Higher-Order Corrections to $M_{H^\pm}$ and $\tilde{t}_i \to \tilde{b}_j H^+$

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1. Motivation
2. Higher-order corrections to $M_{H^\pm}$
3. Higher-order corrections to $\tilde{t}_i \to \tilde{b}_j H^+$
4. Conclusions

1. Motivation

MSSM: Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$ Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

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$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

$\Rightarrow$ m_h, m_H, mixing angle α, $m_{H^\pm}$: no free parameters, can be predicted

In lowest order:

$$m_{H^\pm}^2 = M_A^2 + M_W^2$$

Keep in mind: higher-order corrections

$\Rightarrow$ Test of the model!

Necessary:

– discover the charged Higgs at the LHC or at the ILC

– measure its mass/characteristics at the LHC or at the ILC

– compare with theory prediction for $M_{H^\pm}$/other characteristics

$\tilde{t}/\tilde{b}$ sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^* \tan\beta$):

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ-$\tilde{t}/\tilde{b}$ couplings

$SU(2)$ relation $\Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

2. Higher-order corrections to $M_{H^\pm}$

MSSM: input: M_A and $\tan\beta$

output: neutral and charged Higgs masses, . . .

Tree-level:

$$m^2_{H^\pm} = M^2_A + M^2_W$$

Higher-order: $M^2_{H^\pm}$ is solution of

$$p^2 - m^2_{H^\pm} + \hat{\Sigma}_{H^+H^-}(p^2) = 0$$

with

$$\hat{\Sigma}_{H^+H^-}(p^2) = \Sigma_{H^+H^-}(p^2) + \delta Z_{H^+H^-}(p^2 - m^2_{H^\pm}) - \delta m^2_{H^\pm}$$

The following results are based on/taken from

[M. Frank, T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '10]*

(* in one week? :-)

One-loop (complete):

$$\hat{\Sigma}^{(1)}_{H^+H^-}(p^2) = \Sigma^{(1)}_{H^+H^-}(p^2) + \delta Z^{(1)}_{H^+H^-}(p^2 - m^2_{H^\pm}) - \delta m^{(1)2}_{H^\pm}$$

with

$$\delta Z^{(1)}_{H^+H^-}(p^2) = \sin^2\beta\, \delta Z_{\mathcal{H}_1} + \cos^2\beta\, \delta Z_{\mathcal{H}_2}$$

$$\delta Z_{\mathcal{H}_1} = \delta Z^{\overline{\mathrm{DR}}}_{\mathcal{H}_1} = -\left[\mathrm{Re}\Sigma'_{HH|\alpha=0}\right]^{\mathrm{div}}$$

$$\delta Z_{\mathcal{H}_2} = \delta Z^{\overline{\mathrm{DR}}}_{\mathcal{H}_2} = -\left[\mathrm{Re}\Sigma'_{hh|\alpha=0}\right]^{\mathrm{div}}$$

$$\delta m^{(1)2}_{H^\pm} = \delta M^{(1)2}_W + \delta M^{(1)2}_A$$

$$\delta M^{(1)2}_A = \Sigma^{(1)}_{AA}(M^2_A)$$

Furthermore:

$$m_b \;\to\; \frac{\overline{m}_b}{1 + \Delta_b}$$

$$\Delta_b = \frac{2\alpha_s}{3\,\pi}\, m_{\tilde{g}}\, \mu \,\tan\beta \,\times\, I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}) + \frac{\alpha_t}{4\,\pi}\, A_t\, \mu\, \tan\beta \,\times\, I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu)$$

Two-loop:

leading $\mathcal{O}(\alpha_t \alpha_s)$

- only y_t^2 contributions
- $g, g' \to 0$
- external momentum $\to 0$

$$\hat{\Sigma}^{(2)}_{H^+H^-}(0) = \Sigma^{(2)}_{H^+H^-}(0) - \delta m^{(2)2}_{H^\pm}$$

with

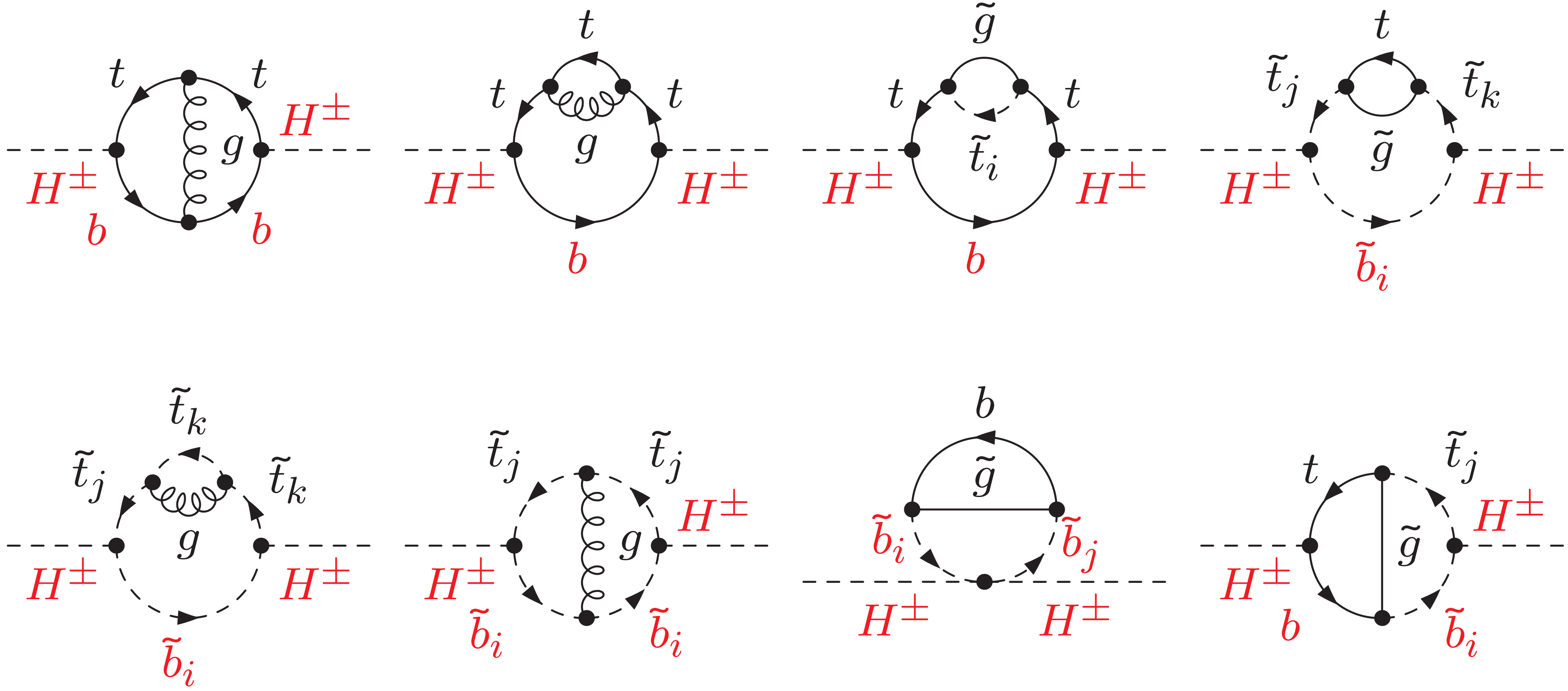
$$\delta Z^{(2)}_{H^+H^-} = 0$$

$$\delta M^{(2)2}_{W} = 0$$

$$\delta m^{(2)2}_{H^\pm} = \delta M^{(2)2}_{A} = \Sigma^{(2)}_{AA}(0)$$

Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



new: $H^{\pm}$ as external Higgs

$\Rightarrow$ $b/\tilde{b}$ enter (even diagrams without $t/\tilde{t}$: $H^+H^-\tilde{b}_i\tilde{b}_j \sim y_t^2$)

