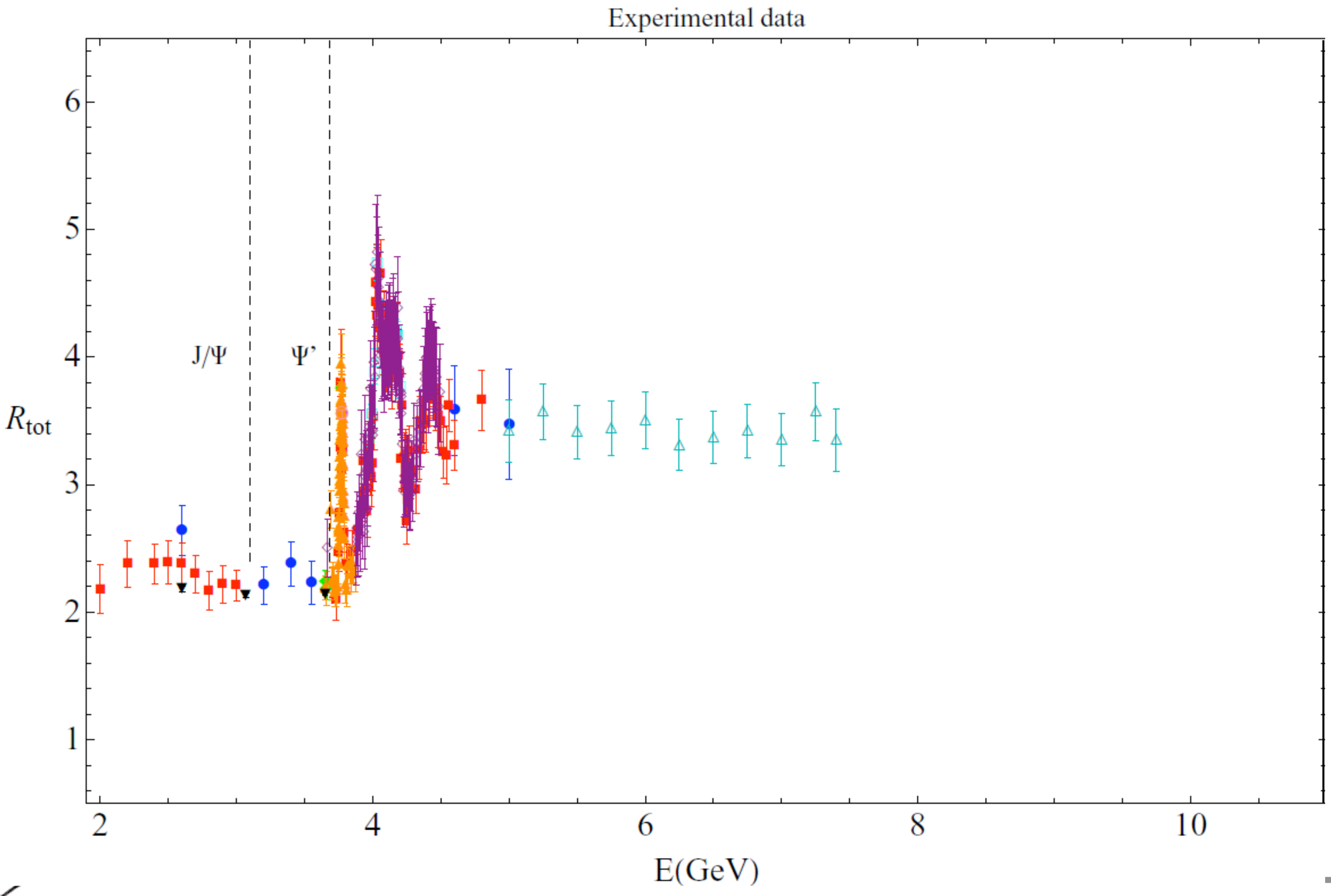
Experimental Data

Sub-threshold and threshold **Chrystal Ball 1986**



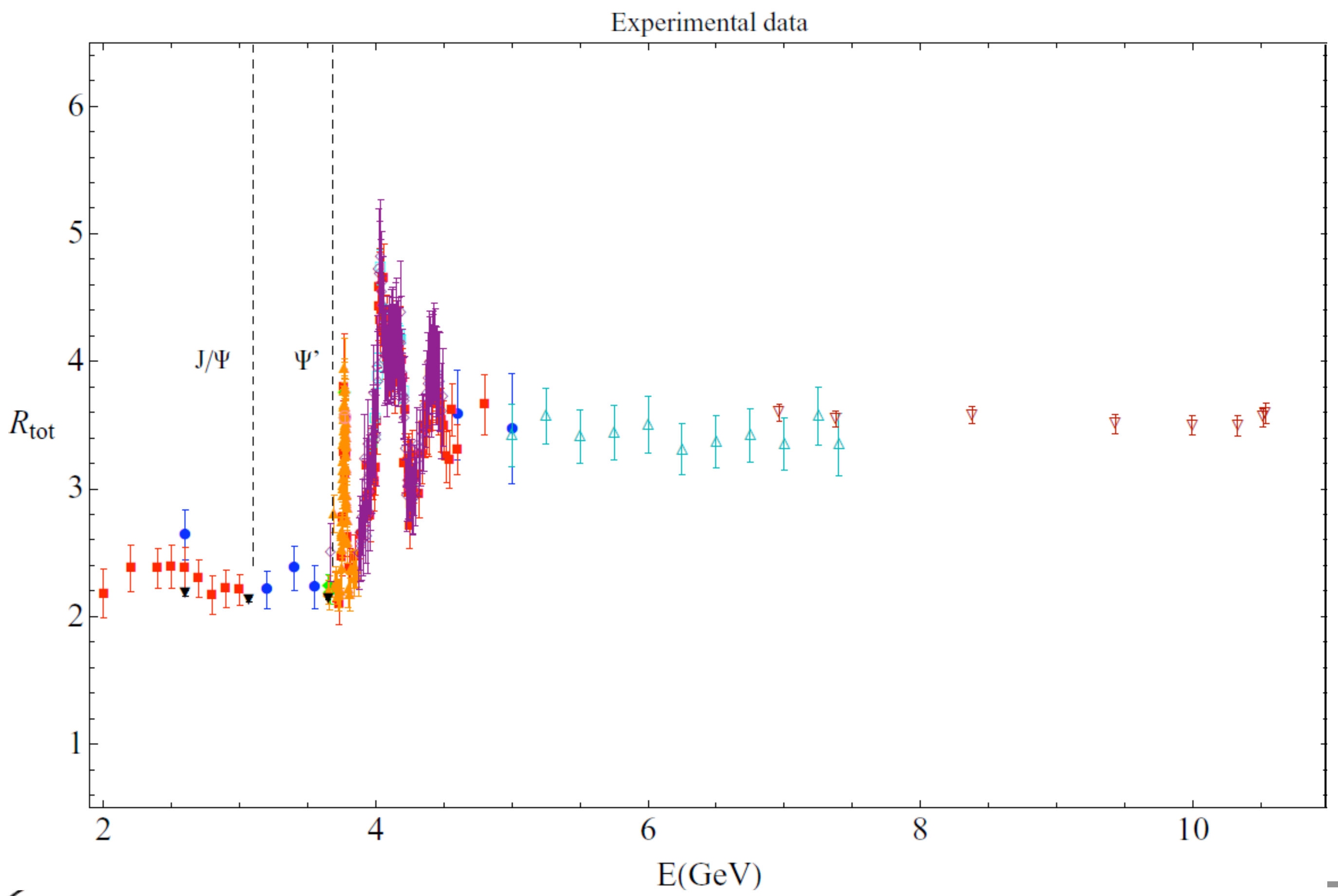
Experimental Data

Gap region **Chrystal Ball 1990**



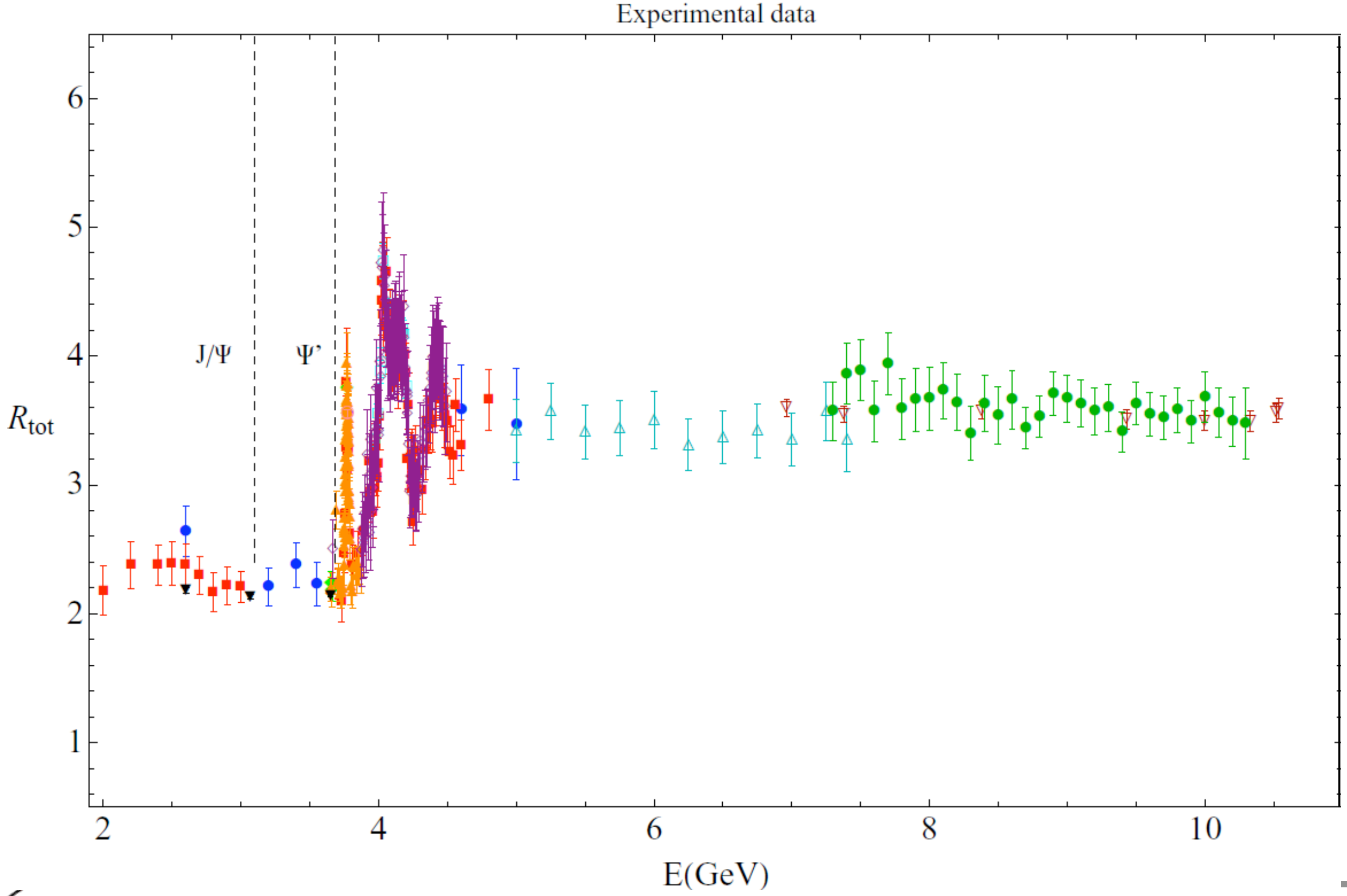
Experimental Data

High energy region **CLEO 2007**



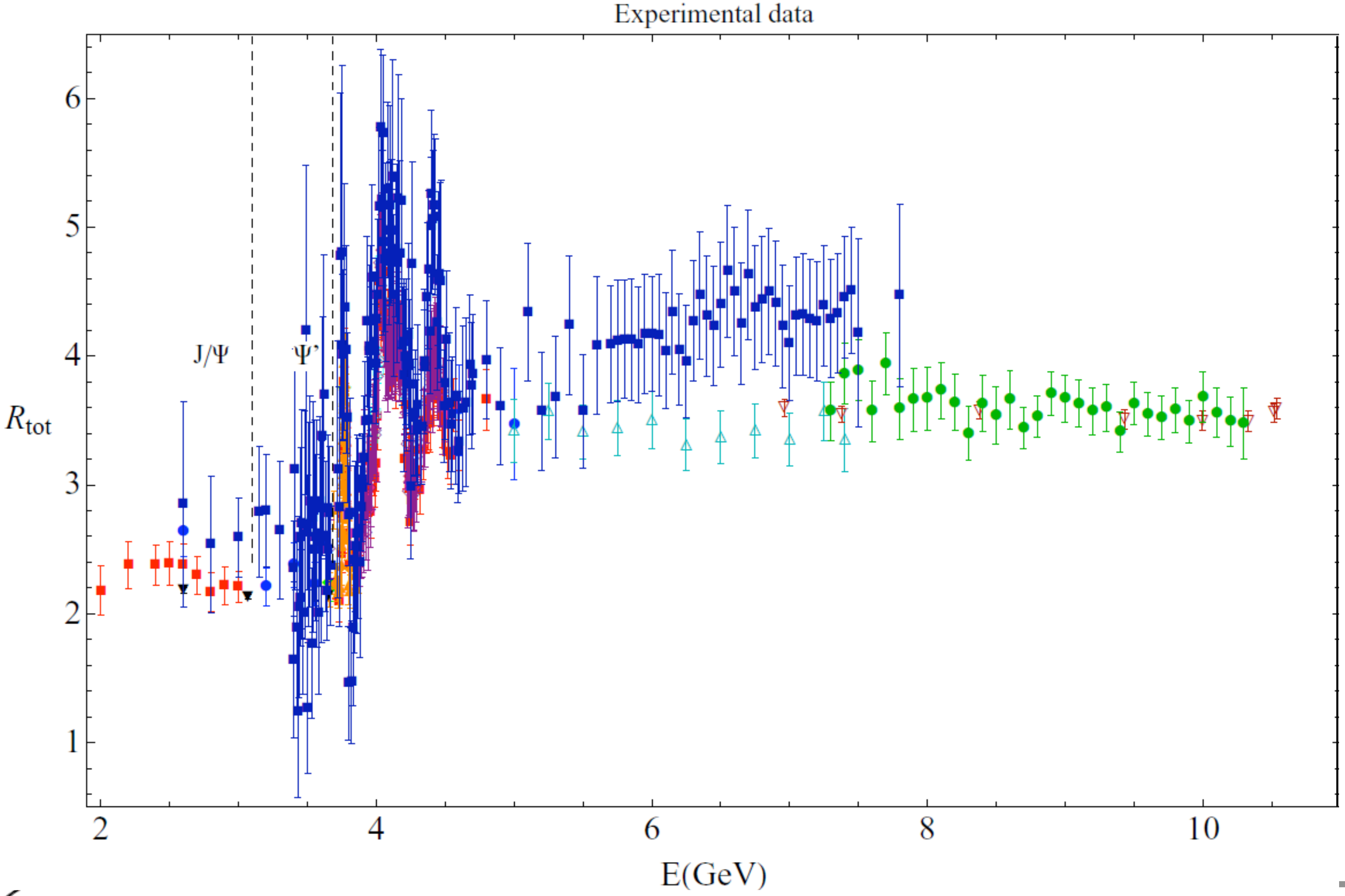
Experimental Data

High energy region **MD-1 1996**



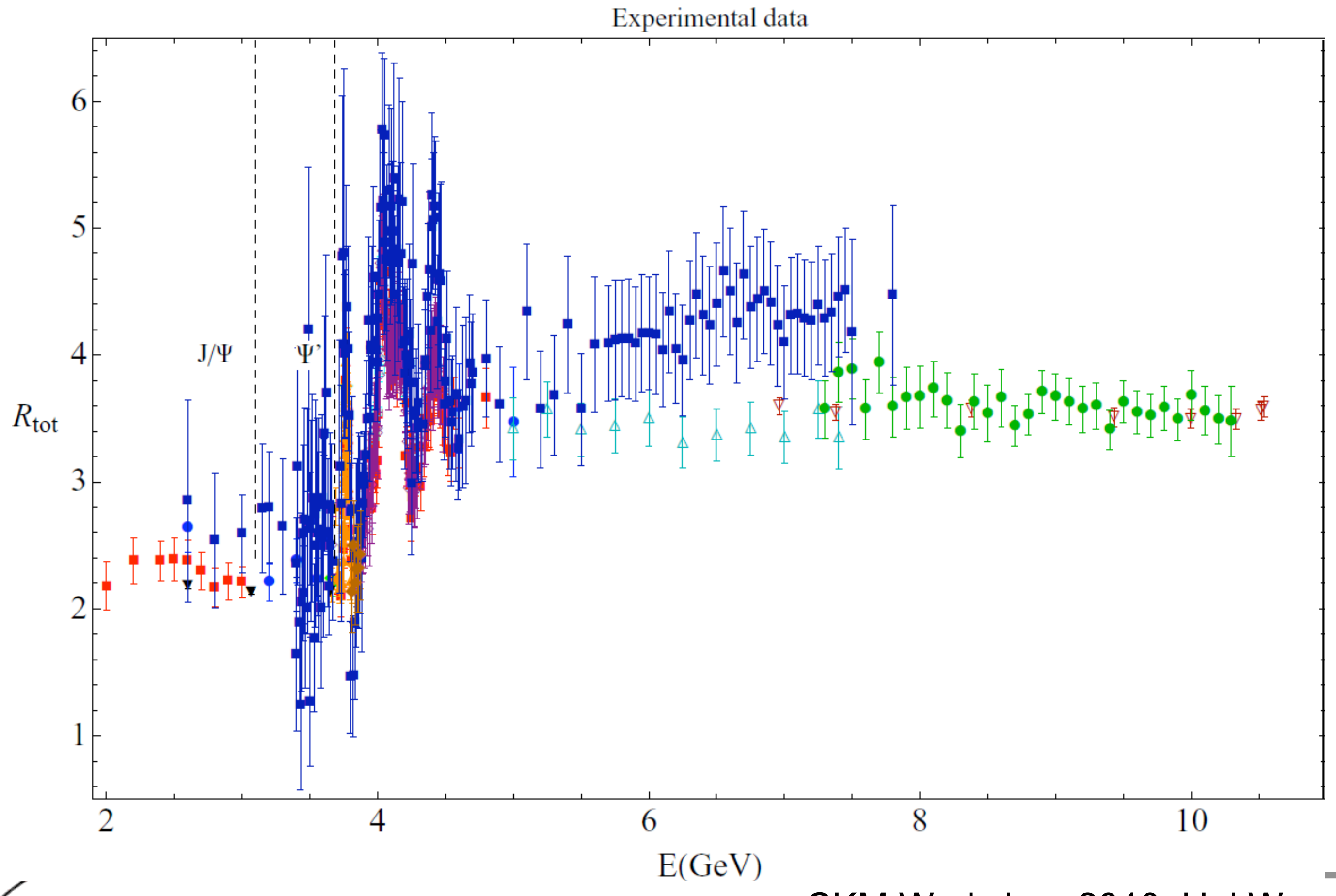
Experimental Data

Threshold and gap regions   Mark-I 1981



Experimental Data

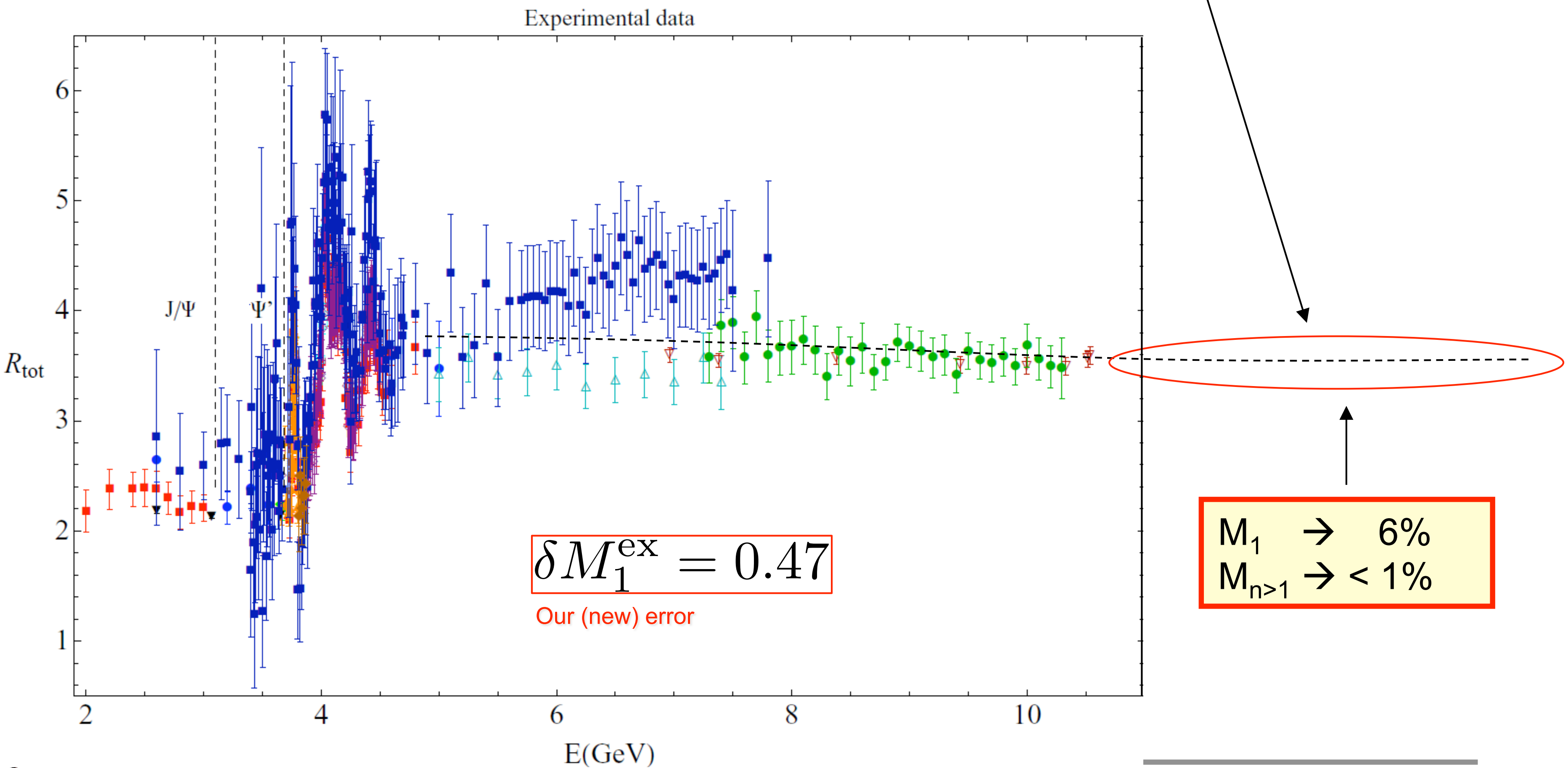
