ISMD Ist, Paris 1970

ANGULAR CORRELATIONS IN THE REACTION $K^+p \rightarrow K^+p\, 2\pi^+\, 2\pi^-$ AT 4.97 GeV/c.

Brussels-CERN Collaboration

(presented by E. de Wolf)

In conclusion we believe that our phenomenological analysis has shown that the angular correlation effect results from an interplay of different phenomena such as peripheral resonance production, decay of resonances, interferences and symmetrization, which all add up to produce the observed effect. Therefore, only a better knowledge of the reaction mechanism will enable a detailed understanding of the GGLP effect.

I. LPS Analysis

(Van Hove 1969)

$$a + b \longrightarrow c_1 + c_2 + \cdots + c_n$$

dimensionality: 3n-5

however: transverse momenta small

masses small

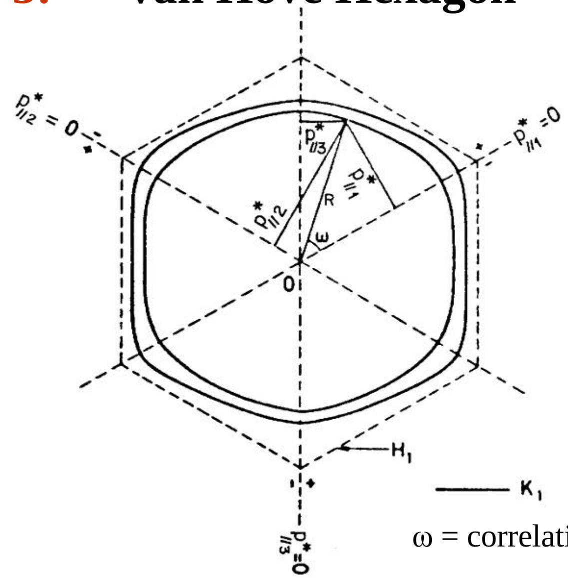
$\Rightarrow$ **restrict ourselves to longitudinal momenta, then**

$$\sum_{i=1}^{n} p^{*}_{\parallel i} = 0$$

$$\sum_{i=1}^{n} | p^{*}_{\parallel i} | = \sqrt{s}$$

define a regular polyhydron H_{n-2}

n=3: Van Hove Hexagon

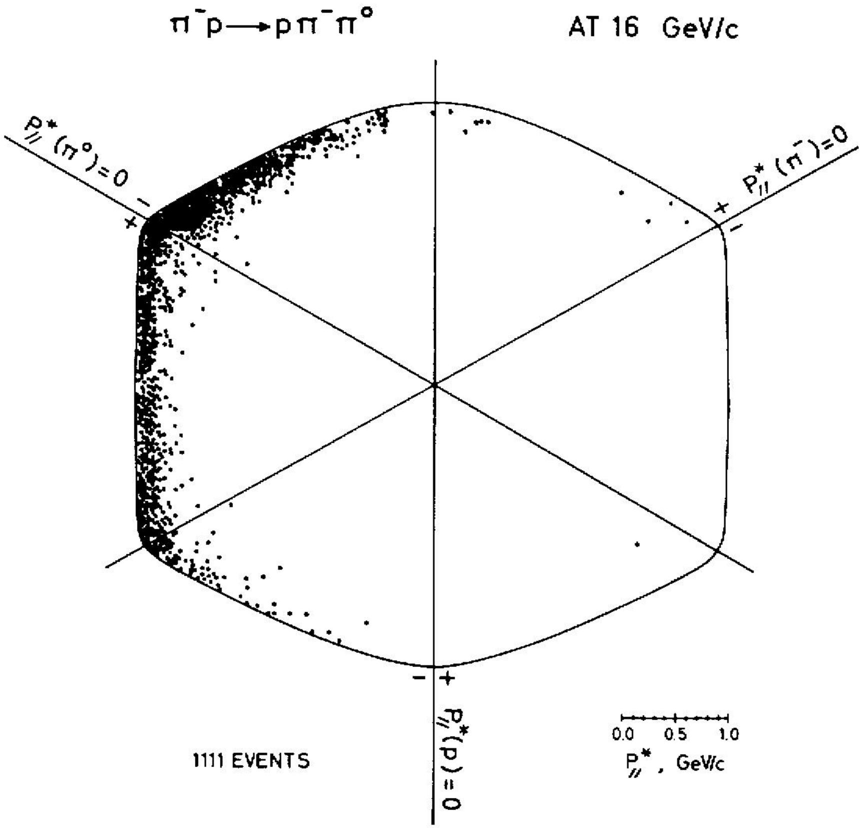


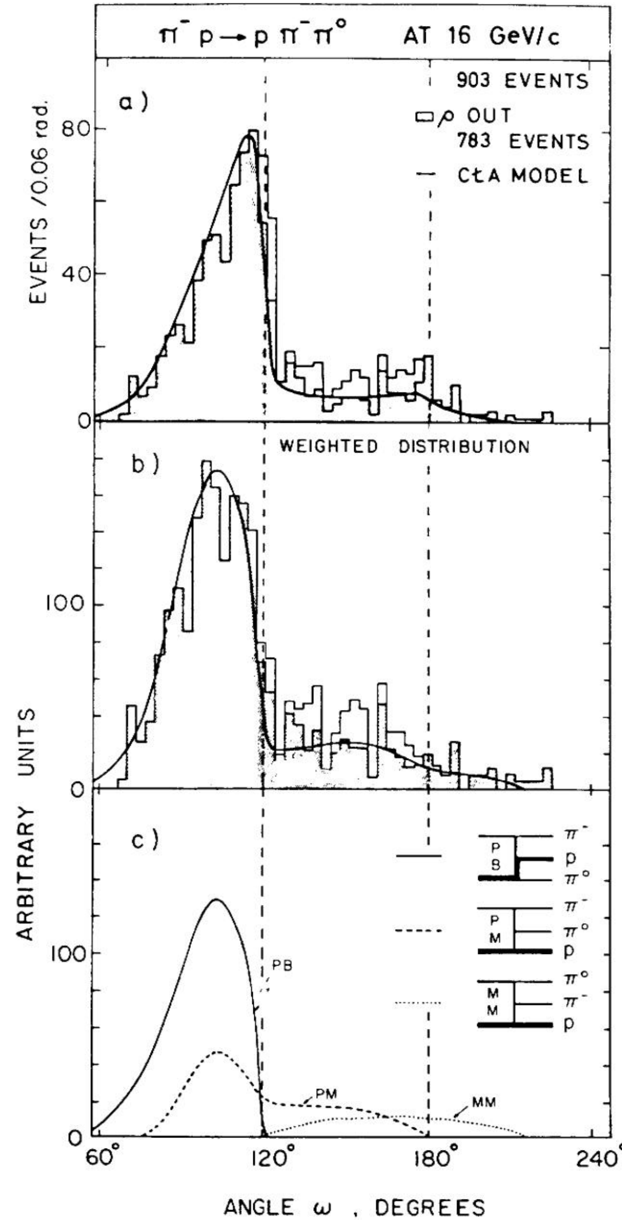
ω = correlation angle

n=4:

by R. Sosnowski

ABBCHLV Coll., 1970





conclude: $p\pi^0$ **most strongly correlated**

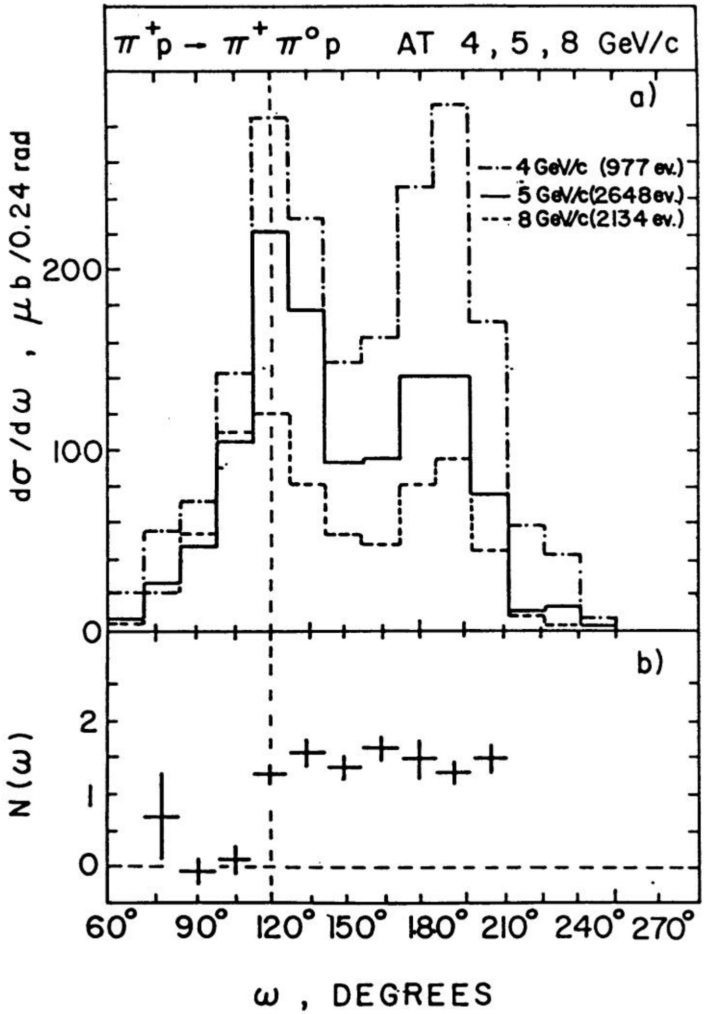
$\pi^-\pi^0$ **correlated, even without ρ**

