F_K/F_π from BMW

[Budapest-Marseille-Wuppertal Collaboration]

Alberto Ramos

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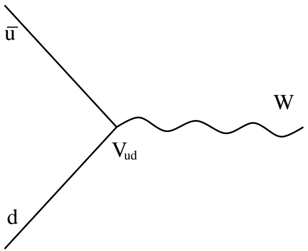
Centre de Physique Théorique (CNRS) – Marseille

6th international workshop on the CKM unitarity triangle

Dürr, Fodor, Hoelbling, Hoffman, Katz, Krieg, Kurth, Lellouch, Lippert, Ramos, Szabo

[Phys.Rev.D 81 (2010)]

Obtaining CKM matrix elements



You get them from quark-W vertex.

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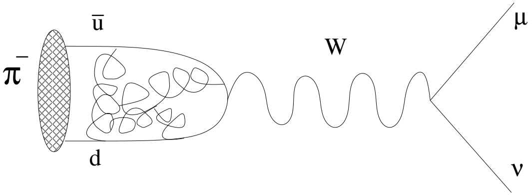

$$\Gamma(\pi \to \mu\nu_\mu) = \frac{G^2 M_\pi m_\mu^2}{8\pi}\left(1 - \frac{m_\mu^2}{M_\pi^2}\right)^2 \times |V_{ud}|^2 \times |F_\pi|^2$$

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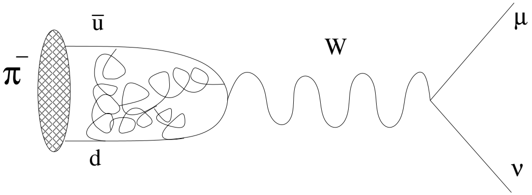

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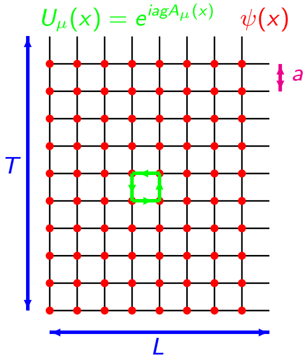
Common structure for weak decays: Product of

- Kinematic factor
- CKM Matrix element
- Non perturbative QCD factor

Lattice QCD in one slide

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



$$\langle O \rangle = \int \mathcal{D}[U]\mathcal{D}\overline{\psi}\mathcal{D}\psi \, O(U,\overline{\psi},\psi) e^{-S_G[U]-S_F[U,\overline{\psi},\psi]}$$
$$= \int \mathcal{D}[U] \, O(U)_{\text{Wick}} e^{-S_G[U]} \det(D)$$

- Compute the integral numerically $\rightarrow$ Monte Carlo sampling of $e^{-S_G[U]}\det(D) \geq 0$.
- Observable computed averaging over samples

$$\langle O \rangle = \frac{1}{N_{\text{conf}}} \sum_{i=1}^{N_{\text{conf}}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\text{conf}}})$$

NOT A MODEL: Lattice QCD IS real world QCD ($a \rightarrow 0$, $L \rightarrow \infty$, ...)

Lattice QCD Timeline

1974 First formulation of a non-Abelian gauge theory on a space-time lattice [Wilson. Phys.Rev D10 (1974)].

1980 First lattice simulation: Pure $SU(2)$ gauge theory in a lattice up to 10^4. [Creutz. Phys.Rev D21 (1980)].

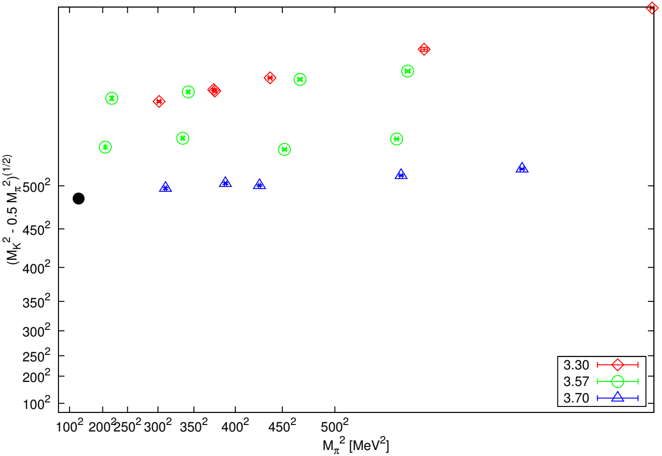
1985 Firsts unquenched simulations: 2×4^3 to 4×8^3 lattices. [Duane, Kogut. Phys.Rev.Lett. 55 (1985)].

