RF Cavity Simulation for SPL

Simulink Model for HP-SPL Extension to LINAC4 at CERN from RF Point of View

Acknowledgement:
CEA team, in particular O. Piquet (simulink model)
W. Hofle, J. Tuckmantel, D. Valuch, G. Kotzian

Presentation Overview

- SPL Characteristics

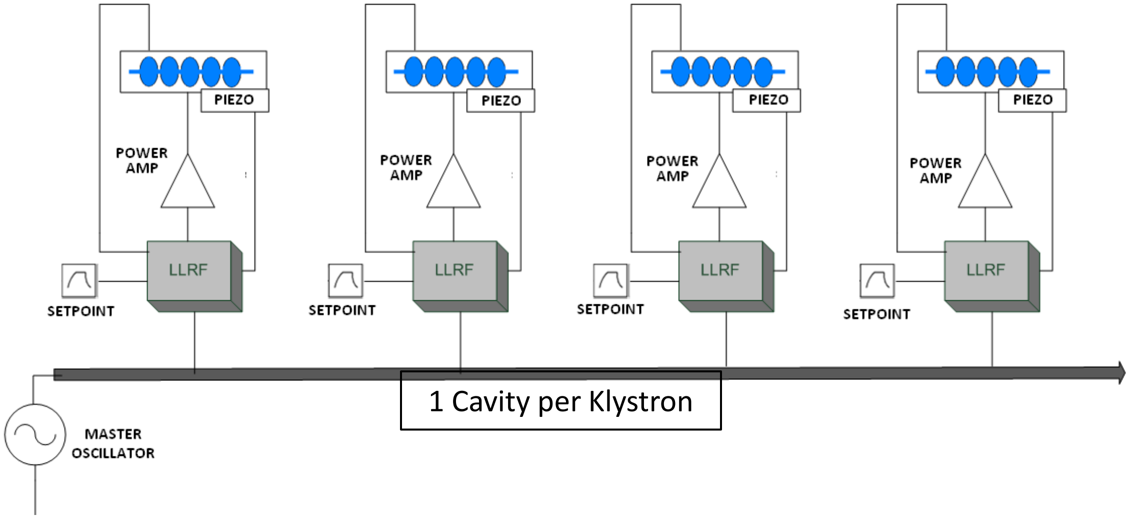
- Single Cavity Model and Simulation Results

- Double Cavity Model and Simulation Results

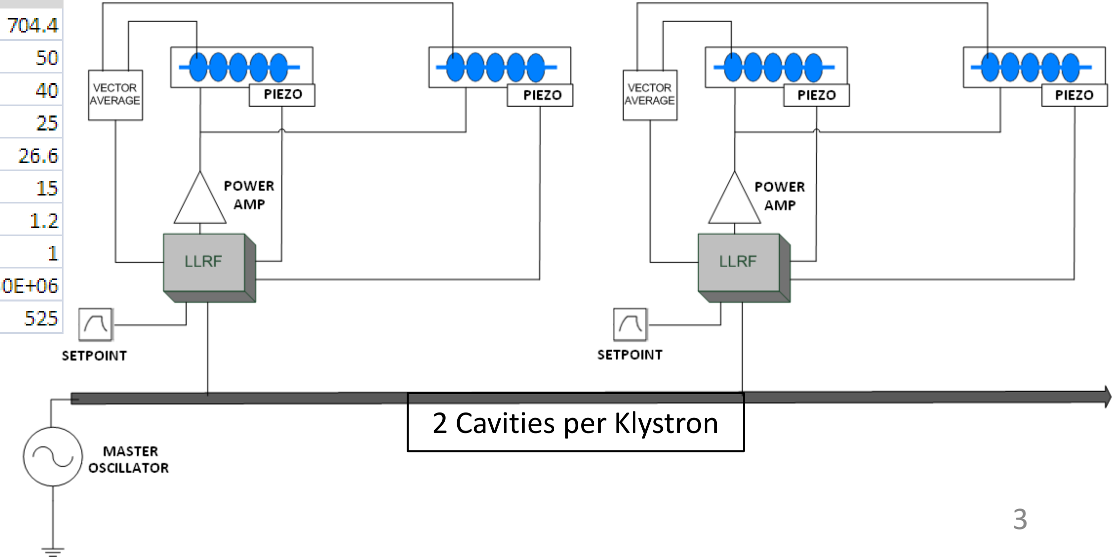
- Error Analysis

SPL High Power Operation

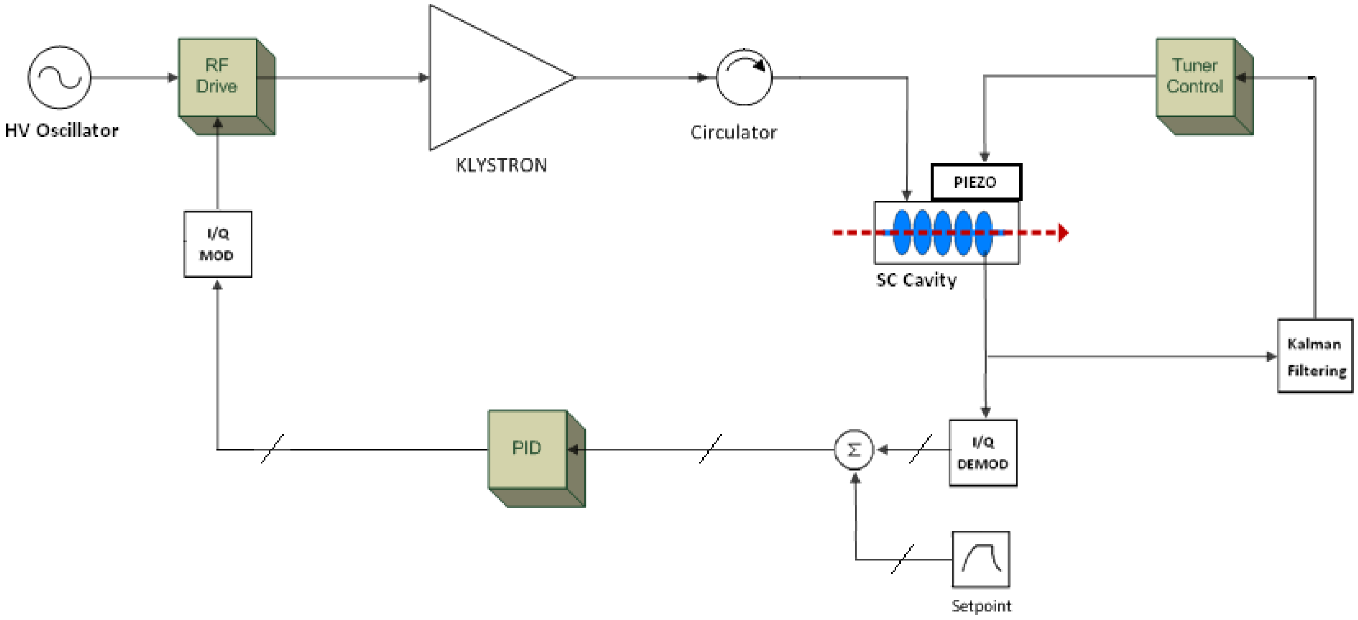
- Possible operation using single, double and four cavities fed by a single power amplifier.



GENERAL PARAMETERS FOR HIGH POWER SPL		
PARAMETER	UNIT	VALUE
Resonant Frequency	MHz	704.4
Repetition Rate	Hz	50
Average Pulse Current	mA	40
Accelerating Field	MV/m	25
Accelerating Voltage	MV	26.6
Beam Synchronous Angle	Degrees (LINAC)	15
Length of Beampulse	ms	1.2
Power Delivered to Beam per Cavity	MW	1
Cavity/Generator Coupling Loaded Quality Factor		1.30E+06
Geometry Factor (R/Q)	Ohm (LINAC)	525



High-Level Diagram of Single Cavity + Control System



$$f_{RF} = 704.4 \text{ MHz}$$

$$I_{b,DC} = 40 \text{ mA}$$

$$\phi_s = 15^\circ \text{ (LINAC)}$$

$$P_b = V_{acc} \times I_{b,DC} \times \cos(\phi_s) = 1.0285 \text{ MW}$$

$$Q_L = 1.3113 \times 10^6$$

$$\frac{R}{Q} = 525 \ \Omega \text{ (LINAC)}$$

$$\tau_{beampulse} = 1.2 \text{ ms}$$

$$\text{rep period} = 20 \text{ ms}$$

$$R_L = 680 \text{ M}\Omega$$

$$I_g = \frac{V_{acc}}{R_L} + I_{b,DC} \cos(\phi_s) = 77.3 \text{ mA}$$

$$\tau_{fill} = \frac{2Q_L}{\omega_{RF}} = 0.5926 \text{ ms}$$

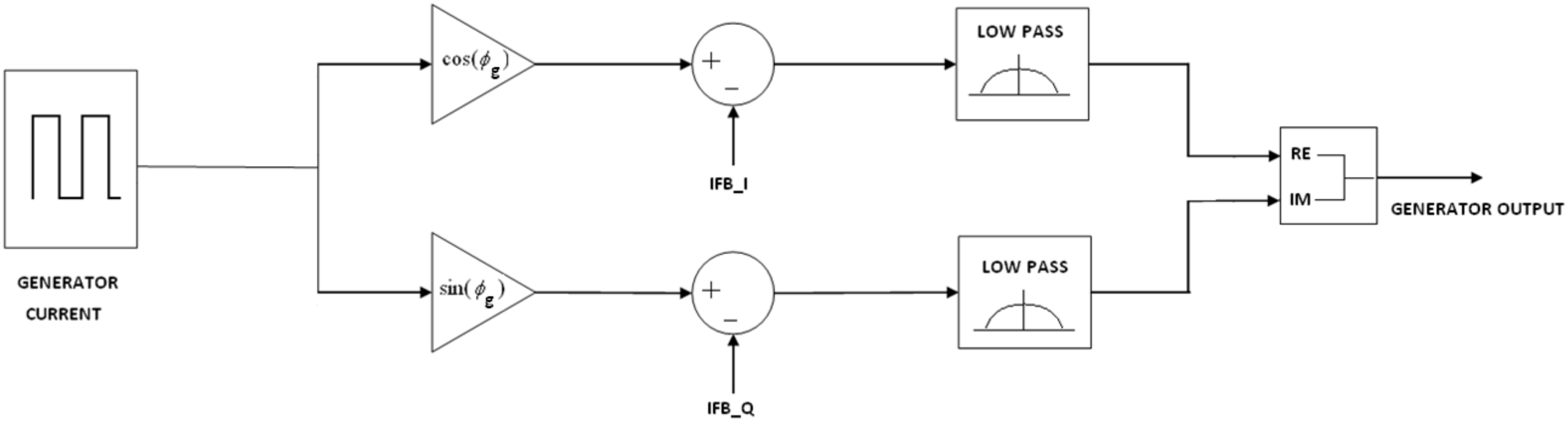
$$\alpha = \frac{I_g}{I_{b,DC} \cos(\phi_s)} = 2$$

$$t_{inj} = \tau_{fill} \ln(\alpha) = 0.4108 \text{ ms}$$

RF Drive and Generator Model

- Generator current modeled as square pulse for the duration of injection + beam pulse time

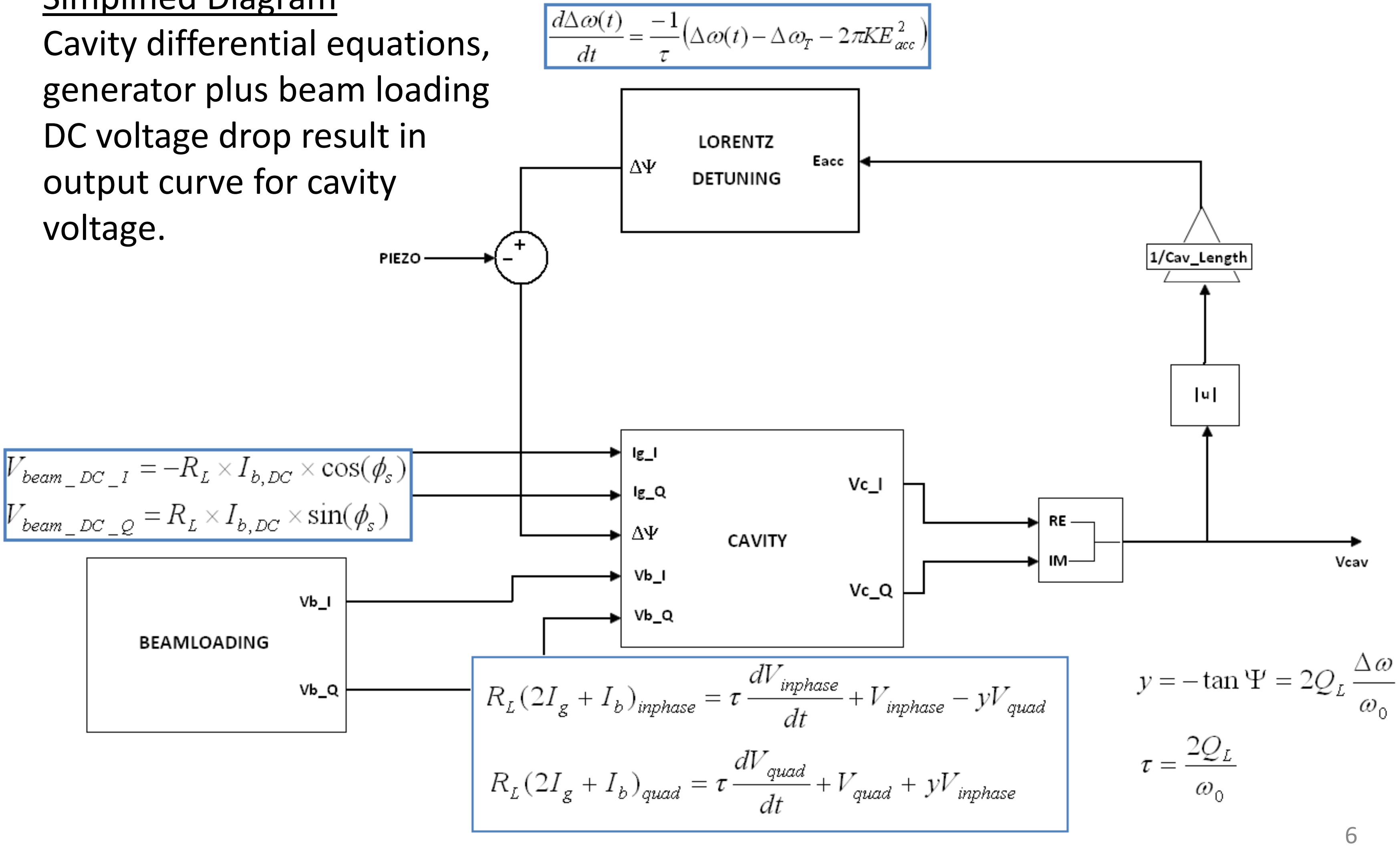
- High bandwidth compared to feedback loop and cavity (1 MHz)



Cavity Model (cont)

Simplified Diagram

Cavity differential equations, generator plus beam loading DC voltage drop result in output curve for cavity voltage.