$p\pi^-$ **much less**

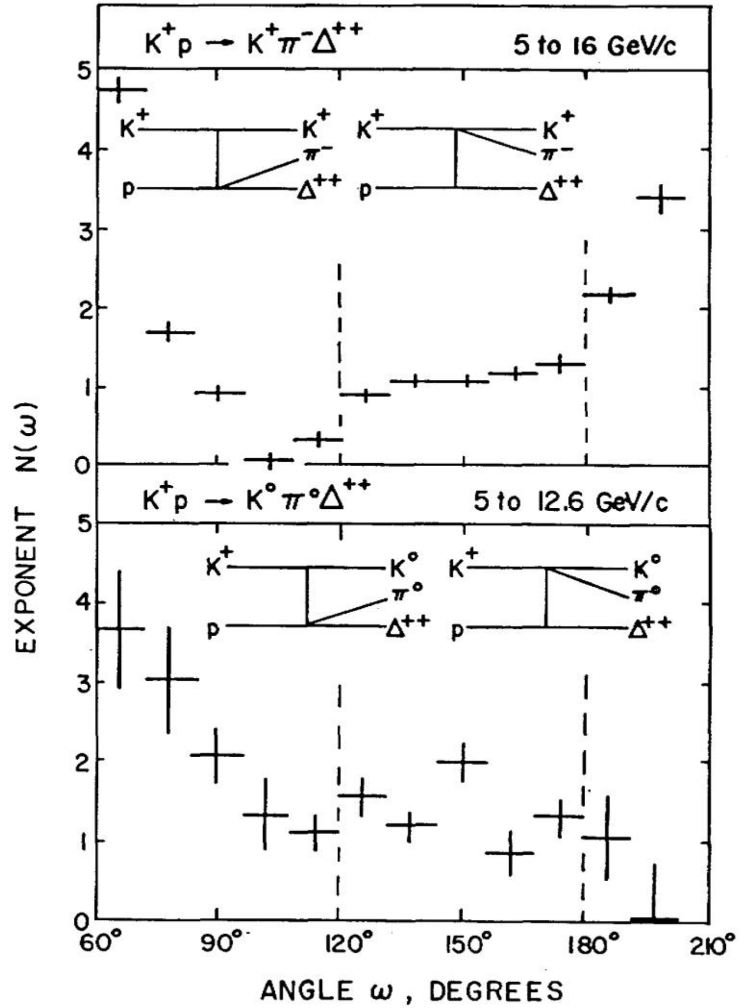
but why?

$\sigma \propto p_{Lab}^{-N}$

BDNPT Coll. , 1971



De Wolf, Verbeure, Czyzewski, 1971

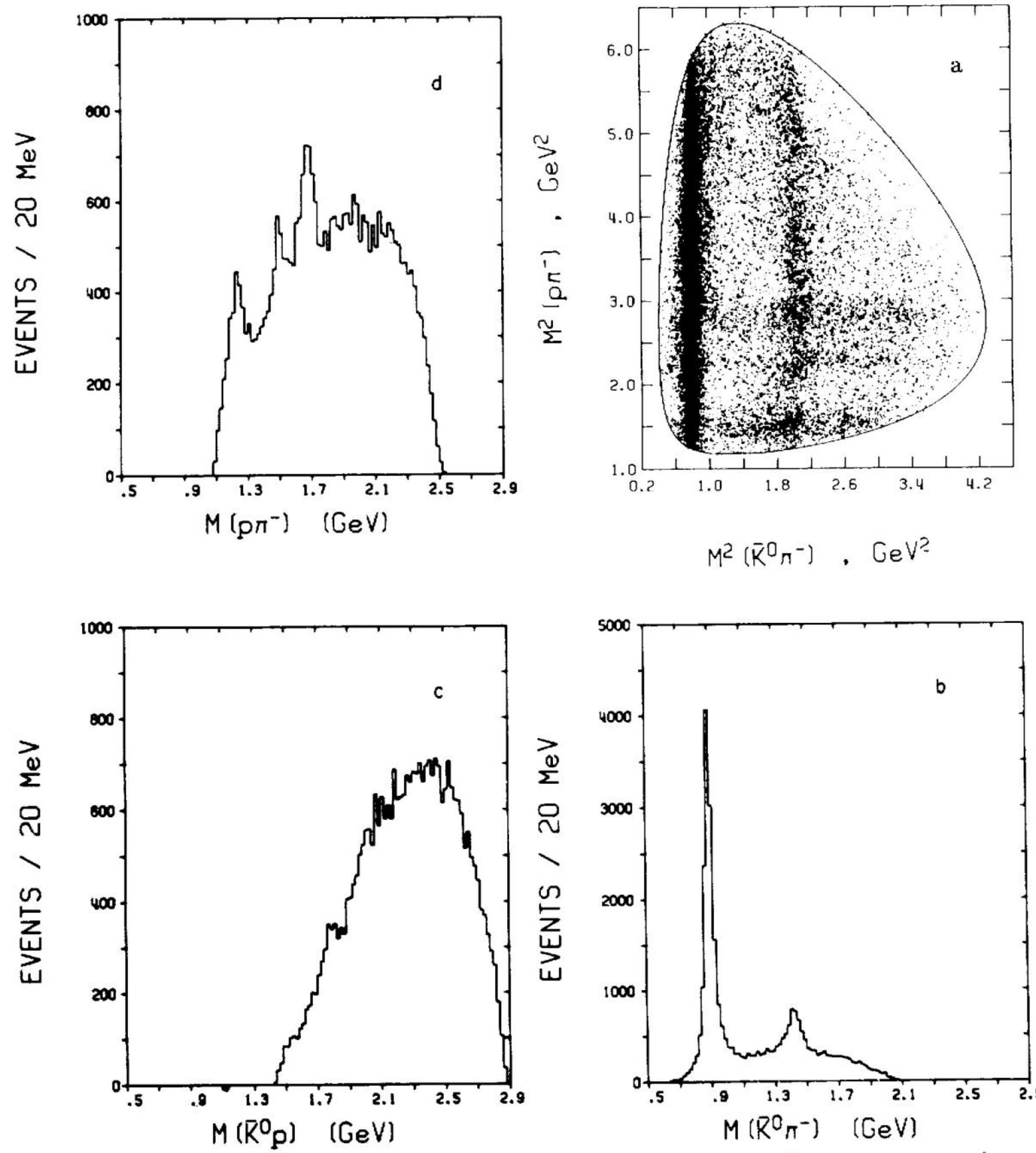


conclude: diffraction dissociation strong, but where is the Δ^{+} resonance?

$K^- p \rightarrow \bar{K}^0 \pi^- p$ 30 000 evts

ACNO Coll.

Engelen et al., 1978



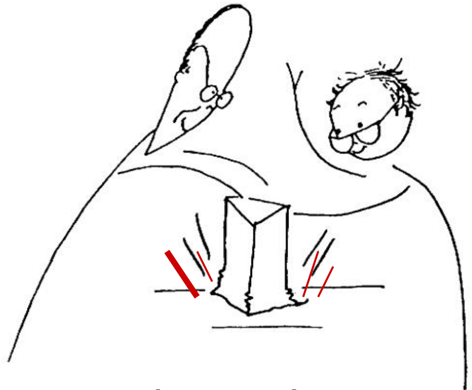
observe:

bands due to various resonances,

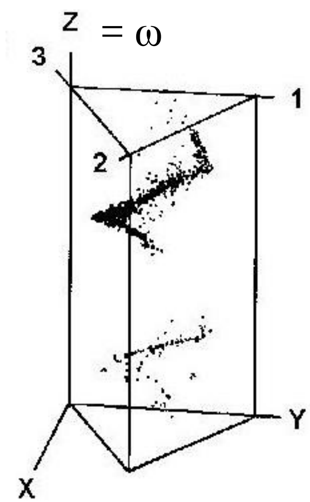
but on large background from other subsystem

<u>Further steps</u>:

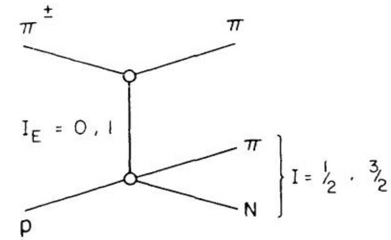
1. Prism Plot *(Brau et al., 1972)*

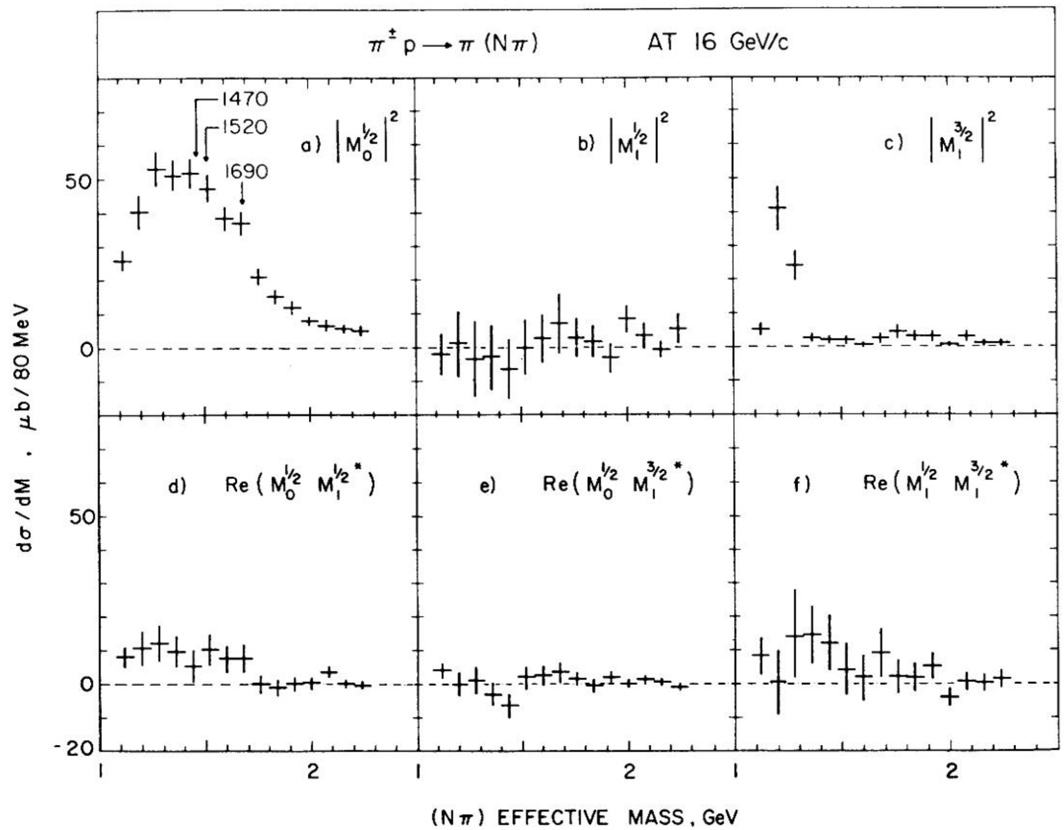


by Suzy Smile



2. Isospin Analysis *(Van Hove et al., 1952)*





still overlap in full phase space,
but allows to study interferences
with the help of quantum numbers!