Threshold and gap regions **Mark-II 1979**



Experimental Data

Perturbation theory

- Only when there is no data: use pert. theory
- Assign a 10% error to reduce model dependence



$\boxed{\delta M_1^{ex} = 0.47}$

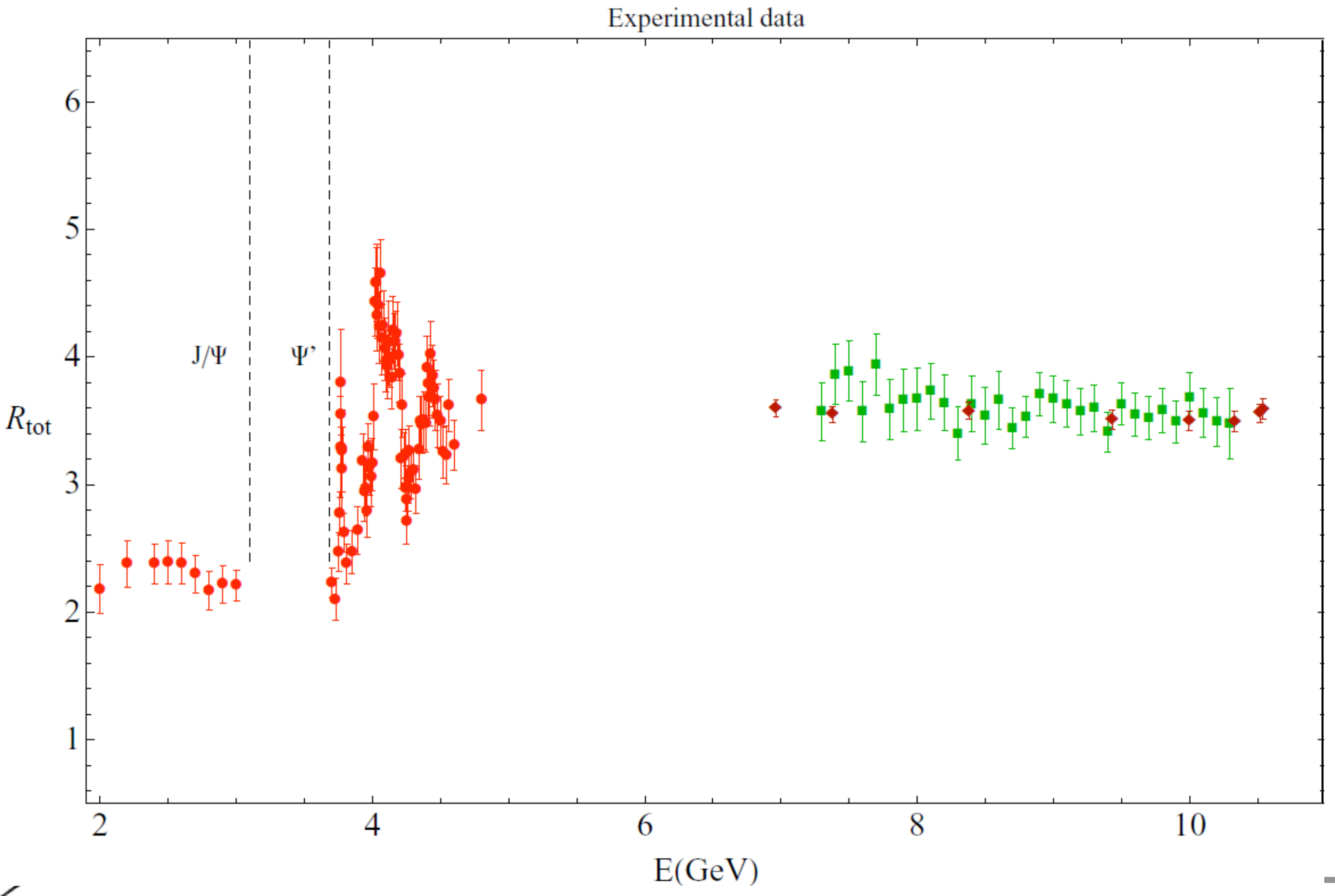
Our (new) error

$M_1 \rightarrow 6\%$
$M_{n>1} \rightarrow < 1\%$

Experimental Data: previous work

Data used in Hoang and Jamin (2004)

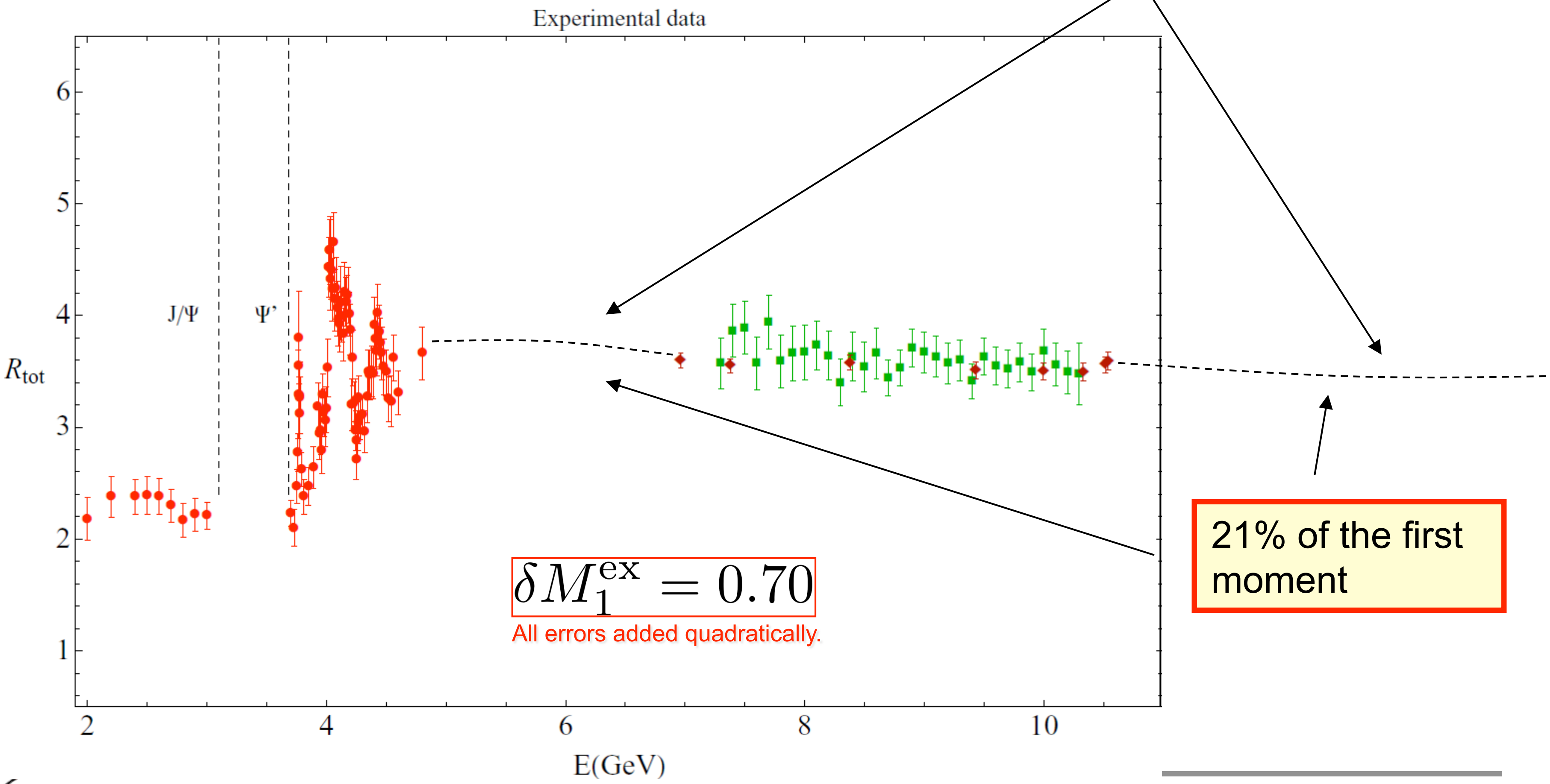
BES 2001
MD-1 1996



Experimental Data: previous work

Data used in Hoang and Jamin (2004)