$\Rightarrow$ renormalization of $b/\tilde{b}$ sector necessary

Numerical results:

$\longrightarrow m_h^{\rm max}$ scenario, with variation of

– M_A : tree-level parameter

– $\tan\beta$: tree-level parameter

– μ : enters via Δ_b

(no-mixing scenario similar)

Experimental resolution:

$M_{H^\pm} = 200$ GeV:

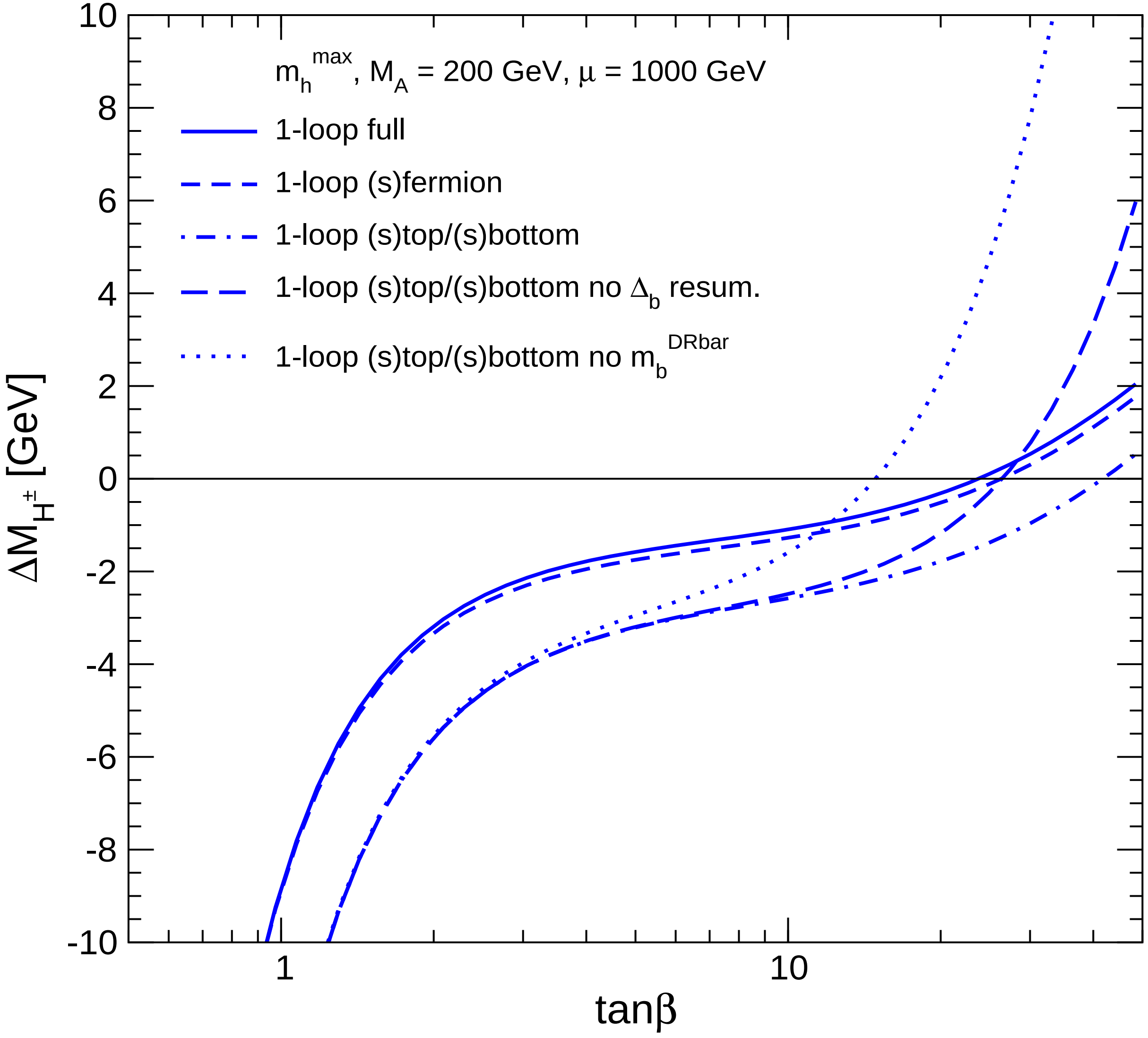
$$\text{LHC}: \Rightarrow \delta M_{H^\pm} \approx 1.5 \text{ GeV}$$

$$\text{ILC}: \Rightarrow \delta M_{H^\pm} \approx 0.5 \text{ GeV}$$

Higher masses:

$$\text{LHC}: \Rightarrow \delta M_{H^\pm} \approx 1 - 2\%$$

1-loop, $\mu = 1000$ GeV, $\tan\beta$ varied:



$t/\tilde{t}/b/\tilde{b}$ important

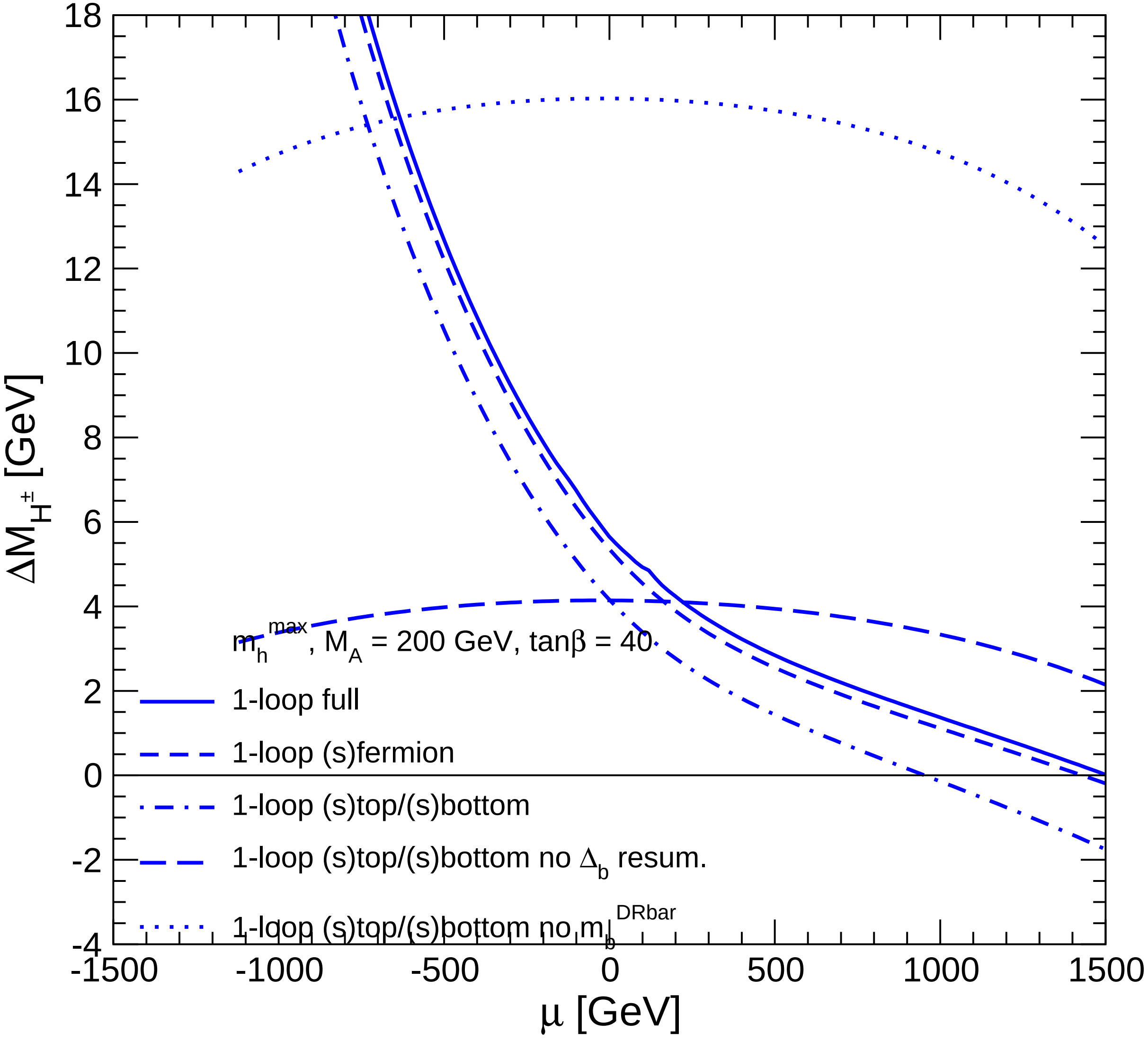
$\overline{m}_b$ important

Δ_b important

non-$t/\tilde{t}/b/\tilde{b}$
$\sim \log(M_{SUSY}/M_W)$
relevant

non-sfermion
corrections small

1-loop, $\tan\beta = 40$, μ varied:



$t/\tilde{t}/b/\tilde{b}$ important

$\overline{m}_b$ important

Δ_b important

non-$t/\tilde{t}/b/\tilde{b}$
$\sim \log(M_{\text{SUSY}}/M_W)$
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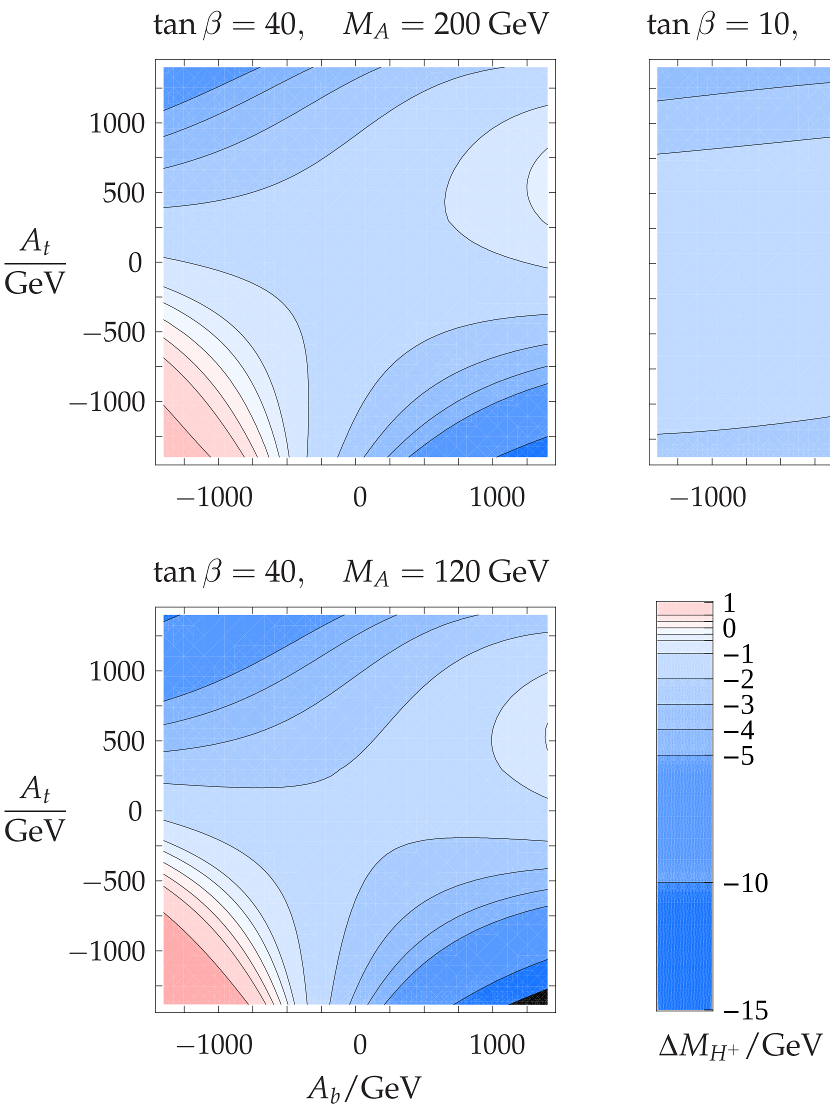
full 1-loop, A_t–A_b plane, $\mu = 1000$ GeV:

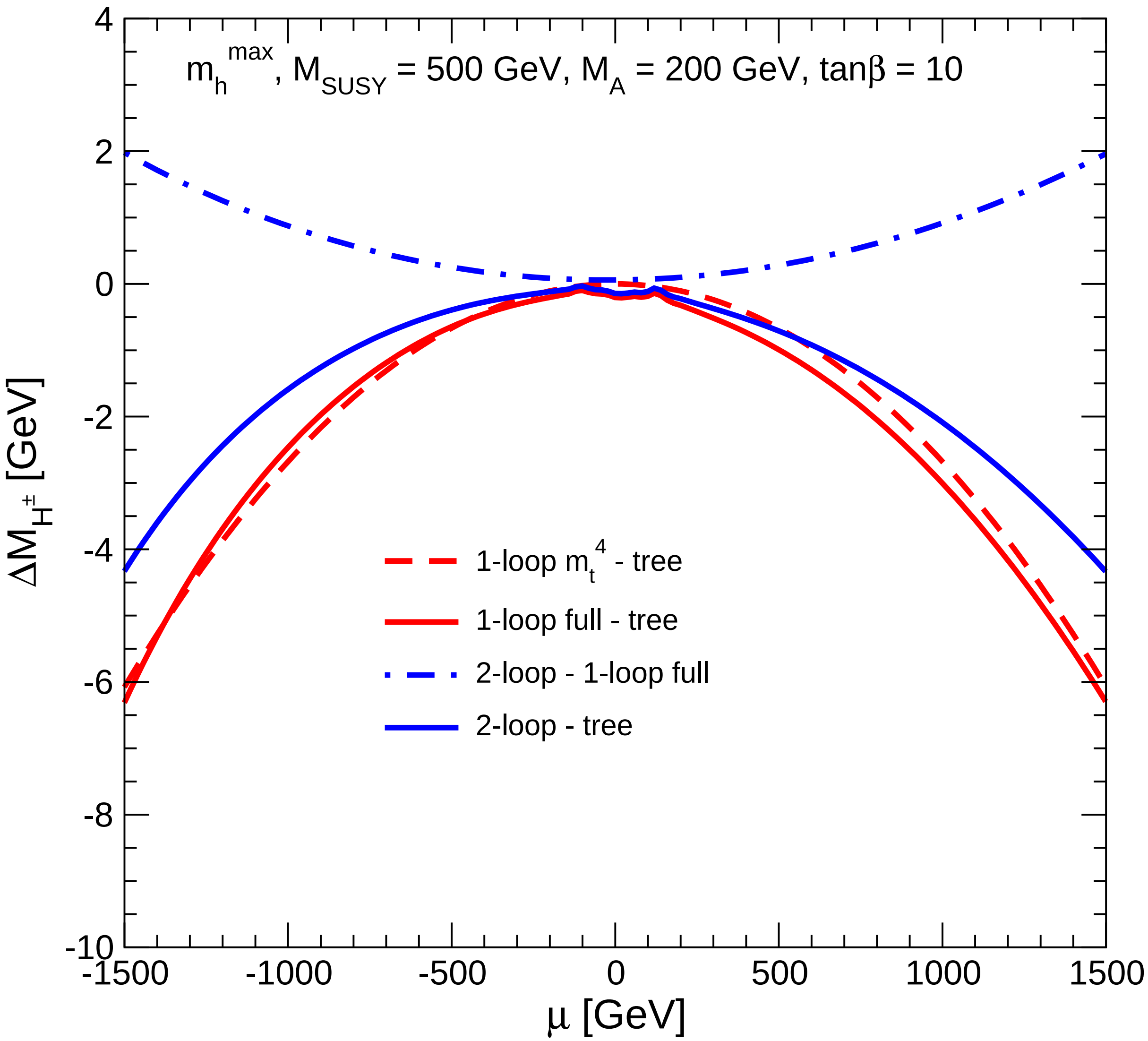


Huge effects possible for

$A_b = -A_t$

(not realized in $m_h^{\rm max}$ or no-mixing scenario)

2-loop $\mathcal{O}(\alpha_t\alpha_s)$, $\tan\beta = 10$, μ varied:

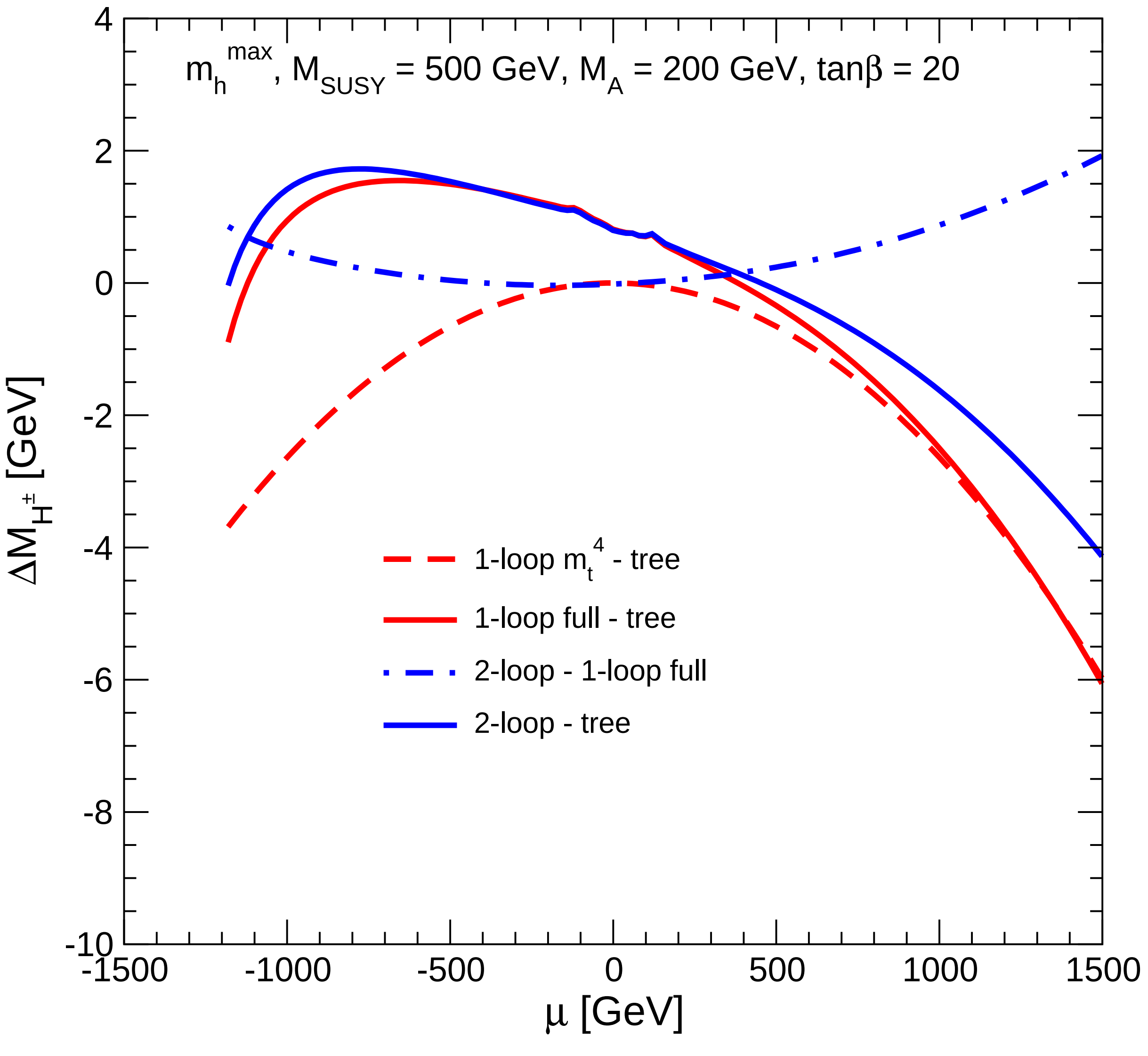


For these parameters:
m_t^4 approximation
very good

2-loop corrections
up to 2 GeV

⇒ LHC relevant

2-loop $\mathcal{O}(\alpha_t\alpha_s)$, $\tan\beta = 20$, μ varied:



For these parameters:
m_t^4 approximation
good for $\mu > 0$

2-loop corrections
up to 2 GeV

$\Rightarrow$ LHC relevant

3. Higher-order corrections to $\tilde{t}_i \to \tilde{b}_j H^+$

cH$^{\pm}$arged 2008:

several working groups were set up:

– search for $H^{\pm} \to$ SUSY at the LHC
  $\longrightarrow$ see Ketevi's talk yesterday

– search for SUSY $\to H^{\pm}$ at the LHC
  i.e. charged Higgs production in SUSY cascades
  $\Rightarrow$ any progress here?

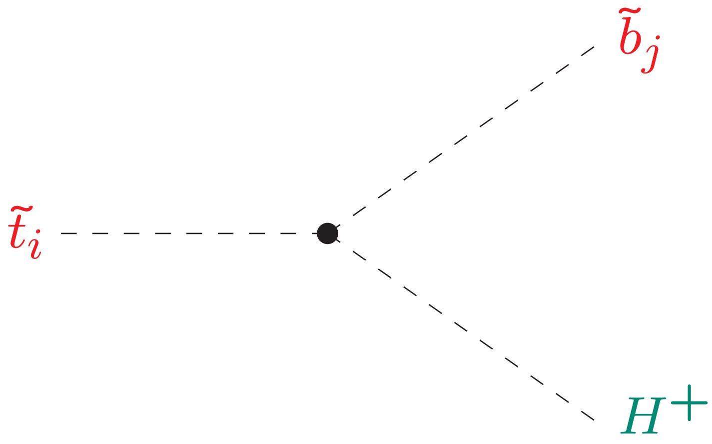
– . . .