3. Partial Wave Analysis

use decay angular distribution

4. Analytical Multichannel Analysis

(Van Hove, 1973; Engelen et al., 1980)

use amplitudes in all variables simultaneously for all possible channels

⇒ **Interferences play an important role**

5. Extension to 4- and even 5-body Final States
(Kittel, Ratti, Van Hove, 1971; De Wolf et al., 1972)

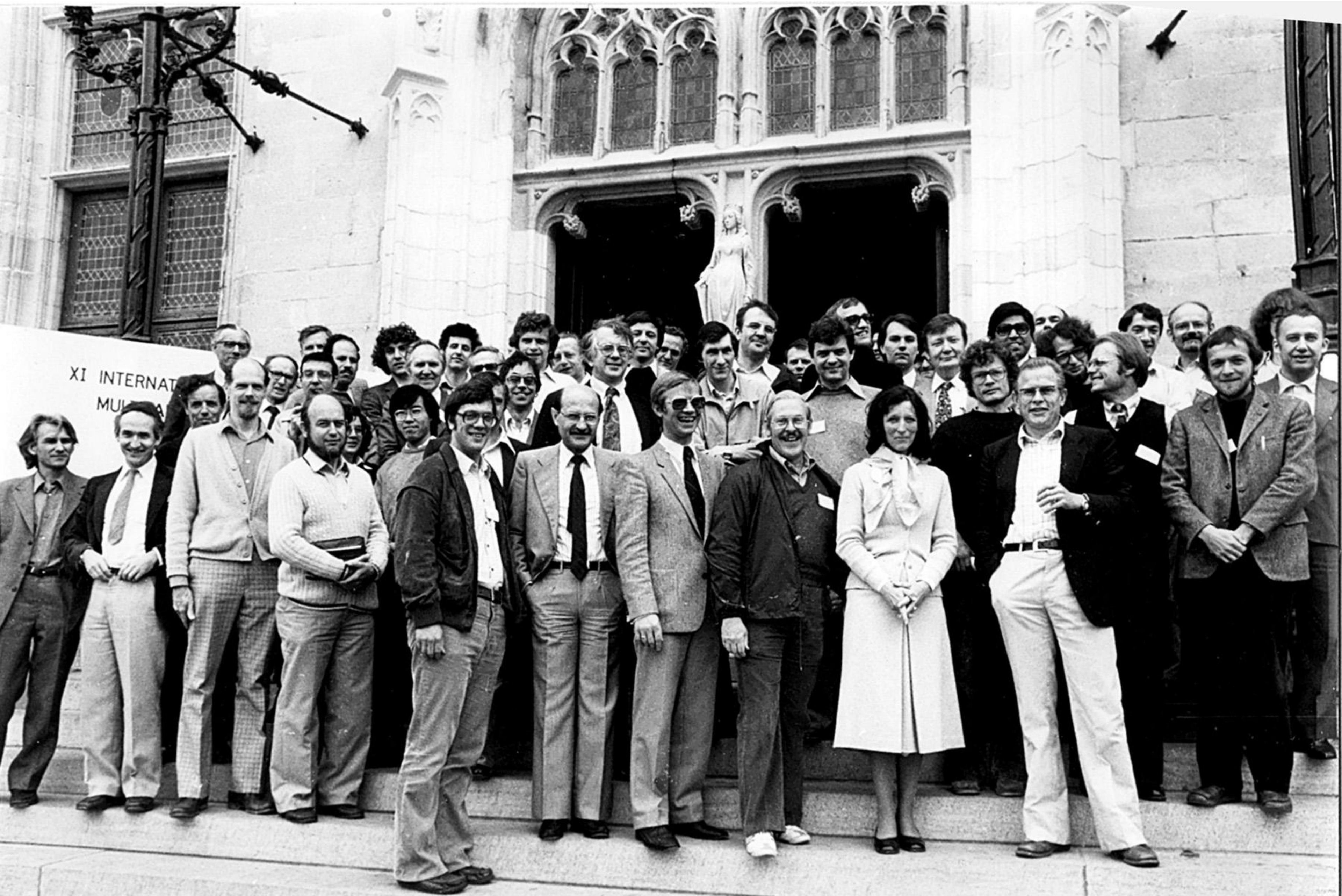
CONCLUDE: Correlation of final state particles due to diffraction dissociation and (interfering) resonances

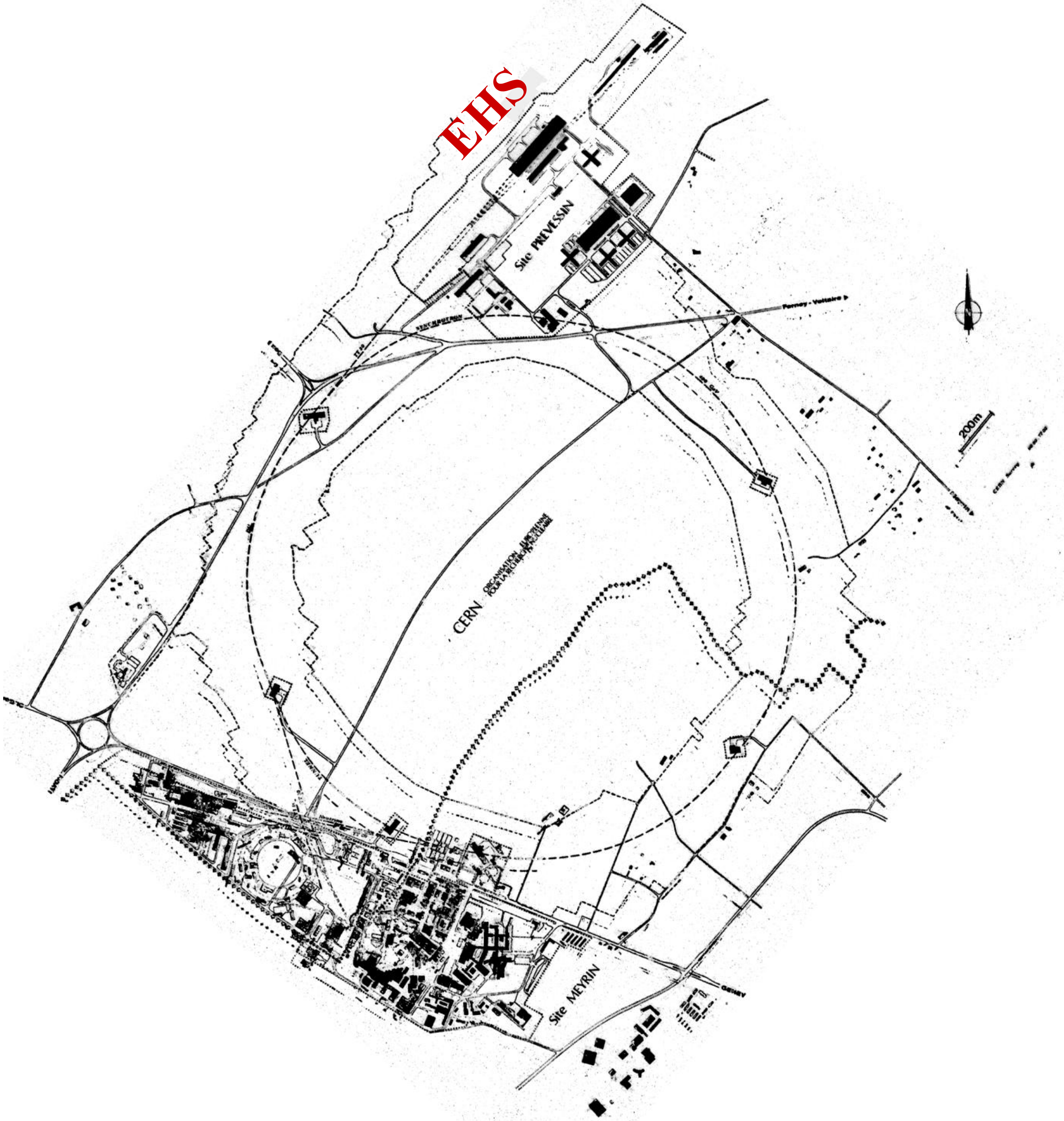
LESSONS:

Analysis ITERATIVE

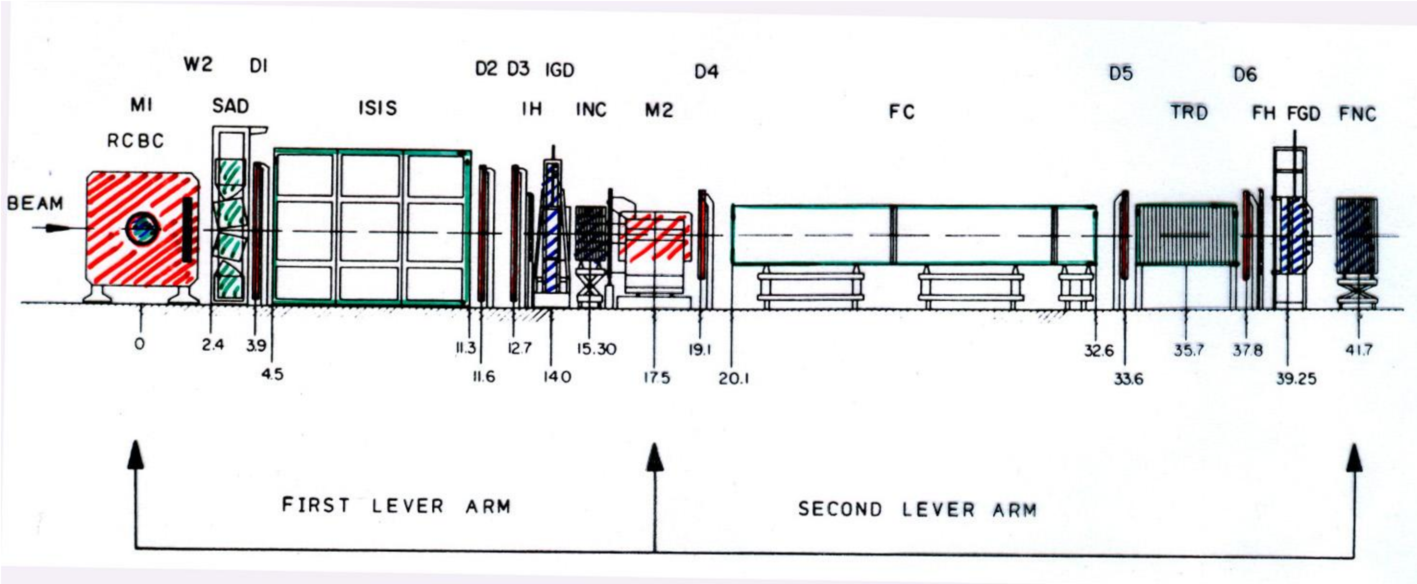
Detector HERMETIC

XIth ISMD, Brugge 1980




EHS
Site PREVESSIN
Ferney-Voltaire
CERN
200m
Site MEYRIN

NA22:



Aachen, Antwerp/Brussels, Berlin (Zeuthen), Helsinki, Krakow, Moscow, Nijmegen, Rio de Janeiro, Serpukhov, Tbilisi, Warsaw, Yerevan

BEAM: $\pi^+/K^+/p$ at 250 GeV/c

TARGET: H_2 200 k events
Al/Au 10 k events

PARTICLES PRODUCED

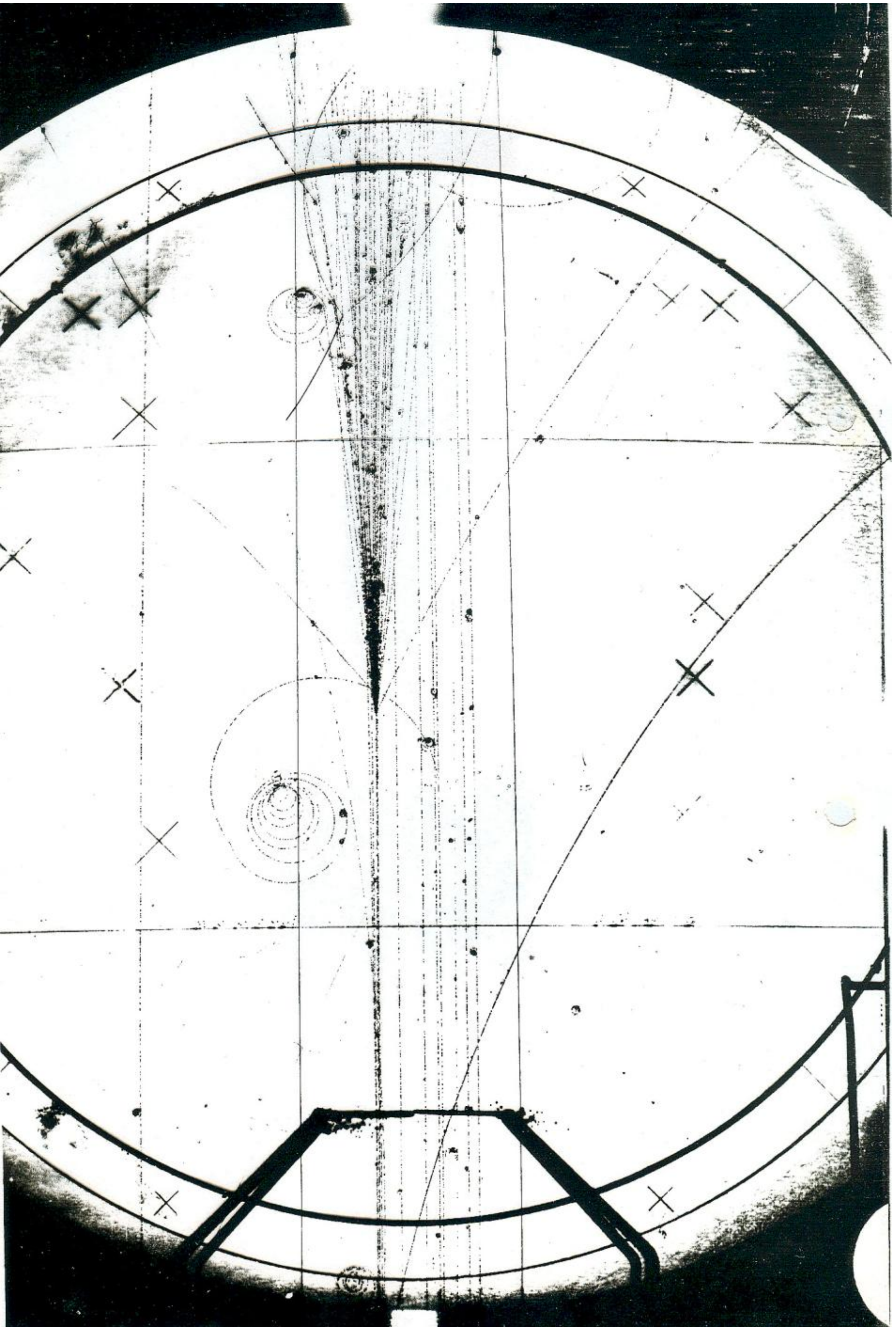
$\pi^+,\ \pi^-$ — momentum spectrometer
$\pi^0 \longrightarrow \gamma\gamma$ — electro-magnetic calorimeter
$K^+, K^-,\ p,\ \bar{p}$ — particle identification

$K^0_S \longrightarrow \pi^+\pi^-$, $\pi^0\pi^0$
$\Lambda^0 \longrightarrow p\pi^-$, $n\pi^0$
— reconstruct from decay

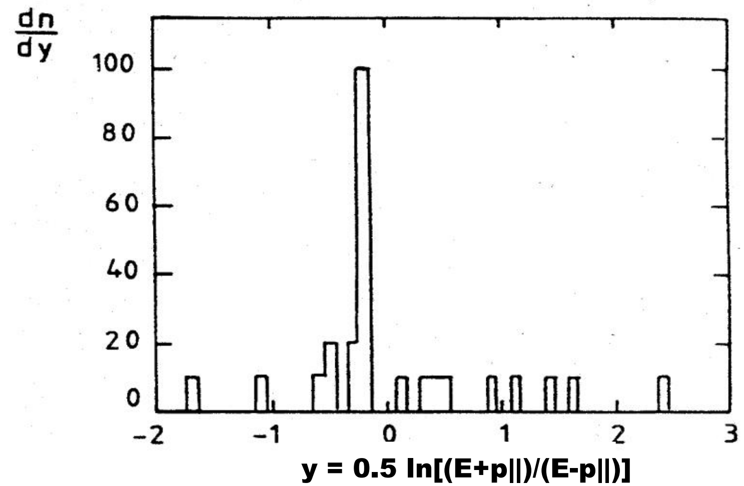
$\Sigma^+ \longrightarrow p\pi^0$, $n\pi^+$
$\Sigma^- \longrightarrow n\pi^-$
$\Sigma^0 \longrightarrow \Lambda\gamma$
— reconstruct from decay

n, K^0_L — hadron calorimeter

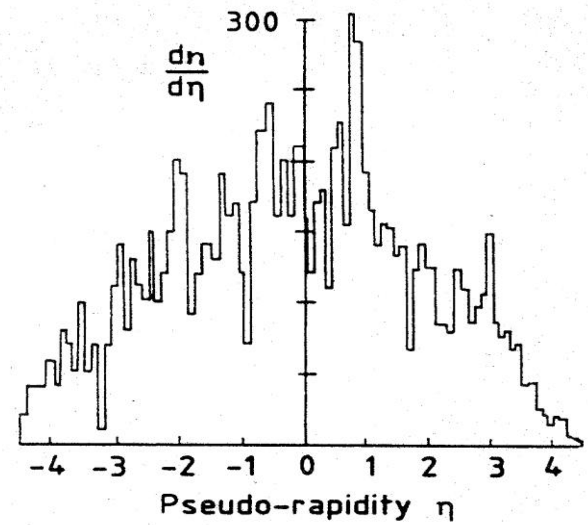
II. Intermittency



NA22 event 1987



JACEE event 1983



Question: statistical or dynamical ?

FORMALISM

INCLUSIVE q-PARTICLE DISTRIBUTION FUNCTIONS:

$$\rho_q(p_1,\ldots,p_q) \equiv \frac{1}{\sigma_{\rm tot}} \frac{\mathrm{d}\sigma(p_1,\ldots,p_q)}{\mathrm{d}p_1\ldots\mathrm{d}p_q}$$

Integration over $\Omega \to$ factorial moments:

$$\int_\Omega \rho_1(p)\mathrm{d}p = \langle n\rangle$$

$$\int_\Omega\int_\Omega \rho_2(p_1,p_2)\mathrm{d}p_1\mathrm{d}p_2 = \langle n(n-1)\rangle$$

$$\int_\Omega \mathrm{d}p_1\ldots\int_\Omega \mathrm{d}p_q\rho_q(p_1,\ldots,p_q) = \langle n(n-1)\ldots(n-q+1)\rangle$$

n = multiplicity

(the angular brackets imply the average over the event ensemble)

CELL-AVERAGED FACTORIAL MOMENTS

Białas + Peschanski, 1986 and 1988

1. Divide phase space volume into M non-overlapping cells Ω_m (e.g. rapidity intervals of size $\delta y = \Delta y/M$)
2. Integrate over phase space cells Ω_m and average

def. : normalized cell-averaged factorial moments:

$$F_q(\delta p) \equiv \frac{1}{M}\sum_{m=1}^{M}\frac{\int_{\Omega_m}\rho_q(p_1,\ldots,p_q)\prod_{i=1}^{q}\mathrm{d}p_i}{\left(\int_{\Omega_m}\rho(p)\mathrm{d}p\right)^q}$$