'90s Quenched lattice QCD reign. Formally large N_c limit of QCD. Error $\sim 1/N_c \approx 30\% \rightarrow$ Uncontrolled systematics.

'00s Unquenched simulation comes up.

Present Start of precision lattice QCD era: Large volumes, physical quark masses, etc. . .

Reaching the physical point



2008 BMW data set.

Error source for a lattice computation

A lattice simulation is different from real QCD only because:

- We use Monte-Carlo methods to estimate the value of the integral. $N_{\text{conf}} < \infty$.
- Some quarks are heavier in the lattice than in real world. $m_{ud} > m_{ud}^{\text{phys}}$, $m_s \gtrsim m_s^{\text{phys}}$.
- Simulate a finite volume limit $L < \infty$
- Need experimetal input to "set the scale".
- Simulate with $a > 0$.
- Correlators receive corrections from excited states $T < \infty$.

$$C(t) \equiv \left(\frac{a}{L}\right)^3 \sum_{\vec{x}} \langle [\bar{d}\gamma_5 u](x)[\bar{u}\gamma_5 d](0)\rangle \xrightarrow{0 \ll t \ll T} \frac{\langle 0|\bar{d}\gamma_5 u|\pi^+(\vec{0})\rangle\langle \pi^+(\vec{0})|\bar{u}\gamma_5 d|0\rangle}{2M_\pi} e^{-M_\pi t}$$

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Control of errors

All the sources of error should be under control.

Marciano's suggestion

Examine the decay ratio

$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu)}{\Gamma(\pi \to \mu\bar{\nu}_\mu)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{M_K(1 - m_\mu^2/M_K^2)^2}{M_\pi(1 - m_\mu^2/M_\pi^2)^2} \left[1 + \frac{\alpha}{\pi}(C_K - C_\pi)\right] \frac{F_K}{F_\pi}$$

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Conclusion

$|V_{us}|$ can be determined with an accurate determination of F_K/F_π.

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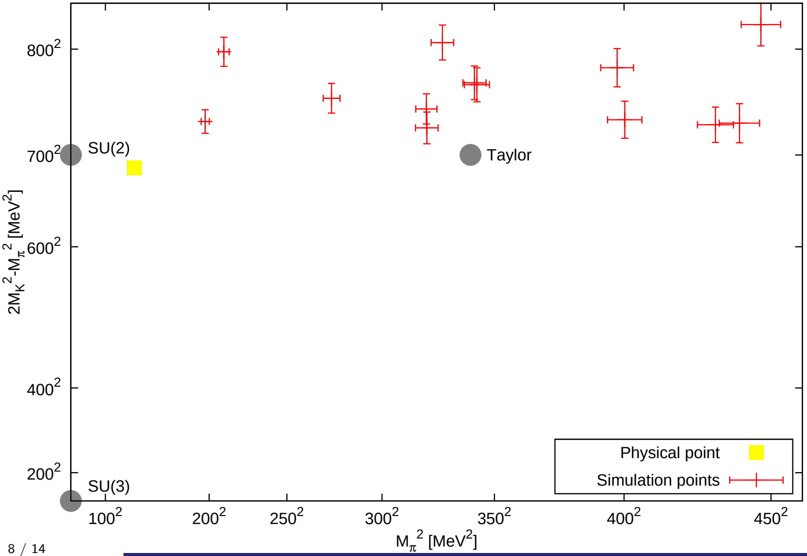
Some "numerology"

- F_K/F_π to 0.4% to match precision of $|V_{ud}|$ contribution to unitarity relation.
- F_K/F_π to 0.25% to match experimental error in $\Gamma(K \to \mu\overline{\nu}_\mu)/\Gamma(\pi \to \mu\overline{\nu}_\mu) \times \left[1 + \frac{\alpha}{\pi}(C_K - C_\pi)\right]^{-1}$.

