MPI model in Herwig++

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Hadron–Hadron and Cosmic Ray Interactions at Multi TeV Energies ECT* Trento, 29 Nov–3 Dec 2010

MPI model with *Manuel Bähr, Jon Butterworth, Mike Seymour*. New studies, new colour reconnection model with *Christian Röhr, Andrzej Siodmok*

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pp Event Generator



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- POWHEG type matching for DY, h⁰, assoc Higgs, available, full truncated showers. More soon.
- Now also MC@NLO. Both methods automatically with new dipole shower.
- CKKW type matching with truncated showers general machinery, tested for e⁺e⁻, DY, γ+j.

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UA5 model (deprecated, only for reference)

Included from Herwig++ 2.0.

[Herwig++, hep-ph/0609306]

- Little predictive power.
- ► Only gets averages right, not large (and interesting!) fluctiations → mini jets.
- ► Was default in fHerwig. Superseded by JIMMY.

[JM Butterworth, JR Forshaw, MH Seymour, ZP C72 637 (1996)]

Semihard UE

▶ Default from Herwig++ 2.1.

[Herwig++, 0711.3137]

• Multiple hard interactions, $p_t \ge p_t^{\min}$. (Bail

[Bähr, SG, Seymour, JHEP 0807:076]

- Similar to JIMMY.
- ► Good description of harder Run I UE data (Jet20).

Underlying event in Herwig++

Semihard+Soft UE

- ▶ Default from Herwig++ 2.3.
- Extension to soft interactions $p_t < p_t^{\min}$.
- Theoretical work with simplest possible extension.

[Bähr, Butterworth, Seymour, JHEP 0901:065]

"Hot Spot" model.

[Bähr, Butterworth, SG, Seymour, 0905.4671]

- No development since then (currently at v2.4.2), but new data. First look.
- New developments in this talk.

[Herwig++, 0812.0529]

This talk

- Constrain parameter space from Tevatron.
- ▶ First look at LHC data.
- How well does it work out of the box?

Mulitple hard interactions



Starting point: hard inclusive jet cross section.

$$\sigma^{\mathrm{inc}}(s;p_t^{\mathrm{min}}) = \sum_{i,j} \int_{p_t^{\mathrm{min}^2}} \mathrm{d}p_t^2 f_{i/h_1}(x_1,\mu^2) \otimes \frac{\mathrm{d}\hat{\sigma}_{i,j}}{\mathrm{d}p_t^2} \otimes f_{j/h_2}(x_2,\mu^2),$$

 $\sigma^{\text{inc}} > \sigma_{\text{tot}}$ eventually (for moderately small p_t^{\min}).



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 $\sigma^{\text{inc}} > \sigma_{\text{tot}}$ eventually (for moderately small p_t^{\min}).

Interpretation: σ^{inc} counts *all* partonic scatters that happen during a single *pp* collision \Rightarrow more than a single interaction.

$$\sigma^{\rm inc} = \bar{n}\sigma_{\rm inel}$$

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number *m* of additional scatters,

$$P_m(\vec{b},s) = \frac{\bar{n}(\vec{b},s)^m}{m!} e^{-\bar{n}(\vec{b},s)}$$

Then we get σ_{inel} :

$$\sigma_{\text{inel}} = \int d^2 \vec{b} \sum_{n=1}^{\infty} P_m(\vec{b}, s) = \int d^2 \vec{b} \left(1 - e^{-\bar{n}(\vec{b}, s)} \right)$$

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Cf. σ_{inel} from scattering theory in eikonal approx. with scattering amplitude $a(\vec{b},s) = \frac{1}{2i}(e^{-\chi(\vec{b},s)} - 1)$

$$\sigma_{\text{inel}} = \int d^2 \vec{b} \left(1 - e^{-2\chi(\vec{b},s)} \right) \qquad \Rightarrow \quad \chi(\vec{b},s) = \frac{1}{2} \bar{n}(\vec{b},s) \; .$$

 $\chi(\vec{b},s)$ is called *eikonal* function.

