Progress in understanding quarkonium polarization measurements

1. Why it is *essential* that we approach the measurement of polarization as a multidimensional problem: we must not average out information!
2. Why using more than one polarization frame in our experimental analysis we can provide clearer and more straightforward physical information
3. How we can perform self-consistency checks using frame-independent relations among the angular parameters

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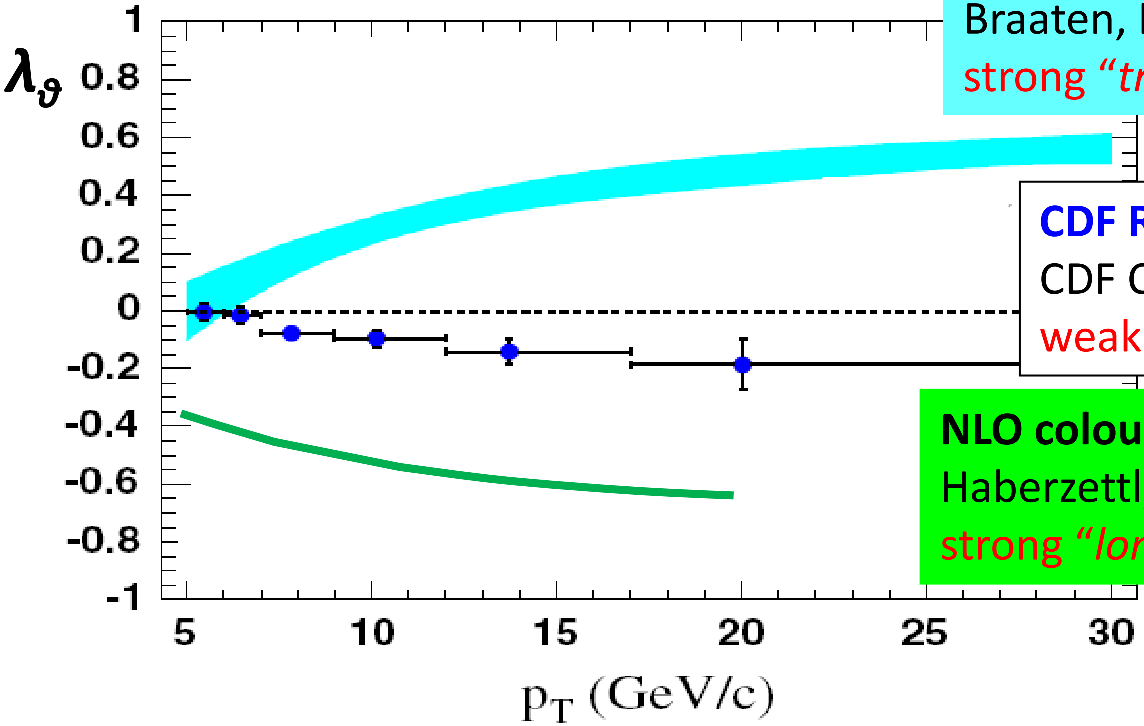
Quarkonium production at the LHC

CERN, February 19th, 2010

The theory "puzzle"



NRQCD factorization: prompt J/ψ
Braaten, Kniehl, Lee, PRD62, 094005 (2000)
strong "*transverse*" polarization

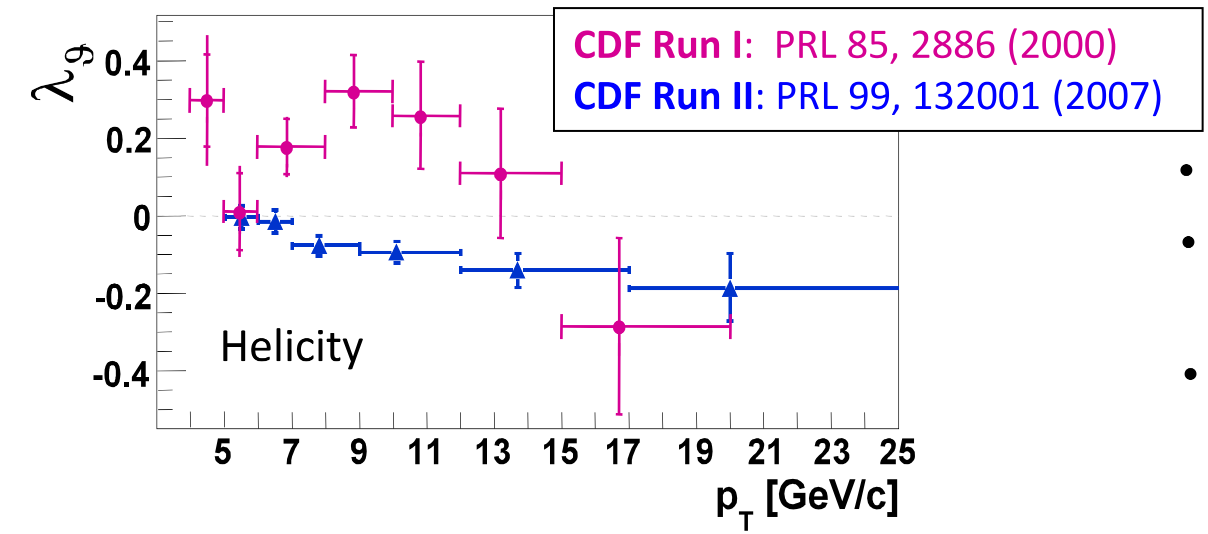
CDF Run II data: prompt J/ψ @1.96TeV
CDF Coll., PRL 99, 132001 (2007)
weak "*longitudinal*" polarization