Chiral extrapolation.



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$$\frac{F_K}{F_\pi} = 1 + \frac{1}{32\pi^2 F_0^2}\left[\frac{5}{4}\mu_\pi^2 - \frac{1}{2}\mu_K^2 - \frac{3}{4}\mu_\eta^2\right] + \frac{4}{F_0^2}L_5(M_K^2 - M_\pi^2)$$

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$$\frac{F_K}{F_\pi} = \frac{F}{\overline{\overline{F}}} \left\{ 1 + \frac{\alpha}{(4\pi F)^2} [(2M_K^2 - M_\pi^2) - (...)_{\rm phys}] \right\} \left\{ 1 + \frac{5}{8}\mu_\pi^2 + \Delta L_4 \frac{M_\pi^2}{(4\pi F)^2} \right\}$$

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Taylor Expansions

$$\frac{F_K}{F_\pi} = A_0 + A_1\Delta_\pi + A_2\Delta_\pi^2 + B_1\Delta_K$$

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Fundamental idea of the analysis.

Use all the approaches:

- Weight them with the quality of fit.

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Finite volume

Lüscher approach and ChPT

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$$\frac{F_K(L)}{F_\pi(L)} = \frac{F_K}{F_\pi}\left\{1 + \left[4 - \frac{3F_\pi}{2F_K}\right]\sum_{n=1}^{\infty}\frac{m(n)}{\sqrt{n}}\,\frac{1}{M_\pi L}\,\frac{M_\pi^2}{(4\pi F_\pi)^2}\,K_1(\sqrt{n}M_\pi L)\right\}$$

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Way of proceed.

- Correct data *before* fitting with the two loop expression.
- Use one loop expression and upper bound to estimate the error.

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- Action formally $\mathcal{O}(a)$ improved, but cutoff effects seem to go with $\mathcal{O}(a^2)$.

Continuum extrapolation

- Estimate them as a flavour breaking term:

$$\propto c \times \begin{cases} a(M_K^2 - M_\pi^2)/\mu_{\mathrm{QCD}} \\ \text{or} \\ a^2(M_K^2 - M_\pi^2) \end{cases}$$

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- Excited states: Use 18 different time intervals to fit the correlators.

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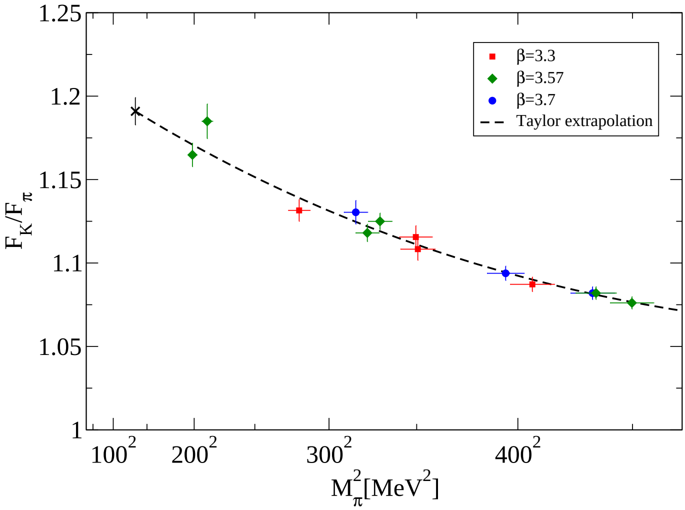
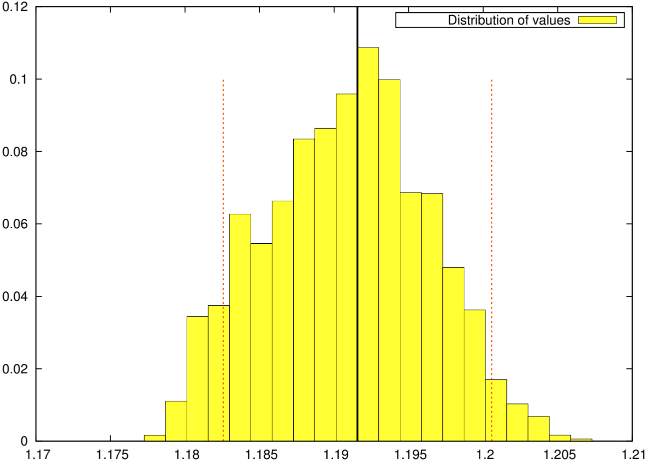
- Scale setting: Use M_π, M_K and either M_Ξ or M_Ω to set the scale.