Work based on

[S.H., H. Rzehak, C. Schappacher '10]

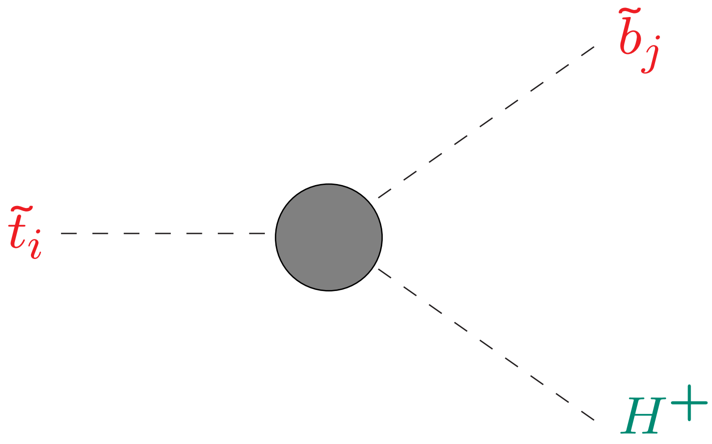
[T. Fritzsche, S.H., H. Rzehak, C. Schappacher, G. Weiglein '10]

Decay of $\tilde{t}_i \to \tilde{b}_j H^+$:



– important decay modes of stops
– A_t and A_b directly enter the vertex
– source of charged Higgs bosons in SUSY cascades at the LHC
– . . .

Decay of $\tilde{t}_i \to \tilde{b}_j H^+$:



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⇒ higher-order corrections important!

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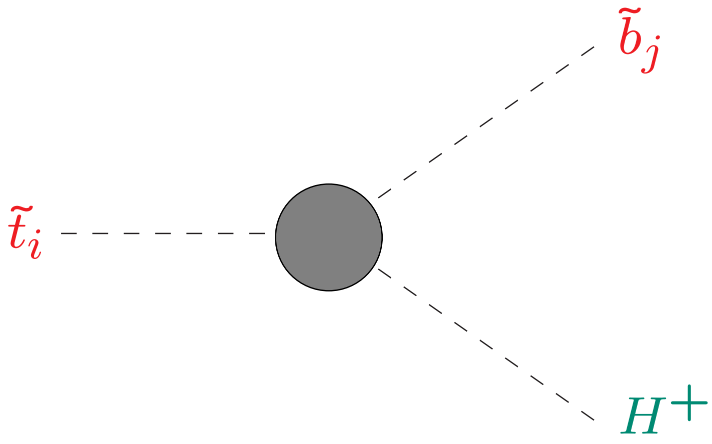

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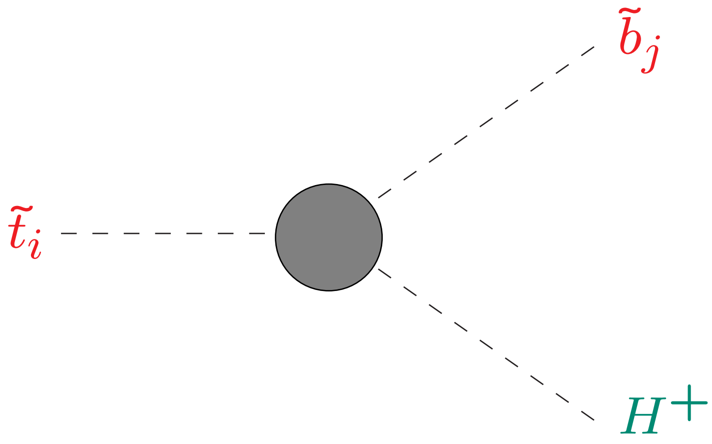
$\Rightarrow$ higher-order corrections important!

$\Rightarrow$ simultaneous renormalization of stop and sbottom sector required!

Decay of $\tilde{t}_i \to \tilde{b}_j H^+$:



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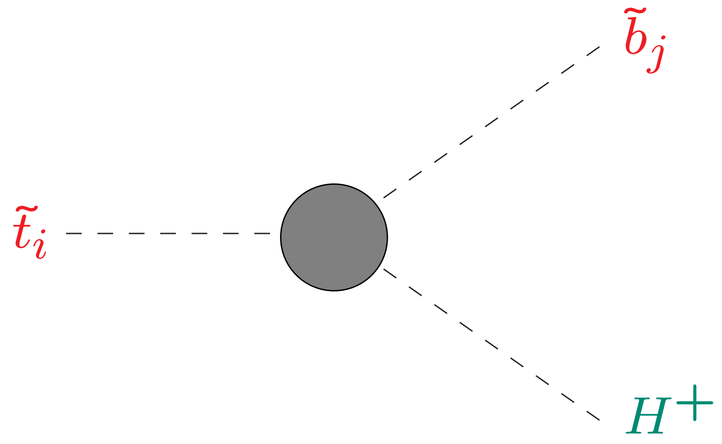
- A_t and A_b directly enter the vertex
- source of charged Higgs bosons in SUSY cascades at the LHC
- ...

⇒ higher-order corrections important!

⇒ simultaneous renormalization of stop and sbottom sector required!

⇒ with on-shell properties for external particles!

Decay of $\tilde{t}_i \to \tilde{b}_j H^+$:



- important decay modes of stops
- A_t and A_b directly enter the vertex incl. complex phases!
- source of charged Higgs bosons in SUSY cascades at the LHC
- . . .

⇒ higher-order corrections important!

⇒ simultaneous renormalization of stop and sbottom sector required!
⇒ including complex phases!

Renormalizations of the $b/\tilde{b}$ sector in the complex MSSM:

scheme	$m_{\tilde{b}_{1,2}}$	m_b	A_b	Y_b	name
analogous to the $t/\tilde{t}$ sector: "OS"	OS	OS	—	OS	RS1
"m_b, A_b $\overline{\text{DR}}$"	OS	$\overline{\text{DR}}$	$\overline{\text{DR}}$	—	RS2
"m_b, Y_b $\overline{\text{DR}}$"	OS	$\overline{\text{DR}}$	—	$\overline{\text{DR}}$	RS3
"m_b $\overline{\text{DR}}$, Y_b OS"	OS	$\overline{\text{DR}}$	—	OS	RS4
"A_b $\overline{\text{DR}}$, ReY_b OS"	OS	—	$\overline{\text{DR}}$	ReY_b: OS	RS5
"A_b vertex, ReY_b OS"	OS	—	vertex	ReY_b: OS	RS6

"— " = dependent parameter

$\Rightarrow$ often very involved analytical dependences

$\to$ more combinations possible

. . . also tested

. . . upcoming results remain unchanged

What is the “preferred” renormalization scheme?

one counterterm is a “dependent” quantity

$\Rightarrow$ no scheme can be identified that shows “good” behavior over the full cMSSM parameter space

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Most "robust" behavior:

– RS2: "m_b, A_b $\overline{\text{DR}}$"
  $\Rightarrow$ problems only for maximal sbottom mixing
– RS6: "A_b vertex, ReY_b OS"
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$\Rightarrow$ we choose RS2: "m_b, A_b $\overline{\text{DR}}$" as our "preferred" scheme

Calculation of partial widths:

– all diagrams created with FeynArts $\rightarrow$ T

$\rightarrow$ model file with all counterterms in the cMSSM

– including all soft/hard QED/QCD diagrams

– further evaluation with FormCalc

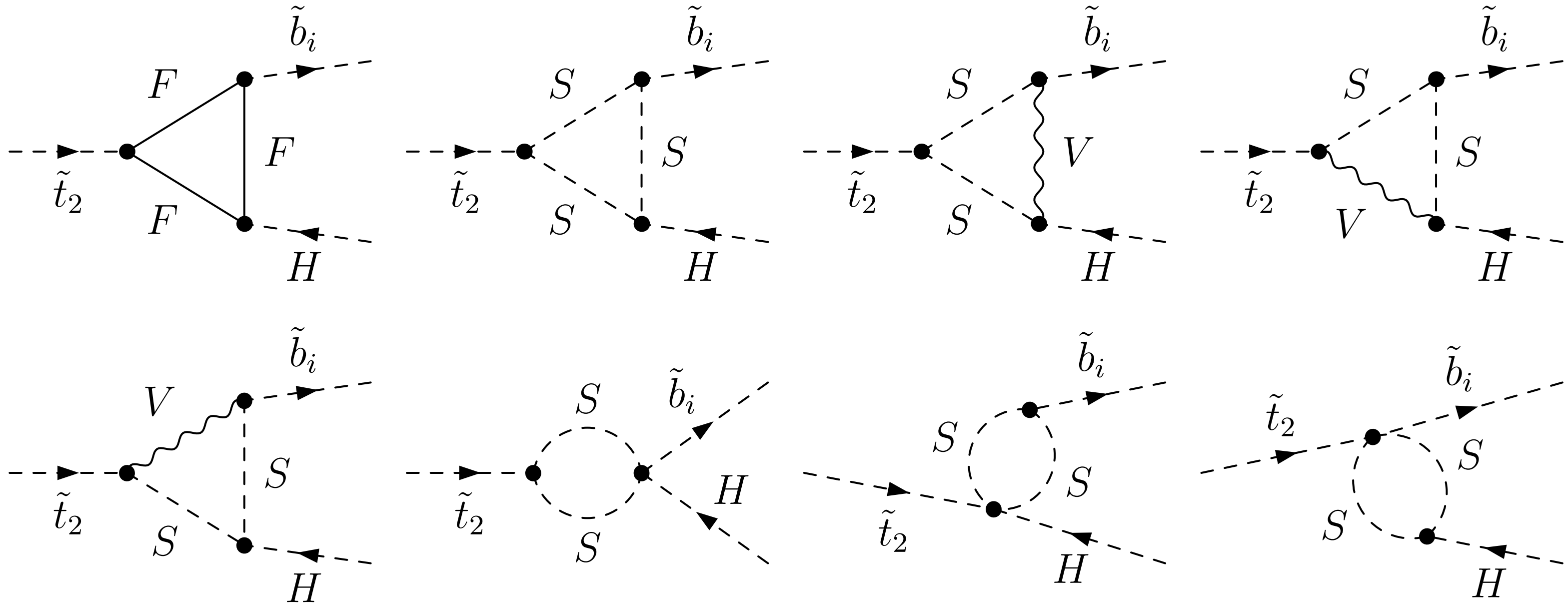
– Dimensional REDuction

– all UV and IR divergences cancel

– results will be included into FeynHiggs (www.feynhiggs.de)

$\rightarrow$ example plots will focus on $\Gamma(\tilde{t}_2 \rightarrow \tilde{b}_1 H^+)$

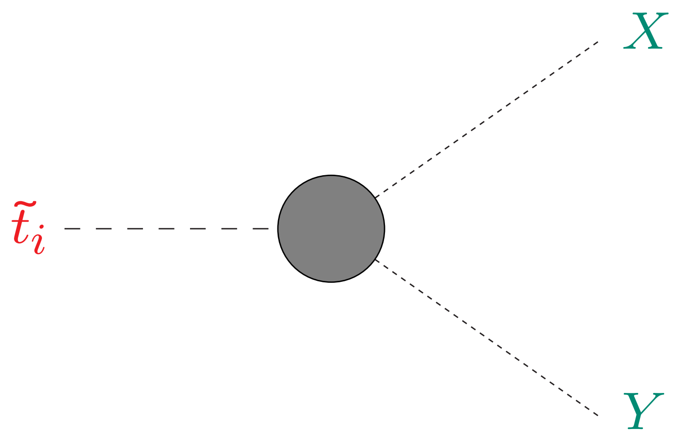
Feynman diagrams for $\tilde{t}_2 \to \tilde{b}_i H^+$



– including W^+–H^+ or G^+–H^+ transition contribution on the external Higgs boson leg

– including all soft/hard QED/QCD diagrams

Needed for BR prediction: all stop decays at 1-loop in the cMSSM:



$\Rightarrow$ (nearly) all sectors of the cMSSM enter as external particles

$\Rightarrow$ (nearly) all sectors of the cMSSM have to be renormalized simultaneously

$\Rightarrow$ (nearly) done . . .

$\Rightarrow$ focus on one-loop corrections to $\Gamma(\tilde{t}_i \to \tilde{b}_j H^+)$

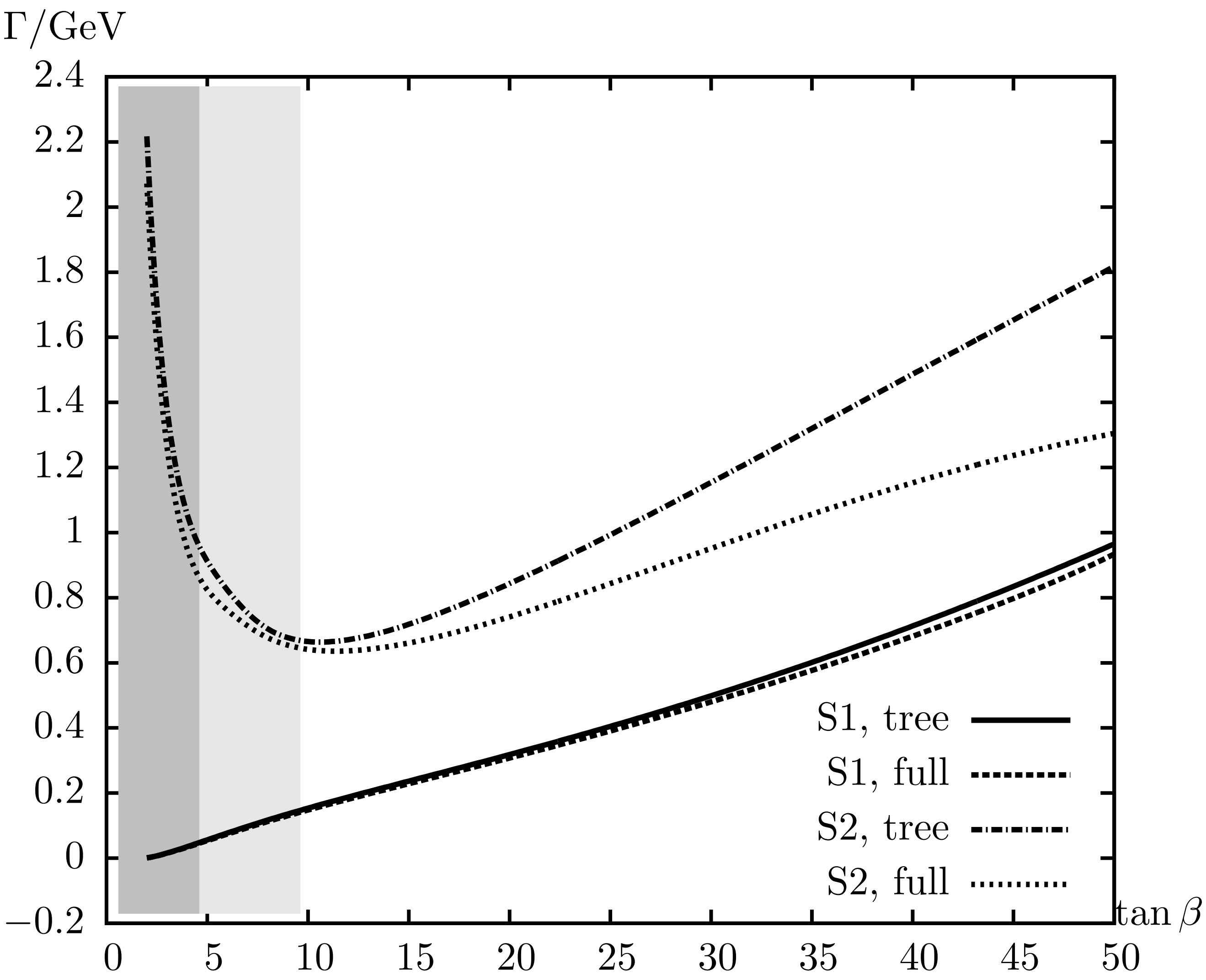
$\Rightarrow$ corrections to $\text{BR}(\tilde{t}_i \to \tilde{b}_j H^+)$ depend on SUSY parameters, kinematically open channels, . . .