- Perturbation theory for gap and h.E.region
- Assigned 10% error



$\boxed{\delta M_1^{\text{ex}} = 0.70}$

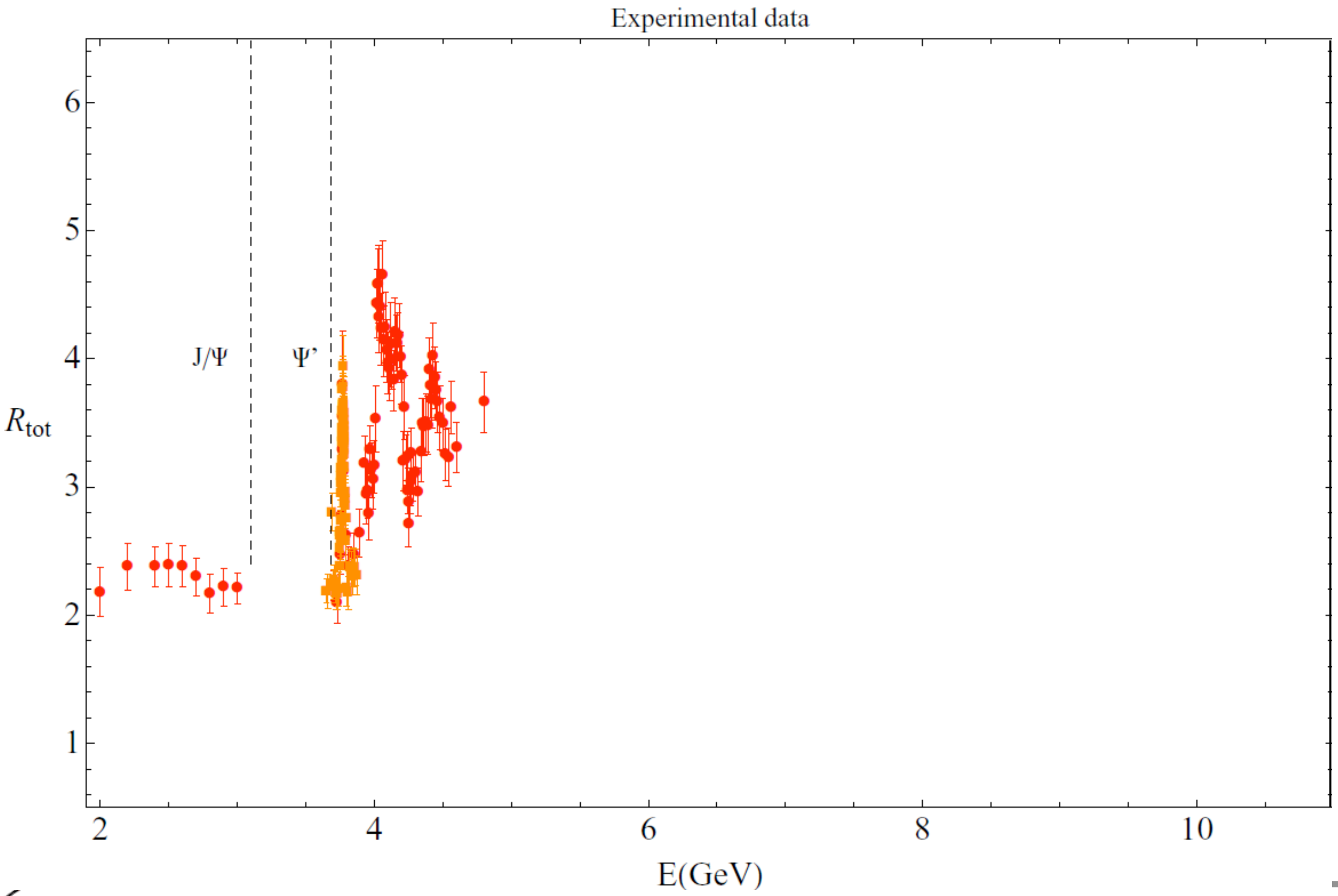
All errors added quadratically.

21% of the first moment

Experimental Data

Data used in Chetyrkin etal (2004, 2005, ...)

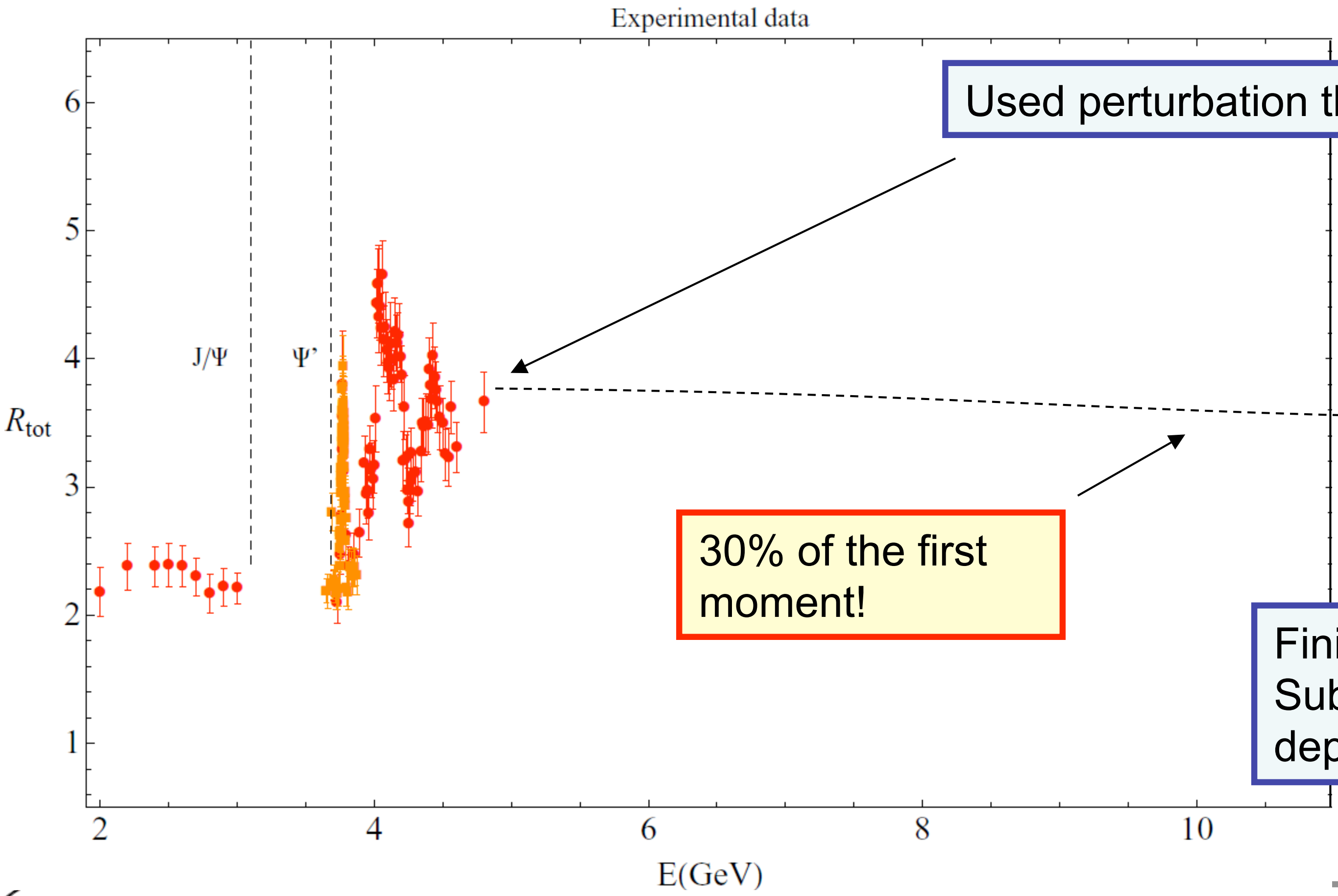
BES 2001
BES 2006 (II)



Experimental Data

Data used in Chetyrkin etal (2004, 2005, ...)

BES 2001
BES 2006 (II)



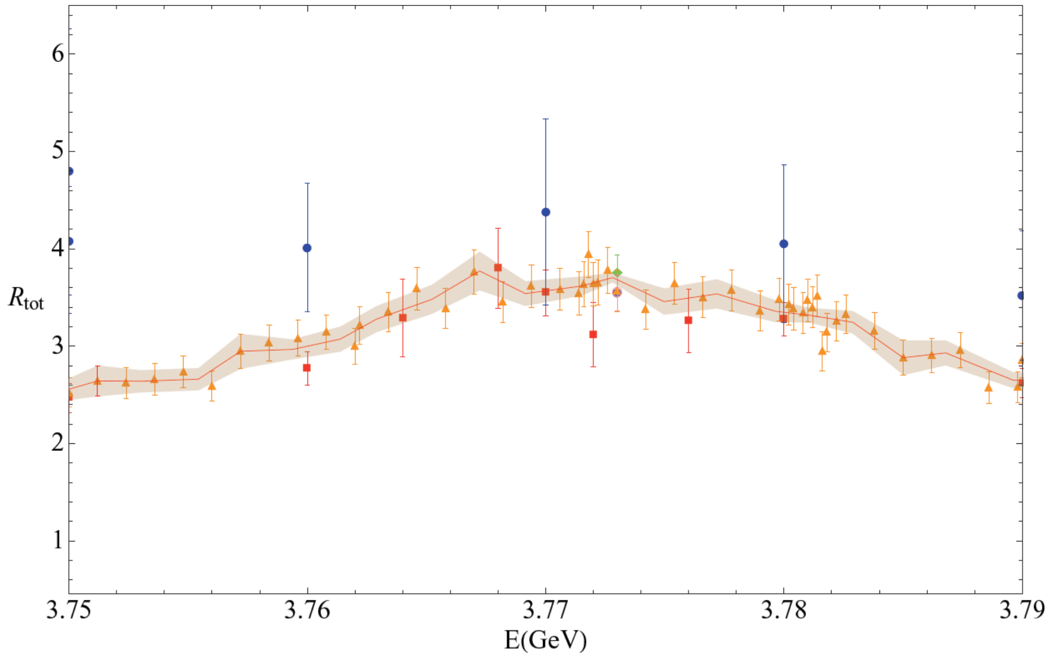
Used perturbation theory + theory errors !

$$\delta M_1^{\mathrm{ex}} = 0.31$$

30% of the first moment!

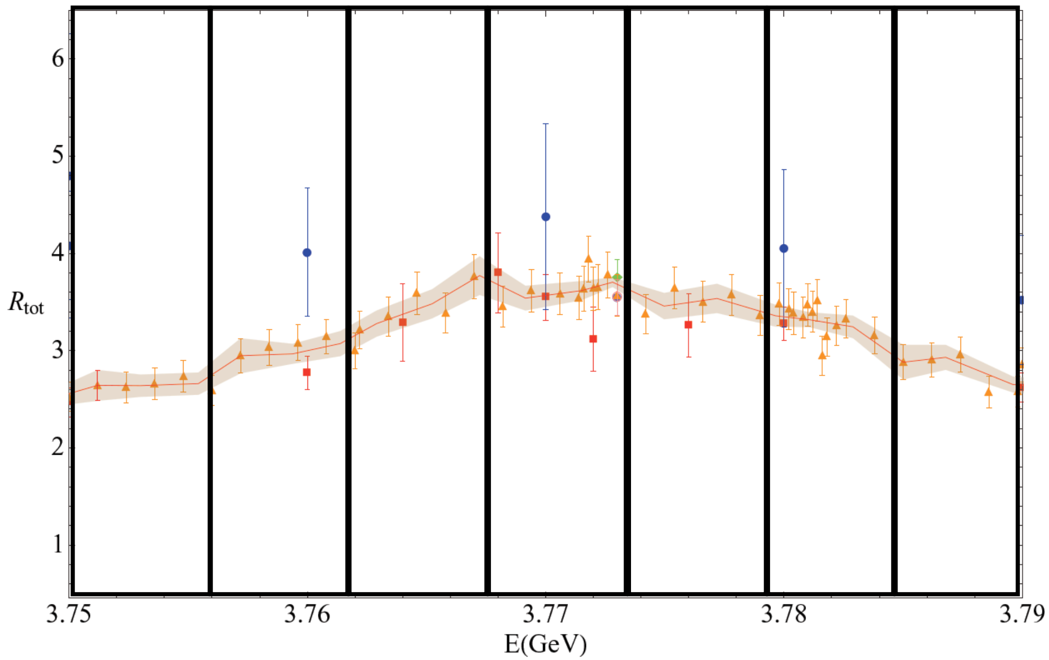
Finite energy sum-rule? Substantial model dependence !!

