Lattice determination and experimental constraints on f_{B_d}, f_{B_s} and ξ

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CKM 2010

Outline

- Motivations
- Lattice implementation of the b-quark
- Lattice results
- New developments
- Status and outlook

Motivations [PDG'10]

Consider the $\mathrm{B_d}$ and $\mathrm{B_s}$ mesons: $\mathrm{B_q} = \bar{q}\gamma_5 b$, where $q = d, s$

Physical eigenstates ${\mathrm{B_q}}^{L,H}$ are related to the flavor eigenstate $\mathrm{B_q}$ through

$$ {\mathrm{B_q}}^{L,H} = p\mathrm{B_q} \pm q\overline{\mathrm{B}}_\mathrm{q} $$

Experimentally one measures Δm_s and Δm_d through B oscillations

$$ \Delta m_\mathrm{q} = m_{{\mathrm{B_q}}^H} - m_{{\mathrm{B_q}}^L} = 2|M_{12}^\mathrm{q}| $$

$|M_{12}^\mathrm{q}|$ is dominated by box diagrams

$$ |M_{12}^\mathrm{q}| = \frac{G_F^2 m_{\mathrm{B_q}} m_\mathrm{W}^2}{12\pi^2} (V_{tb} V_{tq}^*)^2 \eta_B S_0(x_t) f_{\mathrm{B_q}}^2 B_{\mathrm{B_q}} $$

Where the non-perturbative contributions are given by $f_{\mathrm{B_q}}$ and $B_{\mathrm{B_q}}$

$$ \begin{aligned} \langle 0|A_0|\mathrm{B_q}\rangle &= f_{\mathrm{B_q}} m_{\mathrm{B_q}} \\ \langle \overline{\mathrm{B}}_\mathrm{q}|(\bar{b}\gamma_\mu^L q)(\bar{b}\gamma_L^\mu q)|\mathrm{B_q}\rangle &= \frac{8}{3} B_{\mathrm{B_q}} m_{\mathrm{B_q}}^2 f_{\mathrm{B_q}}^2 \end{aligned} $$

Motivations

$$\Delta m_{\mathrm{d}} \propto f^2_{\mathrm{B_d}} B_{\mathrm{B_d}} |V_{\mathrm{td}} V^*_{\mathrm{tb}}|^2 \qquad \text{and} \qquad \Delta m_{\mathrm{s}} \propto f^2_{\mathrm{B_s}} B_{\mathrm{B_d}} |V_{\mathrm{ts}} V^*_{\mathrm{tb}}|^2$$

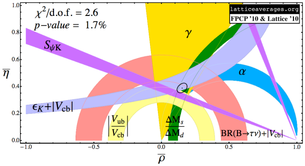
Combine

- Experimental values of Δm_{q}
- Perturbative computation of the short-distance effects
- Non-perturbative computation of the long-distance effects

$\Rightarrow$ Constraints the CKM matrix elements

More accurate: use the ratio

$$\frac{\Delta m_{\mathrm{s}}}{\Delta m_{\mathrm{d}}} = \frac{m_{\mathrm{B_s}}}{m_{\mathrm{B_d}}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2 \qquad /\mathrm{w}\ \xi^2 = \frac{B_{\mathrm{B_s}} f^2_{\mathrm{B_s}}}{B_{\mathrm{B_d}} f^2_{\mathrm{B_d}}}$$



plot from [Laiho, Lunghi, Van de Water]

Other motivations

- $\mathcal{B}_R(\mathrm{B_s} \to \mu^+\mu^-) = F^2_{B_s}(C_{\mathrm{SM}} + \tan^6\beta_{\mathrm{MSSM}})$
- Theoretical uncertainty on the inclusive determination on $|V_{ub}|$ dominated by the one of the b-quark mass $\delta V_{ub}/V_{ub} \sim 4\,\delta m_{\mathrm{b}}/m_{\mathrm{b}}$

  Now $\delta m_{\mathrm{b}} = 40\ \mathrm{MeV} \Rightarrow \delta V_{ub}/V_{ub} = 3.5\%$ [Hitlin et al. 09]

Need to decrease the errors in the b-quark sector, in particular the ones coming from the non-perturbative effects.
$\Rightarrow$ A task for lattice QCD

Unfortunately, with today's computer resources, it is not possible
(Realistic simulations of a b-quark from lattice QCD)*
$\Rightarrow$ Instead one uses lattice effective theory

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* With one recent and noticeable exception, HPQCD with HISQ [HPQCD PRD'10]
not covered in this talk
but was discussed this morning by Junko Shigemitsu