NLO colour-singlet: *direct* J/ψ
Haberzettl, Lansberg, PRL100, 032006 (2008)
strong "*longitudinal*" polarization

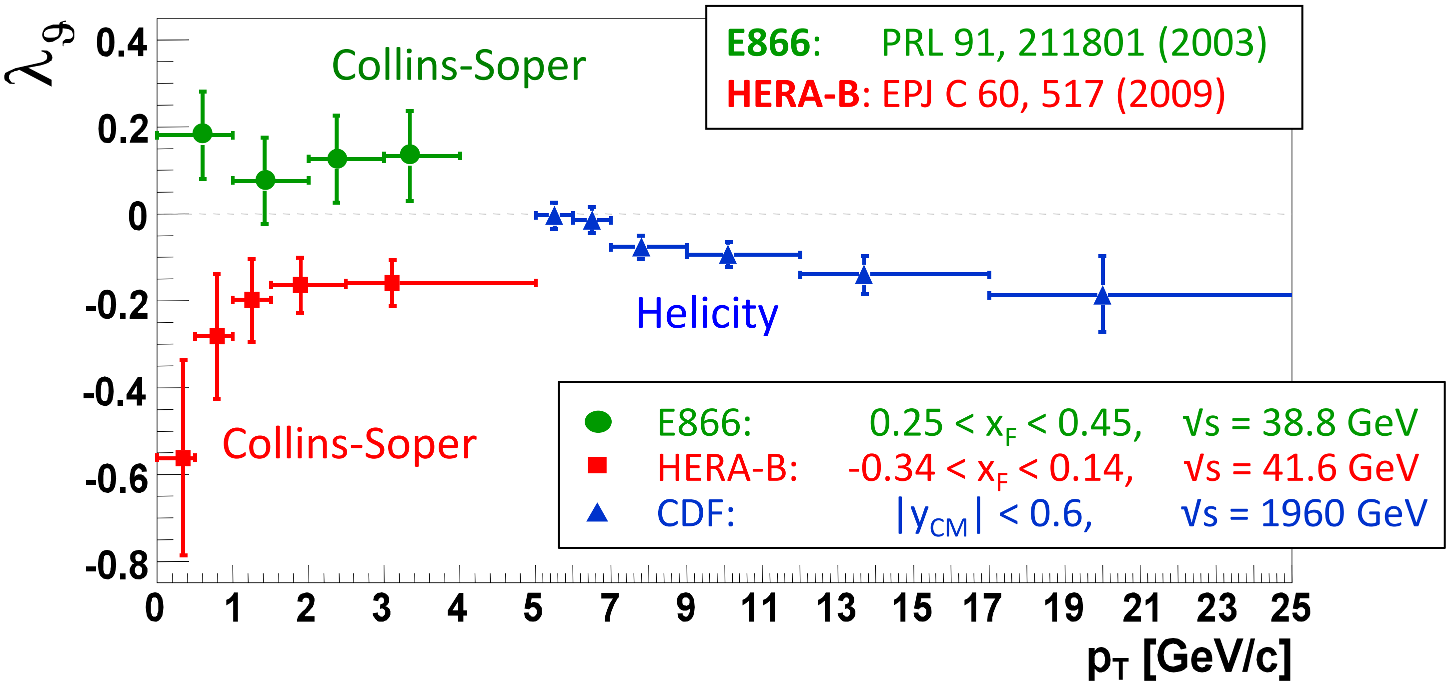
$$\frac{dN}{d\cos\vartheta} \propto 1 + \lambda_\vartheta \cos^2\vartheta$$

ϑ = angle between lepton direction
(in the J/ψ rest frame)
and J/ψ lab direction (helicity axis)