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Calculation of $\bar{n}(\vec{b},s)$ from parton model assumptions:

$$\begin{split} \bar{n}(\vec{b},s) &= L_{\text{partons}}(x_1, x_2, \vec{b}) \otimes \sum_{ij} \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} \\ &= \sum_{ij} \frac{1}{1+\delta_{ij}} \int dx_1 dx_2 \int d^2 \vec{b}' \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} \\ &\times D_{i/A}(x_1, p_t^2, |\vec{b}'|) D_{j/B}(x_2, p_t^2, |\vec{b} - \vec{b}'|) \end{split}$$

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 $\bar{n}($

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$$\Rightarrow \quad \chi(\vec{b},s) = \frac{1}{2}\bar{n}(\vec{b},s) = \frac{1}{2}A(\vec{b})\sigma^{\rm inc}(s;p_t^{\rm min})$$



 \Rightarrow Two main parameters: μ^2 , p_t^{\min} .

Unitarized cross sections



Rick Field's analysis



Semi hard underlying event

Good description of Run I Underlying event data ($\chi^2 = 1.3$).



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So far only hard MPI. Now extend to soft interactions with

 $\chi_{\rm tot} = \chi_{QCD} + \chi_{\rm soft}.$

Similar structures of eikonal functions:

$$\chi_{\text{soft}} = \frac{1}{2} A_{\text{soft}}(\vec{b}) \sigma_{\text{soft}}^{\text{inc}}$$

Simplest possible choice: $A_{\text{soft}}(\vec{b};\mu) = A_{\text{hard}}(\vec{b};\mu) = A(\vec{b};\mu)$. Then

$$\chi_{\rm tot} = rac{A(b;\mu)}{2} \left(\sigma_{\rm hard}^{
m inc} + \sigma_{
m soft}^{
m inc}
ight) \;.$$

One new parameter $\sigma_{\text{soft}}^{\text{inc}}$.

Extending into the soft region

Continuation of the differential cross section into the soft region $p_t < p_t^{\min}$ (here: p_t integral kept fixed)



Exploit knowledge of σ_{tot} in eikonal model:

$$\begin{split} \sigma_{\text{tot}} &= 2 \int d^2 \vec{b} \left(1 - e^{-\chi_{\text{tot}}(\vec{b},s)} \right) \\ &= 2 \int d^2 \vec{b} \left(1 - e^{-\frac{A(\vec{b};\mu)}{2}(\sigma_{\text{hard}}^{\text{inc}} + \sigma_{\text{soft}}^{\text{inc}})} \right) \end{split}$$

 $\sigma_{\rm tot}$ well measured. Fixes $\sigma_{\rm soft}^{\rm inc}$.

Energy extrapolation from Donnachie-Landshoff

- ▶ DL '92 [D&L, PLB296, 227 (1992)]
- DL '92 normalized at TVT
- ▶ DL ′04

[D&L, PLB595, 393 (2004)]

Find constraints on (p_t^{\min}, μ) .



Find constraints on (p_t^{\min}, μ).
 Require $\sigma_{\text{soft}}^{\text{inc}} > 0$ mb, while describing σ_{tot} .



- Find constraints on (p_t^{\min}, μ) .
- Require σ^{inc}_{soft} > 0 mb, while describing σ_{tot}.
- Require elastic *t*-slope,

$$b_{\rm el}(s) = \left[\frac{\rm d}{{\rm d}t} \left(\ln \frac{{\rm d}\sigma_{\rm el}}{{\rm d}t}\right)\right]_{t=0}$$

to be correctly described

$$b_{\rm el}(s) = \int {\rm d}^2 ec b {b^2\over\sigma_{
m tot}} \left[1-{
m e}^{-\chi_{
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 Final state tune of semi-hard MPI (MRST2001)



► What to expect at 14 TeV?