Lattice implementation of the b-quark

Source of errors

- Finite number of statistics.
  Statistical errors decrease $\propto \frac{1}{\sqrt{N_{\rm conf}}}$
- Systematic errors
  - Finite volume
    Contaminations decrease exponentially if $L \gg \frac{4}{M_\pi}$
  - Finite lattice spacing
    Discretizations errors $\propto (am_{\rm q})^\alpha$
    $\Rightarrow$ choose a and m_q such that $(am_q) \ll 1$
  - (Quenched approximation)

Nowadays calculations :

- Disctretization: Wilson, Twisted Mass, Staggered, Domain Wall, Overlap, ...
- Number of dynamical flavor $n_f = 2, n_f = 2+1, n_f = 2+1+1$
- Space extent: ~ 2 fm
- Lattice spacing: $\sim 0.05 - 0.1$ fm
- Quark mass: depends on the action $\sim m_{\rm c} \to m_{\rm s}/8$ (for staggered)

b-quark on the lattice

B meson contains both light and heavy degrees of freedom
$m_s \sim 100\ \mathrm{MeV}$ and $m_b \sim 4\ \mathrm{GeV}$
$\Rightarrow$ need a large volume and a small lattice spacing

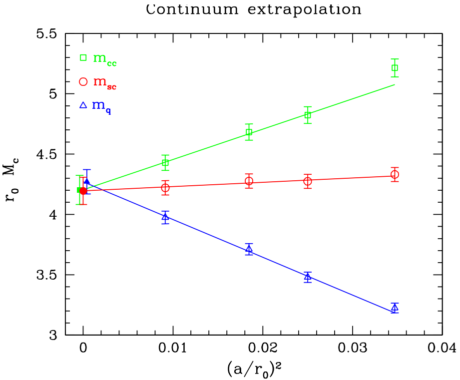
- Discretization errors $\propto (am_\mathrm{q})^\alpha$
  Choose bare heavy quark mass $am_\mathrm{b} \ll 1$, eg $am_\mathrm{b} = 0.1$
  $\Rightarrow$ For a O(a)-improved action, leading discr error $\mathcal{O}(am_\mathrm{b})^2 \sim 1\%$

- Spatial extent $L = aN$. For instance impose $L > 2\ \mathrm{fm}$
  $\Rightarrow$ Requires a large number of points (per space time direction)

$$N > \frac{2\ \mathrm{fm}}{a} = (2\ \mathrm{fm}) \times (10 m_\mathrm{b}) = 80\ \mathrm{GeV\ fm} \sim 400$$

Not doable with nowadays computers $\Rightarrow$ Effective theory

The charm quark



The simulation of the charm mass is just doable ...

... and $M_b \simeq 4M_c$.

Effective theories for heavy quark (I)

Momentum of a heavy quark (inside a hadron) $p = m_Q v + k$
Interaction with light dof $k \sim \Lambda_{\rm QCD} \ll m_Q$
Separate the higher and lower components of the heavy quark, and find an effective Lagrangian (see eg [Grozin '02])

$$\mathcal{L}_{\rm eff}^{\rm heavy} = \bar{\psi}_{\rm h}(x) \left[iv.D + \frac{(iD_\perp)^2}{2m_Q} + \frac{g\sigma.G}{4m_Q} + ... \right] \psi_{\rm h}(x)$$

Different choices of lattice implementation

- Expansion in $\Lambda_{\rm QCD}/m_Q$: HQET
- Expansion in v and $1/am_Q$: NRQCD
- Fermilab Method [El-Khadra et al '96]
- Relativistic heavy quarks [Aoki et al '01, Christ et al, Lin et al '06]
- Recent proposal by ETMC [ETMC '10]

Effective theories for heavy quark (II)

Main advantages and disadvantages of the different methods

method	pros	cons
NRQCD	heavy-heavy possible many results available	non-renormalizable (no continuum limit)
HQET	np renormalization	only heavy-light not many unquenched results
Fermilab	from light to heavy many results available	perturbative matching
RHQ	from light to heavy, np matching	numerically hard

Lattice results

Unquenched results (a selection)

Group	$f_{\mathrm{B_d}}$ (MeV)	$f_{\mathrm{B_s}}$ (MeV)	ξ	n_f	Heavy	Light
FNAL/MILC @ lat 08-09	195(11)	243(11)	1.205(52)	$2+1$	Fermilab	Asqtad
HPQCD PRD 09	190(13)	231(15)	1.258(33)	$2+1$	NRQCD	Asqtad
RBC-UKQCD PRD 10			1.13(12)	$2+1$	Static	Domain Wall
ETMC @lat 09	191(14)	243(13)		2	Stat. +Int.	Twisted Mass
ETMC JHEP 10	194(16)	235(12)		2	Stat +Int. new method	Twisted Mass