Numerical scenarios:

Scen.	$M_{H^\pm}$	$m_{\tilde{t}_2}$	μ	A_t	A_b	M_1	M_2	M_3
S1	150	600	200	900	400	200	300	800
S2	180	900	300	1800	1600	150	200	400

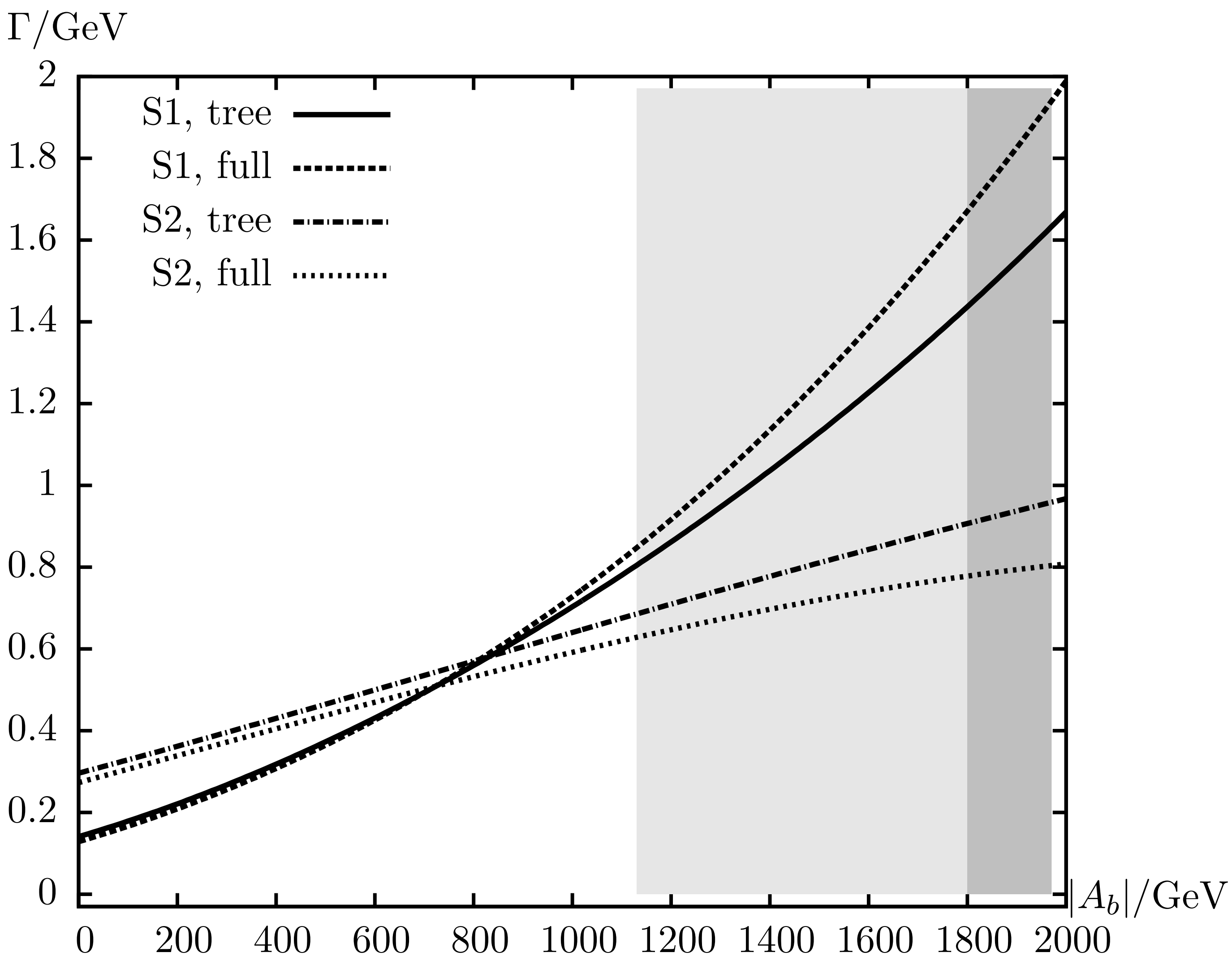
Scen.	$\tan\beta$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$
S1	2	293.391	600.000	441.987	447.168
	20	235.073	600.000	418.824	439.226
	50	230.662	600.000	400.815	449.638
S2	2	495.014	900.000	702.522	707.598
	20	445.885	900.000	678.531	695.180
	50	442.416	900.000	628.615	697.202

$\Gamma(\tilde{t}_2 \to \tilde{b}_1 H^+)$: dependence on $\tan\beta$



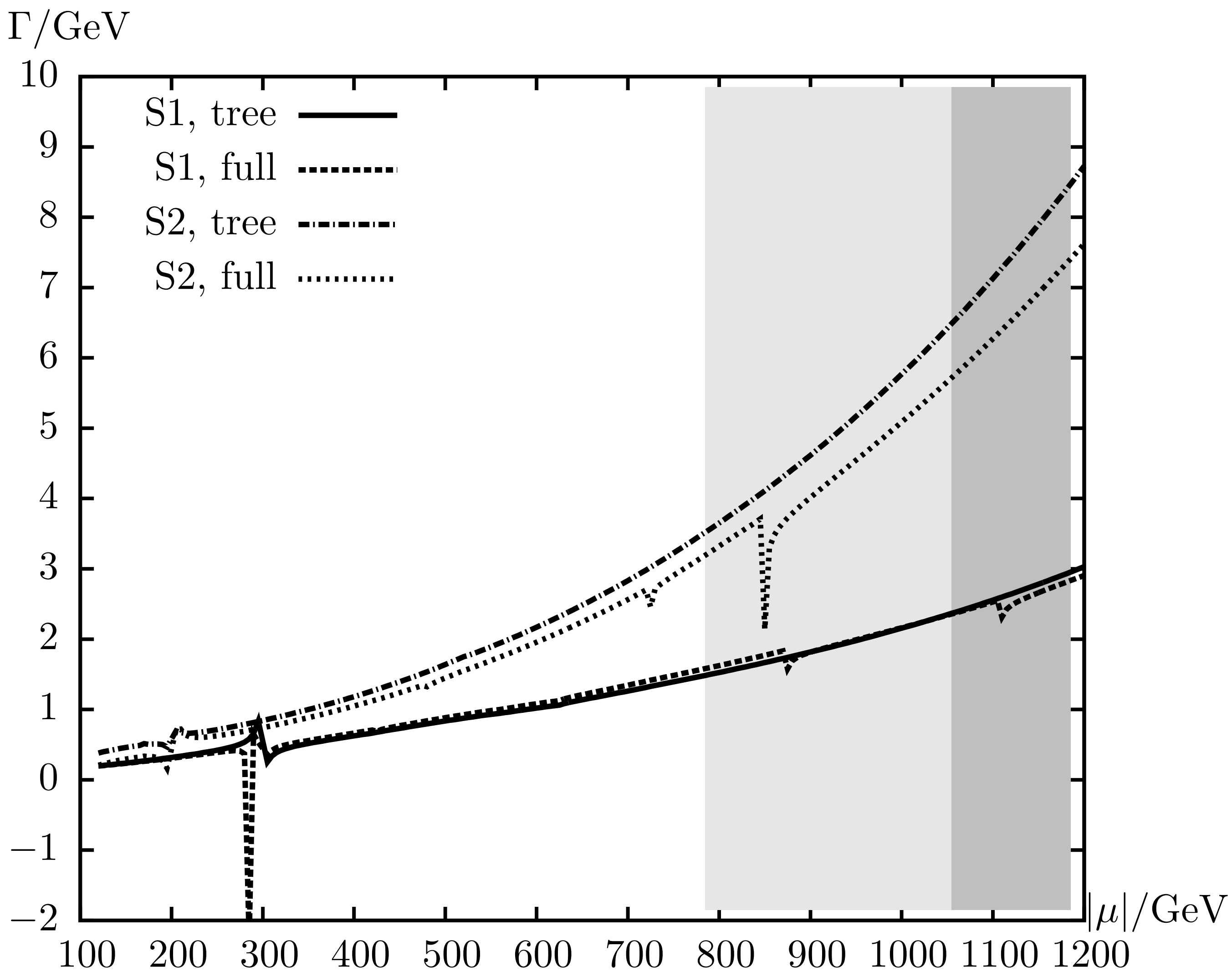
$\Rightarrow$ one-loop corrections under control for all $\tan\beta$ values, up to $\sim 25\%$