Results



In total we have $2 \times 18 \times 7 \times 2 \times 3 = 1512$ Methods.

Results


Distribution of values
0.12
0.1
0.08
0.06
0.04
0.02
0
1.17
1.175
1.18
1.185
1.19
1.195
1.2
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- $$|V_{\mathrm{ud}}|^2 + |V_{\mathrm{us}}|^2 + |V_{\mathrm{ub}}|^2 = 1.0001(9)$$

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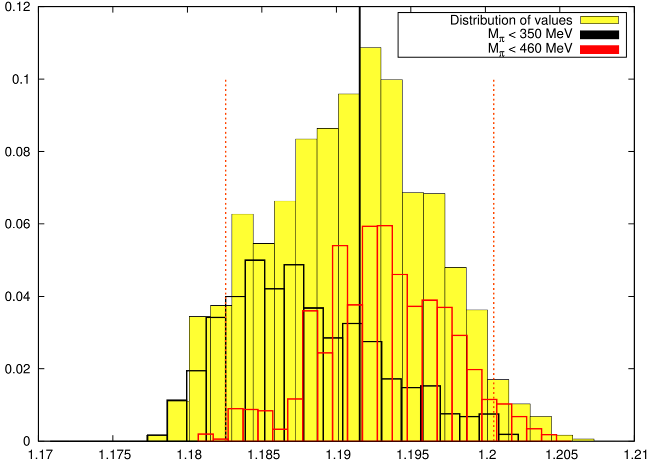

Distribution of values
M_π < 350 MeV
M_π < 460 MeV
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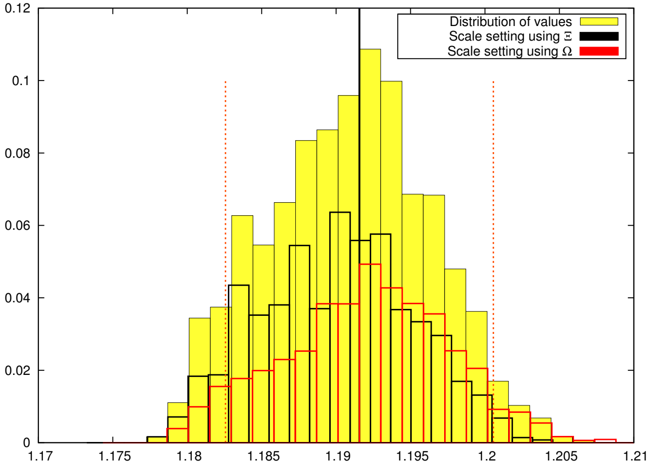