FNAL/MILC

Ongoing project. 3 proceedings: [Evans, El-Khadra, Gámiz @ lat'08], [Evans, El-Khadra, Gámiz, Kronfeld @ lat'09] [Fermilat Lattice and MILC @ lat'09]

- Fermilab and Asqtad (Staggered) $n_f = 2 + 1$
- 2 lattice spacings $a \sim 0.09\ \mathrm{fm}, 0.12\ \mathrm{fm}$
- Space extent $\sim 2.5\ \mathrm{fm}$
- 4 sea quark masses (2 on the finer lattice), 6 valence quark masses

Pros

- Continuum limit exists
- Very Light (up to $m_\mathrm{s}/10$) and dynamical quarks

Cons

- Staggered (?)
- Perturbative matching

MILC has now 2 finer lattices ($a \sim 0.06\ \mathrm{fm}, 0.045\ \mathrm{fm}$)
$\Rightarrow$ Crucial to check the continuum limit

HPQCD

[Gámiz, Davies, Lepage, Shigemitsu, Wingate PRD'09]. On the same MILC ensemble

- NRQCD and Asqtad (Staggered) $n_f = 2 + 1$
- 2 lattice spacings $a \sim 0.09\ \mathrm{fm}, 0.12\ \mathrm{fm}$
- Space extent $\sim 2.5\ \mathrm{fm}$
- 4 sea quark masses (2 on the finer lattice), 6 valence quark masses

Pros

- Very Light (up to $m_\mathrm{s}/10$) and dynamical quarks
- Small statistical errors

Cons

- Staggered (?)
- No continuum limit for NRQCD

MILC has now 2 finer lattices ($a \sim 0.06\ \mathrm{fm}, 0.045\ \mathrm{fm}$)
$\Rightarrow$ Crucial to check the continuum limit

RBC-UKQCD

[RBC-UKQCD PRD'10]

- Static and Domain-Wall $n_f = 2 + 1$
- 1 lattice spacing $a \sim 0.11$ fm
- Space extent ~ 1.8 fm
- 3 sea quark masses

Pros

- Action more realistic (χ^{al} symmetry and dynamical quarks)
- Continuum limit exists

Cons: Expensive

- Expensive (Only one lattice spacing, rather large stat. errors, light masses not very light)
- $1/m$ effects might be significant

$\Rightarrow$ Promising exploratory work

$\Rightarrow$ Theoretically sounds, but one would need other lattice spacing(s), lighter quark masses, and better statistic.

ETMC

[ETMC '09, ETMC '10] .

- Twisted Mass $n_f = 2$
- 4 lattice spacings $a \sim 0.05\ \mathrm{fm}, 0.065\ \mathrm{fm}, 0.085\ \mathrm{fm}, 0.10\ \mathrm{fm}$
- Space extent $\sim 2.5\ \mathrm{fm}$
- Interpolation between several quark masses (heavier than the charm) and a static quark

Pros

- Go beyond the static approximation

Cons

- Isospin symmetry broken

$\Rightarrow$ Promising exploratory work

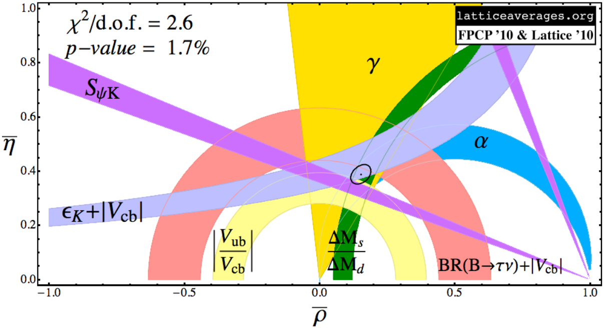
$\Rightarrow$ Agrees with another method proposed by ETMC [ETMC '10]

Impact on the CKM matrix

Impact on the CKM matrix

Lattice average by [Laiho, Lunghi, Van de Water] see www.latticeaverages.org

Only $n_f = 2+1$ results



Degree of tension very sensitive to $|V_{cb}|$

Fit to premature ? Lattice input need to be confirmed
How to use the lattice results in UT fit ? Can one trust the errors ?
See also the work by [CKMfitter] and [UTfit]