Fit Procedure



e.g. [g-2, $\alpha_{\rm em}(M_z)$]
Swartz; Hagiwara etal

Fit Procedure

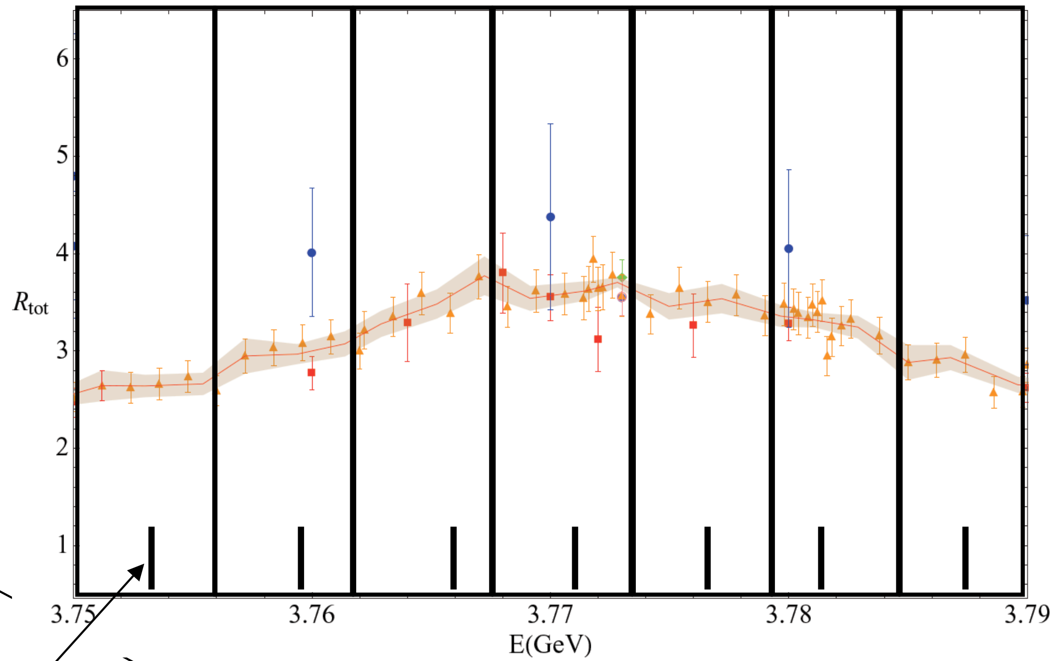


e.g. [g-2, $\alpha_{\rm em}(M_z)$]
Swartz; Hagiwara etal

1. Recluster data. Clusters not necessarily equally sized. Number of clusters and size of cluster according to the structure of the data

Fit Procedure

e.g. [g-2, $\alpha_{\rm em}(M_z)$]
Swartz; Hagiwara etal



Experimental energies

Cluster energy

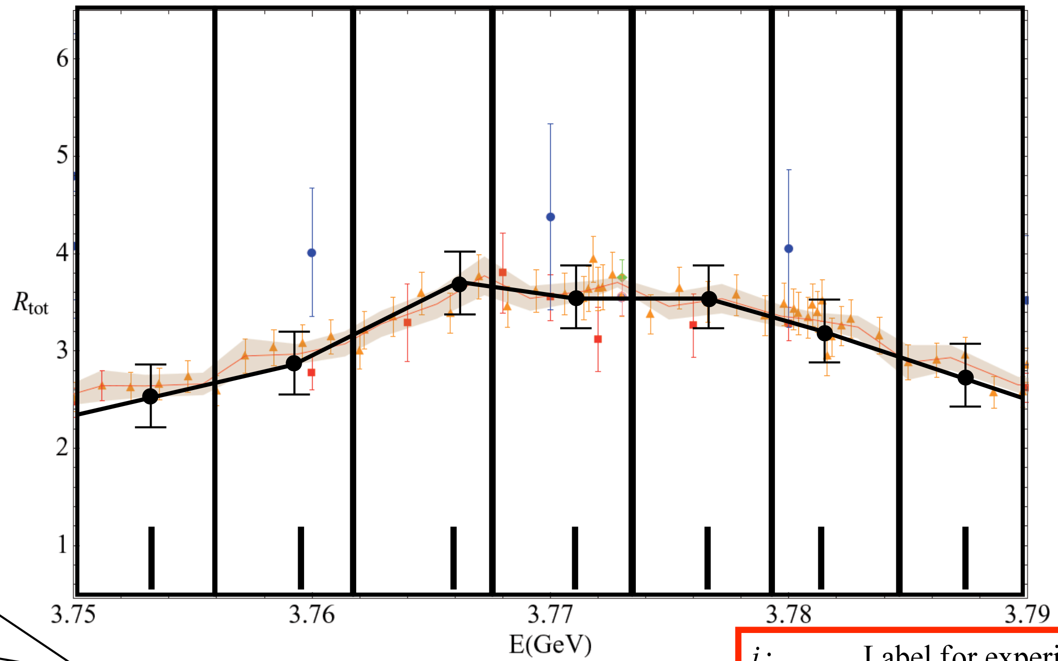
$$E_k = \frac{\displaystyle\sum_{i=1}^{N_{\rm exp}} \sum_{n=1}^{N_{i,k}} \frac{E_{n,k,i}}{\sigma_{n,k,i}^{\rm stat\,2} + \sigma_{n,k,i}^{\rm sys\,2}}}{\displaystyle\sum_{i=1}^{N_{\rm exp}} \sum_{n=1}^{N_{i,k}} \frac{1}{\sigma_{n,k,i}^{\rm stat\,2} + \sigma_{n,k,i}^{\rm sys\,2}}}$$

i :	Label for experiments
$N_{\rm exp}$:	Number of experiments
k :	Label for clusters
$N_{\rm clusters}$:	Number of clusters
n :	Label for data points
$N_{i,k}$:	Number of data points for experiment i in cluster k

2. Calculate the energy of the cluster. One weights the energy of the data points inside the clusters with their errors.

Fit Procedure

e.g. [g-2, $\alpha_{\rm em}(M_z)$]
Swartz; Hagiwara etal



Experimental data

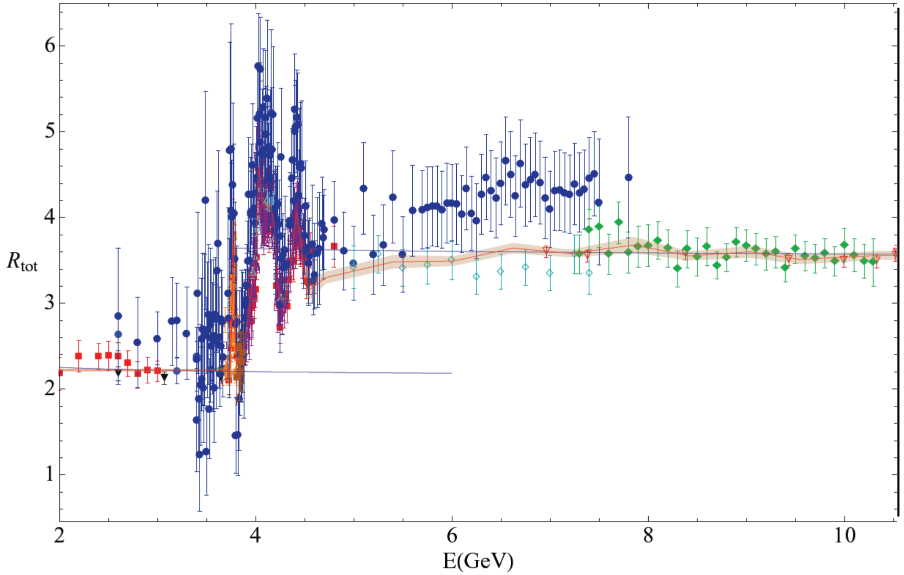
Fit parameters

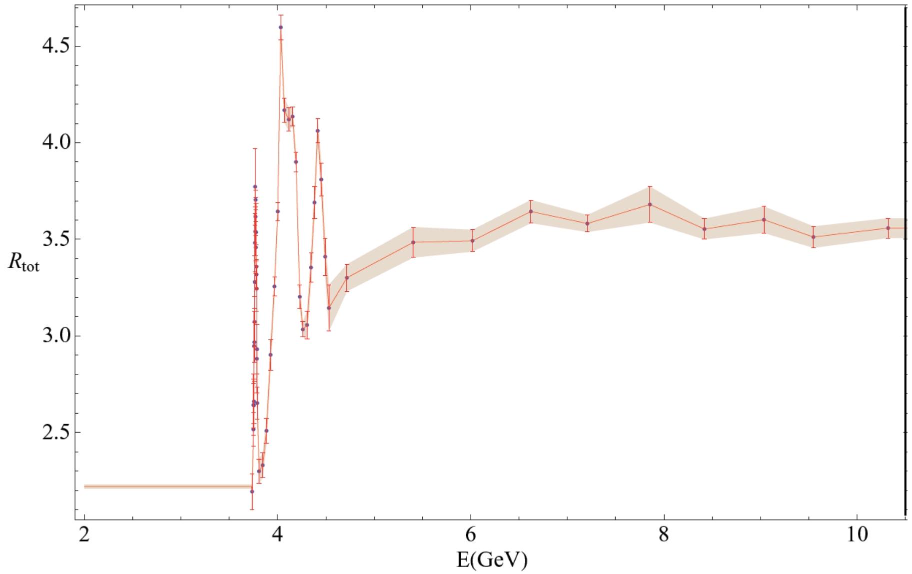
$$\chi^2 = \sum_{i=1}^{N_{\rm exp}} \left(b_i^2 + \sum_{k=1}^{N_{\rm clusters}} \sum_{n=1}^{N_{i,k}} \frac{\left[R_{i,k,n} - \left(1 + b_i \frac{\sigma_{i,k,n}^{\rm sys}}{R_{i,k,n}} \right) R_k \right]^2}{\sigma_{i,k,n}^{\rm stat\ 2}} \right)$$

i :	Label for experiments
$N_{\rm exp}$:	Number of experiments
k :	Label for clusters
$N_{\rm clusters}$:	Number of clusters
n :	Label for data points
$N_{i,k}$:	Number of data points for experiment i in cluster k

3. Fit the value of R for each cluster. Data is allowed to "move" within its systematic error in the same way for each experimental data set. The method renders errors and correlations among various clusters. One can then calculate errors and correlations for the moments.

Fit Procedure: Results





Preliminary results !!!!!

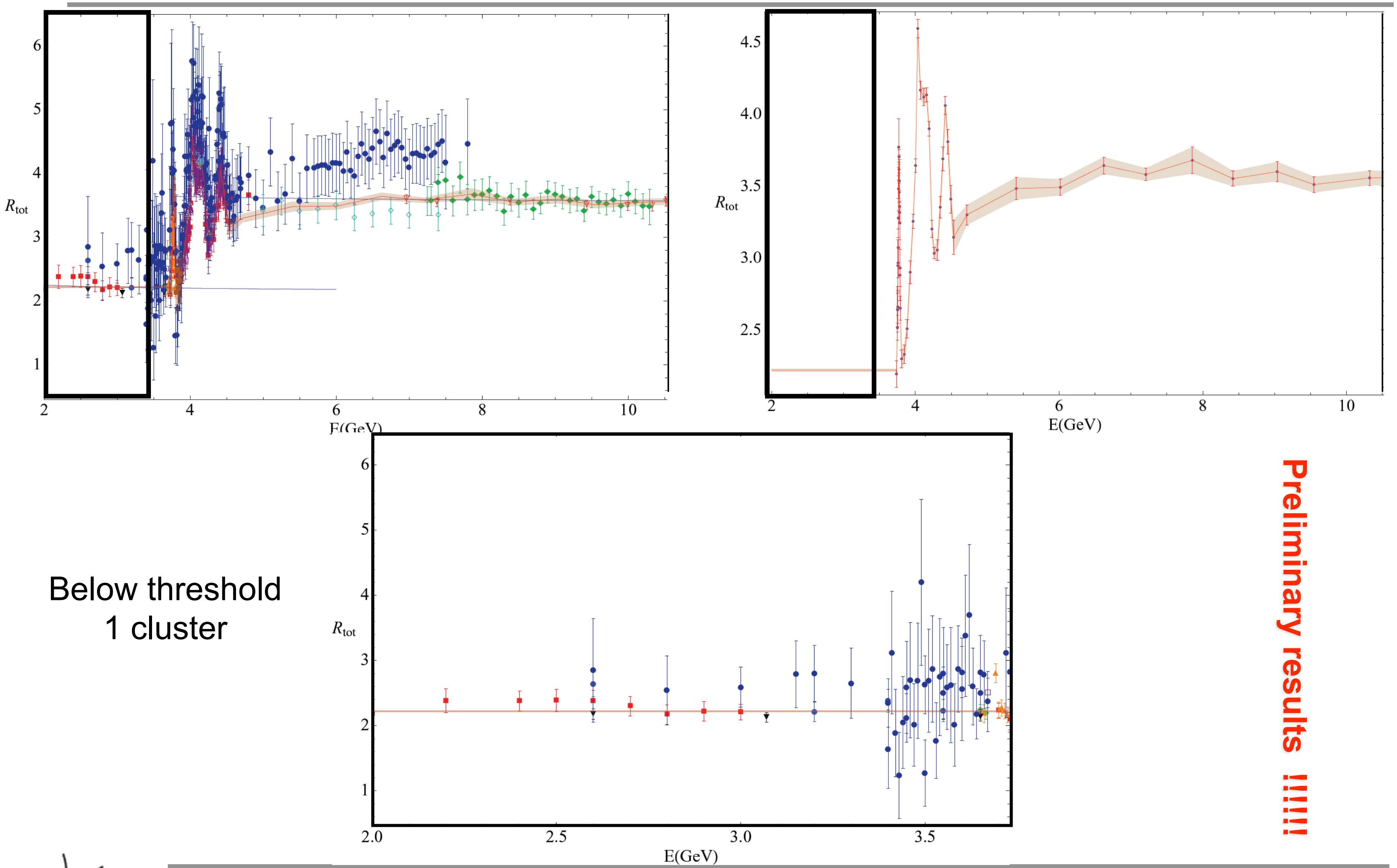
With our fit procedure we are capable of simultaneously determining the R_{uds} background and the R_{cc}

$$R_{\mathrm{tot}} = R_{cc} + R_{uds}$$

$$\frac{\chi^2}{\mathrm{d.o.f.}} = 1.42$$

Excellent quality of the fit !!!