$$\frac{d\Delta\omega(t)}{dt} = \frac{-1}{\tau}\left(\Delta\omega(t) - \Delta\omega_T - 2\pi K E_{acc}^2\right)$$

$$V_{beam\_DC\_I} = -R_L \times I_{b,DC} \times \cos(\phi_s)$$

$$V_{beam\_DC\_Q} = R_L \times I_{b,DC} \times \sin(\phi_s)$$

$$R_L(2I_g + I_b)_{inphase} = \tau \frac{dV_{inphase}}{dt} + V_{inphase} - yV_{quad}$$

$$R_L(2I_g + I_b)_{quad} = \tau \frac{dV_{quad}}{dt} + V_{quad} + yV_{inphase}$$

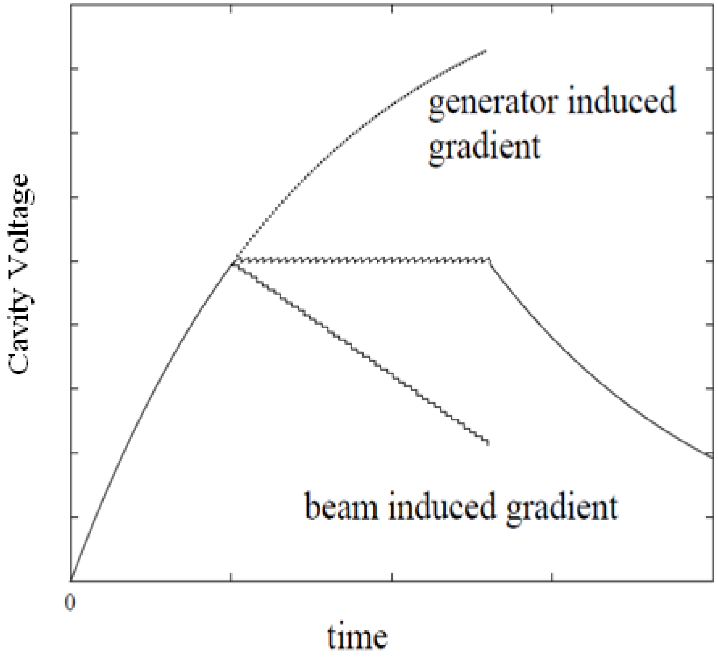
$$y = -\tan\Psi = 2Q_L \frac{\Delta\omega}{\omega_0}$$

$$\tau = \frac{2Q_L}{\omega_0}$$

Beam Loading

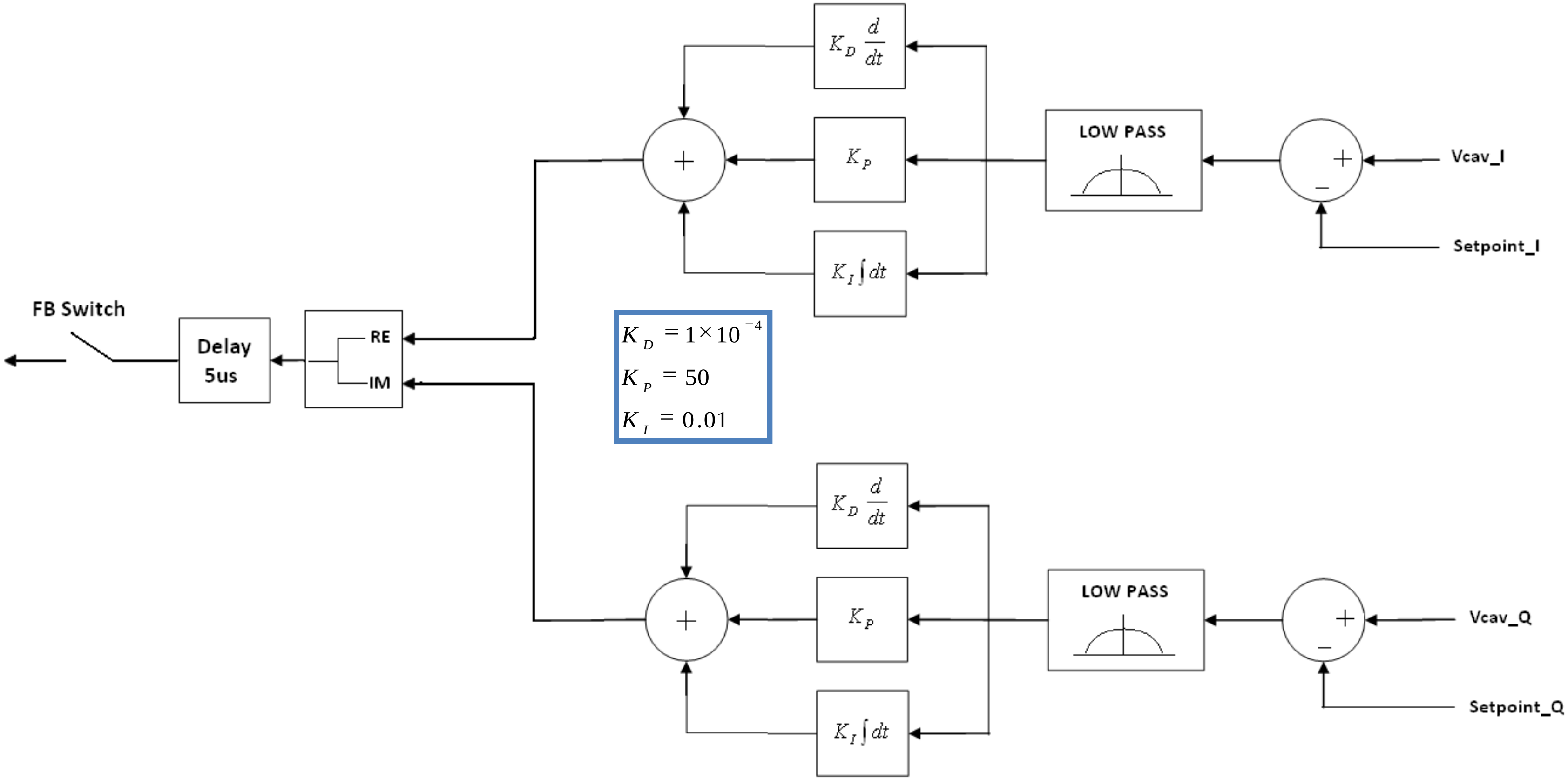
- Infinitely narrow bunches induce instant voltage drops in cavity
- Voltage drop is equal to generator induced voltage between bunches creating flattop operation
- Envelope of RF signal in I/Q

$$\boxed{V_{cav\_bunch} = \omega_{RF} \times \frac{R}{Q}(circuit) \times q_b}$$



RF Feedback

- PID controller
- Limit bandwidth in feedback loop to 100 kHz
- (Klystron bandwidth is 1 MHz)

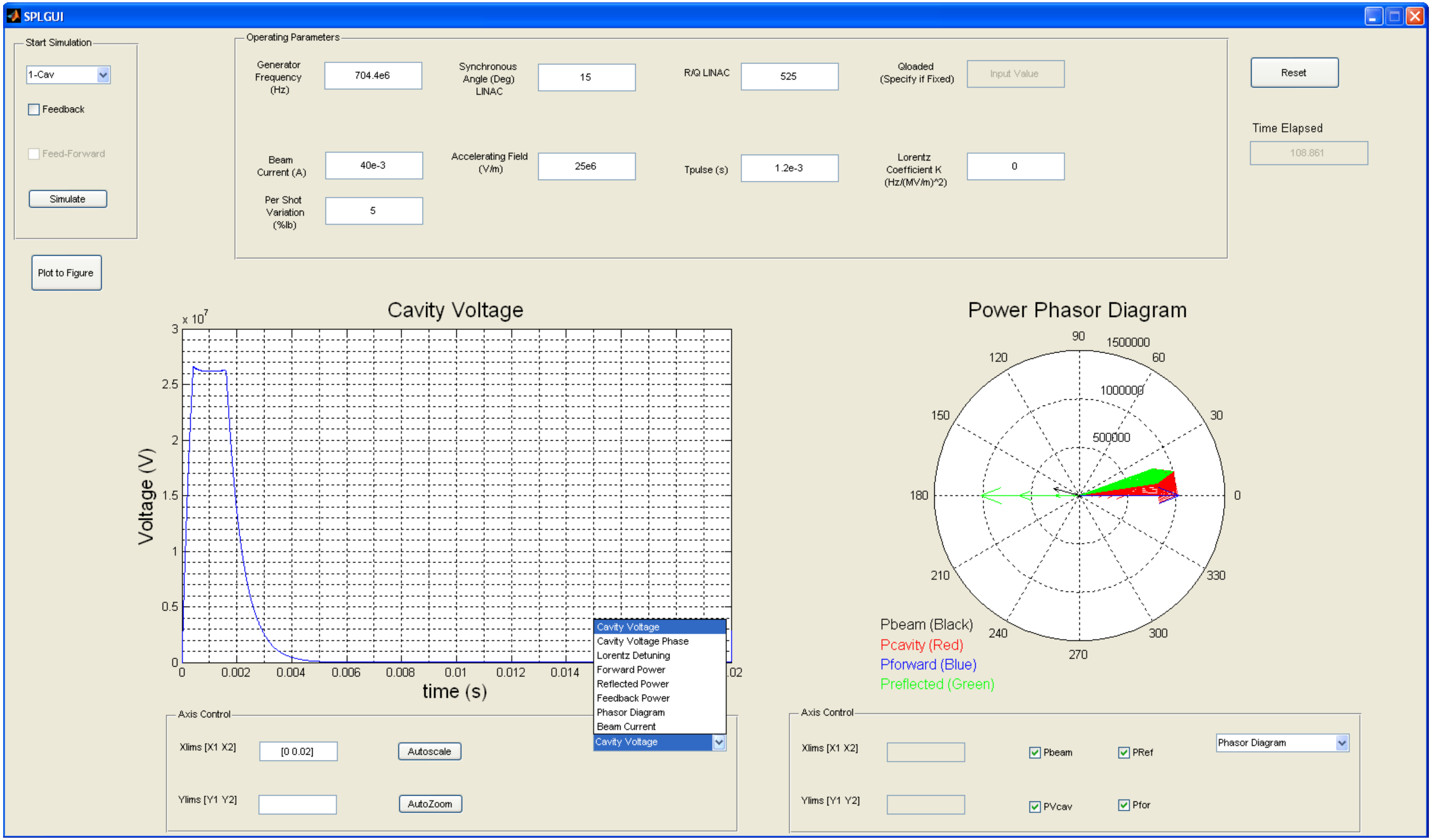


$K_D = 1 \times 10^{-4}$

$K_P = 50$

$K_I = 0.01$

Graphical User Interface



SPLGUI

Start Simulation

1-Cav

☐ Feedback

☐ Feed-Forward

Operating Parameters

Generator Frequency (Hz): 704.4e6

Synchronous Angle (Deg) LINAC: 15

R/Q LINAC: 525

Qloaded (Specify if Fixed): Input Value

Beam Current (A): 40e-3

Accelerating Field (V/m): 25e6

Tpulse (s): 1.2e-3

Lorentz Coefficient K (Hz/(MV/m)^2): 0

Per Shot Variation (%lb): 5

Time Elapsed: 108.861

Axis Control

Xlims [X1 X2]: [0 0.02]

Ylims [Y1 Y2]:

Cavity Voltage

Axis Control

Xlims [X1 X2]:

Ylims [Y1 Y2]:

☑ Pbeam

☑ PRef

☑ PVcav

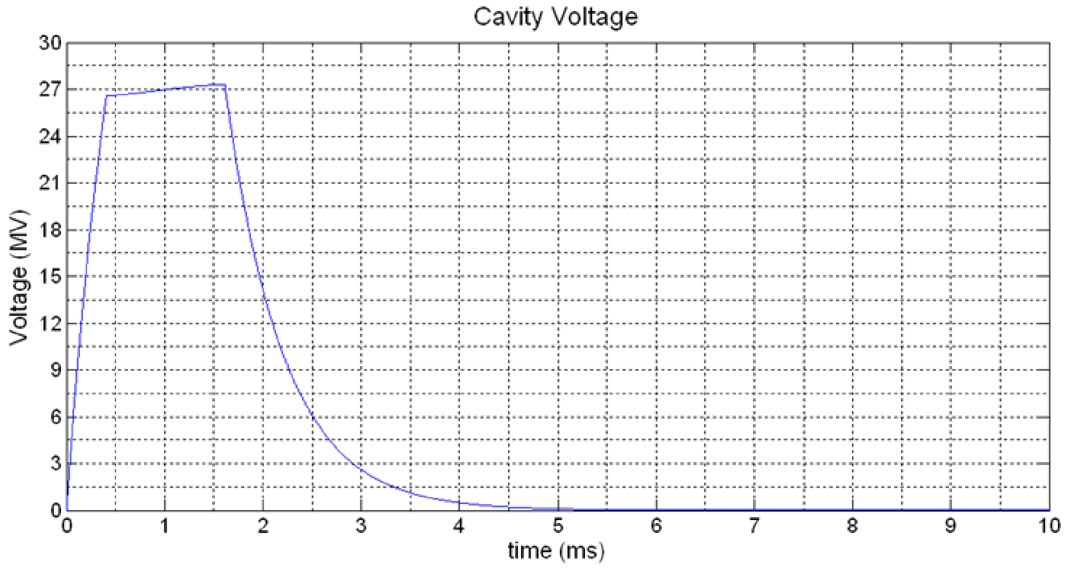
☑ Pfor

Phasor Diagram

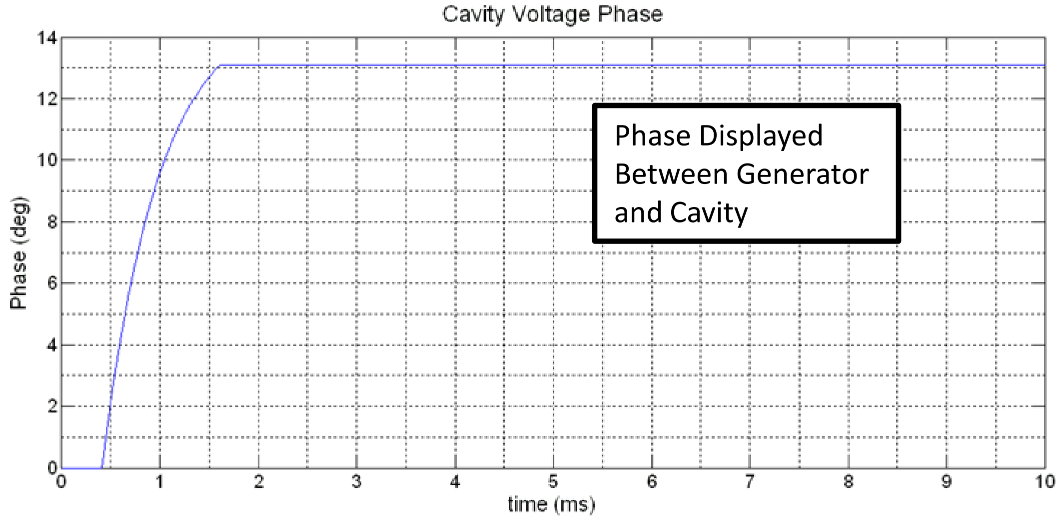
Results

- Cavity Voltage Amplitude and Phase
- Forward and Reflected Power
- Additional Power for Feedback Transients and Control
- Effect of Lorentz Detuning on Feedback Power
- Effect of Source Current Fluctuations
- Mismatched Low-Power Case