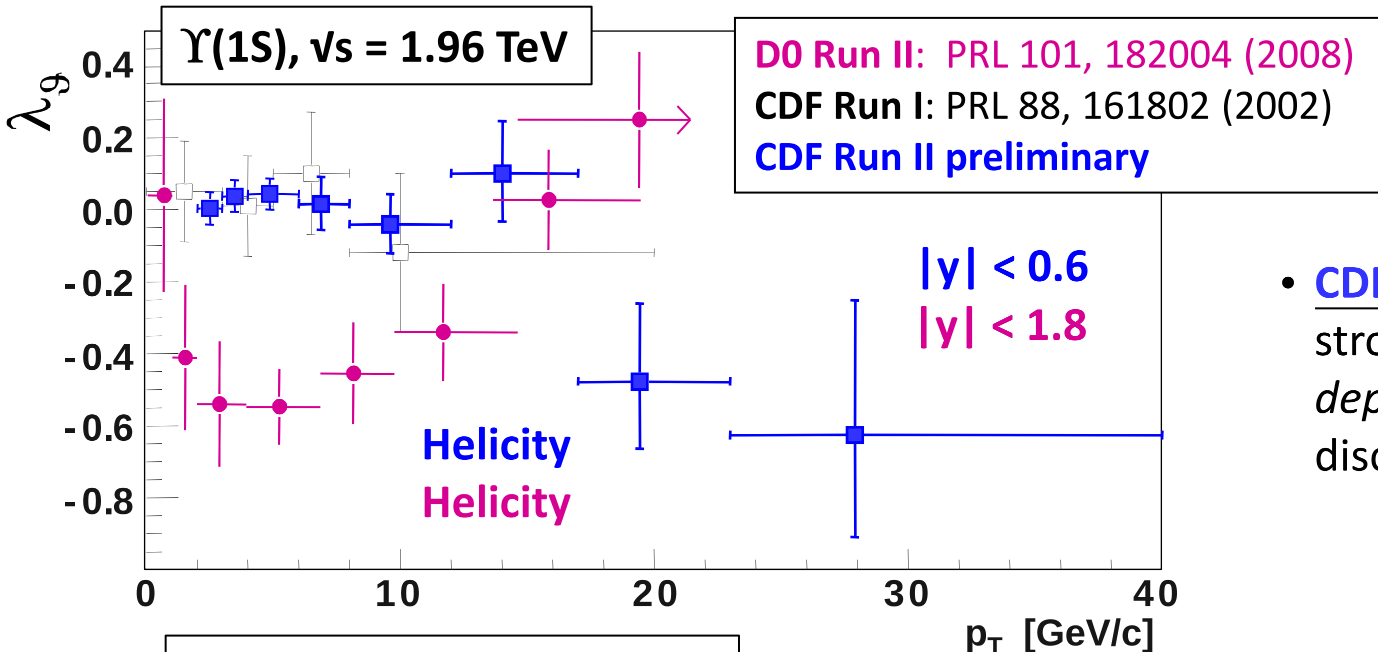
Experimental puzzles: J/ψ



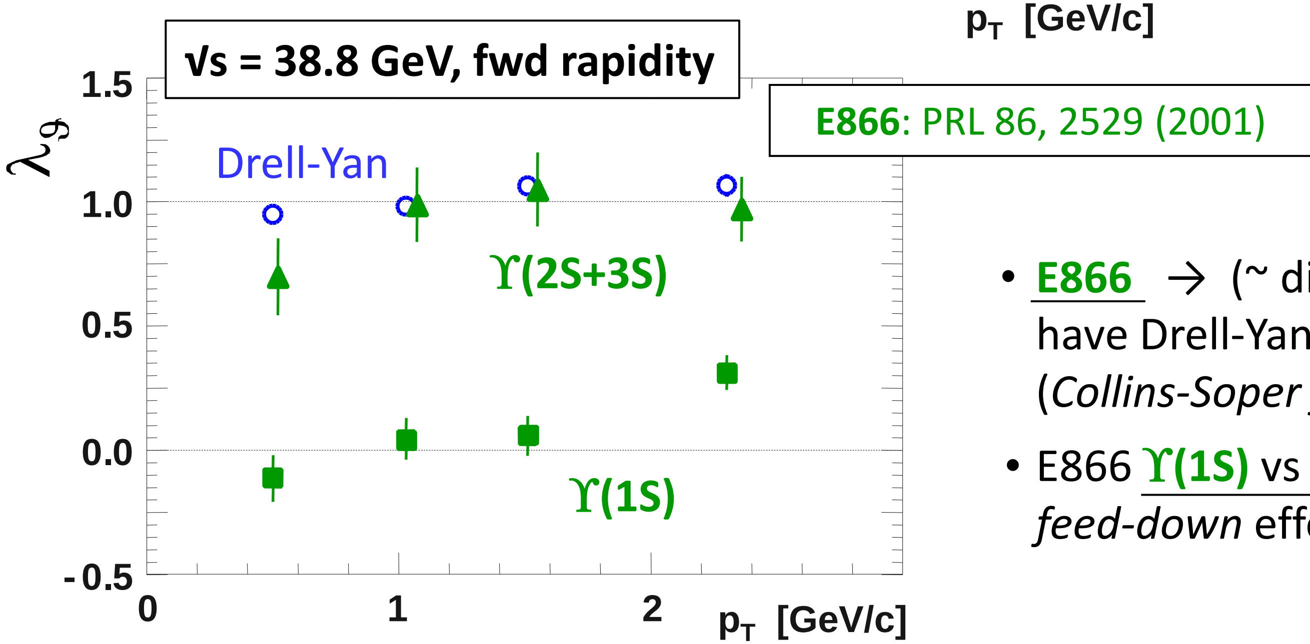
- **CDF** vs **CDF**
- **E866** vs **HERA-B** → there must be a *strong* p_L *dependence*
- **CDF** vs low-p_T → how do the different *frame conventions* affect the comparison?