$\Gamma(\tilde{t}_2 \to \tilde{b}_1 H^+)$: dependence on A_b ($\tan\beta = 20$)



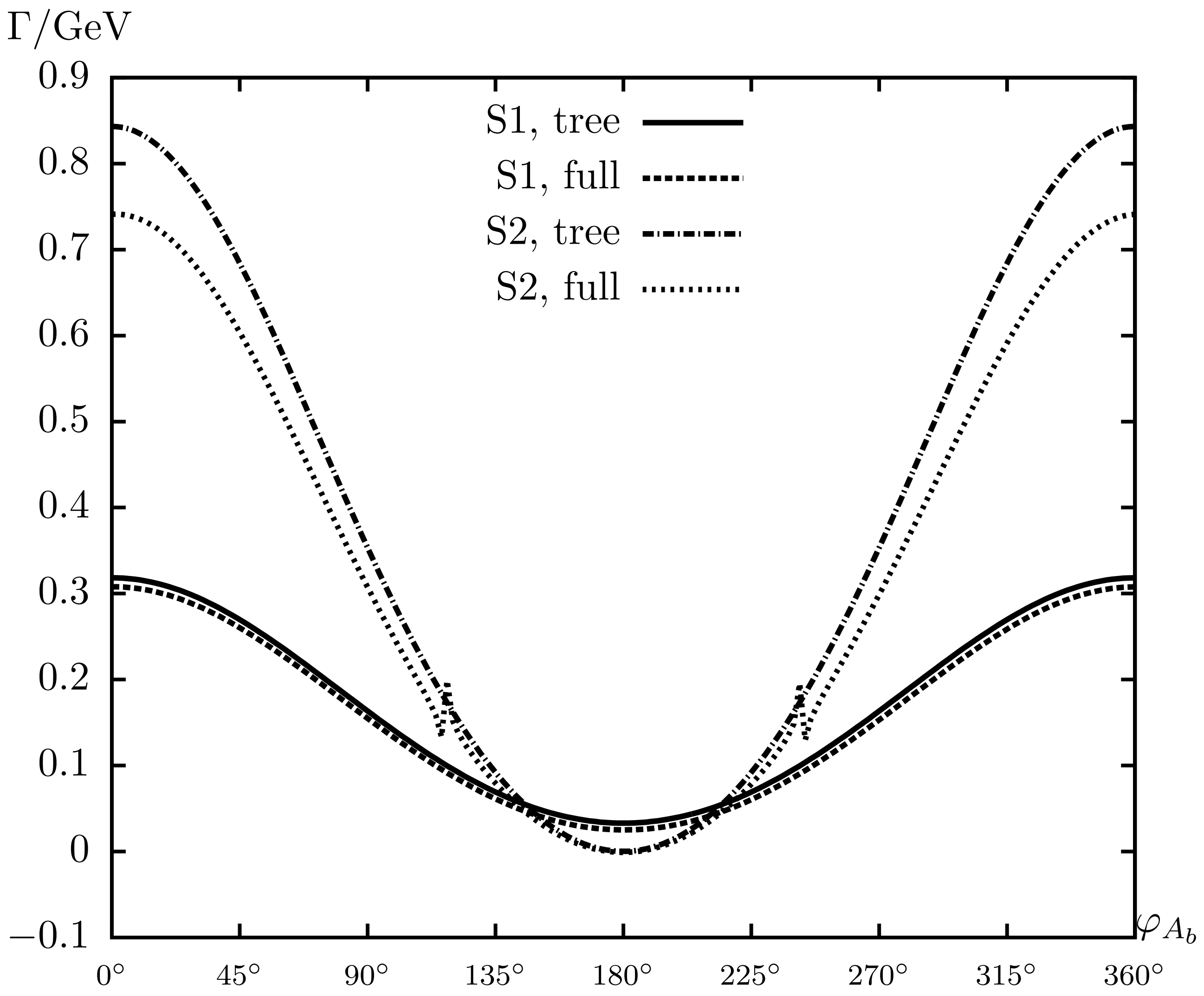
$\Rightarrow$ one-loop corrections under control for all A_b values, up to $\sim 20\%$

$\Gamma(\tilde{t}_2 \to \tilde{b}_1 H^+)$: dependence on μ ($\tan\beta = 20$)



⇒ one-loop corrections under control (but many thresholds)

$\Gamma(\tilde{t}_2 \to \tilde{b}_1 H^+)$: dependence on ϕ_{A_b} ($\tan\beta = 20$)



⇒ one-loop corrections under control except of sharp peaks at $|U_{\tilde{b}_{11}}| \approx |U_{\tilde{b}_{12}}|$

4. Conclusinos

- Charged MSSM Higgs boson:

  mass and couplings predicted in terms of other model parameters
  $\Rightarrow$ test of the model, parameter determination
  $\Rightarrow$ needed for reliable prediction of phenomenology

- Higher-order corrections to $M_{H^\pm}$:
  - 1L: all sectors relevant $\Rightarrow$ full 1L necessary
    Δ_b corrections crucial
  - 2L $\mathcal{O}(\alpha_t\alpha_s)$: $\Delta M_{H^\pm} = 0.5 - 2$ GeV
    important for LHC/ILC precision

  $\Rightarrow$ included in FeynHiggs

- Higher-order corrections to $\tilde{t}_i \to \tilde{b}_j H^+$:
  - many possible ways (renormalizations) for higher-order corrections
  - most "robust": RS2: "m_b, A_b $\overline{\text{DR}}$" $\leftarrow$ preferred scheme
  - 1L corrections under control, up to $\sim 25\%$

  $\Rightarrow$ will be included in FeynHiggs soon