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- $\sigma_{\text{soft}}^{\text{inc}} > 0 \text{ mb. } \sigma_{\text{tot}} \text{ from}$ Regge fit
- Require $\bar{n}_{hard} < 10$
- Require elastic *t*-slope to be correctly described.
 Get range of possible measurements from DL '92 and predictions for b_{el}

[Khoze, Martin, Ryskin, 0710.2494] [Gotsman, Levin, Maor, 0708.1506]


Observations

- $\sigma_{\text{soft}}^{\text{inc}}$ rises artificially fast (expect ~ $s^{0.08}$).
- Forced to have energy dependent parameters (would like to have the choice, i.e. let measurements decide).
- Measurement of $b_{\rm el}$ fixes μ^2 at Tevatron:

 $\mu^2 = 0.56 \pm 0.01 \, \text{GeV}^2$

 $\sigma_{\rm eff}=(\int d^2\vec{b}A^2(b))^{-1}$ as measured by CDF in $\gamma+3j$: $\mu^2=3.0\pm0.5{\rm GeV}^2\;.$

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m eff} = (\int d^2 \vec{b} A^2(b))^{-1}$ as measured by CDF in $\gamma + 3j$: $\mu^2 = 3.0 \pm 0.5 {
m GeV}^2$.

 \rightarrow Relax the constraint of identical overlap functions:

$$A_{\text{soft}}(b) = A(b, \mu_{\text{soft}})$$

If $\mu > \mu_{\text{soft}}$: Hot Spots

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Fix the two parameters μ_{soft} and $\sigma_{\mathrm{soft}}^{\mathrm{inc}}$ in

$$\chi_{\text{tot}}(\vec{b},s) = \frac{1}{2} \left(A(\vec{b};\mu)\sigma^{\text{inc}} \text{hard}(s;p_t^{\min}) + A(\vec{b};\mu_{\text{soft}})\sigma_{\text{soft}}^{\text{inc}} \right)$$

from two constraints. Require simultaneous description of σ_{tot} and b_{el} (measured/well predicted),

$$\begin{split} \sigma_{\text{tot}}(s) &\stackrel{!}{=} 2 \int d^2 \vec{b} \left(1 - e^{-\chi_{\text{tot}}(\vec{b},s)} \right) ,\\ b_{\text{el}}(s) &\stackrel{!}{=} \int d^2 \vec{b} \frac{b^2}{\sigma_{\text{tot}}} \left(1 - e^{-\chi_{\text{tot}}(\vec{b},s)} \right) \end{split}$$

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Tevatron parameter space



LHC parameter space

Same for LHC except for uncertainty in $b_{\rm el}$ and $\sigma_{\rm tot}$.



- So far: only indirect constraints from σ_{tot} and σ_{el} .
- ▶ Now use model in Herwig++ with $\bar{n}(\vec{b},s)$ as input for MPI.
- Remaining free parameters (p_t^{\min}, μ^2) .
- ► Look at χ^2 /dof for Tevatron Run I data in the (p_t^{\min}, μ^2) plane.

Parameter space at Tevatron

χ² for Rick's Run1
 Jet analysis for all regions



Parameter space at Tevatron

- χ² for Rick's Run1 Jet analysis for all regions
- only the transverse region



Detailed look at observables: Transverse Region



Detailed look at observables: Transverse Region



What we have so far:

- Unitarized jet cross sections
- Fulfil constaints from σ_{tot} and σ_{el} .
- Simple model with similar overlap functions.
- ► No additional (explicit) energy dependence.
- Left with freedom in parameter space.

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- Unitarized jet cross sections
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- ► No additional (explicit) energy dependence.
- Left with freedom in parameter space.
- \implies Look at LHC results (900 GeV).
 - ATLAS charged particles in Min Bias.
 - Convenient as the analysis was quickly available in RIVET
 ;-)
 - ► Three points from 'valley' $(p_t^{\min}/\text{GeV}, \mu^2/\text{GeV}^2) = (3.0, 1.0); (4.0, 1.5); (5.0, 2.0)$

First plots against LHC data

Choice of PDF set (CTEQ611 vs MSTW LO** (our default)).



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First plots against LHC data

Colour structure of soft events.