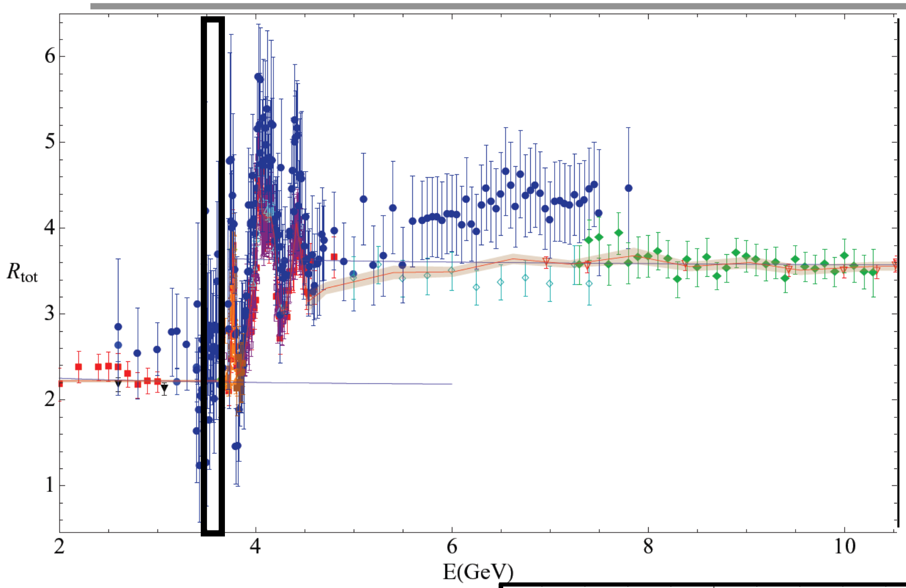
Fit Procedure: Results

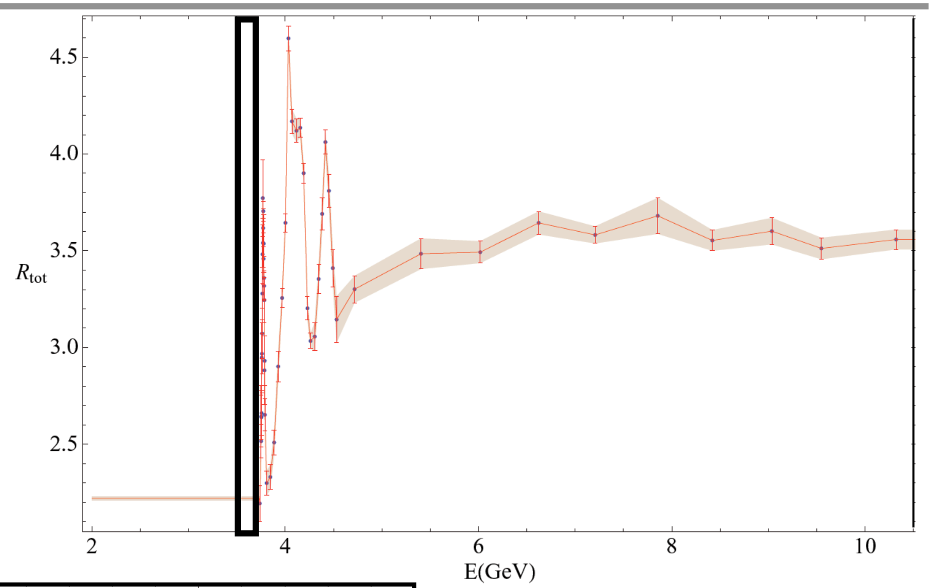


Below threshold
1 cluster

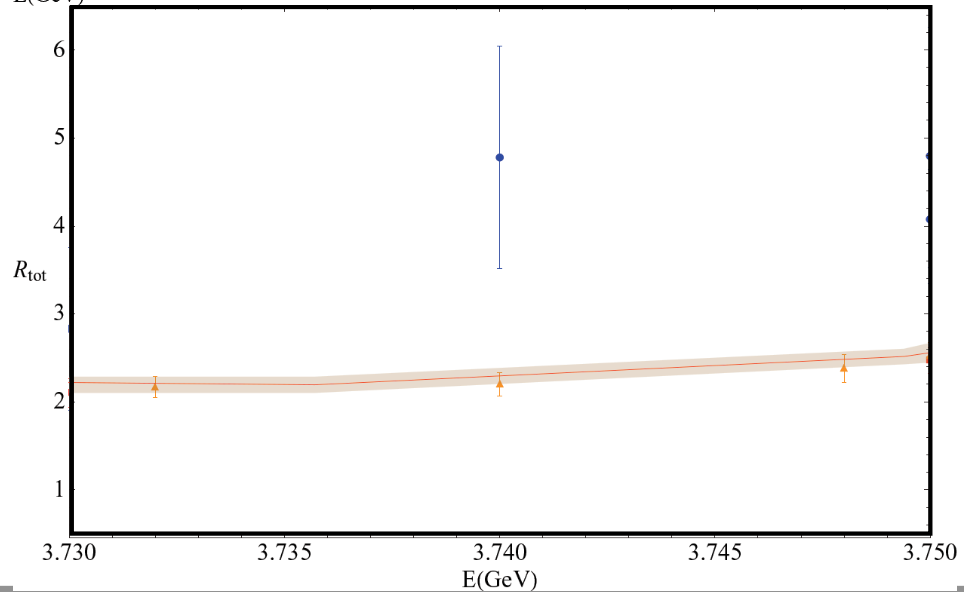
Preliminary results !!!!!

Fit Procedure: Results



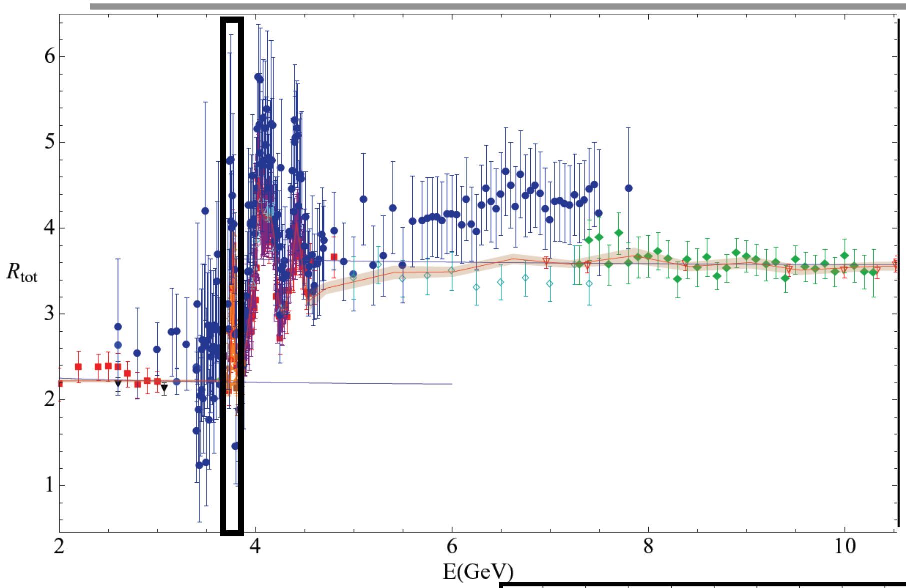


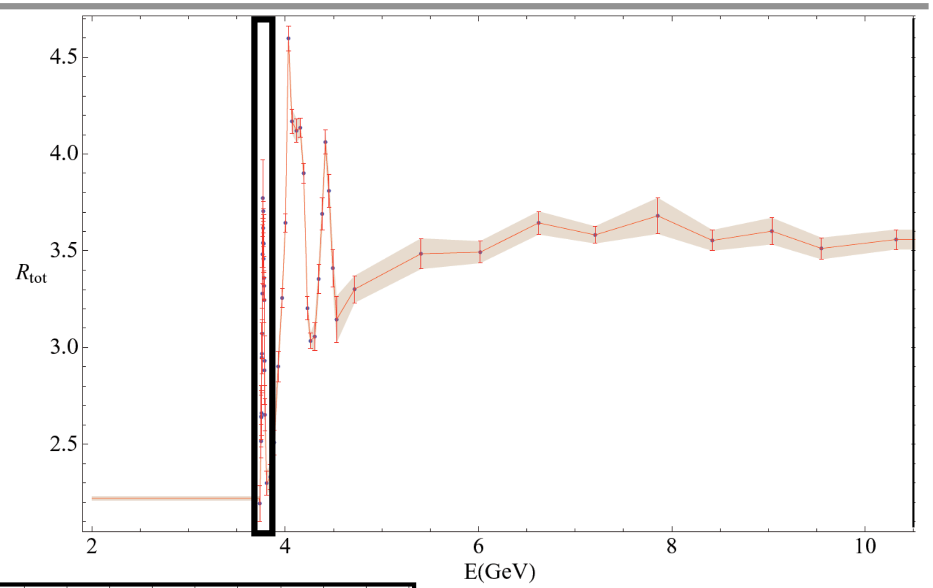
Below resonances
2 clusters v



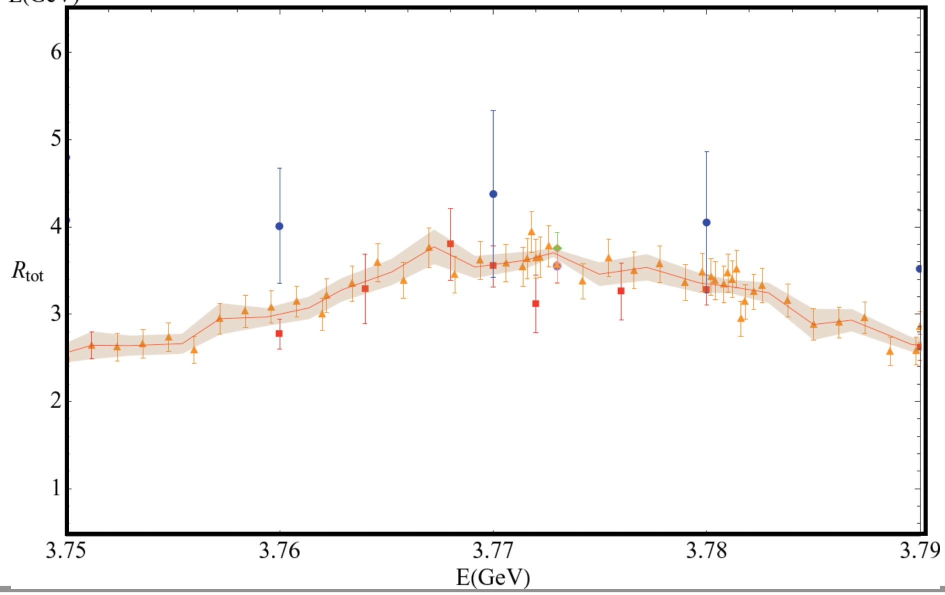
Preliminary results !!!!!

Fit Procedure: Results



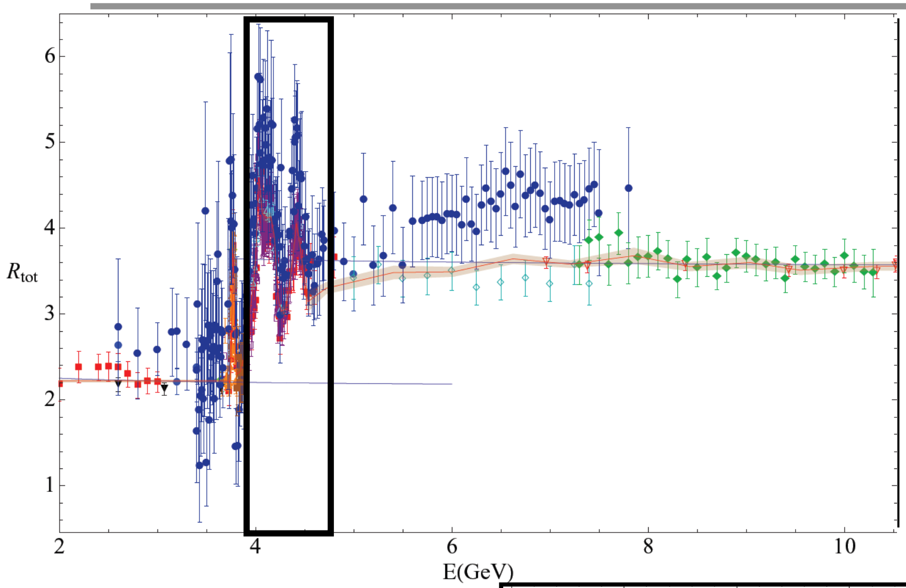


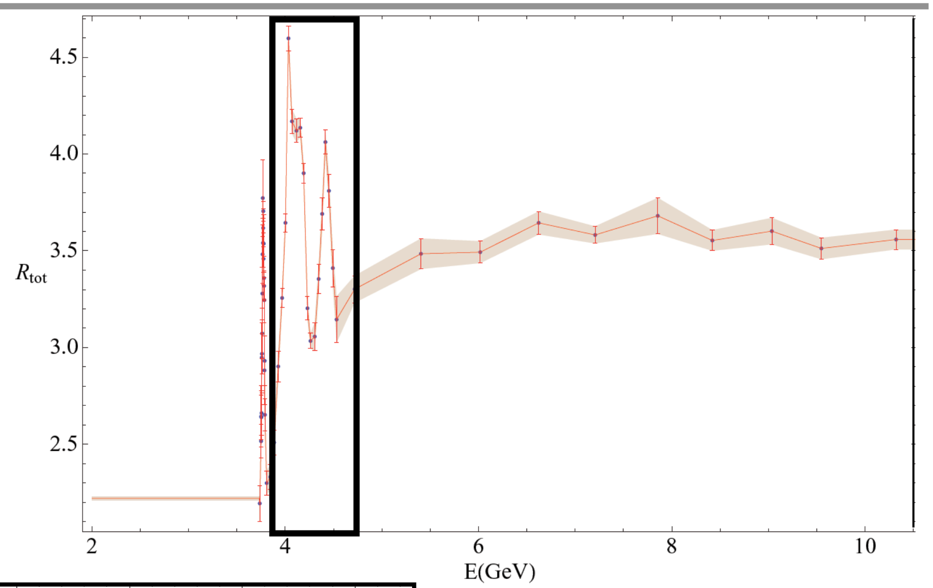
Below resonances
2 clusters v



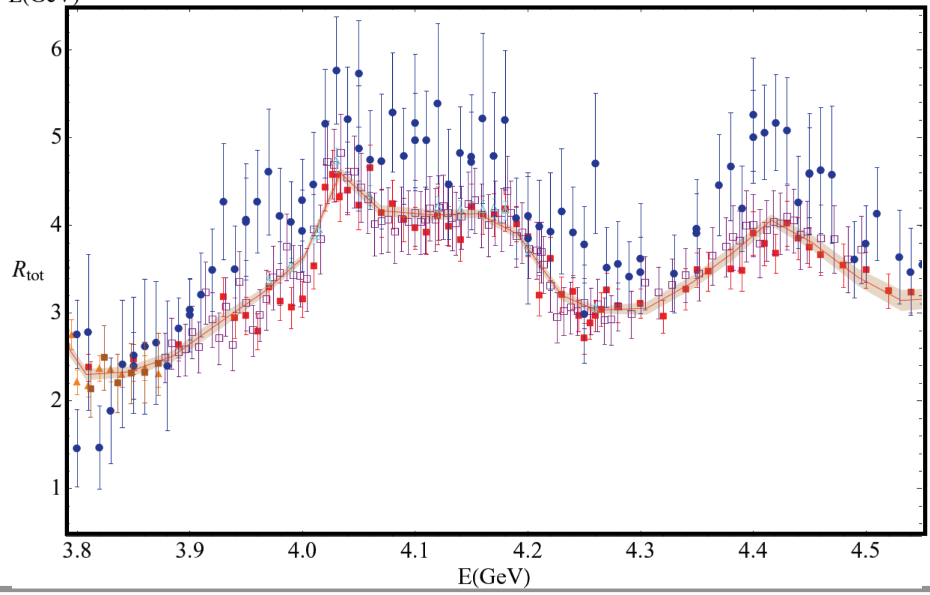
Preliminary results !!!!!