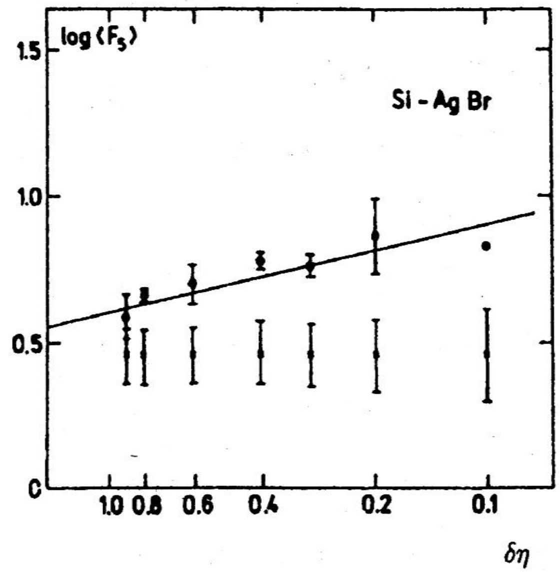
$$\equiv \frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_m(n_m-1)\ldots(n_m-q+1)\rangle}{\langle n_m\rangle^q}$$

n_m = number of particles in cell Ω_m

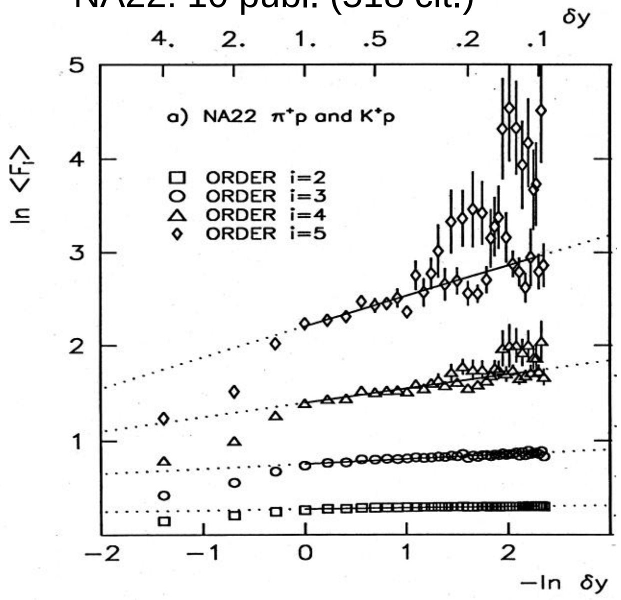
essential properties:

1. Poisson-noise suppression
   (not applicable to ordinary moments $\frac{\langle n^q\rangle}{\langle n\rangle^q}$)
2. $F_q = 1$ for Poisson
3. High-order moments act as filter
   $\Rightarrow$ resolve the high-n_m tail of the multiplicity distribution (particularly sensitive to large density fluctuations)

JACEE event



NA22: 10 publ. (518 cit.)



⇨ **approx. power law**

characterized by:

POWER-LAW SCALING

$$F_q(\delta p) \propto (\delta p)^{-\phi_q}, \ (\delta p \to 0)$$

- scaling law since the ratio at resolutions L and ℓ

$$F_q(\ell)/F_q(L) = (L/\ell)^{\phi_q}$$

only depends on the ratio L/ℓ, but not on L and ℓ, themselves

- The powers ϕ_q (slopes in a double-log plot) are related to the anomalous dimensions

$$\phi_q = (q-1)d_q, \qquad d_q = D - D_q$$

$d_q > 0 \Rightarrow$ fractal structure

Expected from branching or phase transition

multi-fractal (d_q depends on q)

mono-fractal (all d_q the same)

Dremin 1987, 1988

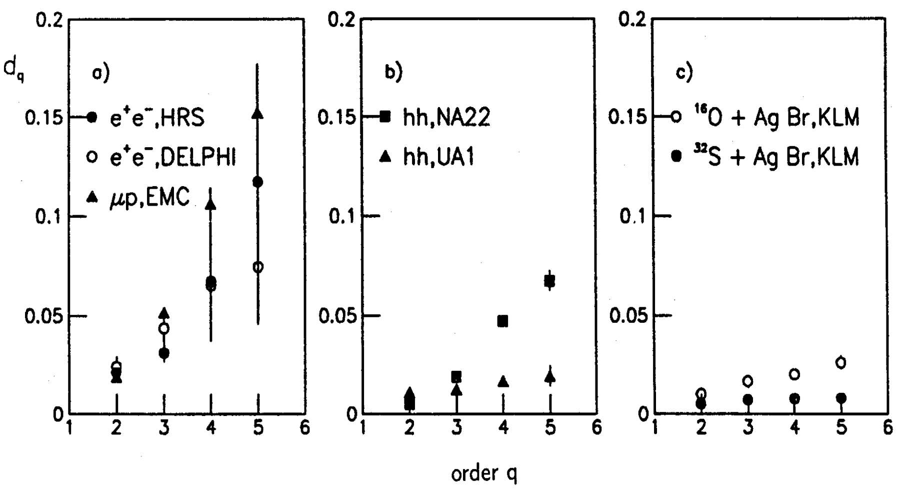
Ochs + Wošiek 1988, 1989

Lipa + Buschbeck 1989

Hwa 1990

Chekanov + Kuvshinov 1994, 1996

Białas 1991



CUMULANT CORRELATION FUNCTIONS

$\rho_q\,(y_1,\ldots,y_q)$ contain:

1. interparticle correlations
2. "trivial" contributions from lower order

$\Rightarrow$ cluster expansion (Mueller 1971):

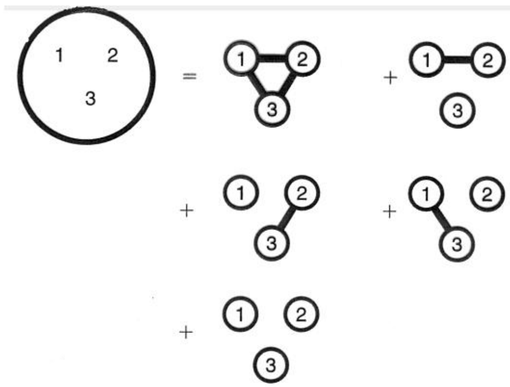
$$\rho_1(1)=C_1(1),$$
$$\rho_2(1,2)=C_1(1)C_1(2)+C_2(1,2),$$
$$\rho_3(1,2,3)=C_1(1)C_1(2)C_1(3)+ +C_1(1)C_2(2,3)+C_1(2)C_2(1,3)+C_1(3)C_2(1,2)+ +C_3(1,2,3);$$
$$\rho_q(1,\ldots,q)=\sum_{\{k_i\}_q}\sum_{\text{perm.}}\underbrace{[C_1()\cdots C_1()]}_{k_1\text{ factors}}\underbrace{[C_2(,)\cdots C_2(,)]}_{k_2\text{ factors}}\cdots \cdots\underbrace{[C_q(,\ldots,)\cdots C_q(,\ldots,)]}_{k_q\text{ factors}}.$$

$k_i=0$ or pos. integer, $\qquad \sum_{i=1}^{n} ik_i=q.$

$\dfrac{q!}{[(q!)^{k_1}(2!)^{k_2}\cdots(q!)^{k_q}]\,k_1!k_2!\cdots k_q!}$ terms in any factor product

$C_q(y_1,\ldots,y_q)$ = (factorial) cumulant correlation functions vanish whenever one of their arguments becomes statistically independent of the others.

def.: "genuine" correlations of order q if $C_q\neq 0$



from inversion:

$$C_2(1,2) = \rho_2(1,2)-\rho_1(1)\rho_1(2)\ ,$$
$$C_3(1,2,3) = \rho_3(1,2,3)-\sum_{(3)}\rho_1(1)\rho_2(2,3)+2\rho_1(1)\rho_1(2)\rho_1(3)\ ,$$
$$C_4(1,2,3,4) = \rho_4(1,2,3,4)-\sum_{(4)}\rho_1(1)\rho_3(2,3,4)-\sum_{(3)}\rho_2(1,2)\rho_2(3,4) +2\sum_{(6)}\rho_1(1)\rho_1(2)\rho_2(3,4)-6\rho_1(1)\rho_1(2)\rho_1(3)\rho_1(4).$$

def.: normalized inclusive densities and correlations

$$R_q(y_1,\ldots,y_q) = \rho_q(y_1,\ldots,y_q)/\rho_1(y_1)\ldots\rho_1(y_q),$$
$$K_q(y_1,\ldots,y_q) = C_q(y_1,\ldots,y_q)/\rho_1(y_1)\ldots\rho_1(y_q).$$

Genuine Higher Order Correlations

A.H. Mueller 1971

Carruthers + Sarcevic 1989; De Wolf 1990

def.: normalized factorial cumulant moments:

$$K_q(\delta p) \equiv \frac{1}{M} \sum_{m=1}^{M} \frac{\int_{\Omega_m} C_q(p_1,\ldots,p_q) \prod_{i=1}^{q} \mathrm{d}p_i}{\left(\int_{\Omega_m} \rho_1(p_1)\mathrm{d}p\right)^q}$$