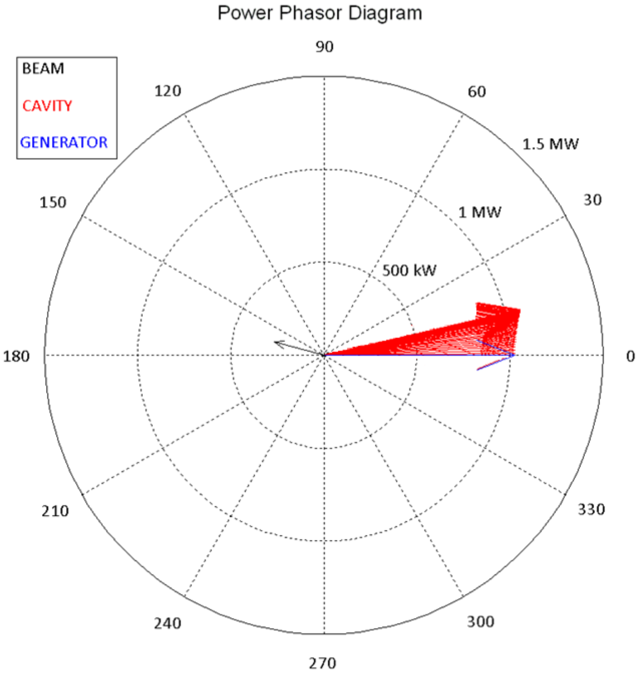
Cavity Voltage and Phase in the Absence of Lorentz Detuning (Open Loop)



Reactive Beamloading Results in Vacc Deviation

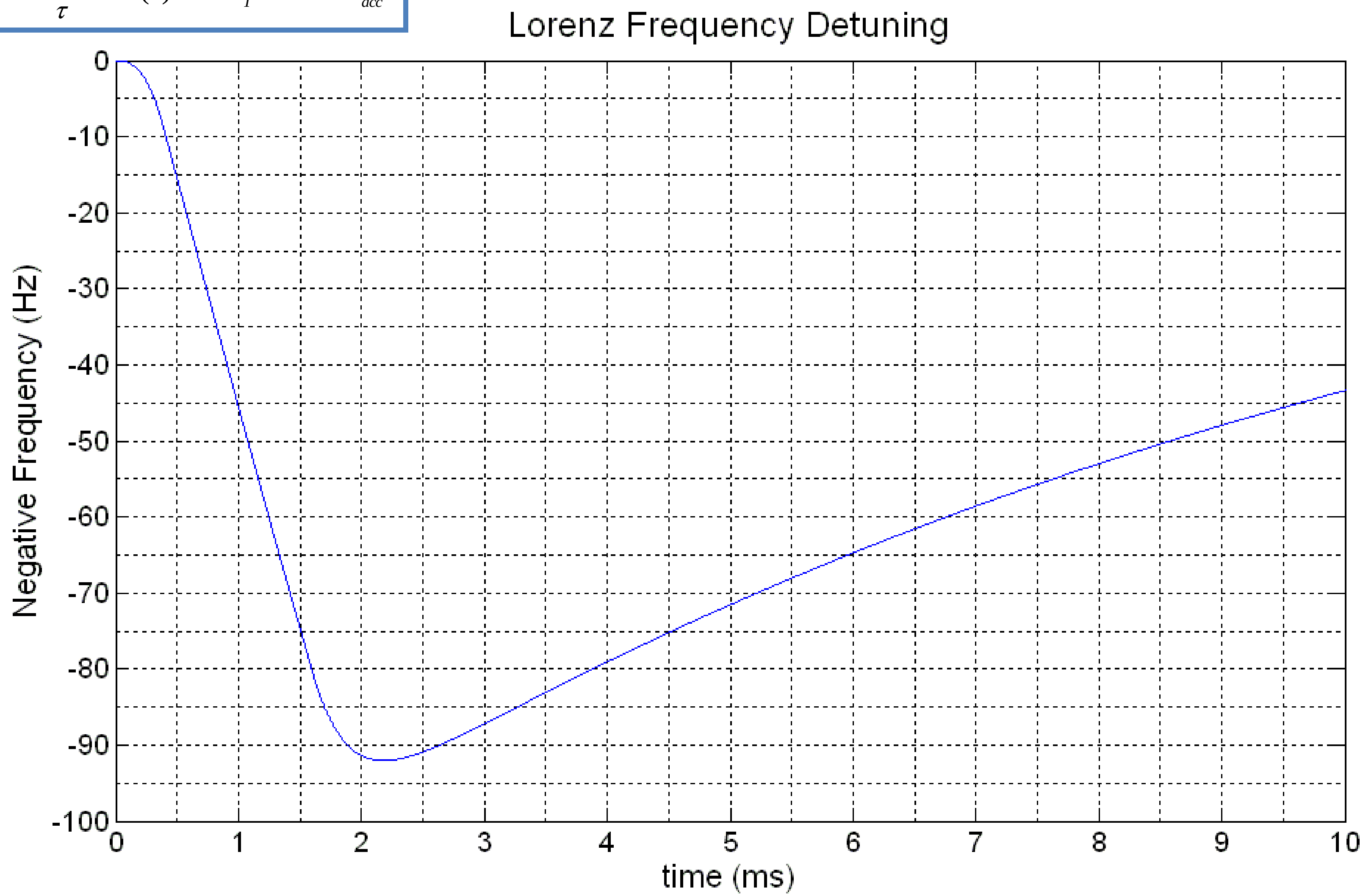


Phase Displayed Between Generator and Cavity

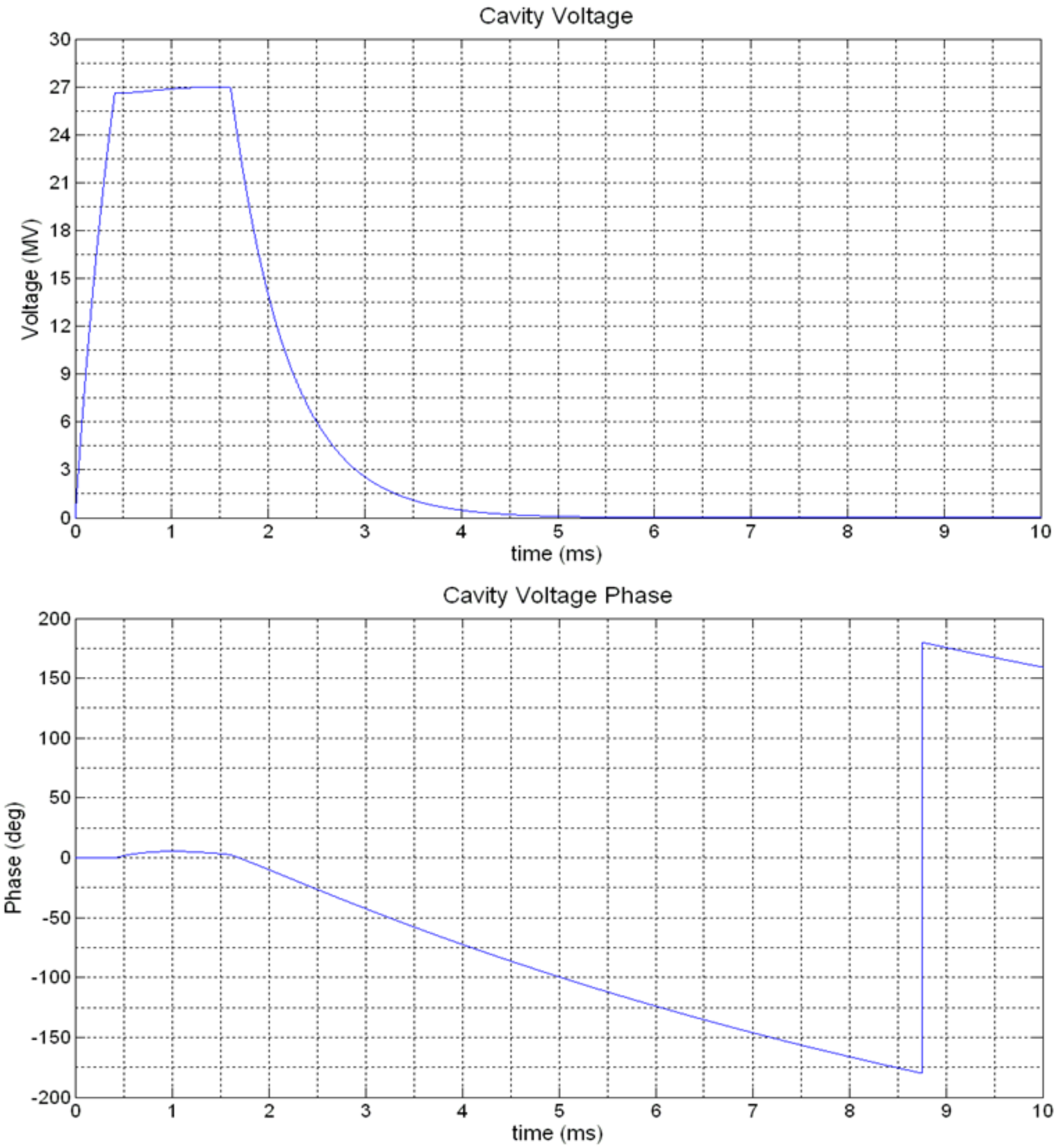


Effect of Lorentz Detuning on Cavity Voltage and Phase (Lorentz Frequency Shift)

$$\frac{d\Delta\omega(t)}{dt} = \frac{-1}{\tau}\left(\Delta\omega(t) - \Delta\omega_T - 2\pi K E_{acc}^2\right)$$



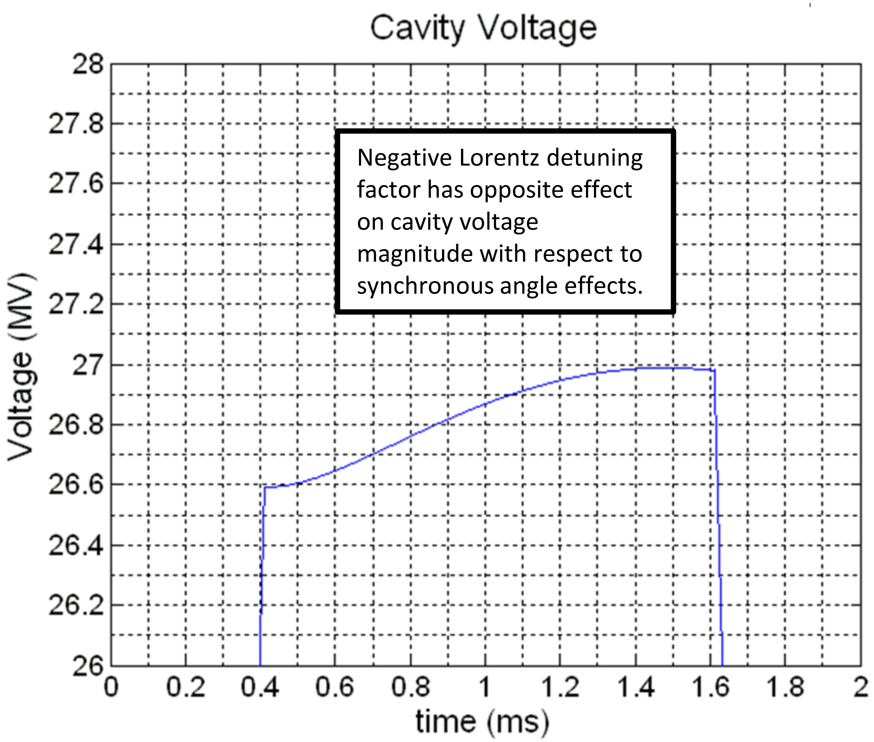
Effect of Lorentz Detuning on Cavity Voltage and Phase (Open Loop)



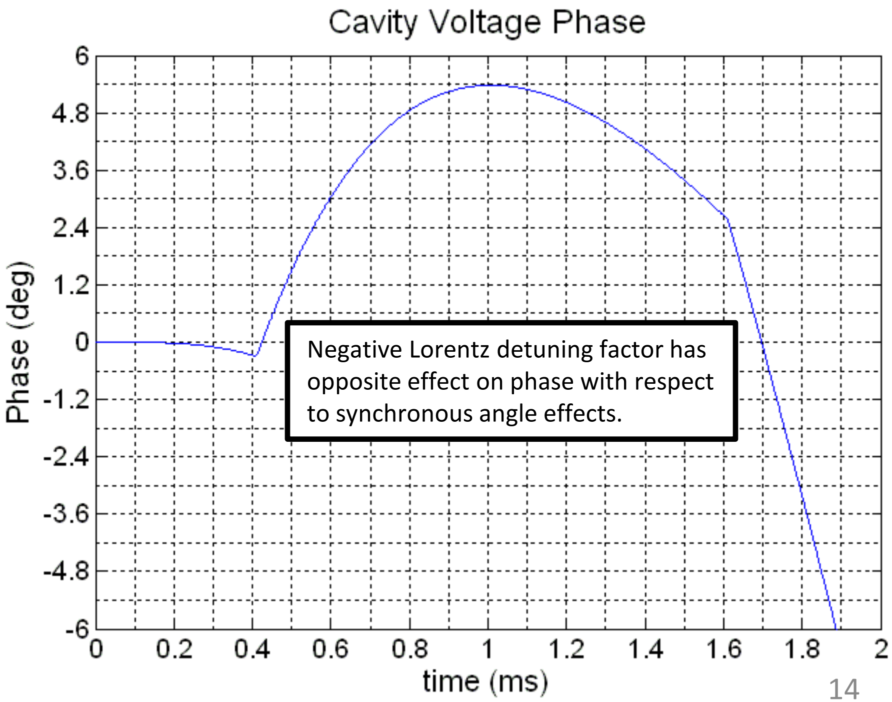
Lorentz effects oppose those of the synchronous angle

Linear phase shift for undriven cavity

Effect of Lorentz Detuning on Cavity Voltage and Phase (Open Loop Close-Up)