Distribution of values
Scale setting using Ξ
Scale setting using Ω
0.12
0.1
0.08
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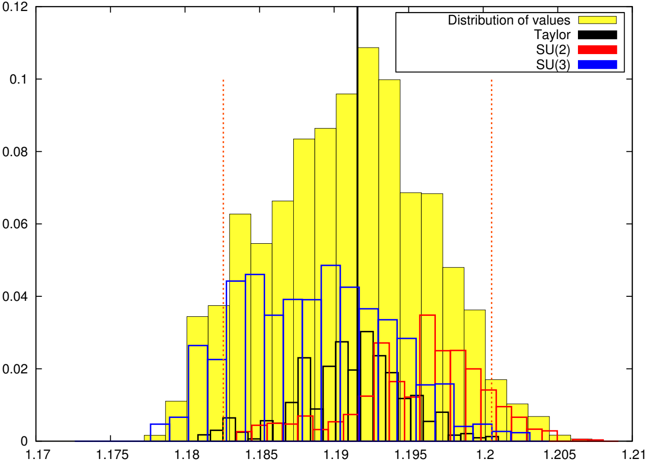

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Distribution of values
Taylor
SU(2)
SU(3)

Results

Source of systematic error	error on F_K/F_π
Chiral Extrapolation:	
- Functional form	3.3×10^{-3}
- Pion mass range	3.0×10^{-3}
Continuum extrapolation	3.3×10^{-3}
Excited states	1.9×10^{-3}
Scale setting	1.0×10^{-3}
Finite volume	6.2×10^{-4}

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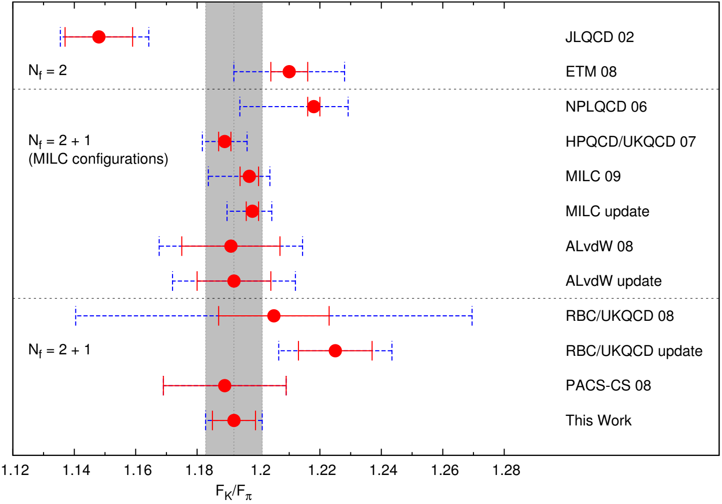

JLQCD 02
ETM 08
NPLQCD 06
HPQCD/UKQCD 07
MILC 09
MILC update
ALvdW 08
ALvdW update
RBC/UKQCD 08
RBC/UKQCD update
PACS-CS 08
This Work
$N_f = 2$
$N_f = 2 + 1$
(MILC configurations)
$N_f = 2 + 1$
1.12
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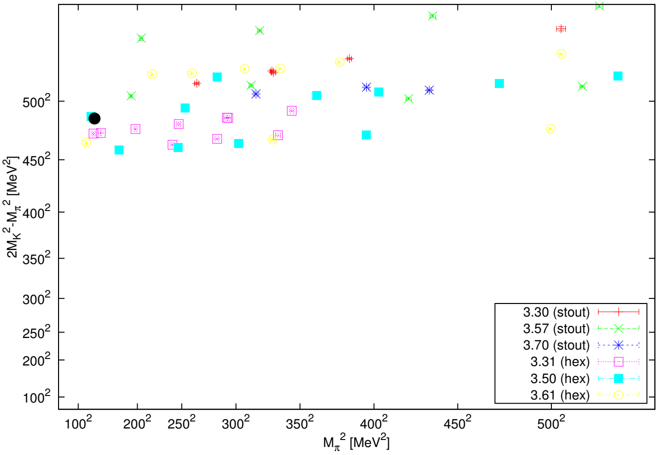
1.22
1.24
1.26
1.28
F_K/F_π

Future

We have generated new ensembles, directly simulating at the physical point, with four values of the lattice spacing. [C. Hoelbling. LAT10]



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- Continuous Evolution for more than 30 years in the lattice. Still lot of work to do, but precision measurements of some quantities is now possible.
- Caution: If strongly coupled quantum field theory play any role (other than QCD) in physics beyond the standard model, we will have a very hard time.