Experimental puzzles: Υ



- **CDF** vs **D0** → may a strong *rapidity dependence* justify the discrepancy?



- **E866** → (~ directly produced) Υ(2S+3S) have Drell-Yan-like polarization (*Collins-Soper frame*!)
- E866 **Υ(1S)** vs **Υ(2S+3S)** → dominant *feed-down* effects for Υ(1S)?

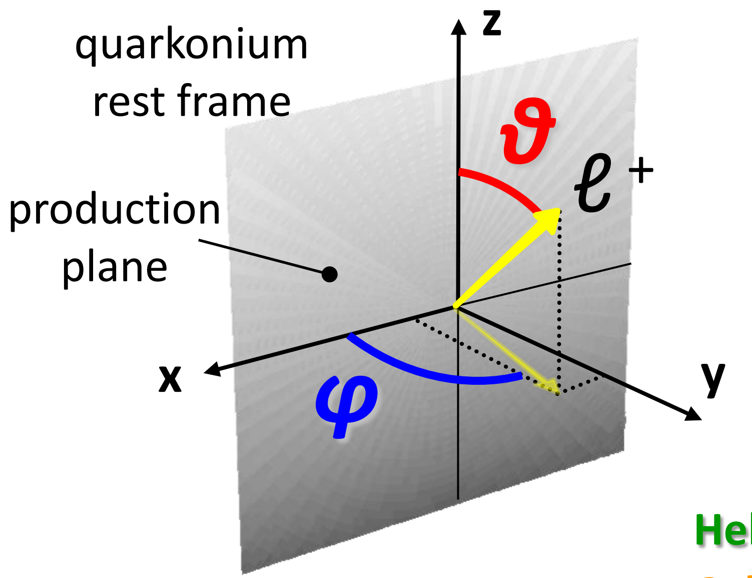
What to improve

This complex and confusing situation can only be clarified by better measurements.

In the new analyses we must avoid the simplifications that make the present results so difficult to be interpreted. We will illustrate the crucial importance and/or the advantages of

- measuring the *full* angular decay distribution, not only the polar anisotropy,
- providing results in at least two polarization frames,
- avoiding averages over large kinematic intervals (e.g. over the whole rapidity range),
- exploiting the existence of frame independent relations

Angles and frames



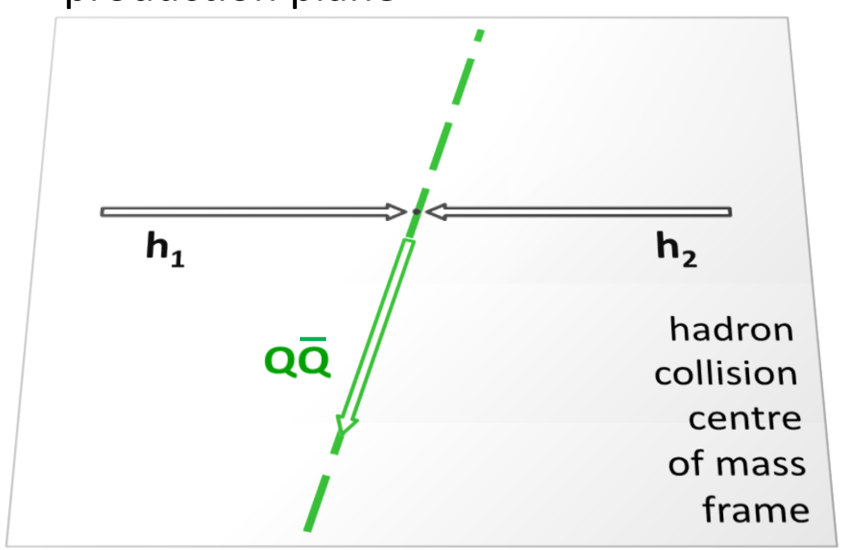
(positive) lepton direction:

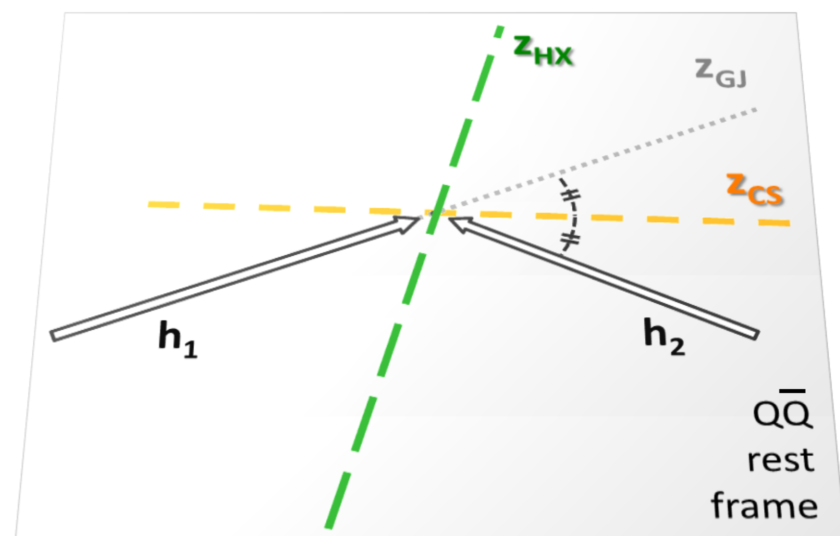
- ϑ wrt a chosen polarization axis (*z*)
- φ wrt the production plane (*xz*)

Helicity axis (HX): quarkonium momentum direction

Collins-Soper axis (CS): beam line direction

production plane





"Unpolarized" J/ψ does not exist

The most general J = 1 state that can be produced in *one elementary subprocess* is represented (wrt the chosen z axis) as a superposition of the three J_z eigenstates:

$$|\psi\rangle = A_0\,|0\rangle + A_{+1}\,|+1\rangle + A_{-1}\,|-1\rangle$$

The general angular distribution of its parity-conserving decay into two fermions is:

$$\boxed{W(\cos\theta,\varphi) \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta\cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi}$$

with $\lambda_\theta = \dfrac{1-3|A_0|^2}{1+|A_0|^2}$ $\quad \lambda_\varphi = \dfrac{2\,\mathrm{Re}\,A^*_{+1}A_{-1}}{1+|A_0|^2}$ $\quad \lambda_{\theta\varphi} = \dfrac{\sqrt{2}\,\mathrm{Re}[A^*_0(A_{+1}-A_{-1})]}{1+|A_0|^2}$

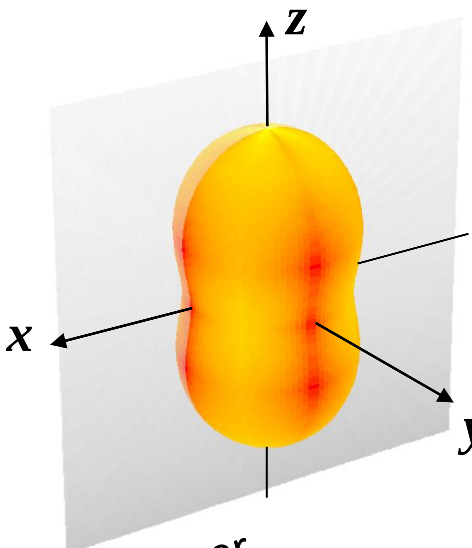
There is no combination of A_0, A_{+1} and A_{-1} (with $\sum_m |A_m|^2 = 1$) **such that the angular distribution is *isotropic* ($\lambda_\vartheta = \lambda_\varphi = \lambda_{\vartheta\varphi} = 0$).** Only a fortunate *mixture of subprocesses* in peculiar kinematic conditions (or randomization effects) can lead to a cancellation of all three *measured* anisotropy parameters.

→ Polarization is a "necessary" property of J = 1 quarkonia.
An accurate knowledge of the *net* polarization of the observed sample is indispensable also for absolute cross-section determinations, because the quarkonium acceptance depends strongly on the dilepton decay kinematics.

more later

The observed "polarization" depends on the frame

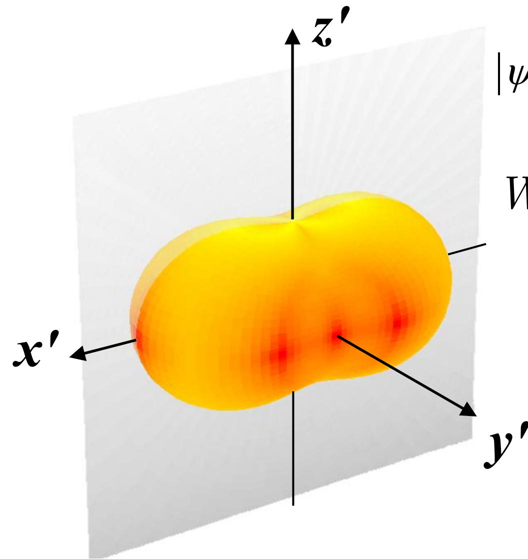
For $|p_L| \ll p_T$ the CS and HX frames differ by a rotation of 90º