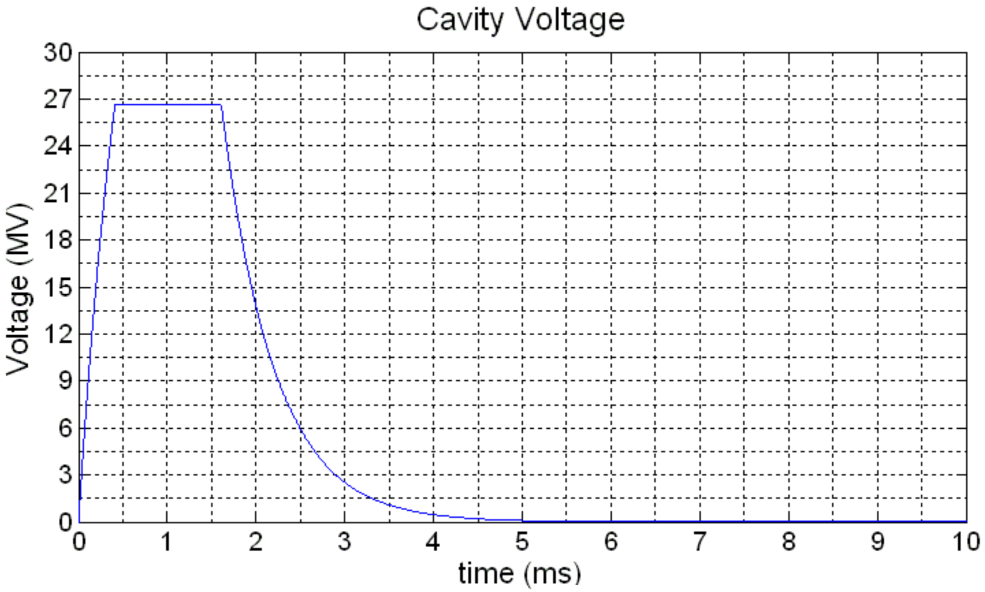

Cavity Voltage

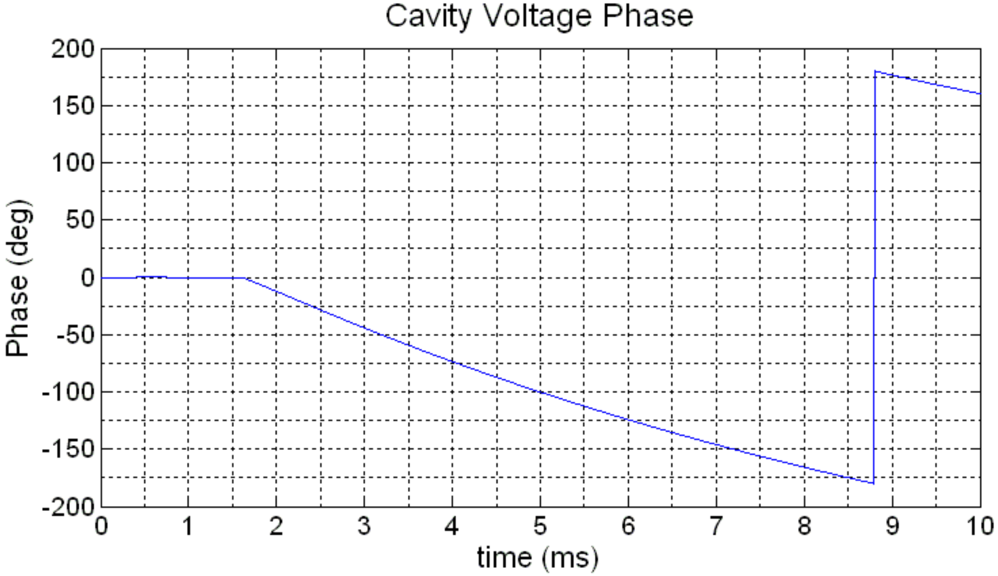
Negative Lorentz detuning factor has opposite effect on cavity voltage magnitude with respect to synchronous angle effects.


Cavity Voltage Phase

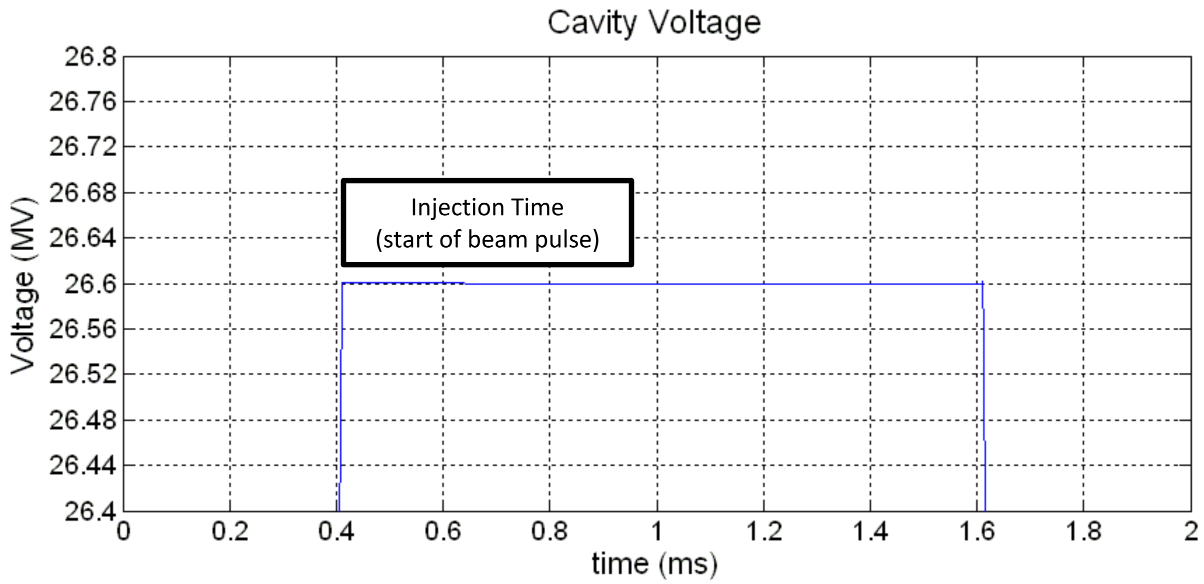
Negative Lorentz detuning factor has opposite effect on phase with respect to synchronous angle effects.

Cavity Voltage and Phase With Lorentz Detuning (Closed Loop Performance of Fast Feedback)

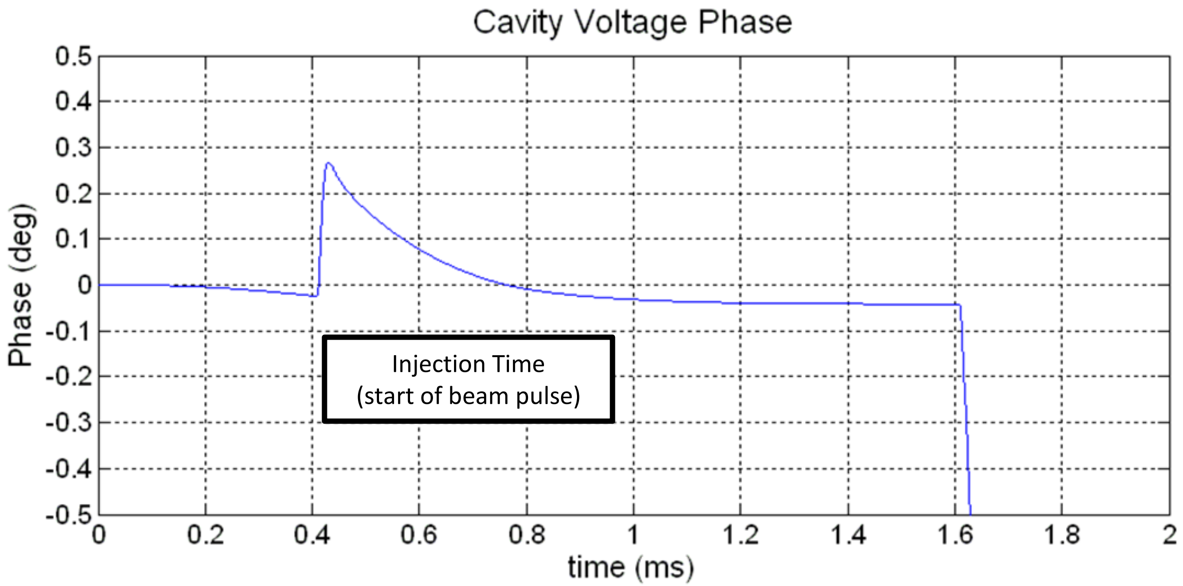




Cavity Voltage and Phase Close-up

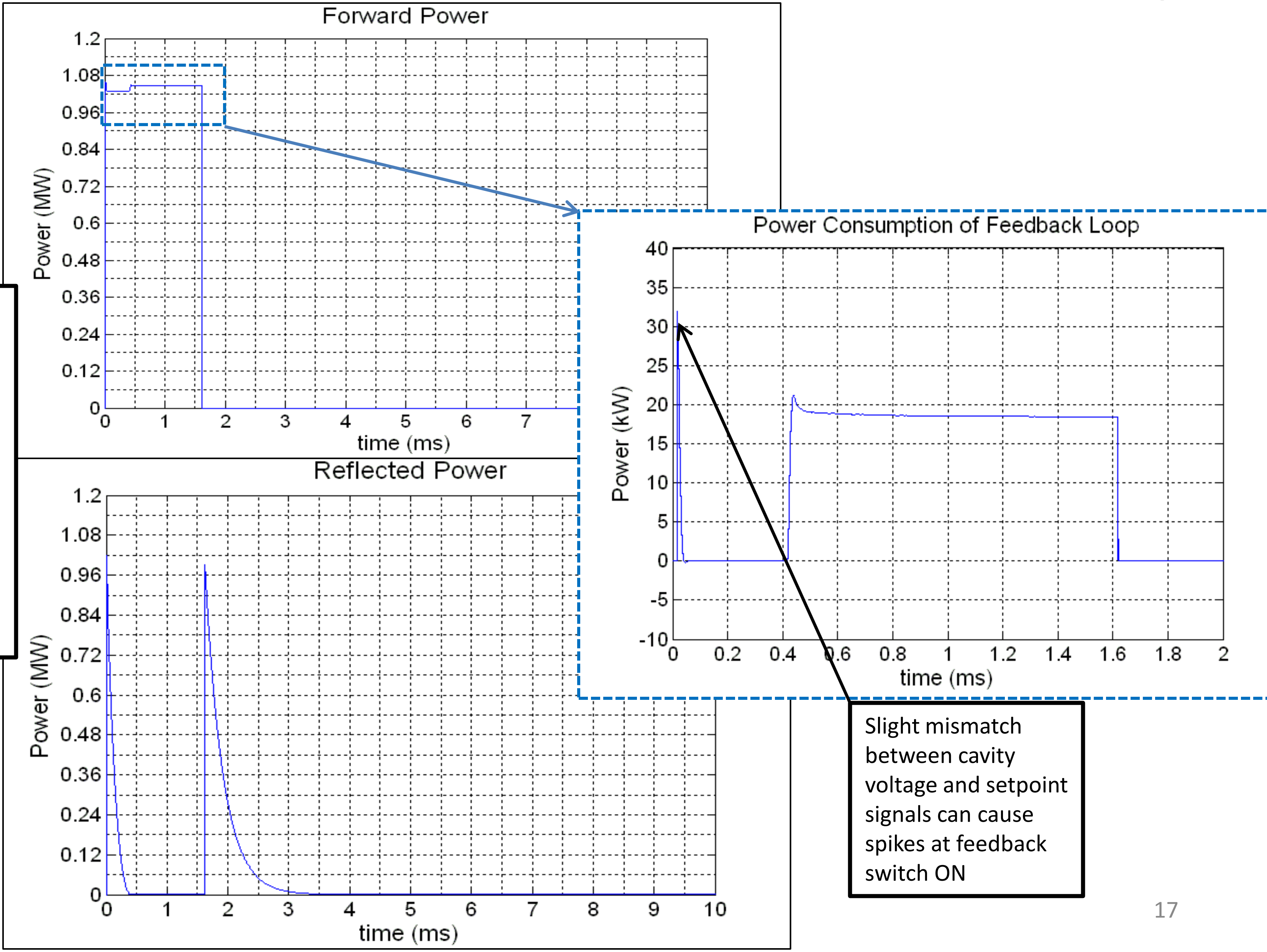


Injection Time
(start of beam pulse)



Injection Time
(start of beam pulse)

Forward and Reflected Power without Lorentz Detuning

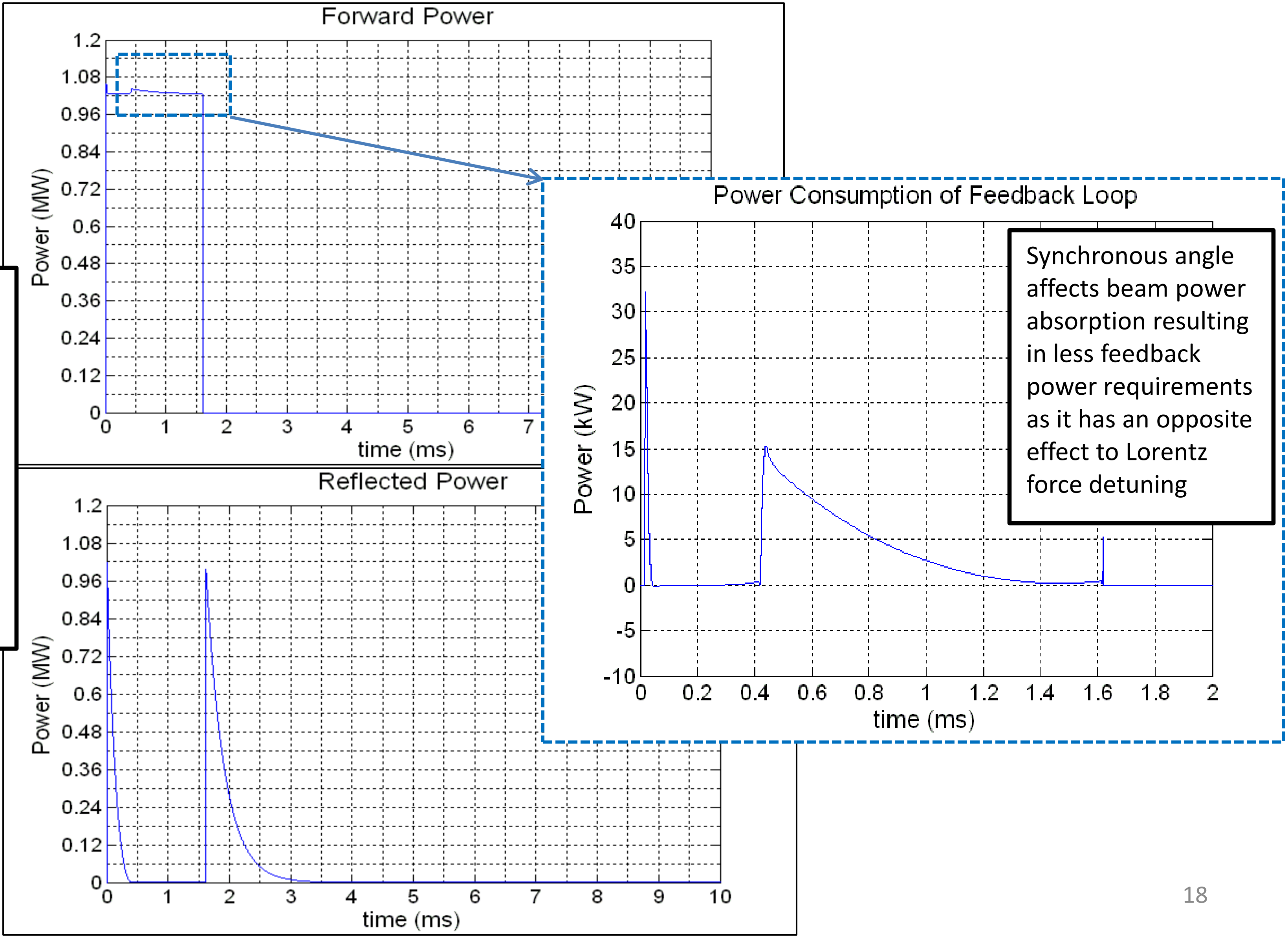


Oscillations due to transients during closing of the feedback loop and beam loading. Feedback loop is closed 10 us after generator pulse and open 10 us after end of beam loading.