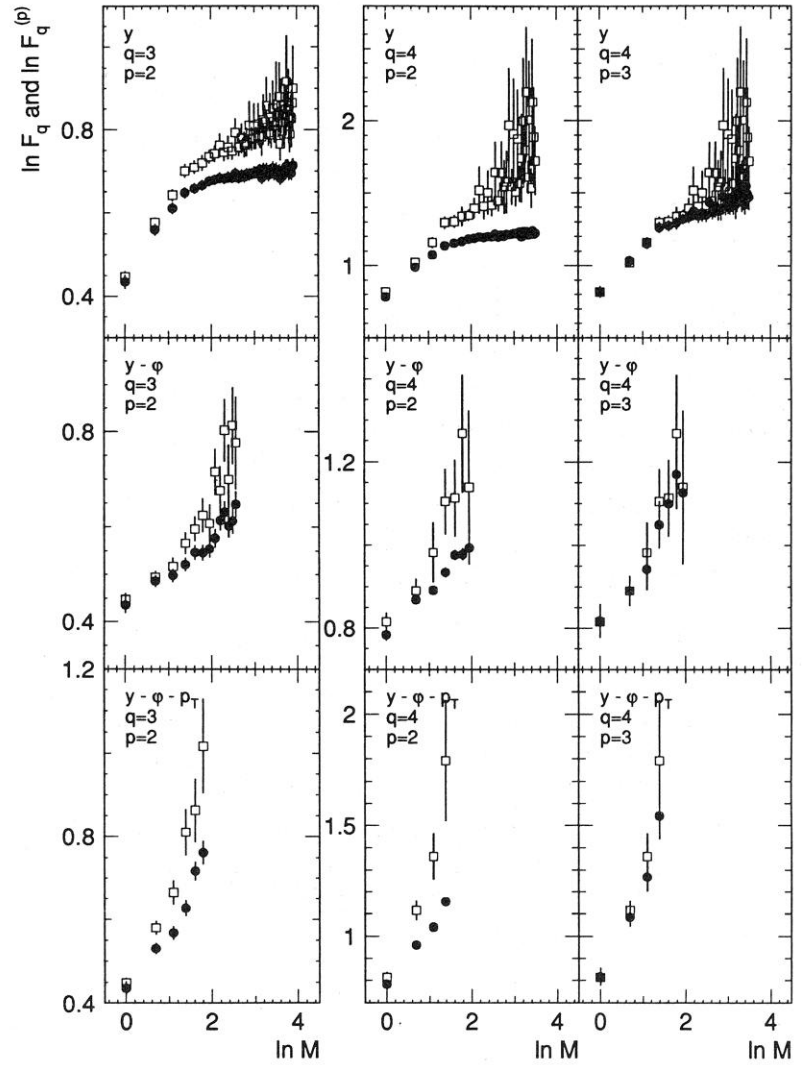
show the genuine correlations

related to the factorial moments

$$\begin{aligned} F_2 &= 1 + K_2 \\ F_3 &= 1 + 3K_2 + K_3 \\ F_4 &= 1 + 6K_2 + 3\overline{K_2^2} + 4K_3 + K_4 \end{aligned}$$

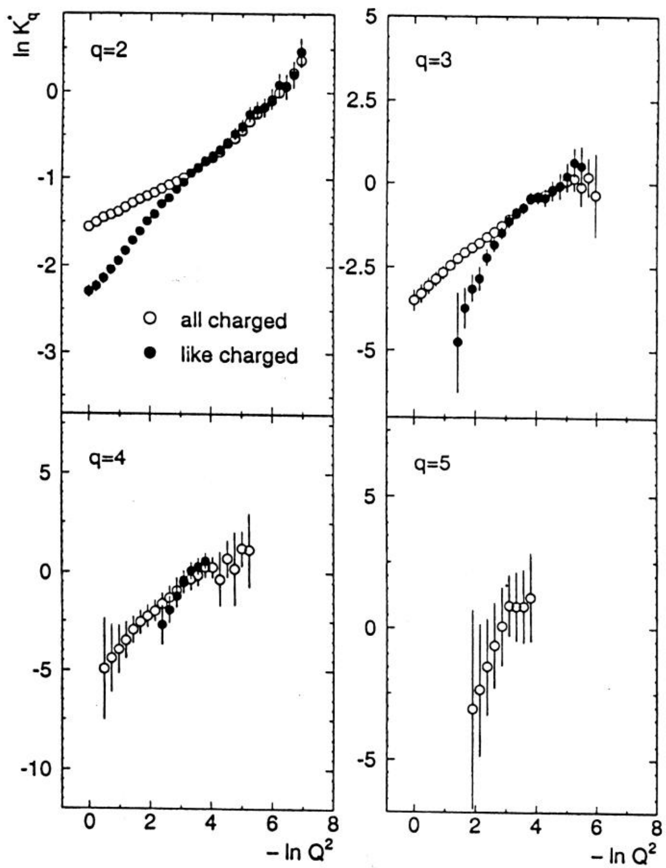
$$(\overline{AB} \equiv \sum_m A_m B_m / M)$$

NA22, 1993

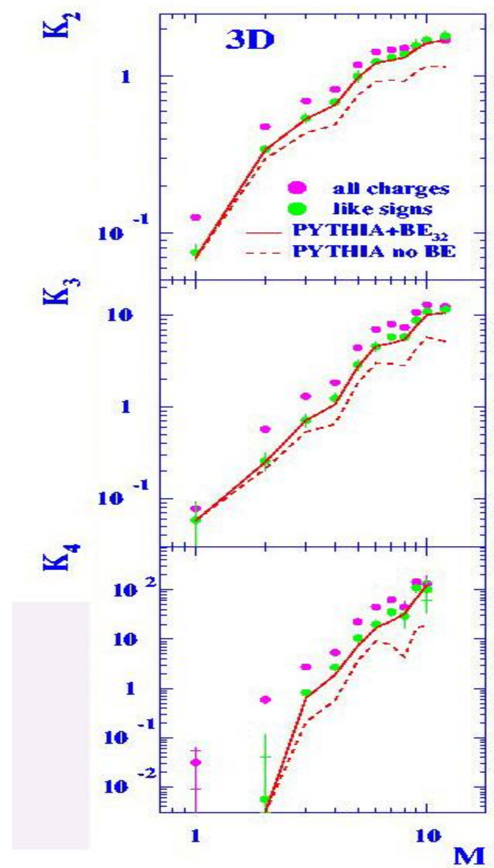


genuine higher order correlations exist

NA22, 1993 with star integral
(Dremin 1988, Carruthers 1991)



De Wolf + Sarkisyan 2001;
OPAL 2001



⇨ **BE correlations (at small Q^2)** **but non-Gaussian!!!**
QCD branching (at large Q^2) **need LHC results**

Gustafson + Nilsson (1991), Ochs + Wošiek (1992,1993, 1995)
Dokshitzer + Dremin (1993), Brax, Meunier + Peschanski (1994)

Remarks:

1. Fiałkowski (1991,1994):

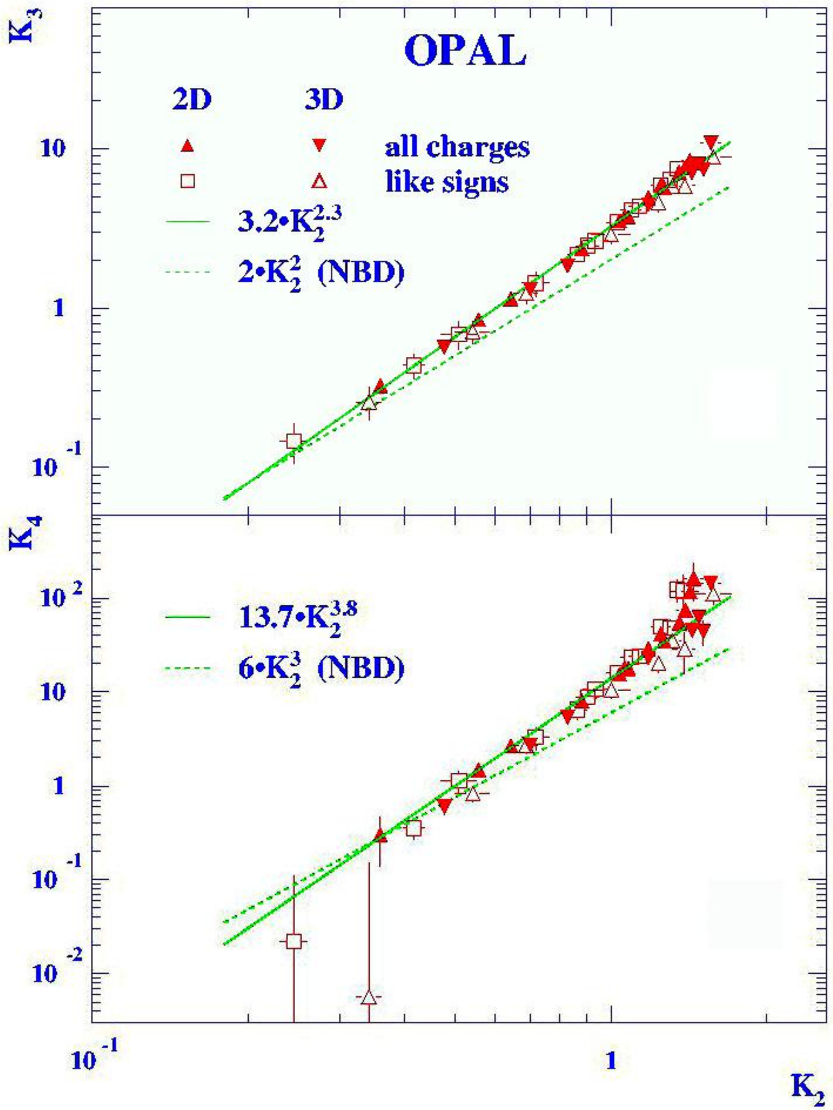
Universal slope for all types of reactions in K_2
(except perhaps e^+e^-)

(very remarkable, deserves further studies at LEP and LHC)

2. Ochs-Wošiek Plot (1988, 1989)

Universal relation between F_q and F_2

(very remarkable, deserves further studies at LEP and LHC)



XXIIIrd ISMD, Aspen 1993

Nijmegen Work-shop 1996

by Suzy Smile

XXVIIth ISMD, Frascati 1997

III. Inter WW BEC in

$$e^+e^- \to W^+W^- \to \bar{q}_1\bar{q}_2 q_3 q_4 \to \text{hadrons}$$

near threshold

The METHOD

(S. Chekanov, E. De Wolf, W.Kittel (1999))

$$\rho_1(p_1) = \frac{1}{N_{ev}} \frac{dN_1(p_1)}{dp_1} \Rightarrow \int \rho_1(p_1) dp_1 = \langle n \rangle$$

$$\rho_2(p_1,p_2) = \frac{1}{N_{ev}} \frac{dN_2(p_1,p_2)}{dp_1 dp_2} \Rightarrow \int \rho_2(p_1,p_2) dp_1 dp_2 =$$

$$\langle n_1(n_2 - \delta_{12}) \rangle$$

If independent decay:

$$\rho_1^{WW}(1) = \rho_1^{W^+}(1) + \rho_1^{W^-}(1)$$

$$\rho_2^{WW}(1,2) = \rho_2^{W^+}(1,2) + \rho_2^{W^-}(1,2) + 2\rho_1^{W^+}(1)\rho_1^{W^-}(2)$$

$$\downarrow$$

$$2\rho_{mix}^{W^+W^-}$$

define:

1. Difference:

$$\Delta\rho(\pm,\pm) \equiv \rho_2^{WW}(\pm,\pm) - 2\rho_2^{W}(\pm,\pm) - 2\rho_{mix}^{W^+W^-}(\pm,\pm)$$

$$\Delta\rho(+,-) \equiv \quad (+,-) - \quad (+,-) - \quad (+,-)$$

where

$$\rho_2^{W}(\pm,\pm) \equiv \rho_2^{W^+}(\pm,\pm) = \rho_2^{W^-}(\pm,\pm)$$

$$\rho_2^{W}(+,-) \equiv \quad (+,-) = \quad (+,-)$$

$$\Rightarrow \text{look for deviation from } \Delta\rho = 0$$

to extract BE effect: $\delta\rho = \Delta\rho(\pm,\pm) - \Delta\rho(+,-)$

advantages:

- rigorous mathematical basis
- direct access to inter-W correlations (without the need for models)
- integral = shift in 2^{nd}-order moment
- easily generalized to higher orders

A distribution of final-state particles produced in four-jet WW decay in a phase-space domain Ω is fully determined by the generating functional

$$\mathcal{R}^{\mathrm{WW}}[u(p)] = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{\Omega} \rho^{\mathrm{WW}}(p_1, p_2, \ldots, p_n) u(p_1) \ldots u(p_n) \prod_{i=1}^{n} \mathrm{d}p_i, \qquad (1)$$

where $\rho^{\mathrm{WW}}(p_1, p_2, \ldots, p_n)$ is the n-particle inclusive distribution with p_i being the 4-momentum of ith particle. The inclusive densities can be recovered from the functional differentiation of (1)

$$\rho^{\mathrm{WW}}(p_1, p_2, \ldots, p_n) = \partial^n \mathcal{R}^{\mathrm{WW}}[u(p)]/\partial u(p_1) \ldots \partial u(p_n) \mid_{u=0} . \qquad (2)$$

Since high-order inclusive densities contain redundant information from lower-order densities, it is advantageous to consider the n-particle (factorial) cumulant correlation functions $C^{\mathrm{WW}}(p_1, p_2, \ldots, p_n)$ which are obtained from the generating functional

$$\mathcal{G}^{\mathrm{WW}}[u(p)] = \ln \mathcal{R}^{\mathrm{WW}}[u(p)], \qquad (3)$$

so that

$$C^{\mathrm{WW}}(p_1, p_2, \ldots, p_n) = \partial^n \mathcal{G}^{\mathrm{WW}}[u(p)]/\partial u(p_1) \ldots \partial u(p_n) \mid_{u=0} . \qquad (4)$$

Analogously, one can define the generating functionals for the final-state hadrons in two-jet WW decay,

$$\mathcal{R}^{\mathrm{W}}[u(p)] = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{\Omega} \rho^{\mathrm{W}}(p_1, p_2, \ldots, p_n) u(p_1) \ldots u(p_n) \prod_{i=1}^{n} \mathrm{d}p_i, \tag{5}$$

$$\mathcal{G}^{\mathrm{W}}[u(p)] = \ln \mathcal{R}^{\mathrm{W}}[u(p)] \tag{6}$$

with $\rho^{\mathrm{W}}(p_1, p_2, \ldots, p_n)$ being the n-particle inclusive density for two-jet WW decay.

e.g. $\rho(1,2) = \frac{1}{N_{\mathrm{ev}}} \frac{\mathrm{d}n_{\mathrm{pairs}}}{\mathrm{d}Q_{12}}$, $\int_Q \rho(1,2)\mathrm{d}Q_{12} = \langle n_1(n_2 - \delta_{12}) \rangle$ $Q_{12} = \sqrt{-(p_1 - p_2)^2}$

Let us consider an uncorrelated WW decay scenario. In this we assume that each W boson showers and fragments into final-state hadrons without any reference to what is happening to the other. In this case $\mathcal{R}^{\mathrm{WW}}[u(p)]$ is the product of the generating functionals for the two-jet WW decay of differently charged W's

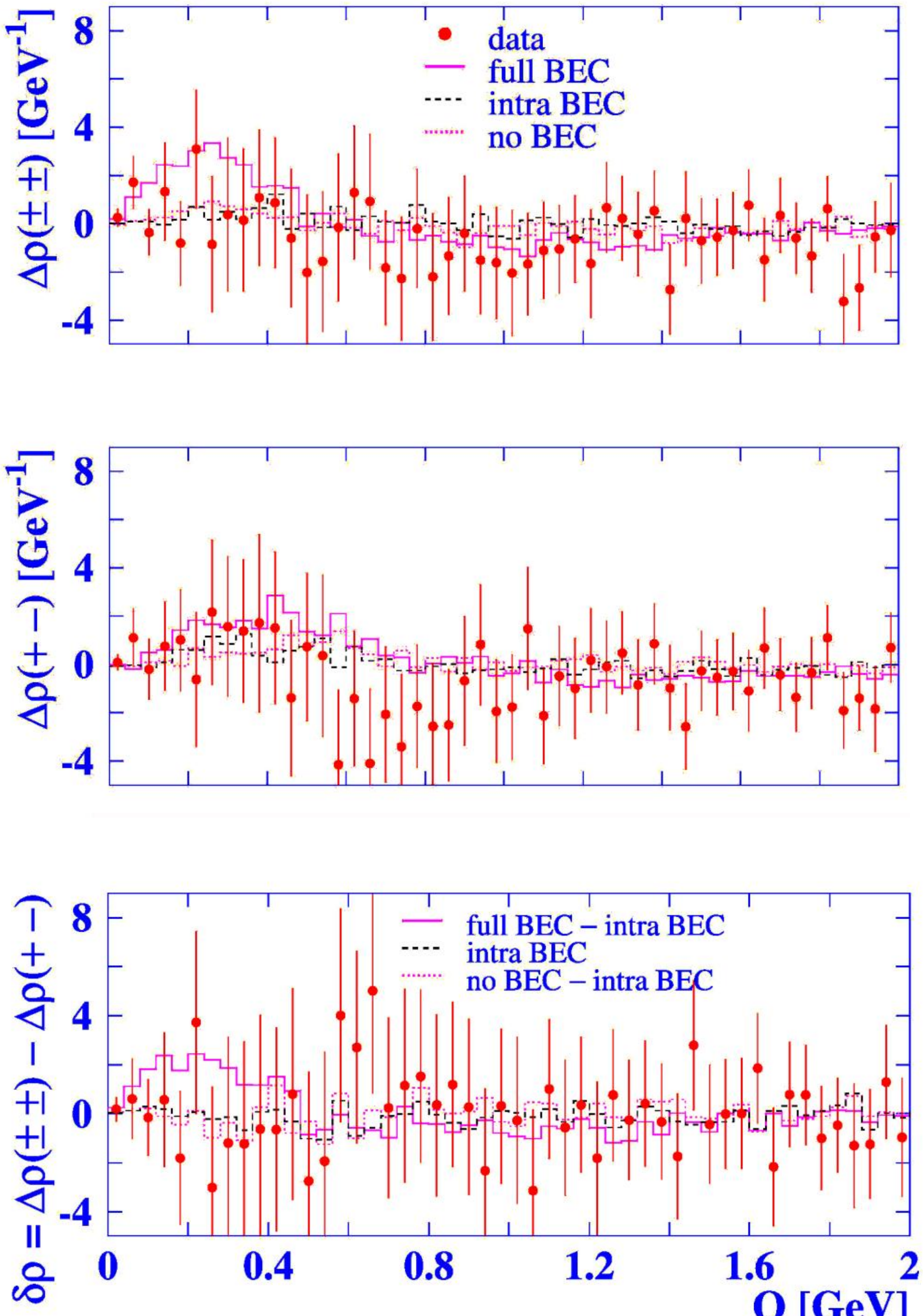
$$\mathcal{R}^{\mathrm{WW}}[u(p)] = \mathcal{R}^{\mathrm{W}^+}[u(p)]\, \mathcal{R}^{\mathrm{W}^-}[u(p)]. \tag{7}$$

In terms of the generating functionals for the correlation functions, this can be represented as follows

$$\mathcal{G}^{\mathrm{WW}}[u(p)] = \mathcal{G}^{\mathrm{W}^+}[u(p)] + \mathcal{G}^{\mathrm{W}^-}[u(p)]. \tag{8}$$

XXXth ISMD, Tihany 2000

XXXth ISMD, Tihany 2000

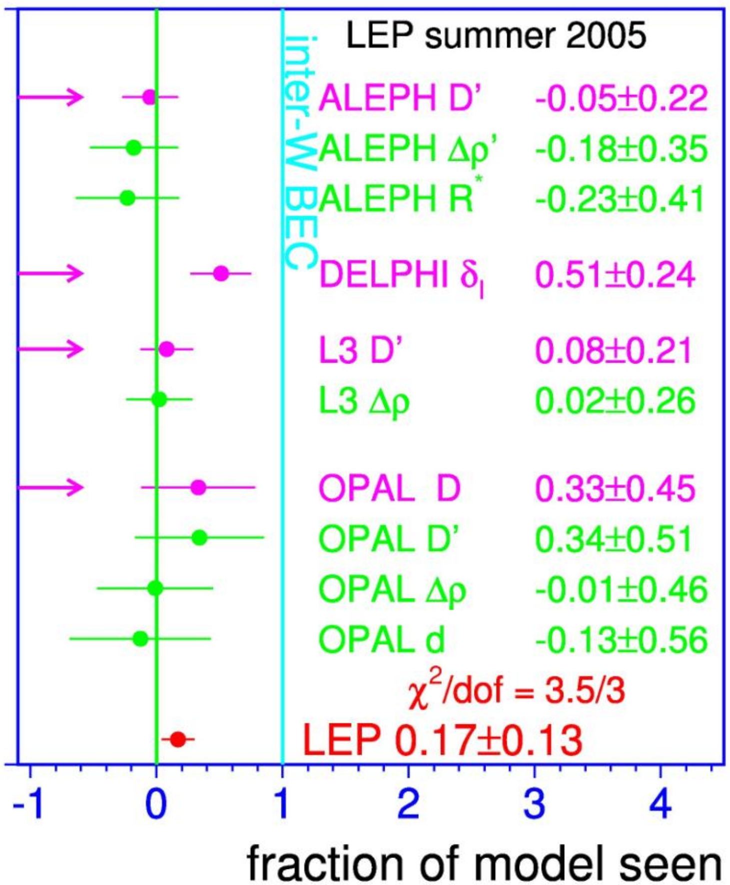


Sarkisyan,
De Wolf

OPAL

(2004)