Recent developments in lattice heavy quark physics

Recent development

- HPQCD direct simulation of a (almost) b-quark (HISQ) [HPQCD PRD'10]
- ETMC Interpolation between charm and static [ETMC@lat09], and new method proposed in [ETMC JHEP'10]
- Relativistic heavy quark [RBC-UKQCD]
- Alpha Collaboration : non-perturbative HQET

In the next couple of slides, I am going to talk about the last point
Apologies to the other collaborations

Non-perturbative HQET at the $1/m$ order

ALPHA Collaboration

B. Blossier, M. Della Morte, P. Fritzsch, J. Heitger, G. von Hippel, F. Knechtli, B. Leder, T. Mendes, S. Schaefer, F. Virotta, H. Simma, R. Sommer, N. Tantalo

HQET at zero velocity on the lattice

The static part is given by the Eichten-Hill action [Eichten & Hill 90]

$$S_{\rm stat} = a^4 \sum_x \overline{\psi}_{\rm h}(x) D_0 \psi_{\rm h}(x)$$

with $P_+\psi_{\rm h} = \psi_{\rm h}\,, \quad \overline{\psi}_{\rm h} P_+ = \overline{\psi}_{\rm h}\,, \quad P_+ = \frac{1}{2}(1+\gamma_0)$

The static energy contains a linear divergence ($\propto 1/a$) which is absorbed by $m_{\rm bare}$

$$m_{\rm B} = E^{\rm stat} + m_{\rm bare}$$

The $1/m$ corrections are the kinetic and chromomagnetic terms

$$\mathcal{O}_{\rm kin} = -\overline{\psi}_{\rm h}(\mathbf{D}^2)\psi_{\rm h} \qquad \mathcal{O}_{\rm spin} = -\overline{\psi}_{\rm h}(\sigma\cdot\mathbf{B})\psi_{\rm h}$$

with coefficient $\omega_{\rm kin}, \omega_{\rm spin}$ $\Rightarrow$ Classically $\omega_{\rm kin} = \omega_{\rm spin} = 1/(2m)$

HQET coefficients $m_{\rm bare}, \omega_{\rm kin}, \omega_{\rm spin}$ are determined non-perturbatively $\Rightarrow$ renormalizability

Example: the B meson mass [Heitger & Sommer 03]

In infinite volume $m_{\rm B} = m_{\rm bare} + E^{\rm stat}$, where $m_{\rm bare}$ cancels the $1/a$ divergence

- Simulate QCD in small volume $L_1 \sim 0.5\,\mathrm{fm}$ with $am_{\rm b} \ll 1$.

  Compute a "finite volume mass" $\Gamma(L_1, m_{\rm q}) = \lim_{a\to 0} \Gamma(L_1, m_{\rm q}, a)$

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- Compute the corresponding quantities ($E^{\rm stat}$) in the effective theory for various a's.

  Impose the matching $\Gamma^{\rm QCD}(L_1, m_{\rm q}) = m_{\rm bare}(m_{\rm q}, a) + \Gamma^{\rm stat}(L_1, a)$

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Example: the B meson mass [Heitger & Sommer 03]

In infinite volume $m_\mathrm{B} = m_\mathrm{bare} + E^\mathrm{stat}$, where m_bare cancels the $1/a$ divergence

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  Compute a "finite volume mass" $\Gamma(L_1, m_\mathrm{q}) = \lim_{a\to 0} \Gamma(L_1, m_\mathrm{q}, a)$

- Compute the corresponding quantities (E^stat) in the effective theory for various a's.

  Impose the matching $\Gamma^\mathrm{QCD}(L_1, m_\mathrm{q}) = m_\mathrm{bare}(m_\mathrm{q}, a) + \Gamma^\mathrm{stat}(L_1, a)$

- Perform another simulation of HQET, with the same a's but in a larger volume, for example $L_2 = 2L_1$, and compute $\Gamma^\mathrm{stat}(L_2, a)$

- To obtain the meson mass in the volume L_2, compute:

$$
\begin{aligned}
\Gamma(L_2, m_\mathrm{q}) &= \lim_{a\to 0} \left(\Gamma^\mathrm{stat}(L_2, a) + m_\mathrm{bare}(m_\mathrm{q}, a)\right) \\
&= \lim_{a\to 0} \left(\Gamma^\mathrm{stat}(L_2, a) - \Gamma^\mathrm{stat}(L_1, a)\right) + \Gamma^\mathrm{QCD}(L_1, m_\mathrm{q})
\end{aligned}
$$