Slight mismatch between cavity voltage and setpoint signals can cause spikes at feedback switch ON

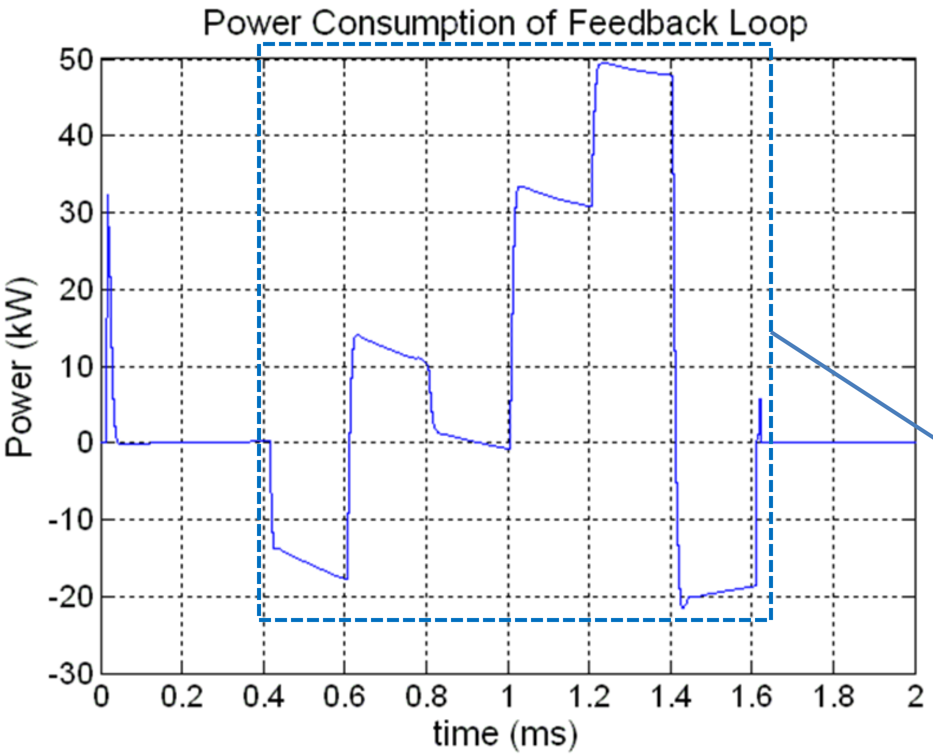
Forward and Reflected Power and Feedback Power Consumption with Lorentz Force detuning

Oscillations due to transients during closing of the feedback loop and beam loading. Feedback loop is closed 10 us after generator pulse and open 10 us after end of beam loading.

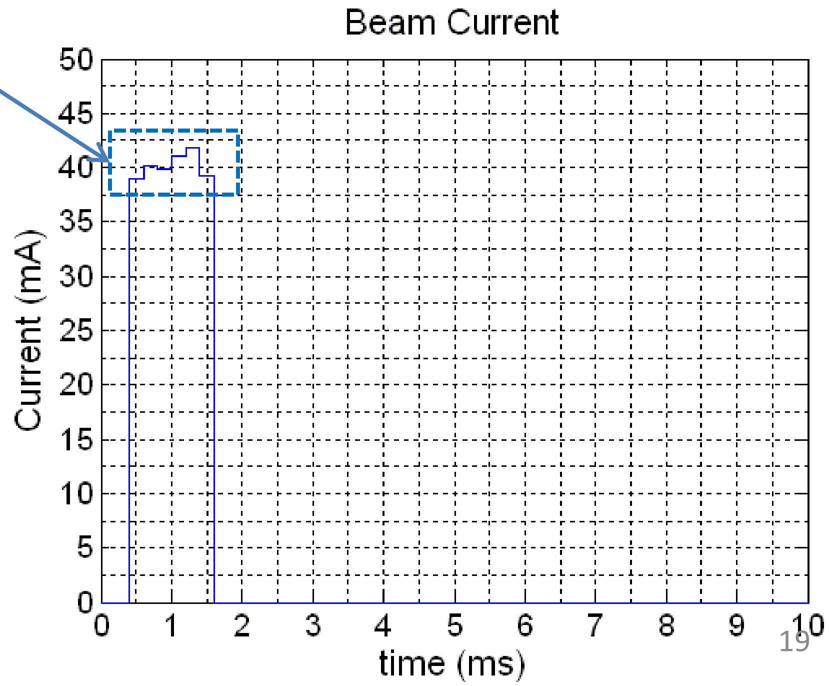


Synchronous angle affects beam power absorption resulting in less feedback power requirements as it has an opposite effect to Lorentz force detuning

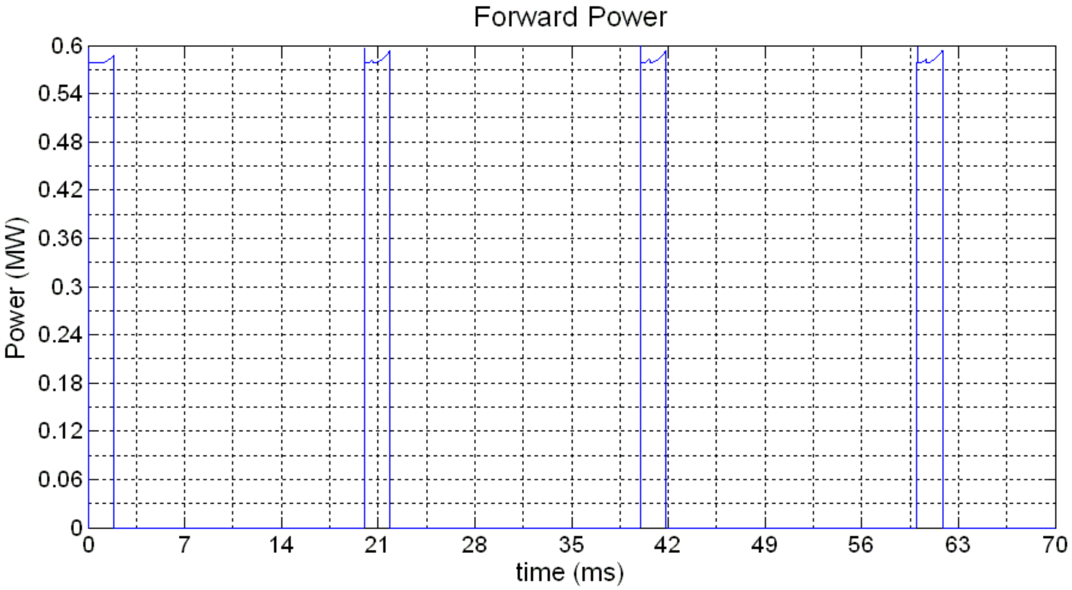
Effects of Source Beam Current Variation

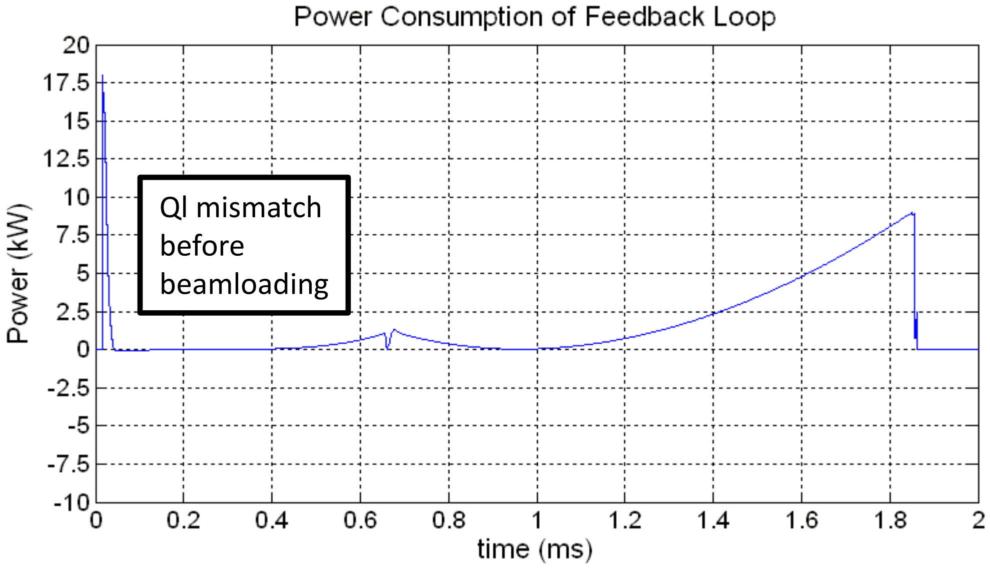


5% variation in Ib for 40mA case requires approximately 60kW of feedback power.



SPL Low Power Operation (Power Analysis)





Ql mismatch before beamloading

$$I_{\mathrm{b,DC}} \cong 20 \ \mathrm{mA}$$

$$P_{\mathrm{b}} = V_{\mathrm{acc}} \times I_{\mathrm{b,DC}} \times \cos(\phi_{\mathrm{s}}) \cong 514 \ \mathrm{kW}$$

$$Q_{\mathrm{L,\,fixed}} = \frac{V_{acc}}{\frac{R}{Q} \times I_{b,\mathrm{40\,mA}} \times \cos(\phi_s)} \cong 1.3113 \times 10^{6}$$

$$I_g = \frac{V_{acc}}{R_L} + I_{b,DC}\cos(\phi_s) = 58 \ \mathrm{mA}$$

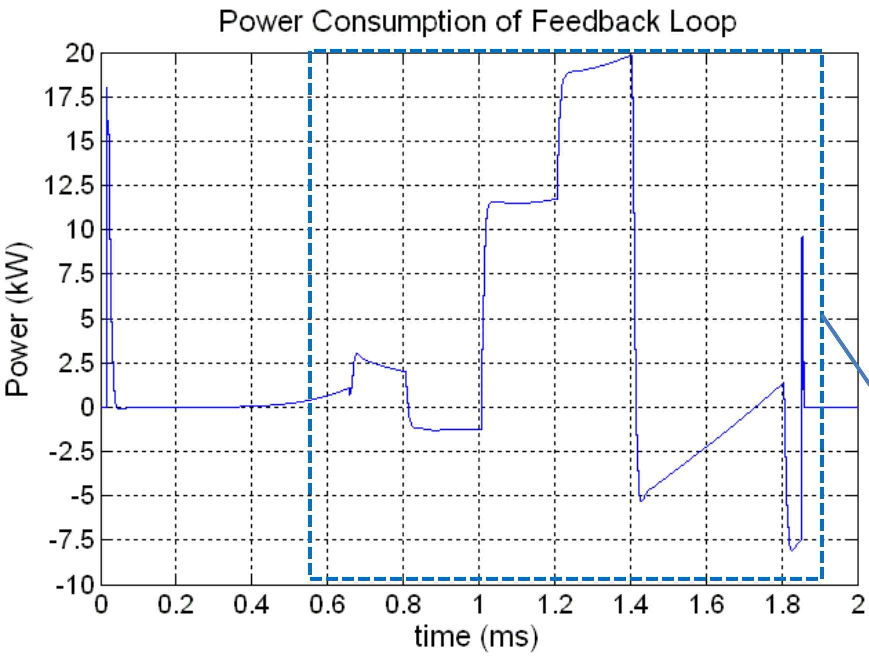
$$\alpha = \frac{I_g}{I_{b,DC}\cos(\phi_s)} = 3$$

$$\tau_{fill} = \frac{2Q_L}{\omega_{RF}} = 0.5926 \ \mathrm{ms}$$

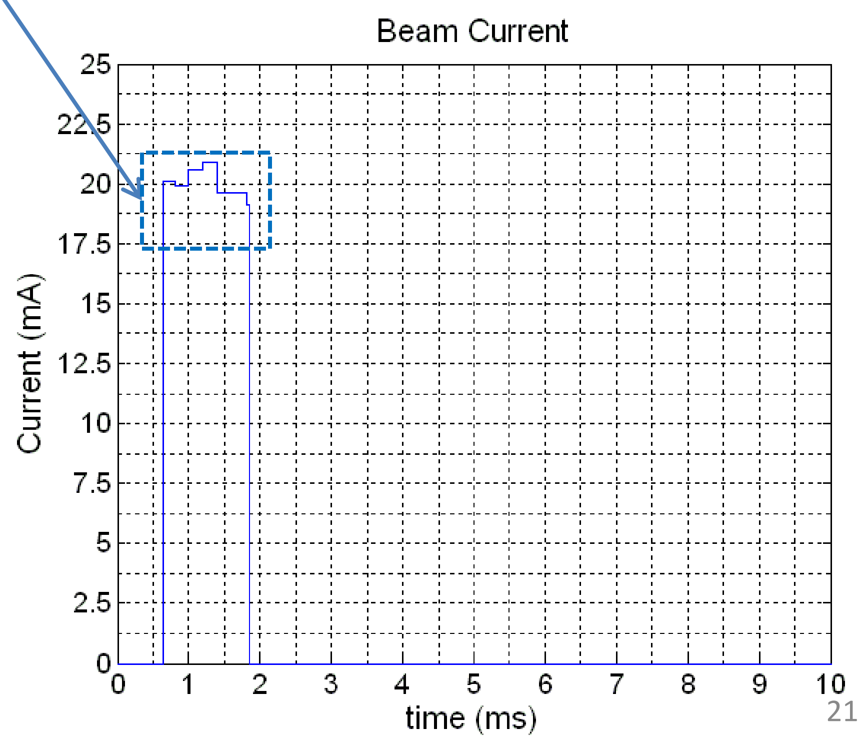
$$t_{inj} = \tau_{fill}\ln(\alpha) = 0.6510 \ \mathrm{ms}$$