Fit Procedure: Results



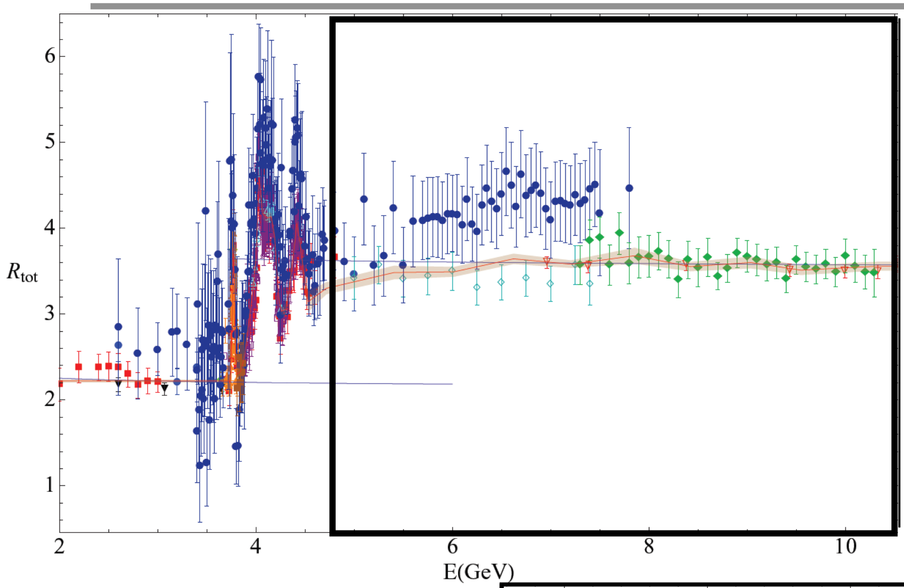


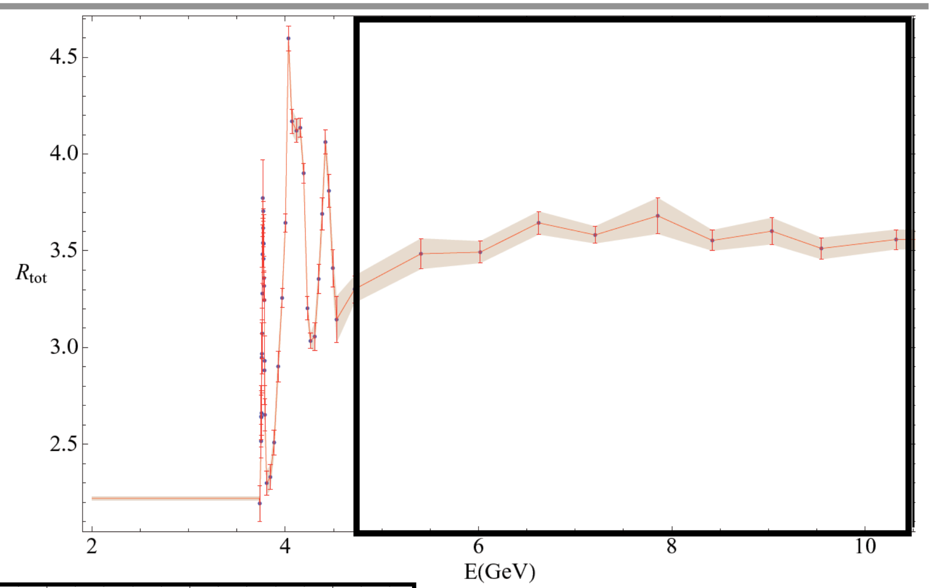
Second resonance
20 clusters



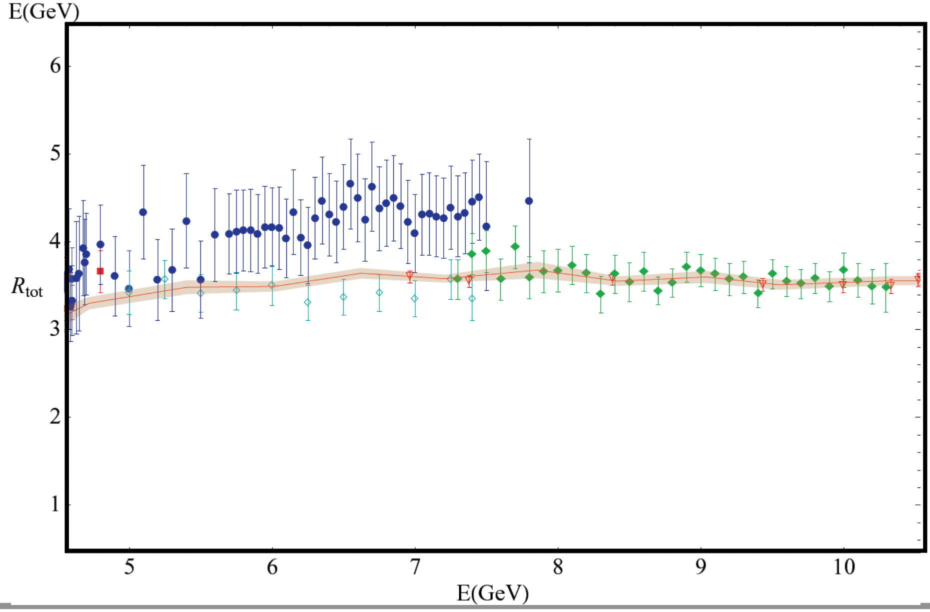
Preliminary results !!!!!

Fit Procedure: Results





Continuum data
10 clusters



Preliminary results !!!!!

Fit Procedure: Results

Dehnadi, Mateu, Zebarjad, AHH

Straightforward numerical procedure:

stat = statistical + uncor. systematic

sys = correlated systematic

Preliminary results !!!!!!

Prediction for moments $M_n = m_n 10^{n-2}\ \mathrm{GeV}^{n+1}$
$M_1 = 21.40 \pm 0.22_{\mathrm{stat}} \pm 0.41_{\mathrm{sys}}$ $M_2 = 14.84 \pm 0.18_{\mathrm{stat}} \pm 0.28_{\mathrm{sys}}$ $M_3 = 13.06 \pm 0.19_{\mathrm{stat}} \pm 0.24_{\mathrm{sys}}$ $M_4 = 12.47 \pm 0.19_{\mathrm{stat}} \pm 0.23_{\mathrm{sys}}$

Error in M_1 from unkown continuum where 10% theory error is used: 0.13

This is an acceptable model-dependence !!

We also predict correlations among the various moments, useful for simultaneous fits.

$$\begin{pmatrix} 0.0484139 & 0.035155 & 0.0338914 & 0.0340731 \\ 0.035155 & 0.033846 & 0.0340239 & 0.0348448 \\ 0.0338914 & 0.0340239 & 0.0348369 & 0.0359718 \\ 0.0340731 & 0.0348448 & 0.0359718 & 0.0372943 \end{pmatrix}_{\mathrm{stat}} + \begin{pmatrix} 0.170001 & 0.115712 & 0.100738 & 0.0956423 \\ 0.115712 & 0.080453 & 0.0693155 & 0.0653107 \\ 0.100738 & 0.069316 & 0.0593267 & 0.0556966 \\ 0.095642 & 0.065311 & 0.0556966 & 0.0521944 \end{pmatrix}_{\mathrm{sys}}$$

Fit Procedure: Results

Straightforward numerical procedure:

Dehnadi, Mateu, Zebarjad, AHH

Preliminary results !!!!!!

Prediction for moments $M_n = m_n 10^{n-2}\ \mathrm{GeV}^{n+1}$
$M_1 = 21.40 \pm 0.22_{stat} \pm 0.41_{sys}$ $M_2 = 14.84 \pm 0.18_{stat} \pm 0.28_{sys}$ $M_3 = 13.06 \pm 0.19_{stat} \pm 0.24_{sys}$ $M_4 = 12.47 \pm 0.19_{stat} \pm 0.23_{sys}$

Error in M_1 from unkown continuum where 10% theory error is used: 0.13

This is an acceptable model-dependence !!

- different correlation models
- modified χ^2-functions
- modified sets of experimental data (redundant data)

Changes within errors.

Sum Rule Error Analysis

How to obtain reliable error estimate:

→ Use differerent types of perturbative expansions

"Fixed order"

$$M_n = \frac{1}{\left[4m(\mu_m)^2\right]^n} \sum_{i=0} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum C_i^{a,b} \log^a\left[\frac{m(\mu_m)}{\mu_m}\right] \log^a\left[\frac{m(\mu_m)}{\mu_\alpha}\right] = M_n^{\exp}$$

"Expanded"

$$(M_n)^{\frac{1}{2n}} = \frac{1}{2m(\mu_m)} \sum_{i=0} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \tilde{C}_i^{a,b} \log^a\left[\frac{m(\mu_m)}{\mu_m}\right] \log^a\left[\frac{m(\mu_m)}{\mu_\alpha}\right] = \left(M_n^{\exp}\right)^{\frac{1}{2n}}$$

"Iterative"

$$m_0 = \left(\frac{M_n^{\exp}}{2C_{n,0}}\right)^{\frac{1}{2n}} = \frac{\left(M_n^{\exp}\right)^{\frac{1}{2n}}}{2\tilde{C}_{n,0}}$$

$$m(\mu_m) = m_0 \left\{1 + \sum_{i=1} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \hat{C}_{n,i}^{a,b} \log^a\left[\frac{m_0}{\mu_m}\right] \log^a\left[\frac{m_0}{\mu_\alpha}\right]\right\}$$

Sum Rule Error Analysis

How to obtain reliable error estimate:

→ Use differerent types of perturbative expansions

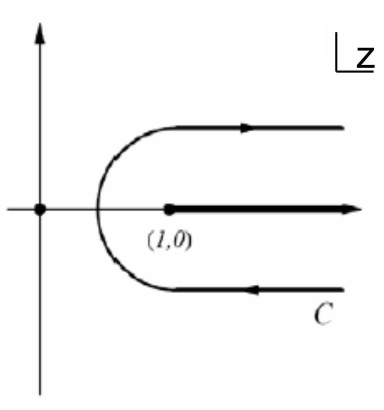
"Contour-"Improved" "

$$\mu_\alpha^2 \to \mu_\alpha^2 (1-z) \qquad z = \frac{q^2}{4m^2}$$

Residual dependence on μ_α

Jamin, AHH (2004)

Reweights threshold versus continuum effects



Moments become residually dependent on the scheme for $\Pi(0)$.

at $\mathcal{O}(\alpha_s^n)$: terms $\propto \alpha_s^{m>n+1}$

Sum Rule Error Analysis

How to obtain reliable error estimate:

→ Use differerent types of perturbative expansions

"Fixed order"

$$M_n = \frac{1}{\left[4m(\mu_m)^2\right]^n} \sum_{i=0} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum C_i^{a,b} \log^a\left[\frac{m(\mu_m)}{\mu_m}\right] \log^a\left[\frac{m(\mu_m)}{\mu_\alpha}\right] = M_n^{\exp}$$

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$$(M_n)^{\frac{1}{2n}} = \frac{1}{2m(\mu_m)} \sum_{i=0} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \tilde{C}_i^{a,b} \log^a\left[\frac{m(\mu_m)}{\mu_m}\right] \log^a\left[\frac{m(\mu_m)}{\mu_\alpha}\right] = \left(M_n^{\exp}\right)^{\frac{1}{2n}}$$

residual μ_α and μ_m dependence
due to truncation of α series

"Iterative"

$$m_0 = \left(\frac{M_n^{\exp}}{2C_{n,0}}\right)^{\frac{1}{2n}} = \frac{\left(M_n^{\exp}\right)^{\frac{1}{2n}}}{2\tilde{C}_{n,0}}$$

$$m(\mu_m) = m_0 \left\{1 + \sum_{i=1} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \sum \hat{C}_{n,i}^{a,b} \log^a\left[\frac{m_0}{\mu_m}\right] \log^a\left[\frac{m_0}{\mu_\alpha}\right]\right\}$$

Sum Rule Error Analysis

How to obtain reliable error estimate:

→ Dependence of $\overline{m}_c(\overline{m}_c)$ on μ_α and μ_m .

"Fixed order" *"Expanded"* *"Iterative"* *"Contour-"Improved" "*

For each expansion method:

1. Determine $\overline{m}_c(\mu_m)$ for a given choice of $\alpha_s(\mu_\alpha)$.
2. Determine $\overline{m}_c(\overline{m}_c)$ using the RGE's.
3. Result: $\overline{m}_c(\overline{m}_c)[\mu_\alpha, \mu_m]$

Relativistic Sum Rule Analysis

Dehnadi, Mateu, Zebarjad, AHH
(our first attempt)

Playing with different choices:

fixed order

1) use $\alpha_s(\mu)$ and $\overline{m}_Q(\overline{m}_Q)$, $\mu = 2 - 4\ \mathrm{GeV}$

contour improved

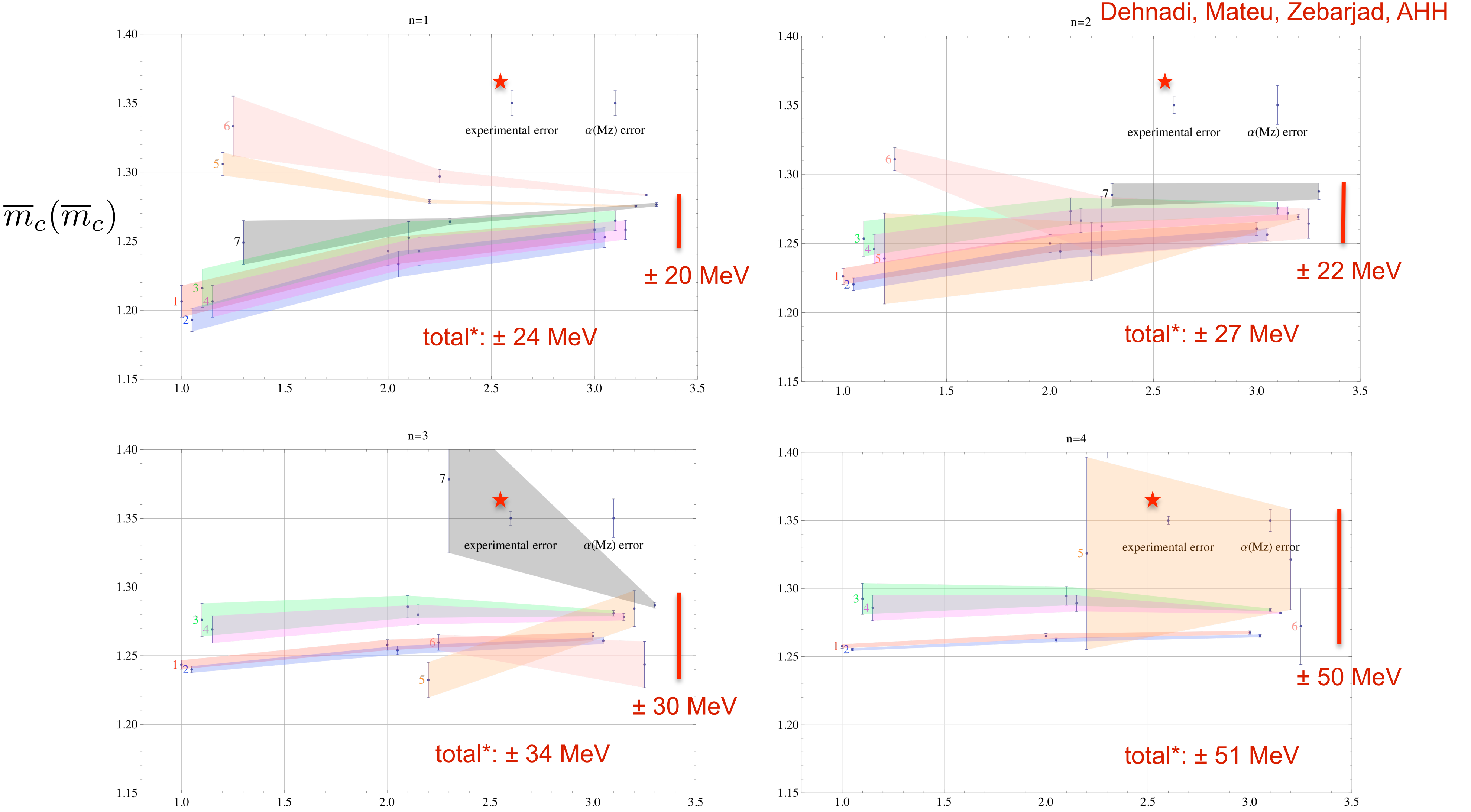
2) use $\alpha_s(\xi s)$ and $\overline{m}_Q(\overline{m}_Q)$, $\xi = 1 - 3$
3) use $\alpha_s(\xi s v^2)$ and $\overline{m}_Q(\overline{m}_Q)$ with $\Pi(0)^{\overline{\mathrm{MS}}}$, $\xi = 1 - 3$
4) use $\alpha_s(\xi s v^2)$ and $\overline{m}_Q(\overline{m}_Q)$ with $\Pi(0)^{\mathrm{onshell}}$, $\xi = 1 - 3$