$$|\psi\rangle = |\pm 1\rangle$$

$$W \propto 1 + \cos^2\theta$$

90º

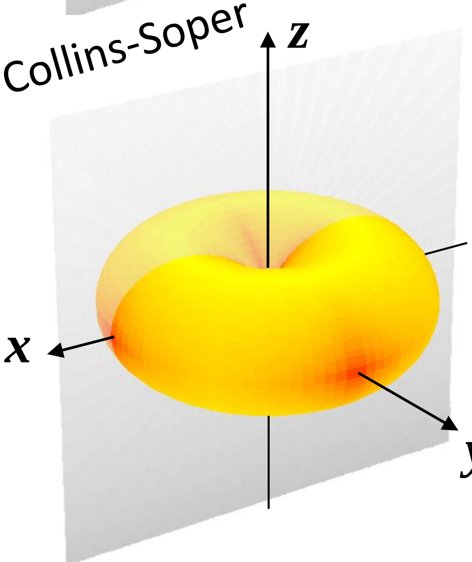


$$|\psi\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle \mp \frac{1}{\sqrt{2}}|0\rangle$$

$$W \propto 1 - \tfrac{1}{3}\cos^2\theta + \tfrac{1}{3}\sin^2\theta\cos 2\varphi$$

Collins-Soper

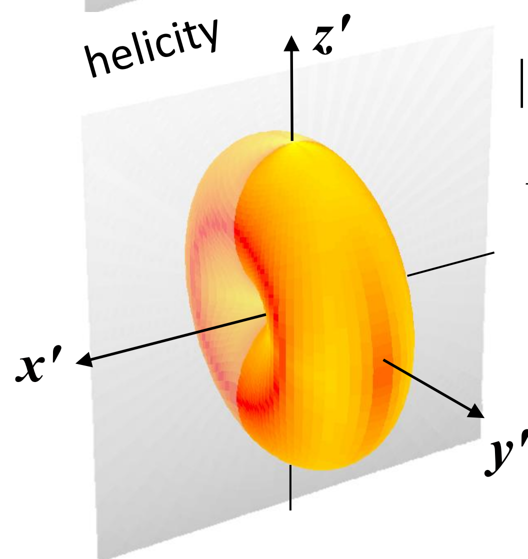
helicity



$$|\psi\rangle = |0\rangle$$

$$W \propto 1 - \cos^2\theta$$

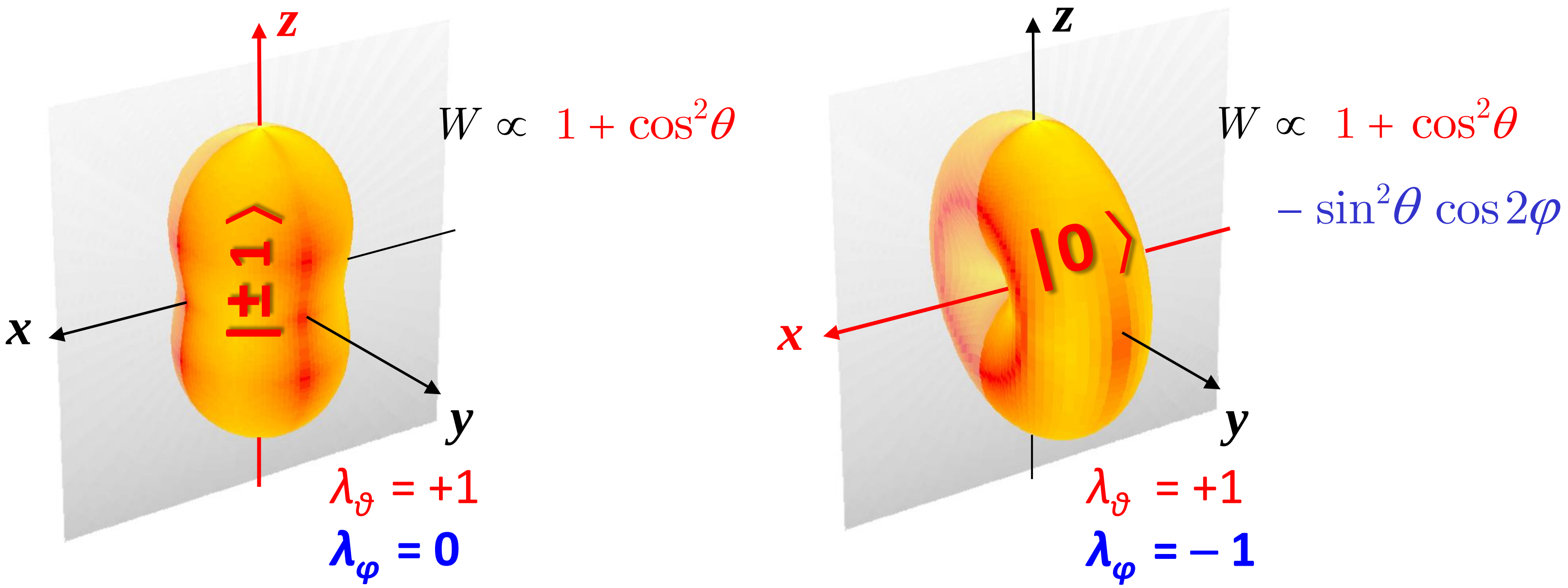
90º



$$|\psi\rangle = \frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$$