$$P_{fwd} = \frac{1}{4}R_L\left|I_g\right|^2 = 578 \ \mathrm{kW}$$

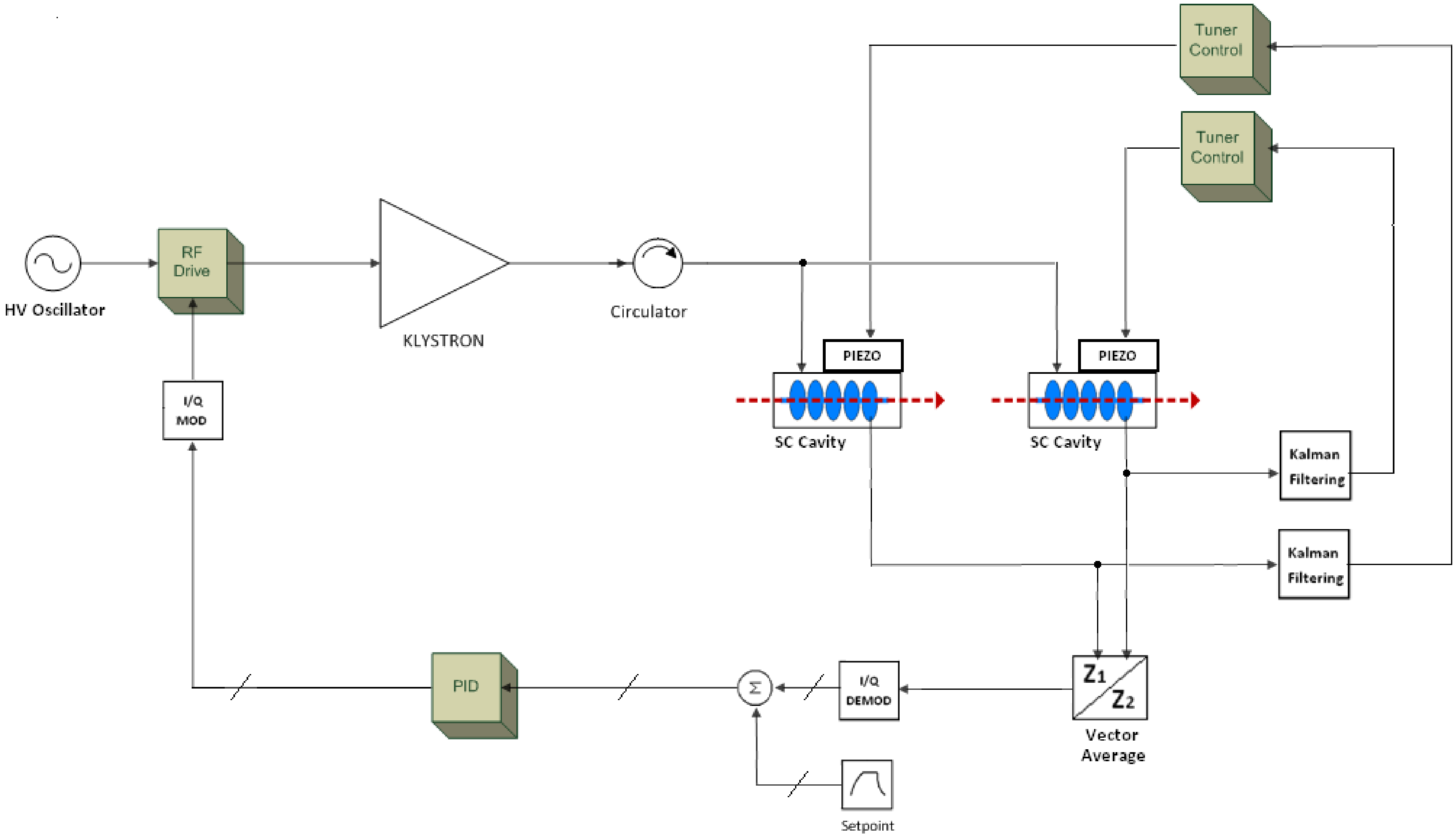
Effects of Source Beam Current Variation



5% (1mA) results in approx. 20kW FB power increase



High Level Diagram for Dual Cavity + Control System



Kalman Filtering

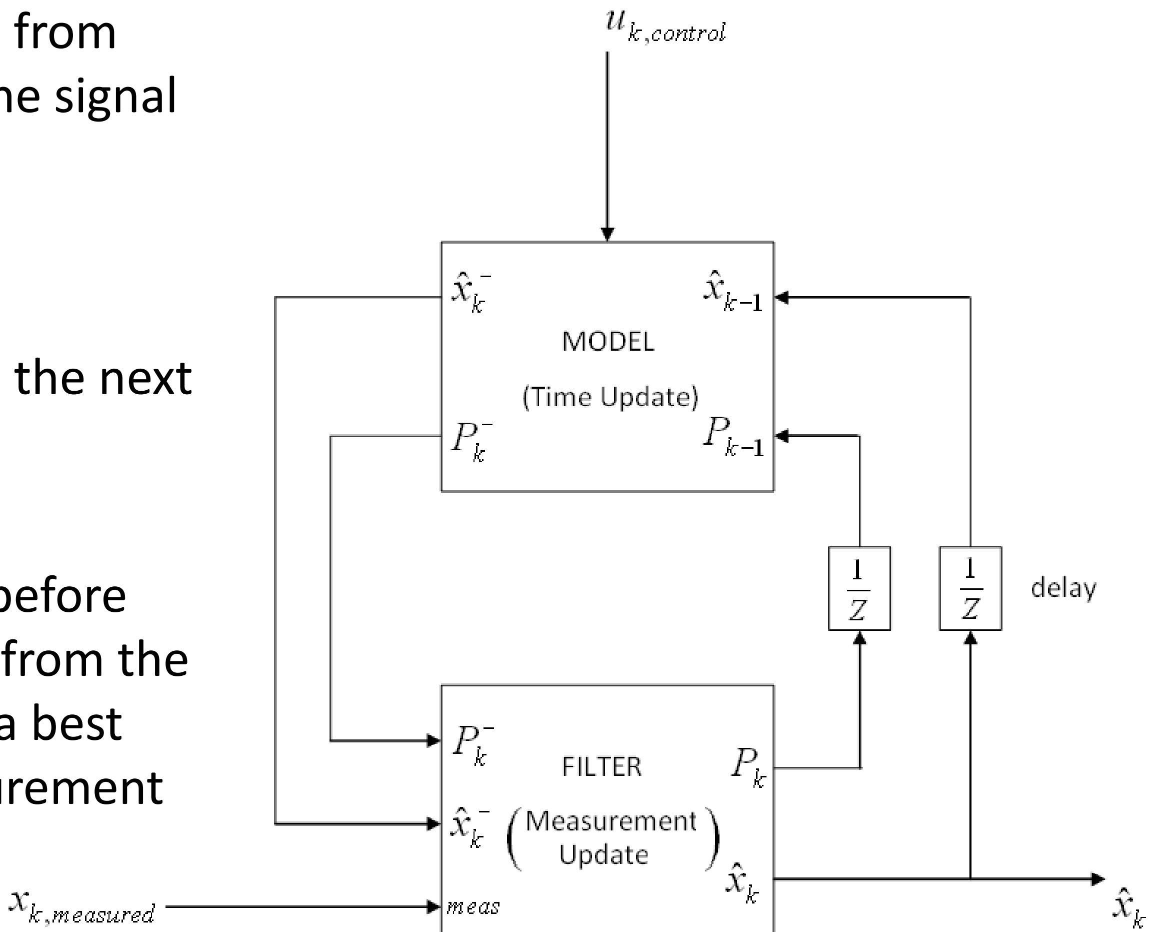
- Proposed to obtain accurate output from noisy measurement (e.g cavity voltage)

- Uses model estimates + real measurements to find best output fit

- Useful for accurate feed-forward scheme for piezo-electric tuning of resonant cavities in the presence of Lorentz force

Kalman Filter Operation

- Model takes information from last estimated value of the signal of interest and its error covariance matrix
- Control signal conditions the next estimate
- Filter takes the a priori (before measurement) estimate from the model and then creates a best estimate with the measurement information



$$\dot{V}_{RE} + \omega_{1/2} V_{RE} + \Delta\omega V_{IM} = R_L \omega_{1/2} I_{RE}$$

$$\dot{V}_{IM} + \omega_{1/2} V_{IM} - \Delta\omega V_{RE} = R_L \omega_{1/2} I_{IM}$$

Kalman Filter Operation (Cont…)

- R and Q are the measurement and model noise covariances and can be viewed as tuning parameters of the filter

- The time step dt (sampling rate) is used to characterize the discrete filter from continuous linear differential equations

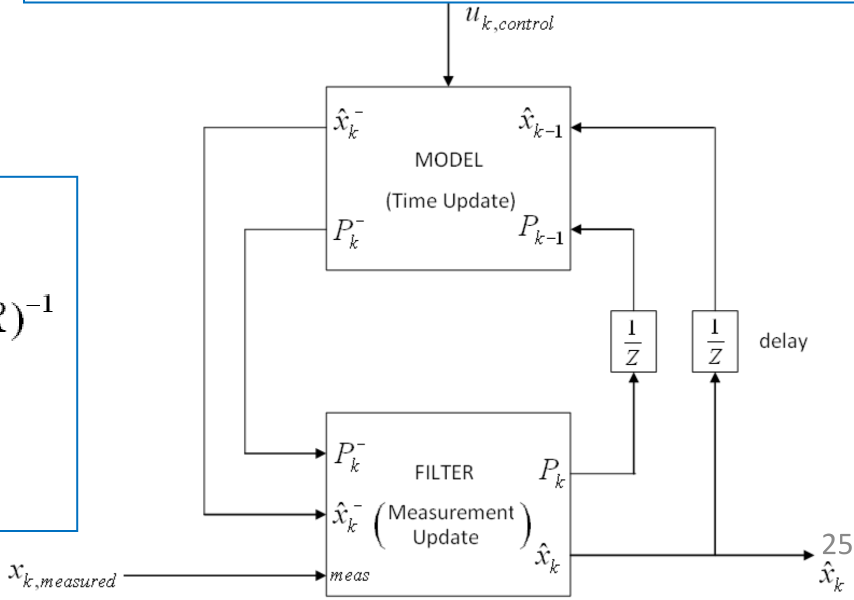
$$x(t_{i+1},t_i)=\Phi(t_{i+1},t_i)x(t_i)+\mathrm{B}(t_{i+1},t_i)u(t_i)$$

$$z(t_i)=Hx(t_i) \qquad H=\mathrm{I}$$

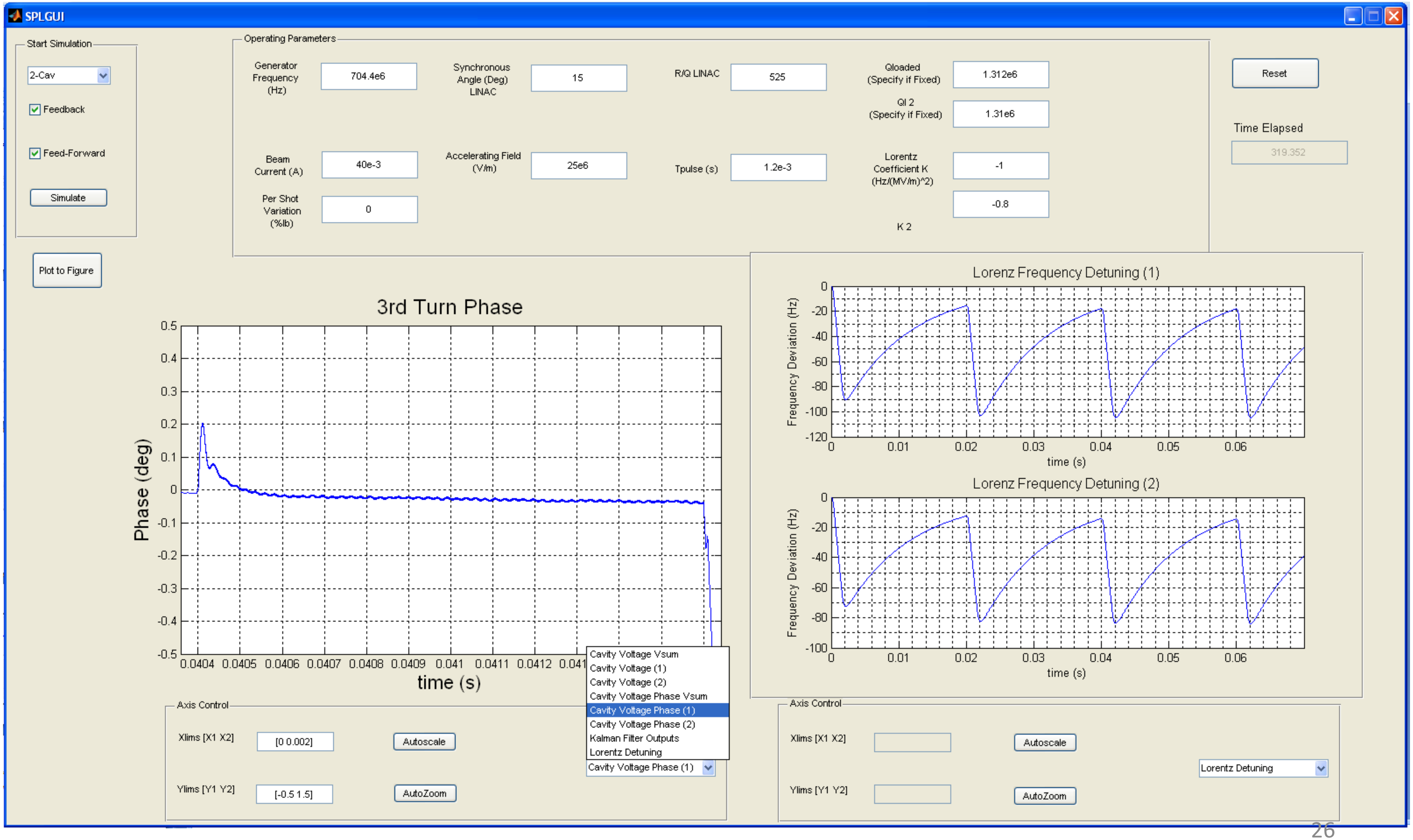
$$x(t_i)=\begin{bmatrix} V_I(t_i) \\ V_Q(t_i) \end{bmatrix} \qquad u(t_i)=\begin{bmatrix} I_I(t_i) \\ I_Q(t_i) \end{bmatrix}$$

$$\Phi=\mathrm{I}+\begin{bmatrix} \frac{-\omega_0}{2Ql} & -\Delta\omega(t) \\ \Delta\omega(t) & \frac{-\omega_0}{2Ql} \end{bmatrix}dt \qquad \mathrm{B}=\begin{bmatrix} \frac{\omega_o Rl}{2Ql} & 0 \\ 0 & \frac{\omega_o Rl}{2Ql} \end{bmatrix}dt$$

Time Update	Measurement Update
$\hat{x}_k^- = \Phi\hat{x}_{k-1} + B_d u_{k-1}$	$K_k = P_k^- H^{\mathrm{T}}(HP_k^- H^{\mathrm{T}} + R)^{-1}$
$P_k^- = \Phi P_{k-1}\Phi^{\mathrm{T}} + Q_d$	$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$
	$P_k = (\mathrm{I} - K_k H)P_k^-$



2-Cavity GUI



SPLGUI

Start Simulation: 2-Cav

☑ Feedback

☑ Feed-Forward

Operating Parameters

Generator Frequency (Hz): 704.4e6

Synchronous Angle (Deg) LINAC: 15

R/Q LINAC: 525

Qloaded (Specify if Fixed): 1.312e6

Ql 2 (Specify if Fixed): 1.31e6

Beam Current (A): 40e-3

Accelerating Field (V/m): 25e6

Tpulse (s): 1.2e-3

Lorentz Coefficient K (Hz/(MV/m)^2): -1

K 2: -0.8

Per Shot Variation (%Ib): 0

Time Elapsed: 319.352

Axis Control

Xlims [X1 X2]: [0 0.002]

Ylims [Y1 Y2]: [-0.5 1.5]