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- Not so nice.
- Despite very good agreement with Rick Field's CDF UE analysis.
- Observe sensitivity to colour structure.

Sensitivity to parameter

colourDisrupt = P(disrupt colour lines)

(as opposed to hard QCD).





Extend cluster hadronization:

 QCD parton showers provide *pre-confinement* ⇒ colour-anticolour pairs



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- ► CR in the cluster hadronization model: allow *reformation* of clusters, *e.g.* (*il*) + (*jk*)



Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* ⇒ colour-anticolour pairs
- \blacktriangleright \rightarrow clusters
- ► CR in the cluster hadronization model: allow *reformation* of clusters, *e.g.* (*il*) + (*jk*)
- Allow CR if the cluster mass decreases,

$$M_{il} + M_{kj} < M_{ij} + M_{kl},$$

Accept alternative clustering with probability *p*_{reco} (model parameter) ⇒ this allows to switch on CR smoothly

- Hadronization sensitive to CR model.
- Proper study requires re-tune to LEP data.
- Used PROFESSOR.







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- Hadronization sensitive to CR model.
- Proper study requires re-tune to LEP data.
- Used PROFESSOR.
- ► Agreement on same level as w/o CR model.

A quick look at CDF data



Sensitivity different for the two observables.

Comparison with MinBias ATLAS data



- Proper comparison: lack of diffraction in Herwig++!
- ATLAS $N_{ch} \ge 6$ analysis.
- Unfortunately: plots public, data not.
- We use EasyNData P. Uwer [arXiv:0710.2896]
- Read off error \ll error bars.

Comparison with MinBias ATLAS data (900 GeV)



Comparison with MinBias ATLAS data (900 GeV)



Comparison with MinBias ATLAS data (900 GeV)



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Comparison with MinBias ATLAS data (7 TeV)



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Comparison with MinBias ATLAS data (7 TeV)



Comparison with MinBias ATLAS data (7 TeV)



Underlying Event (900 GeV)



Underlying Event (900 GeV)


Underlying Event (7 TeV)

 $N_{\rm ch}$ /StdDev transverse vs $p_t^{\rm lead}$ /GeV.



Dip at 2 GeV: mistake in data analysis?

Preliminary results

- ► 900 GeV MB/UE $p_t^{\min} = 2.6 \text{ GeV}, \mu^2 = 1.1 \text{ GeV}^2, p_{\text{reco}} = 0.48, p_{\text{disrupt}} = 0.43$
- ► 7 TeV MB $p_t^{\min} = 5.2 \text{ GeV}, \ \mu^2 = 1.8 \text{ GeV}^2, \ p_{\text{reco}} = 0.55, \ p_{\text{disrupt}} = 0.68$
- ► 7 TeV UE $p_t^{\min} = 3.2 \text{ GeV}, \ \mu^2 = 0.81 \text{ GeV}^2, \ p_{\text{reco}} = 0.61, \ p_{\text{disrupt}} = 0.34$

- ▶ MPI UE/Min Bias model in Herwig++.
- Close connection to σ_{tot} and σ_{el} via unitarization.
- Exploited to constrain free parameters and used Run I data.
- First look at LHC data within these constraints.
- Need colour reconnection model.
- ► First tunes to 900 GeV and 7 TeV Min Bias (N_{ch} ≥ 6) data give good results.
- Non–diffractive physics under good control

Open questions

- Treatment of remnant pdfs too naive?
- More involved overlap function? With Energy dependent parameters?
- Understanding of colour reconnection?

More to come:

- Model for diffraction.
- Further checks of consistency.
- ▶ In future release Herwig++ 2.5 (out very soon).
- Better look at energy dependence.
- Universal tune of UE parameters?

Stay tuned!

Extra slides

Example, outdated

Smooth interplay between shower and hadronization.



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Example, outdated

$$R_n = \sigma(n\text{-jets})/\sigma(\text{jets})$$

 $R_6 = \sigma(> 5\text{-jets})/\sigma(\text{jets})$





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0.45

0.35

0.15