  (note that the divergence cancels out in the difference)

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\end{aligned}$$

  (note that the divergence cancels out in the difference)

- Re-iterate until the volume is large enough

Strategy [Heitger & Sommer 03]

For an observable Φ with an additive renormalization

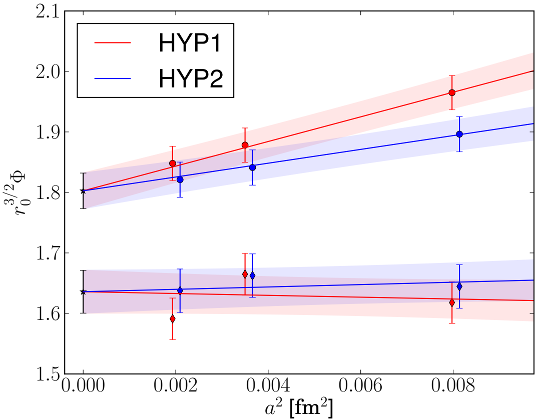
$$\begin{aligned}
\Phi(L_\infty, m_\mathrm{q}) \quad &= \quad \lim_{a\to 0}\left[\Phi^{\mathrm{HQET}}(L_\infty, a) - \Phi^{\mathrm{HQET}}(L_2, a)\right] && a \sim 0.1\ \mathrm{fm} \\
&+ \quad \lim_{a\to 0}\left[\Phi^{\mathrm{HQET}}(L_2, a) - \Phi^{\mathrm{HQET}}(L_1, a)\right] && a \sim 0.05\ \mathrm{fm} \\
&+ \quad \lim_{a\to 0} \Phi^{\mathrm{QCD}}(L_1, m_\mathrm{q}, a) && a \sim 0.025\ \mathrm{fm}
\end{aligned}$$

This can easily be extended beyond the static approximation
$\rightarrow$ Define a vector of observables $\mathbf{\Phi}$, vector of matching parameters ω, . . .
e.g. the matching looks like

$$\mathbf{\Phi}(L, m_\mathrm{q}) = \lim_{a\to 0}\left[\phi(L,a)\,\omega(m_\mathrm{q}, a)\ + \eta(L,a)\right]$$

Example: f_{B_s} for $n_f = 0$

- Large volume matrix element extracted by solving a GEVP
- HQET parameters determined non-perturbatively



Lattice HQET (I)

- It is theoretically sound (continuum limit, renormalizable)
- Non perturbative renormalization is possible.
- The $1/m$ terms are accessibles
- The large volume matrix element and energies can be accurately computed with the GEVP method
- Is has been tested in the quenched approximation

Lattice HQET (II)

- $n_f = 0$
  - ▷ b-quark mass [Alpha '06]

$$m_{\rm b}(m_{\rm b}) = \underbrace{4.350(64)}_{\rm static}\ {\rm GeV}\ \underbrace{-0.049(29)}_{{\rm O}(\Lambda^2/m_{\rm b})}\ {\rm GeV} + {\rm O}(\Lambda^3/m_{\rm b}^2)$$

  - ▷ I. HQET parameters [Alpha '10]
  - ▷ II. Spectoscopy [Alpha '10]
  - ▷ III. Decay constant [Alpha submitted in June]

$$F_{\rm B_s}^{\rm stat} = 229 \pm 6\ {\rm MeV} \qquad F_{\rm B_s}^{\rm stat+1/m} = 219 \pm 8\ {\rm MeV}$$

- $n_f = 2$
  - ▷ HQET parameter : almost finished
  - ▷ Large volume part: very preliminary results (1 lattice spacing)

$$m_{\rm b}(m_{\rm b})^{\rm stat} = 4.255(25)(50)(??) \qquad m_{\rm b}(m_{\rm b})^{\rm HQET} = 4.276(25)(50)(??)$$

Summary and outlook

Summary and outlook

▷ Lattice B-physics is becoming a high precision field. Some collaboration claim to obtain very precise results.

▷ They are working hard to estimate the various errors.

▷ Still some systematic errors are very hard to control (rooting from staggered, or lack of continuum limit for NRQCD , . . .).

▷ To constraint the UT, FNAL/MILC and HPQCD resulst need to be confirmed by non-staggered computations.

▷ Some alternatives already exist (Twisted mass + Static, Domain Wall + Static), but still at an early stage .

▷ Some new developments are very interesting (RHQ, NP-HQET, ETMC methods, . . .)

New unquenched results are coming . . .