Cavity Voltage Phase (1)

Axis Control

Xlims [X1 X2]:

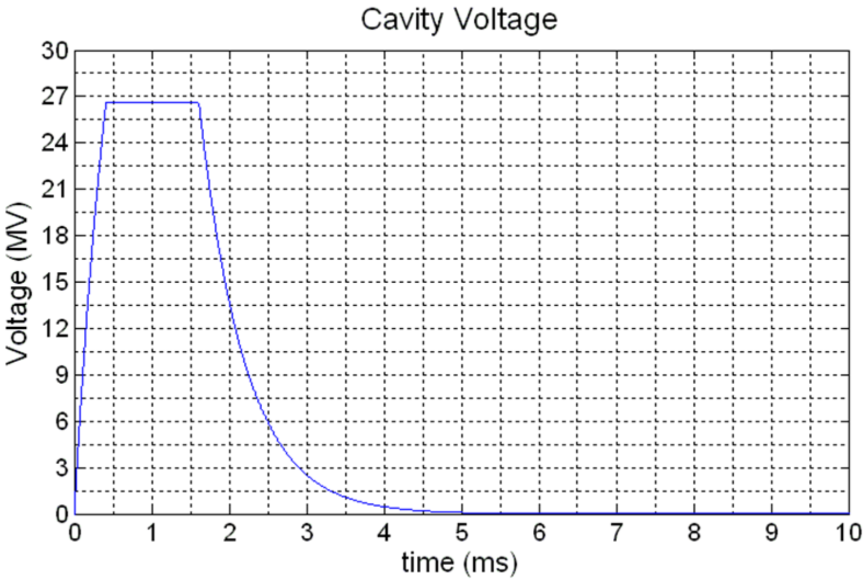
Ylims [Y1 Y2]:

Lorentz Detuning

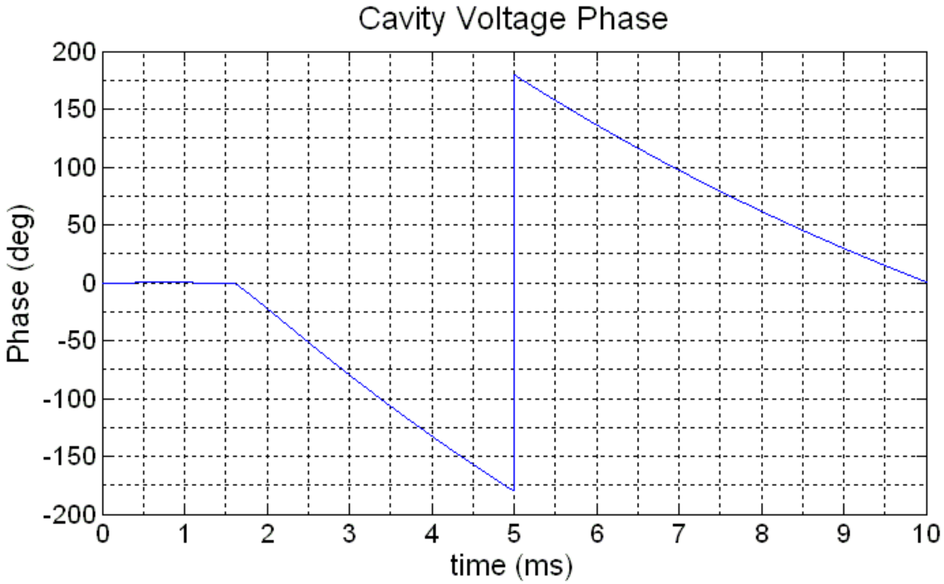
Results

- Cavity Phase Variation Without Feed-Forward
- Effects of Adaptive Feed-Forward
- Effects of Loaded Quality Factor Variation

Vcav Magnitude and Phase for Dual Cavity Case (K=-1 and -0.8)

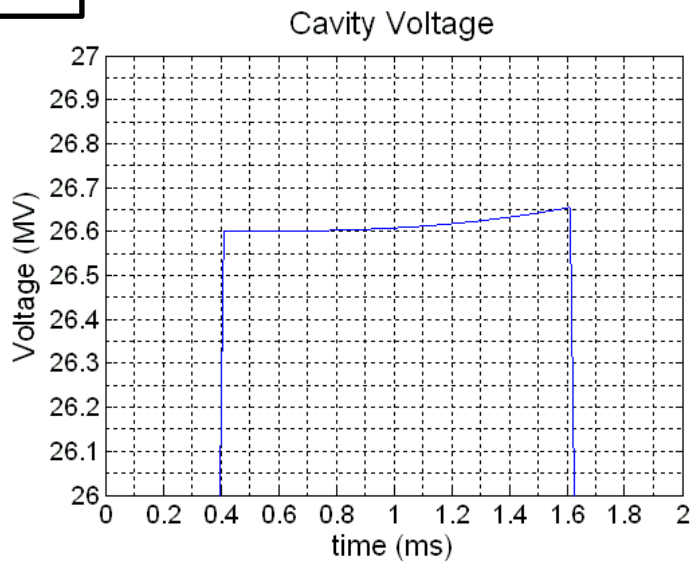


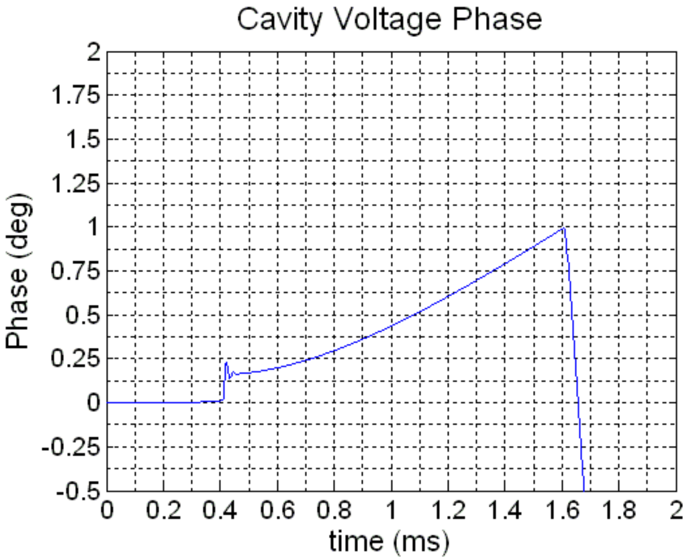
Voltage magnitude and phase of vector average



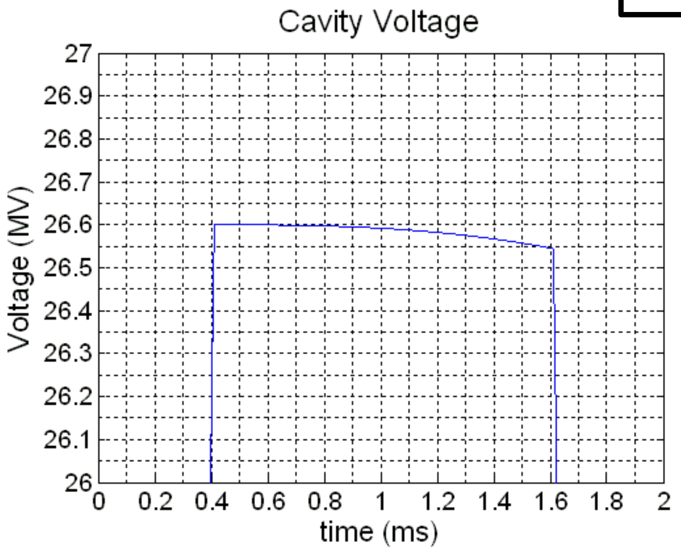
Vcav Magnitude and Phase for Dual Cavity Case (Without Feed-Forward)

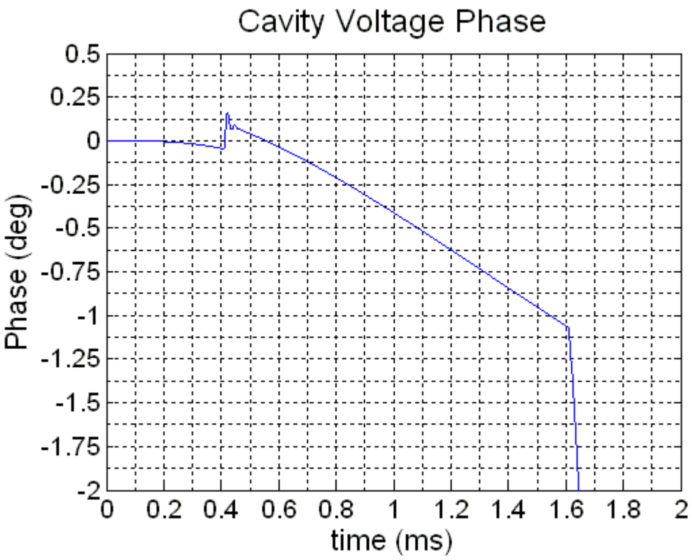
K=-0.8





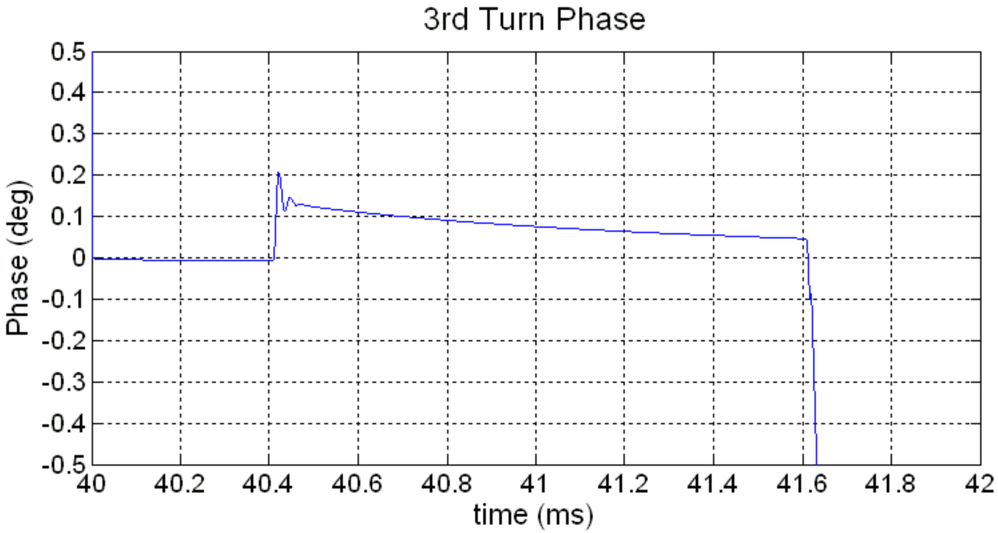
K=-1



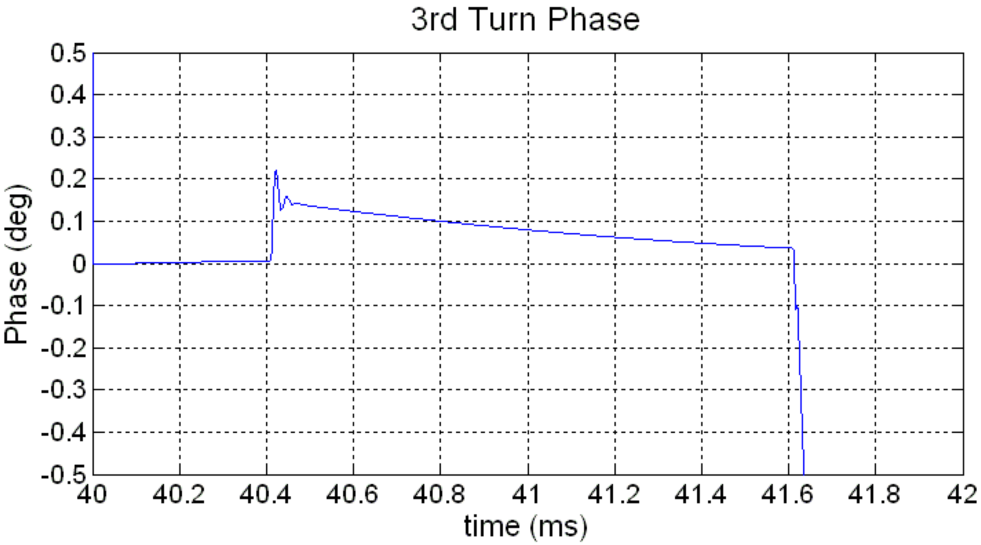


Vcav Magnitude and Phase for Dual Cavity Case (With Feed-Forward)

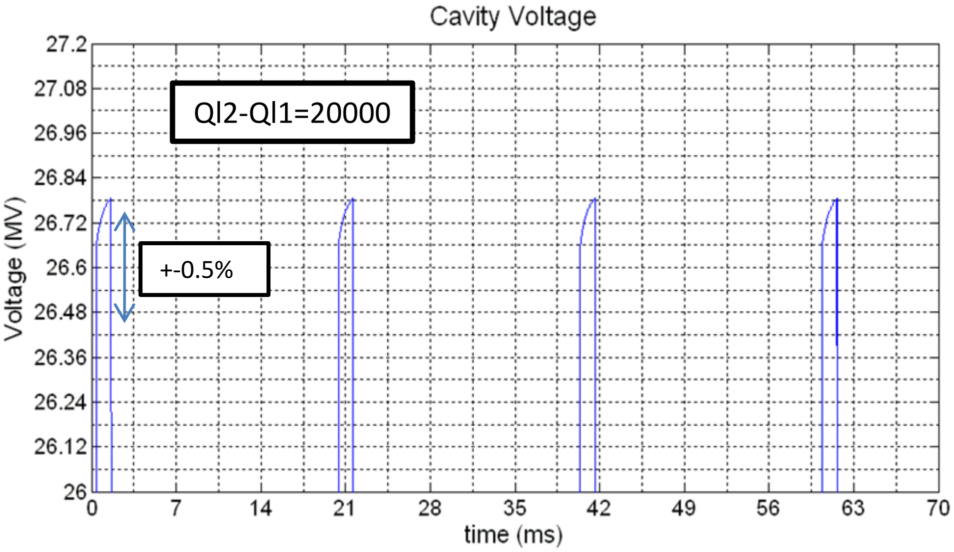
Cavity 1 (K=-1)



Cavity 2 (K=-0.8)