$$W \propto 1 + \cos^2\theta - \sin^2\theta\cos 2\varphi$$

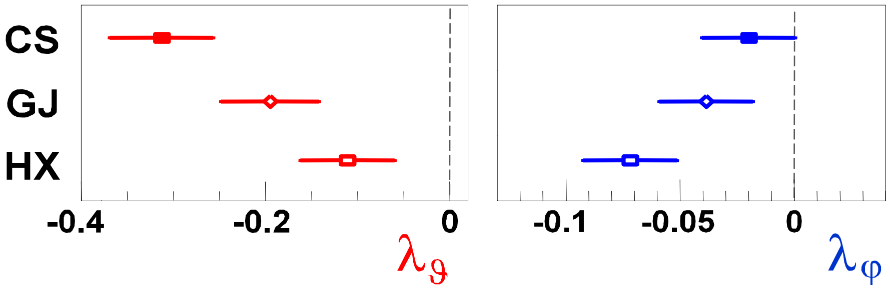
The azimuthal anisotropy is not a detail



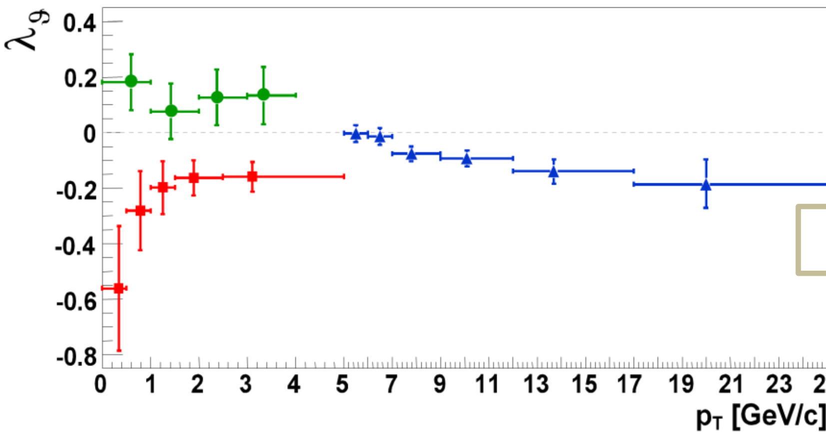
These two decay distributions are indistinguishable when the azimuthal dependence is integrated out. But they correspond to opposite *natural* polarizations, which can only be originated by completely different production mechanisms.

In general, measurements not reporting the azimuthal anisotropy provide an incomplete physical result. Their fundamental interpretation is impossible (relies on arbitrary assumptions).