fixed order

5) use $\alpha_s(\mu)$ and $\overline{m}_Q(\mu)$, $\mu = 2 - 4\ \mathrm{GeV}$ ← Karlsruhe method
6) use $\alpha_s(2\mu)$ and $\overline{m}_Q(\mu)$, $\mu = 2 - 4\ \mathrm{GeV}$
7) use $\alpha_s(1/2\mu)$ and $\overline{m}_Q(\mu)$, $\mu = 2 - 4\ \mathrm{GeV}$

Charm mass: new results

Dehnadi, Mateu, Zebarjad, AHH

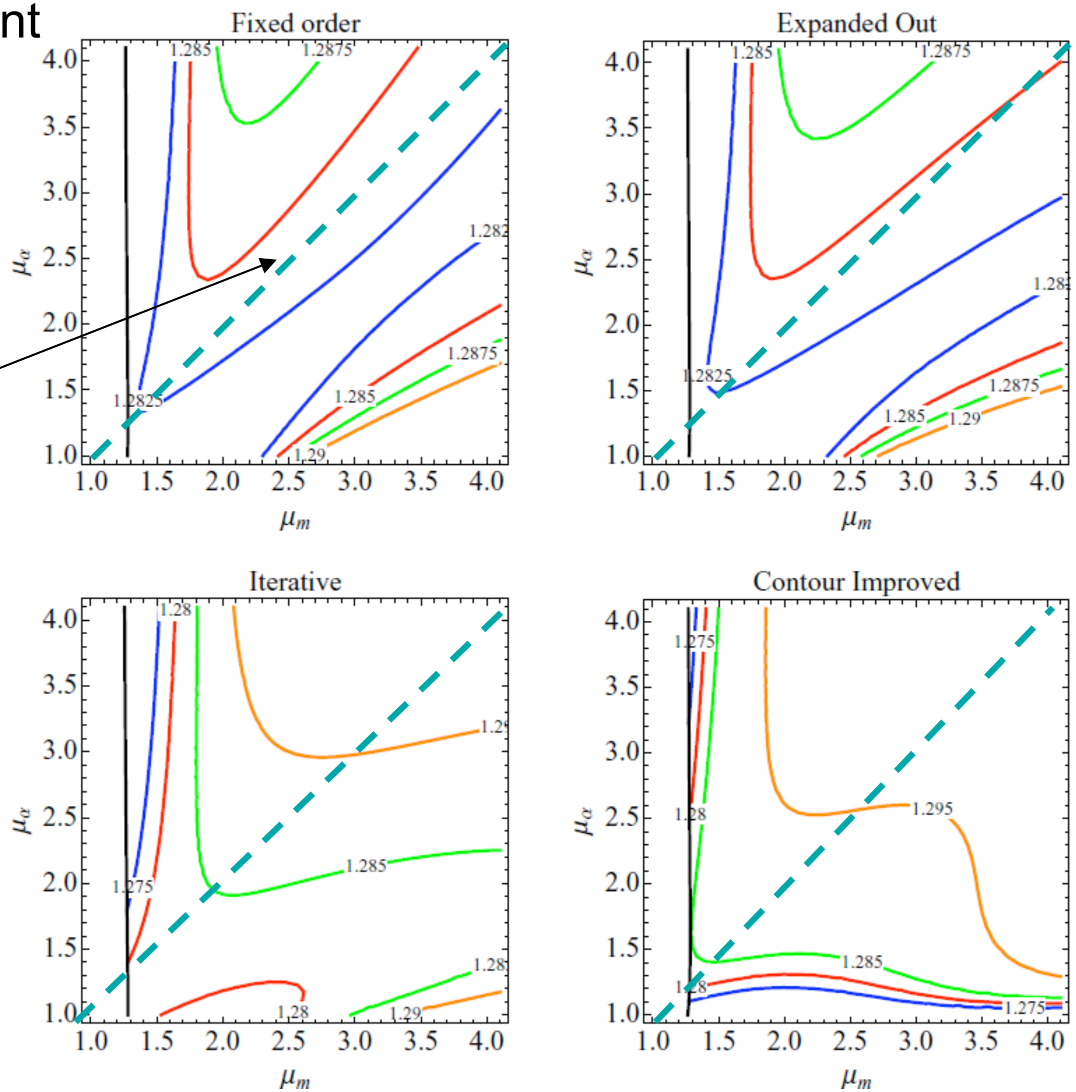


*: if continuum "experimental" errors of Karlsruhe group are adopted

Sum Rule Error Analysis

Contours of constant

$\overline{m}_c(\overline{m}_c)[\mu_\alpha, \mu_m]$



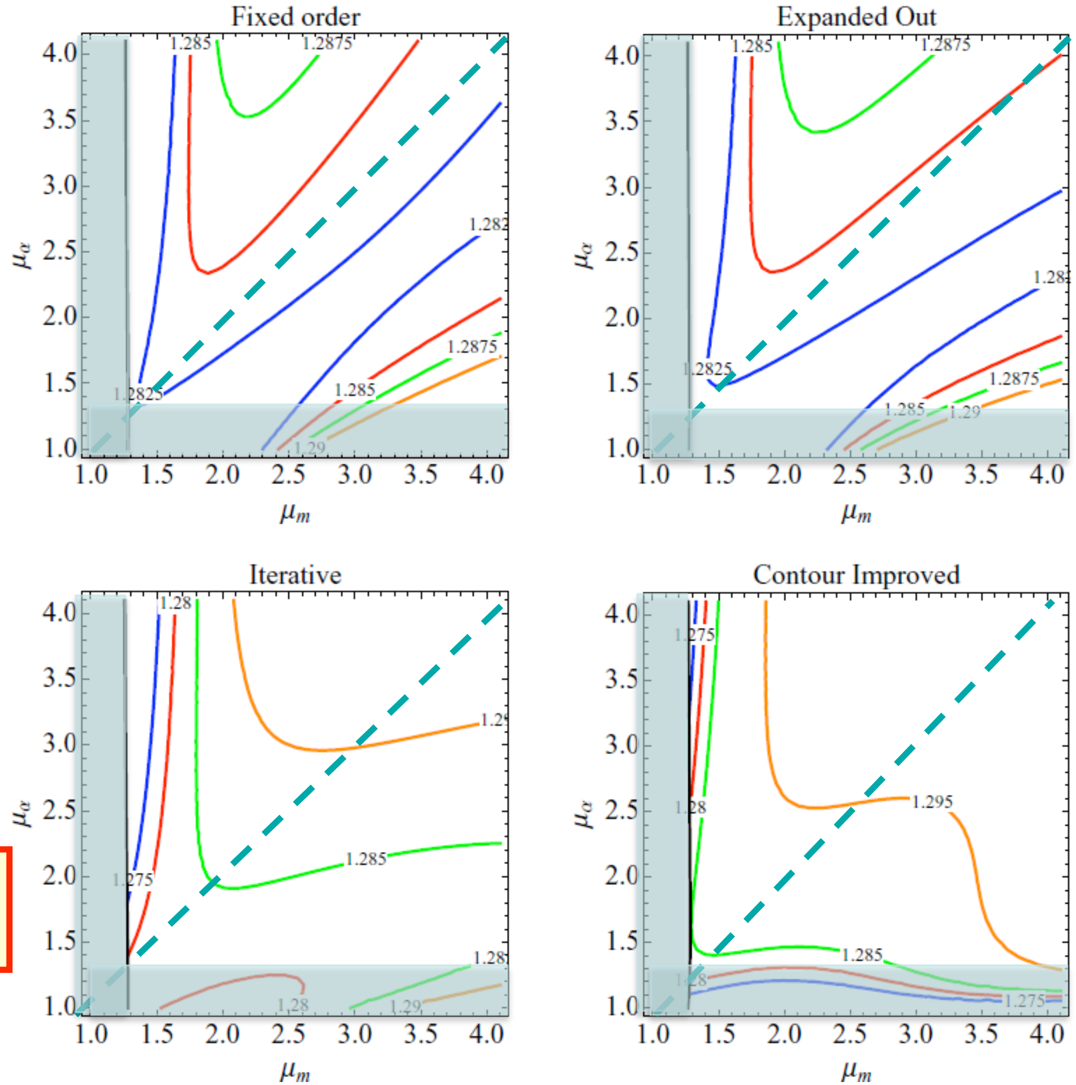
Dehnadi, Mateu, Zebarjad, AHH

Chetyrkin etal used $\mu_\alpha = \mu_m$ and empoyed "Fixed-Order" method

Just happens to be along a contour line!

Sum Rule Error Analysis

Dehnadi, Mateu, Zebarjad, AHH



Exclude regions with

$\mu_m, \mu_\alpha < m(m)$

Use:

$m(m) \le \mu_m, \mu_\alpha \le 4\,\mathrm{GeV}$

Sum Rule Error Analysis

$\mu_m = 3\,\mathrm{GeV}$

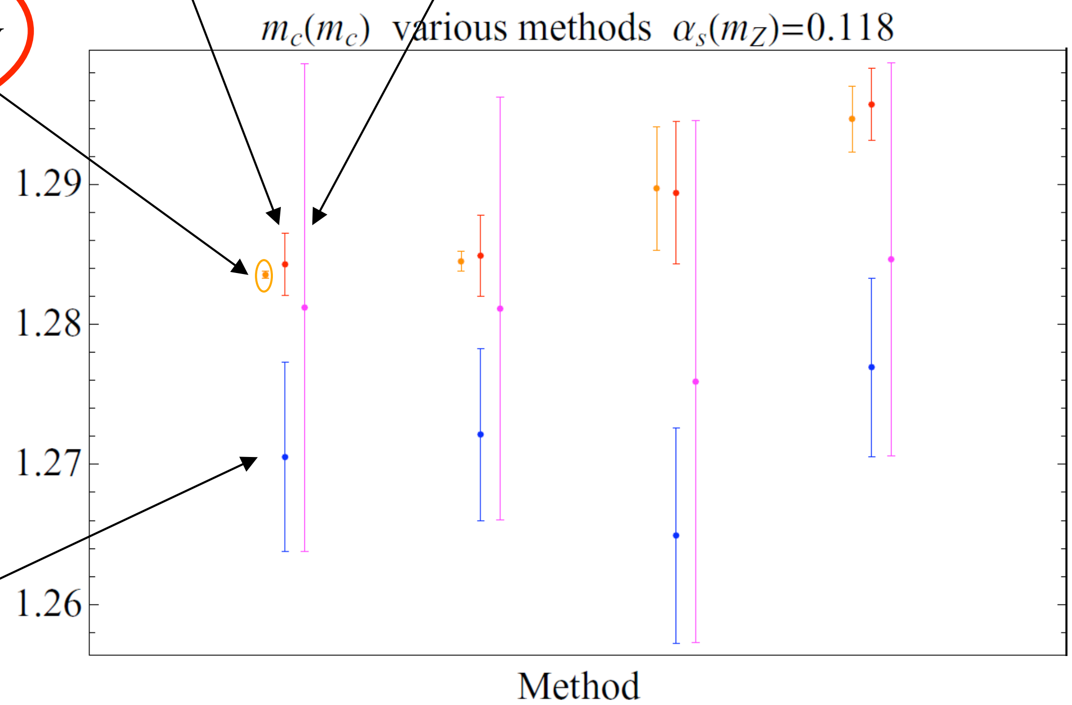
$2\,\mathrm{GeV} \leq \mu_\alpha \leq 4\,\mathrm{GeV}$

Our error analysis:

Double variation $m(m) \leq \mu_m, \mu_\alpha \leq 4\,\mathrm{GeV}$

Chetyrkin etal.