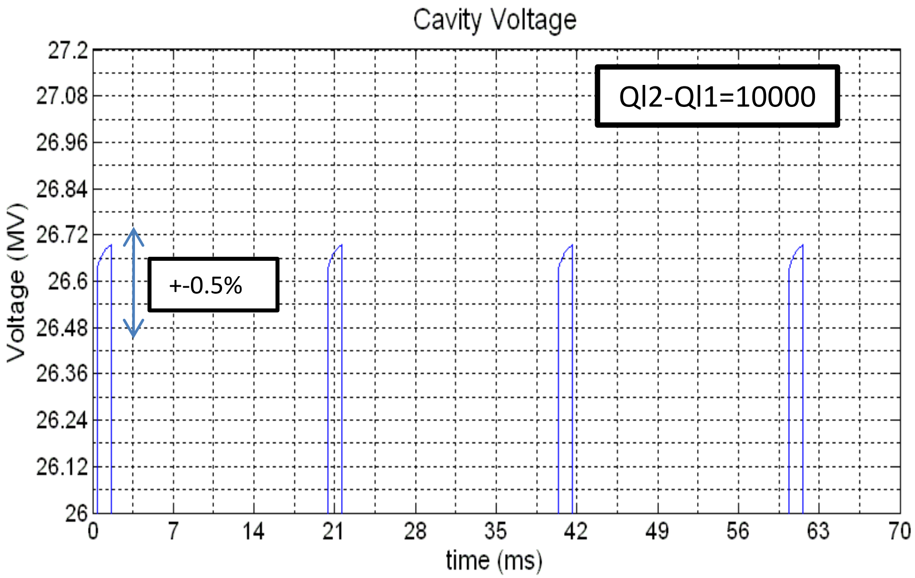


Loaded Quality Factor Fluctuation Effects on Cavity Voltage Magnitude



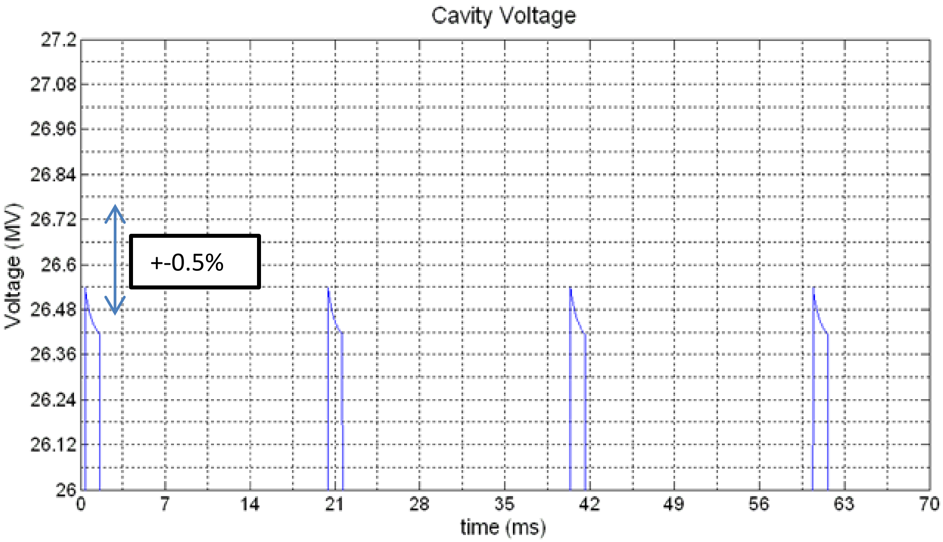
Ql2-Ql1=20000

+-0.5%

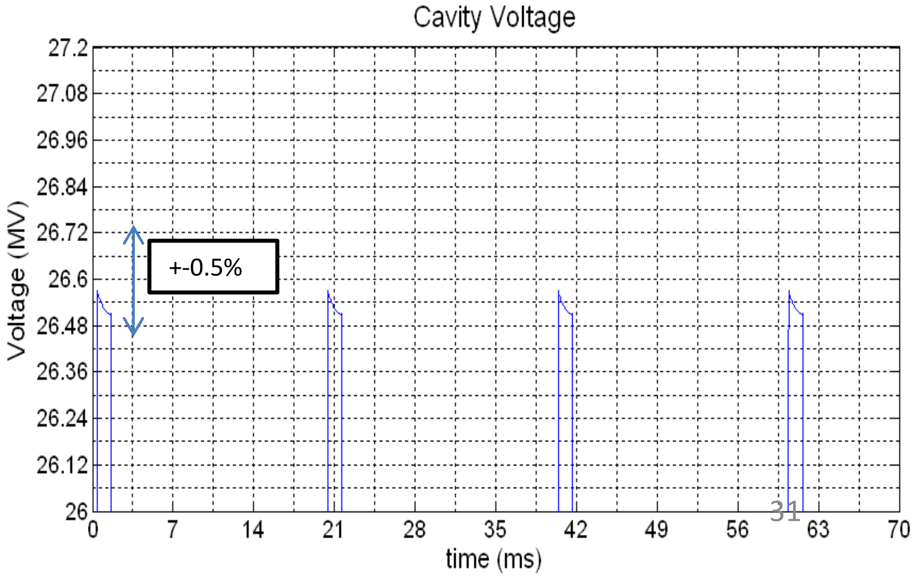


Ql2-Ql1=10000

+-0.5%



+-0.5%



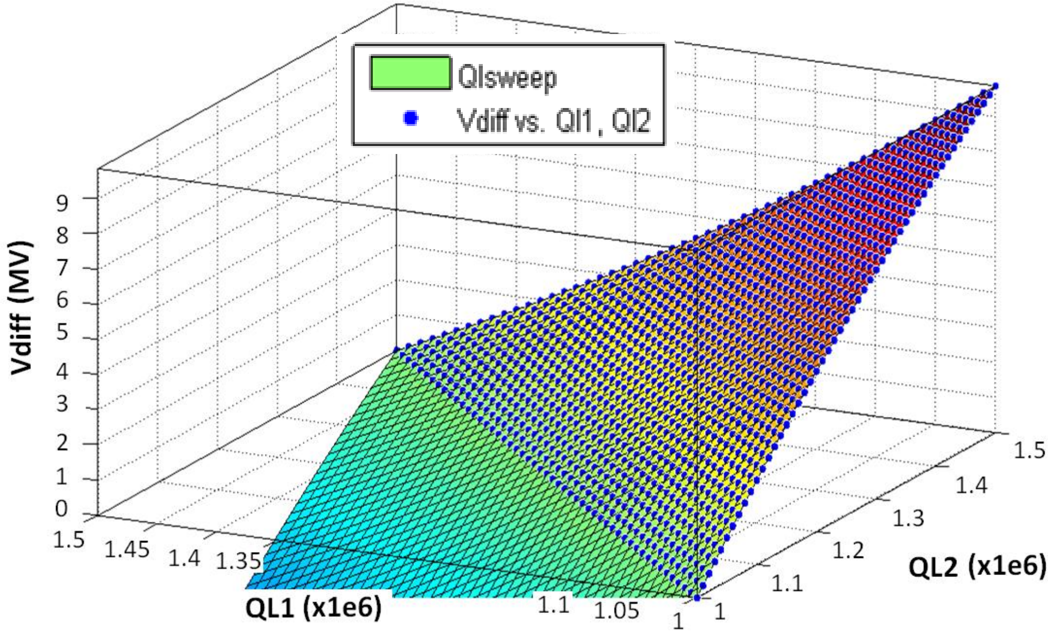
+-0.5%

Error Analysis

- Vector average is maintained within specifications with RF feedback loop, but individual cavities deviate depending on their parameters.
- Characterize deviation of cavity voltage with variations in loaded quality factor and lorentz detuning coefficients
- Curves fitted for difference in cavity voltage magnitude and phase between 2 cavities controlled by a single RF feedback loop, centered at nominal accelerating voltage magnitude and phase.

Effects of Varying Loaded Quality Factor on Cavity Voltage Magnitude

p00=1.107e+006
p10=65.53
p01=-68.21
p20=-2.827e-005
p11=2.417e-006
p02=2.801e-005
p30=4.567e-012
p21=1.135e-012
p12=-2.098e-012
p03=-4.177e-012



$$V_{Diff} = f(Q_{L1}, Q_{L2}) = f(x, y) = V_{c2} - V_{c1}$$

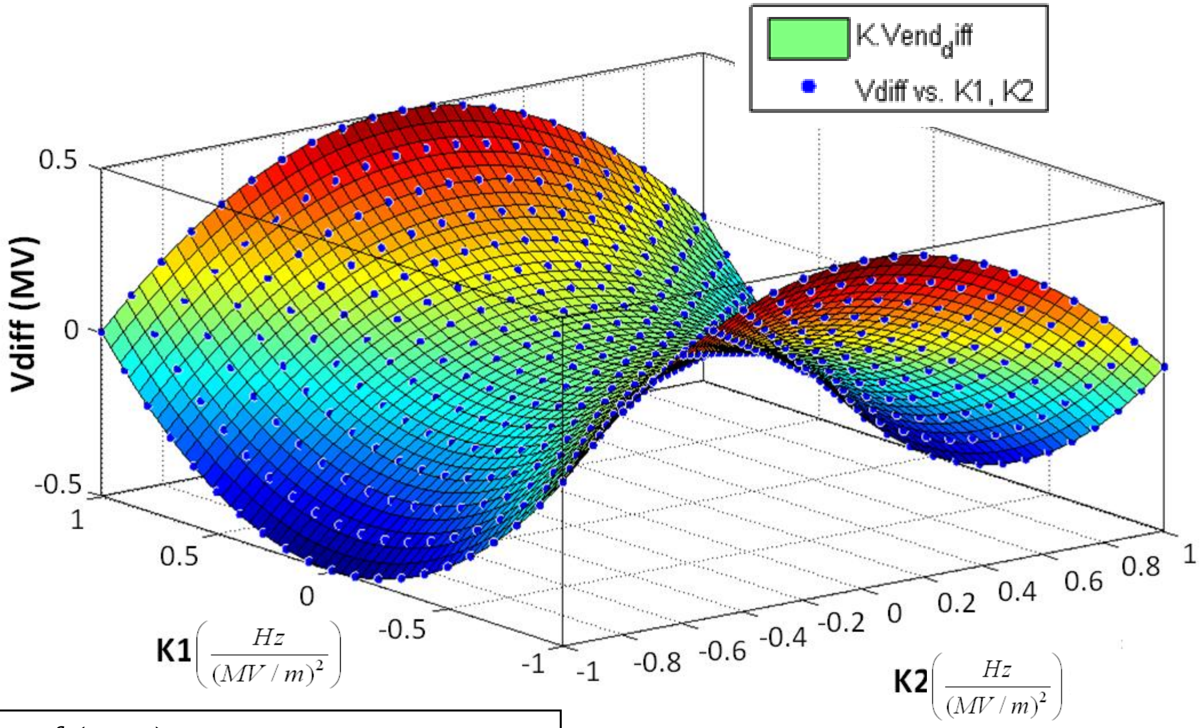
$$V_{c1} = V_{acc} + \frac{V_{Diff}}{2}$$

$$V_{c2} = V_{acc} - \frac{V_{Diff}}{2}$$

$$V_{Diff}(x,y) = p00 + p10\,x + p01\,y + p20\,x^2 + p11\,xy + p02\,y^2 + p30\,x^3 + p21\,x^2 y + p12\,xy^2 + p03\,y^3$$

Effects of Varying Lorentz Detuning Coefficient on Cavity Voltage Magnitude

p00=-1.774e-010
p10=-8.149e+013
p01=8.149e+013
p20=-5.016e+029
p11=-9.525e+013
p02=5.016e+029



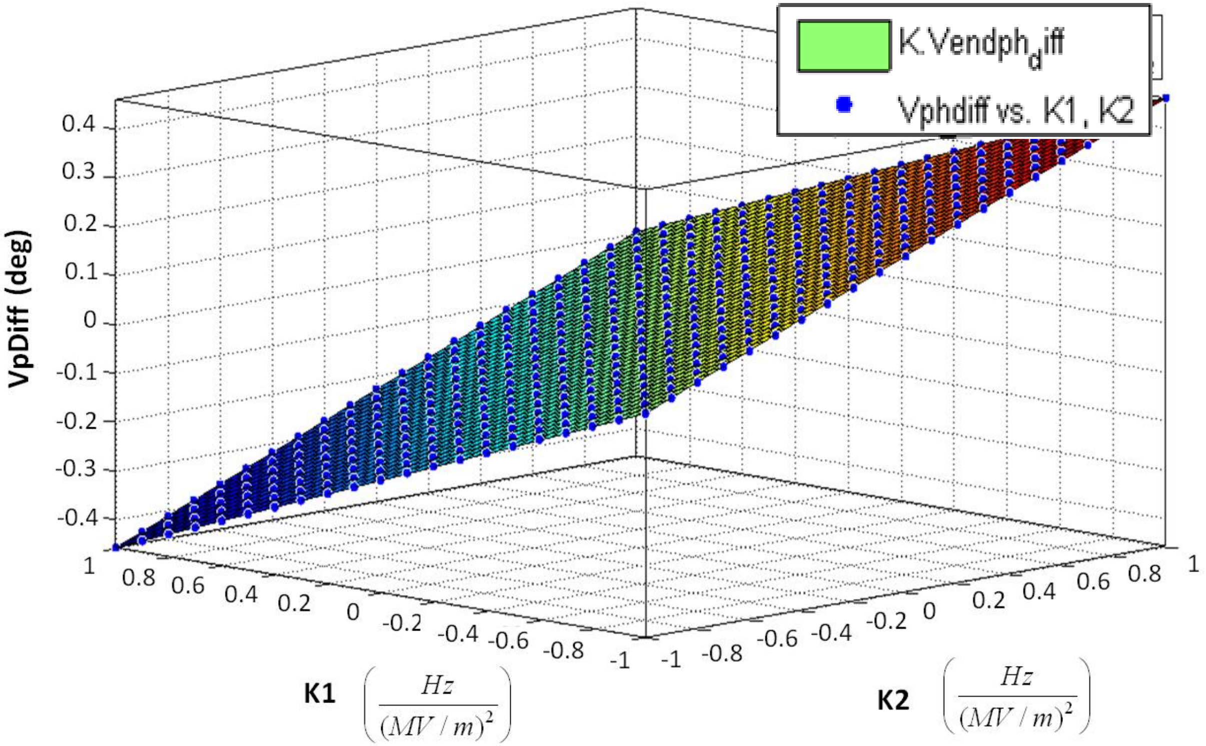
$$V_{Diff} = V_{c2} - V_{c1} = f(K_1, K_2) = f(x, y)$$

$$V_{c1} = V_{acc} + \frac{V_{Diff}}{2}$$

$$V_{c2} = V_{acc} - \frac{V_{Diff}}{2}$$

$$V_{Diff}(x,y) = p00 + p10\,x + p01\,y + p20\,x^2 + p11\,xy + p02\,y^2$$

Effects of Varying Lorentz Detuning Coefficient on Cavity Voltage Phase



p00=-9.52e-018
p10=2.291e+011
p01=-2.291e+011

$$V_{Diff} = V_{c2} - V_{c1} = f(K_1, K_2) = f(x, y)$$

$$V_{c1} = V_{acc} + \frac{V_{Diff}}{2}$$

$$V_{c2} = V_{acc} - \frac{V_{Diff}}{2}$$

$$V_{Diff}(x,y) = p00 + p10\,x + p01\,y + p20\,x^2 + p11\,xy + p02\,y^2$$

In Summary…

- In order to cater for the needs of project specifications in constant need of revision, a high flexibility simulation model developed
- Flexible graphical user interface allows for efficient handling of simulation data
- 1, 2 and 4 cavities can be observed from RF point of view under a wide set of circumstances
- Can estimate practical issues that can arise during development of a real LLRF system in terms of power, stability of accelerating field and technology necessary for operation

Next Step

- Include and quantify effect from finite transit time for low beta cavities.
- Characterize power amplifier nonlinearities
- Characterize the behavior of the piezo-electronic tuner within the control loop.