ALEPH: B. Pietrzyk, F. Martin
DELPHI: N. Van Remortel, Š. Todorova, F. Verbeure, J. D'Hondt
L3: J. van Dalen, W. Kittel, W.Metzger
OPAL: E. Sarkisyan, E.A. De Wolf



2. Quotient:

$$D(\pm,\pm) = \frac{\rho_2(\pm,\pm)^{\mathrm{WW}}}{2\rho_2^{\mathrm{W}}(\pm,\pm) + 2\rho_{\mathrm{mix}}^{\mathrm{W^+W^-}}(\pm,\pm)}$$

$$D(+,-) = \frac{\rho_2(+,-)^{\mathrm{WW}}}{2\rho_2^{\mathrm{W}}(+,-) + 2\rho_{\mathrm{mix}}^{\mathrm{W^+W^-}}(+,-)}$$

$\Rightarrow$ look for a deviation from $D = 1$

3. Double ratio:

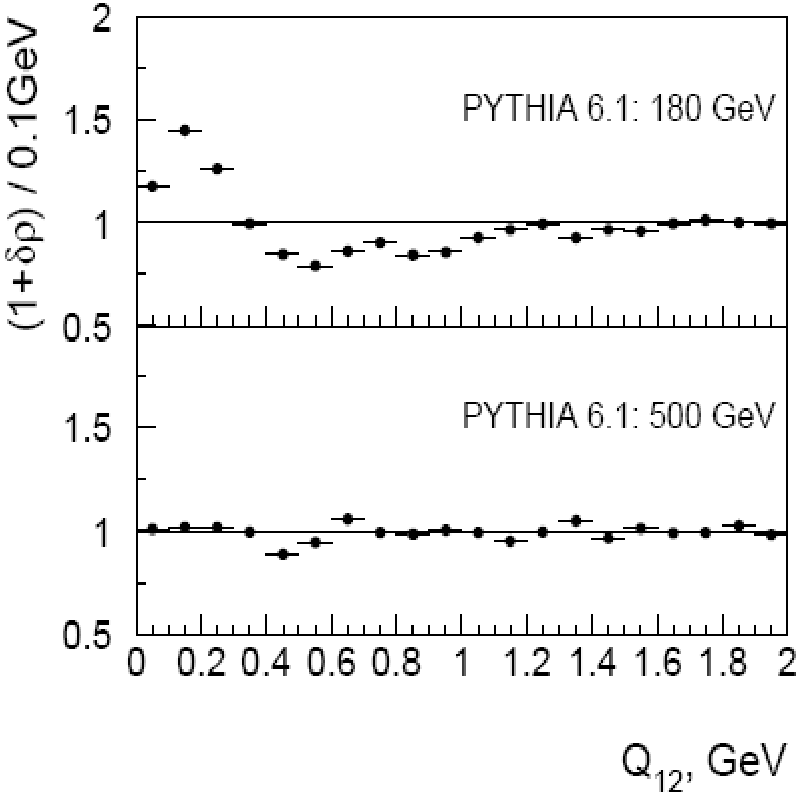
$$D'(\pm,\pm) = \frac{D(\pm,\pm)}{D(\pm,\pm)_{\mathrm{MC,no\ BE}}}$$

$$D'(+,-) = \frac{D(+,-)}{D(+,-)_{\mathrm{MC,no\ BE}}}$$

to abandon non-BE correlations
detector effects etc.

Chekanov, De Roeck, De Wolf (2000)

$$\delta\rho = \rho^{WW}(\pm,\pm) - 2\rho^{W}(\pm,\pm) - \rho^{WW}(+,-) + 2\rho^{W}(+,-)$$



➡ **Less overlap at 500 GeV !**

De Wolf (2001)

Extension of mathematical formalism:

intra-W correl.

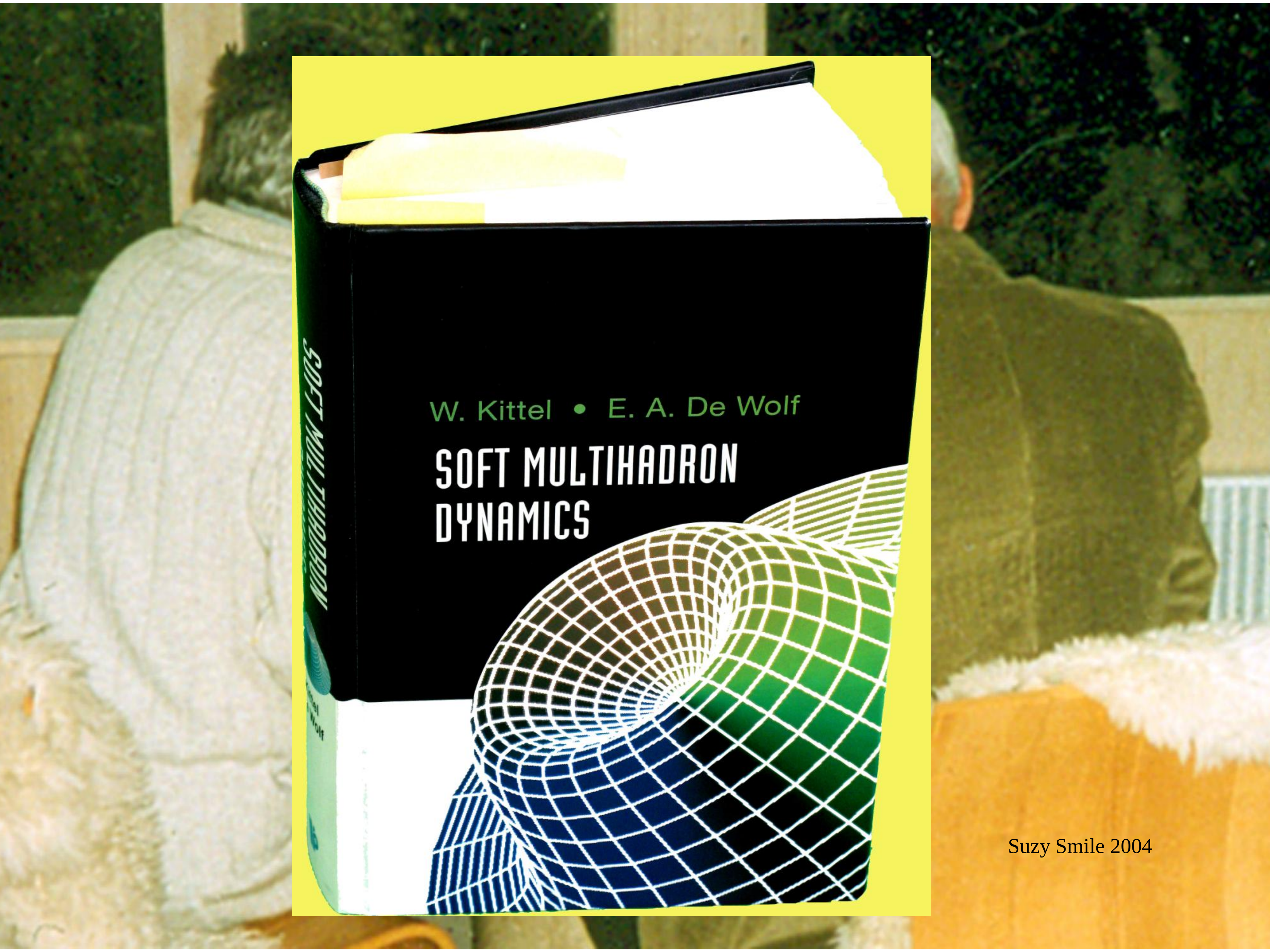
inter-W correl. $\delta_I(Q)$

overlap $g(Q)$

➡ **not all observables optimal !**

$g(Q)$ **important to select a sensitive quantity and check influence of cuts**

best in class: $\Delta\rho$ **(and** $\delta\rho$**)**


W. Kittel • E. A. De Wolf
SOFT MULTIHADRON
DYNAMICS
SOFT MULTIHADRON
Suzy Smile 2004

ISMD Ist, Paris 1970

ANGULAR CORRELATIONS IN THE REACTION $K^+p \rightarrow K^+p\, 2\pi^+\, 2\pi^-$ AT 4.97 GeV/c.

Brussels-CERN Collaboration

(presented by E. de Wolf)

In conclusion we believe that our phenomenological analysis has shown that the angular correlation effect results from an interplay of different phenomena such as peripheral resonance production, decay of resonances, interferences and symmetrization, which all add up to produce the observed effect. Therefore, only a better knowledge of the reaction mechanism will enable a detailed understanding of the GGLP effect.

Questions

- Elongation ($r_{in}/r_L < 1$)
  Q_{inv} versus directional dependence

- $r_{out} \cong r_{in}$
  Boost invariance

- m_T dependence (also in e^+e^-) factor 0.5 from m_π to 1GeV.
  Space-momentum correlation

- non-Gaussian behavior
  Edgeworth, power law, Lévy-stability
  Connection to intermittency

- 3-particle correlations
  Phase versus higher-order suppression
  Strength parameter λ

- **Source image reconstruction**

- **Overlapping systems (WW, 3-jet, nuclei)**
  HBT versus string

- **Dependence on type of collision**
  (no, except for heavy nuclei)

- **Energy (virtuality) dependence**
  (no, except for r_L)

- **Multiplicity Dependence**
  r increases
  λ decreases

- **effect on multiplicity and single-particle distribution**


by Suzy Smile 2004