$2\,\mathrm{GeV} \leq (\mu_\alpha = \mu_m) \leq 4\,\mathrm{GeV}$



$\mu_m = m(m)$

$2\,\mathrm{GeV} \leq \mu_\alpha \leq 4\,\mathrm{GeV}$

"Fixed order" *"Expanded"* *"Iterative"* *"Contour-"Improved" "*

Relativistic Sum Rule Analysis

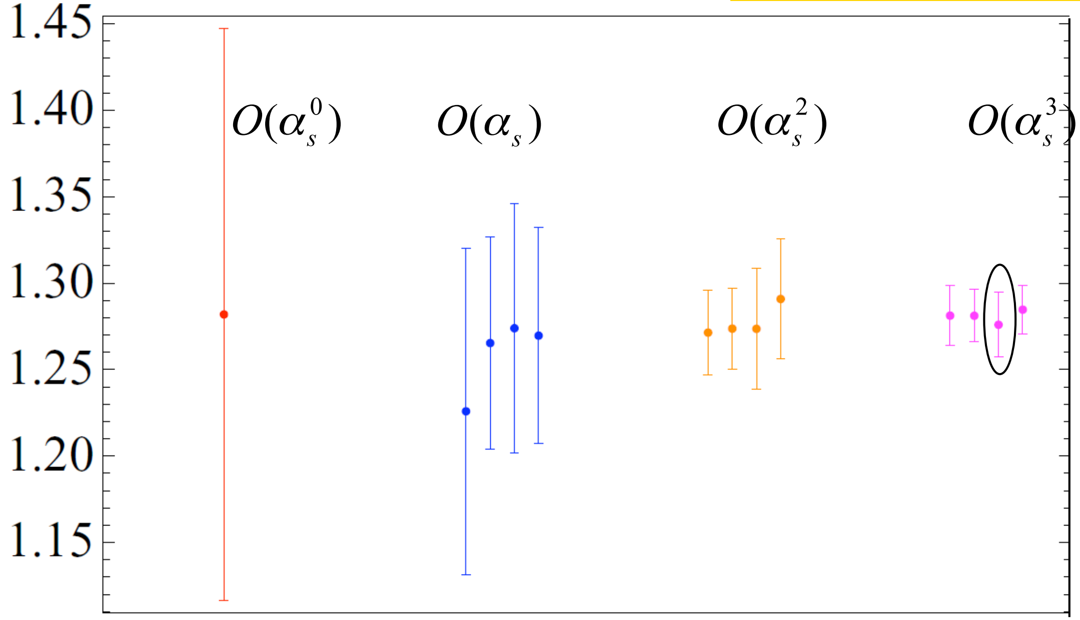
Perturbation theory is in good shape !

Chetyrkin etal.

n	$m_c(3\,\mathrm{GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

- $m_c(m_c) = 1279 \pm 13\,\mathrm{MeV}$

$m_c(m_c)$ Double μ variation $\alpha_s(m_Z) = 0.1180 \pm 0.0020$



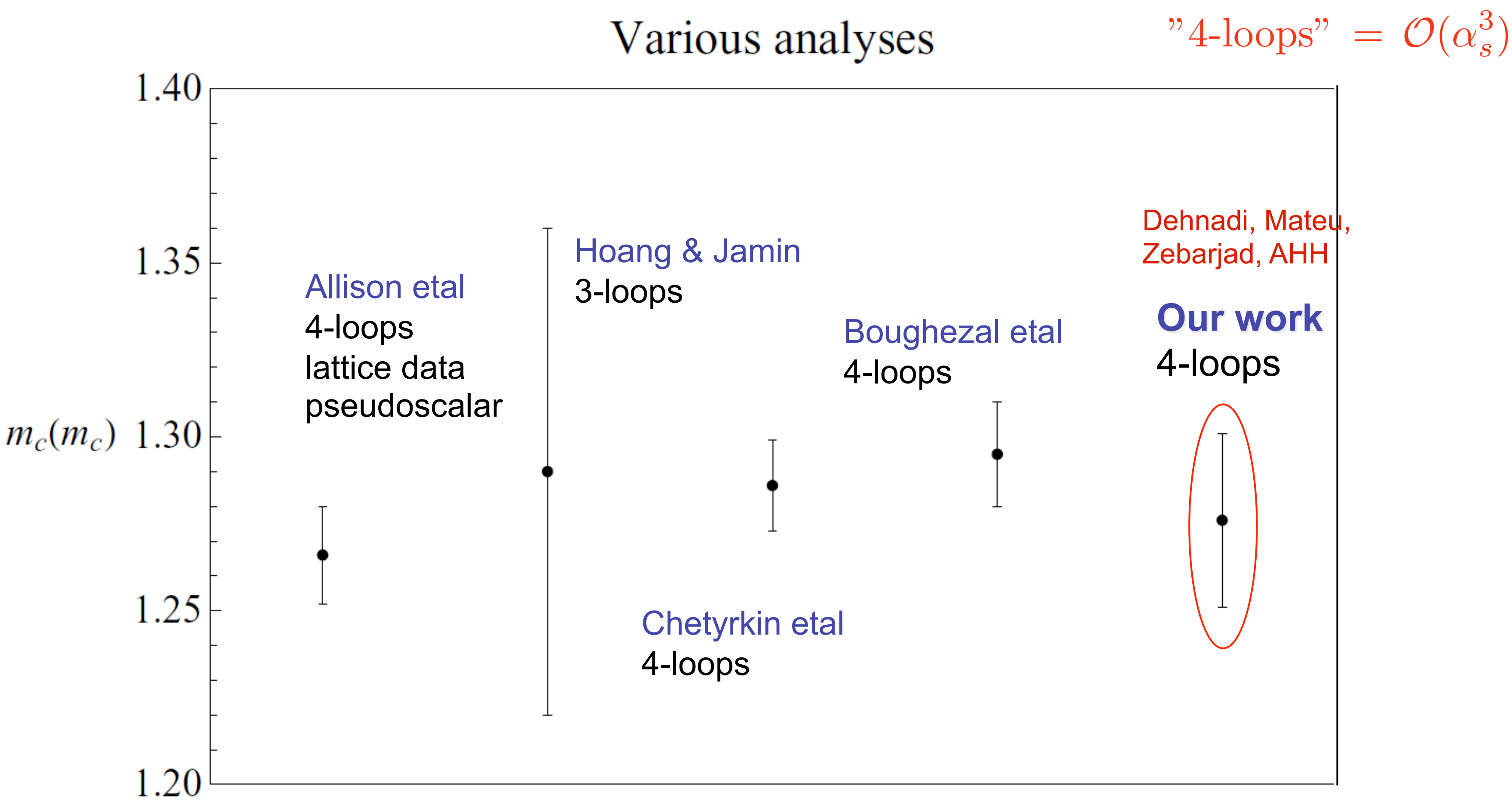
Dehnadi, Mateu, Zebarjad, AHH

Using double variation all methods have similar values and errors

Our result (preliminary):

$$m_c(m_c) = 1.2759 \pm 0.0186_{th} \pm \underbrace{0.0067_{stat} \pm 0.0125_{sys}}_{0.014} \pm 0.0076_{\alpha} \pm 0.0001_{\langle GG \rangle}$$
$$= 1.2759 \pm 0.0247$$

Comparison



Various analyses
"4-loops" $= \mathcal{O}(\alpha_s^3)$
1.40
1.35
$m_c(m_c)$ 1.30
1.25
1.20
Allison etal
4-loops
lattice data
pseudoscalar
Hoang & Jamin
3-loops
Chetyrkin etal
4-loops
Boughezal etal
4-loops
Dehnadi, Mateu, Zebarjad, AHH
Our work
4-loops

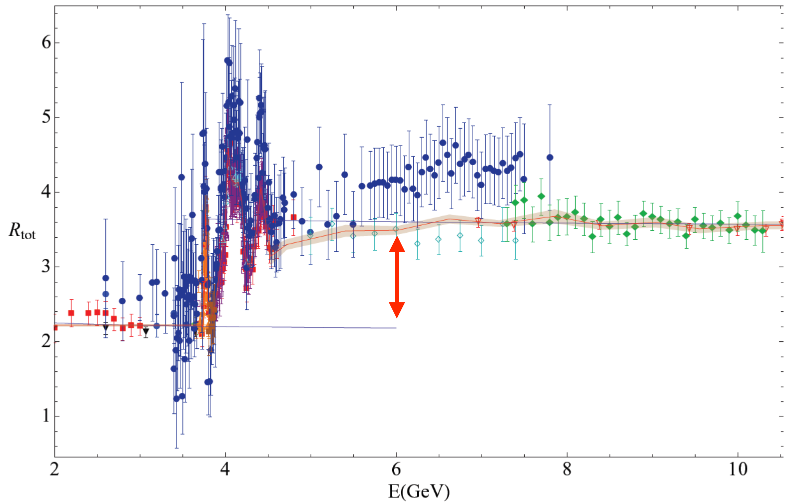
Is that it ???

Reminder: Short-distance masses do in general still have an ambiguity that is parametrically of $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{m_Q}\right)$

$$\begin{aligned} \text{top:}&\ 0.5 - 1\ \text{MeV} \\ \text{bottom:}&\ 20 - 50\ \text{MeV} \\ \text{charm:}&\ 60 - 150\ \text{MeV} \end{aligned}$$

Are we missing anything?

Is our determination of the experimental charm cross section sound ??



Is it ok to eyeball the charm cross section for non-resonance contributions?

Is that it ???

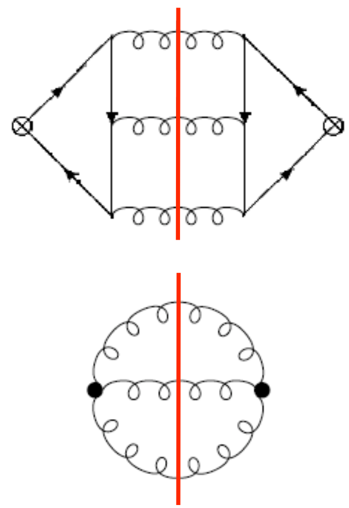
Reminder: Short-distance masses do in general still have an ambiguity that is parametrically of $\mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{m_Q}\right)$

$$\begin{aligned} \text{top:}\ & 0.5 - 1\ \text{MeV} \\ \text{bottom:}\ & 20 - 50\ \text{MeV} \\ \text{charm:}\ & 60 - 150\ \text{MeV} \end{aligned}$$

Are we missing anything?

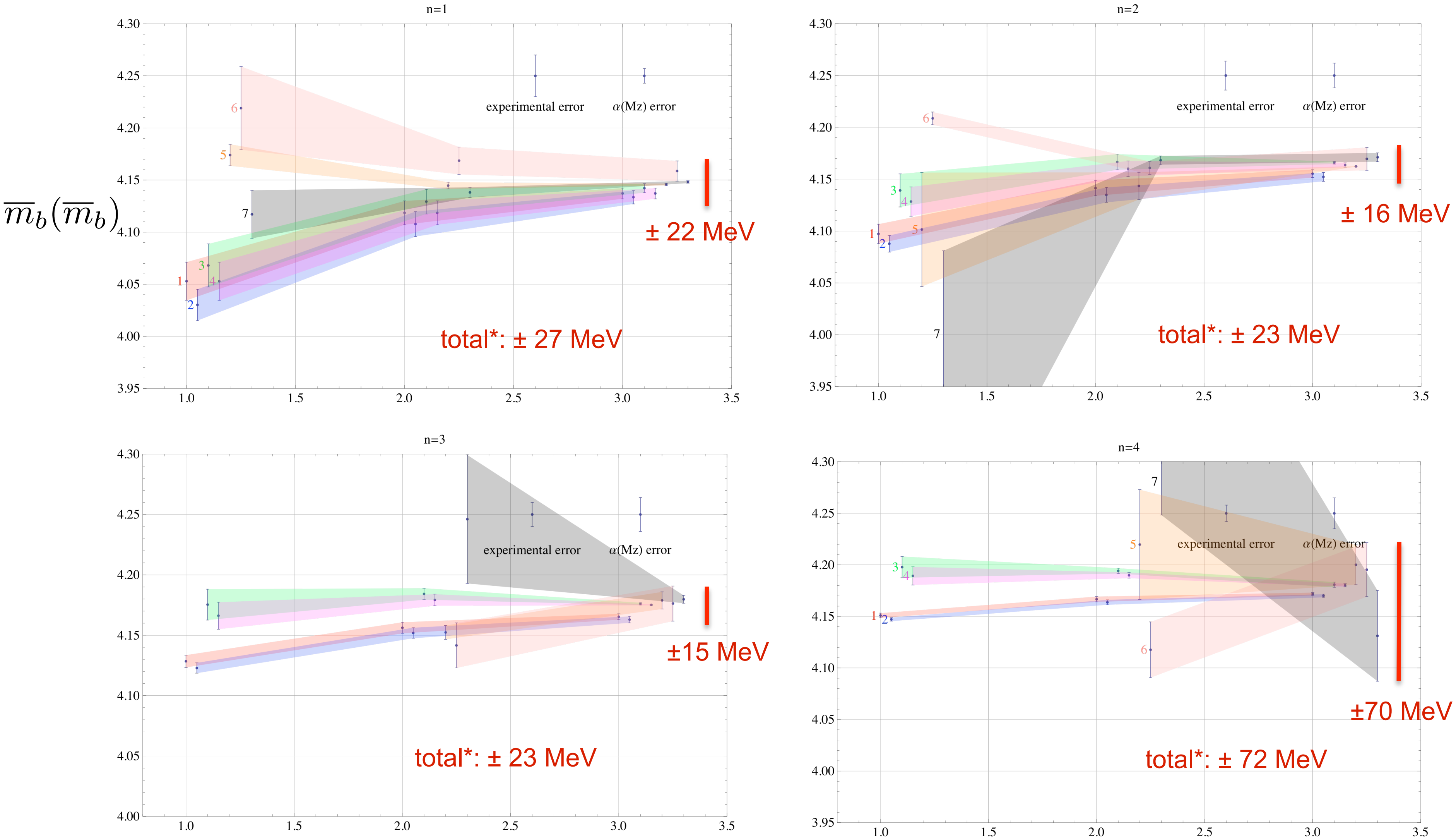
Massless cut diagram contribution at $\mathcal{O}(\alpha_s^3)$

Groote, Pivovarov



- Contains a contribution belonging to ψ resonances (hadronic decay)
- Contains a contribution belonging to non-charm hadron production
- How to separate both contributions theoretically ??
- Does this affect the validity of the classic SVZ-OPE ??
- Do we need NRQCD to address this issue ?? ($1/m_c$-expansion)
- Not clear whether this issue constitute another sizeable theory error

Remarks on Bottom



Remarks on Bottom

Chetyrkin etal.

n	$m_b(10\,\text{GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

- $m_\text{b}(m_\text{b}) = 4163 \pm 16\,\text{MeV}$

- Perturbative error also O(10) times larger than stated by Chetyrkin etal.
- Contribution from unmeasured continuum regions in R(had) much larger than for charm. Experimental moments with negligible theory modeling error more difficult compare to charm.
- Problem of separating R(bottom) from R(non-bottom) remains, like for the charm case. BUT for bottom it is more of an issue because Q_b= -1/3 (vs. Q_c= 2/3).
- Problem of diagrams with massless cuts like for the charm case remains.

Final numbers: work in progress

Conclusions & Outlook

General remarks:

- Accurate and reliable values for charm and bottom masses are needed
- MSbar charm mass should be a good scheme for almost all applications.
- Relativistic e+e- sum rules are a precision tool (low n values only!)

Results of this work:

- Reduction of model dependence in experimental moments M_n to negligible level.
  - Combination of all data through clustered χ^2-fit procedure that accounts for experimental correlations.
- Double variation of μ_α and μ_m required to achieve reliable perturbative uncertainties.
- There are still theory issues one should understand that are not accounted for in the theory error of our result !!!!!!!!!!!

Our result (preliminary):

$$m_c(m_c) = 1.2759 \pm 0.0186_{th} \pm 0.0067_{stat} \pm 0.0125_{sys} \pm 0.0076_{\alpha} \pm 0.0001_{\langle GG \rangle}$$
$$= 1.2759 \pm 0.0247$$