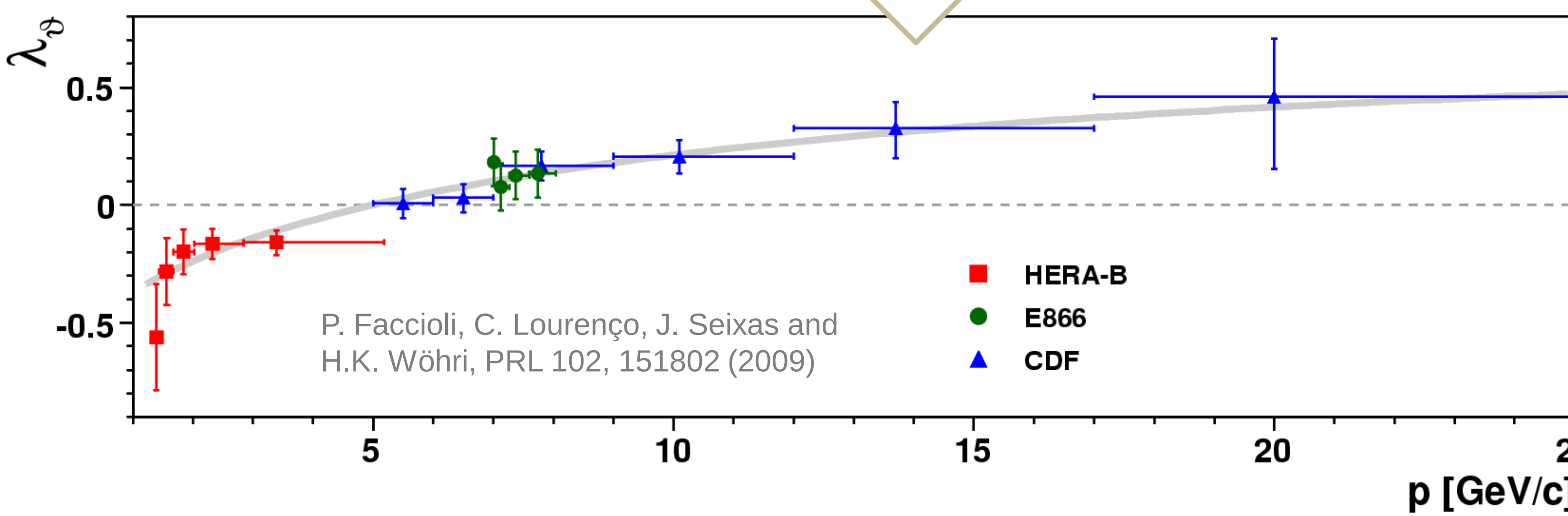
A possible hypothesis about CDF's J/ψ



HERA-B has shown that low-p_T J/ψ's (at fixed-target energies) are **naturally polarized in the Collins-Soper frame** (**most significant λ_ϑ** and **purely polar anisotropy, λ_φ = 0**).



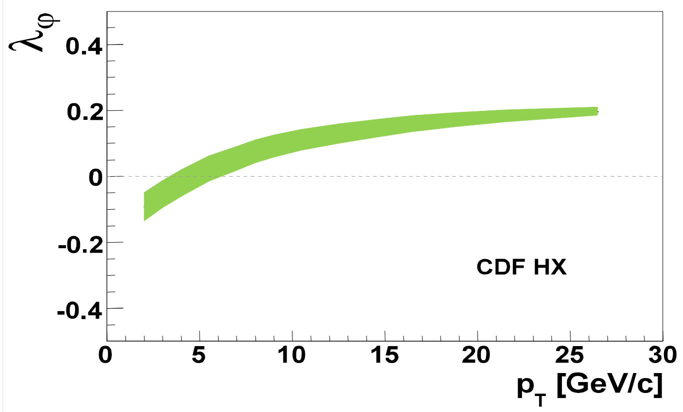
If we assume that this continues to be valid up to collider energies, we can translate the CDF points from the helicity frame to the Collins-Soper frame and recognize a smoothly varying polarization from low to high quarkonium *momentum*.



Message nº1

Today, we are allowed to make the speculation in the previous slide because CDF has not reported the azimuthal anisotropy.

We have assumed that $\lambda_\varphi = 0$ in the CS frame, automatically implying that a significant value of λ_φ should be measured in the HX frame:



By measuring also λ_φ CDF will remove this ambiguity of interpretation.

Measure the *full* angular decay distribution, not only the polar anisotropy.

Reference frames are not all equally good

How the anisotropy parameters transform from one frame to another depends explicitly on the production kinematics.

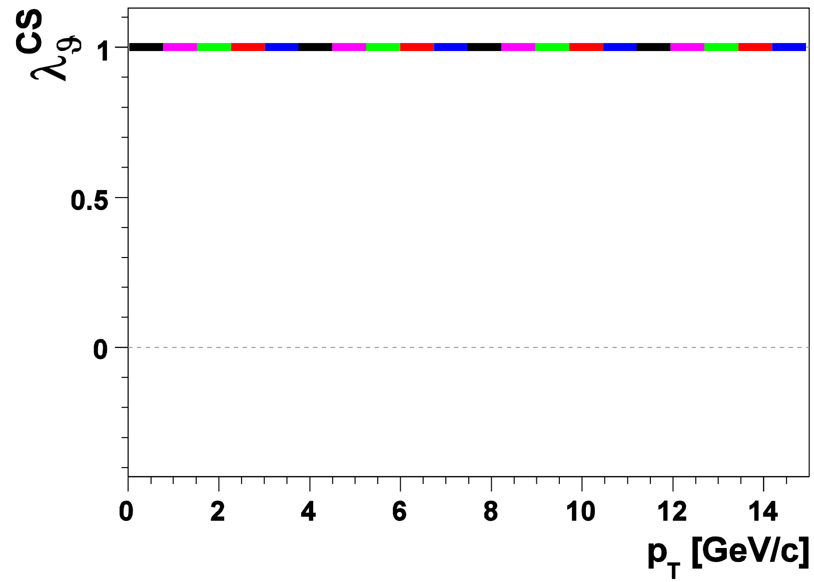
Example: **how would different experiments observe a Drell-Yan-like decay distribution** ["naturally" of the kind **$1 + \cos^2\vartheta$** in the Collins-Soper frame – see e.g. E866's Υ result] **with an arbitrary choice of the reference frame?**

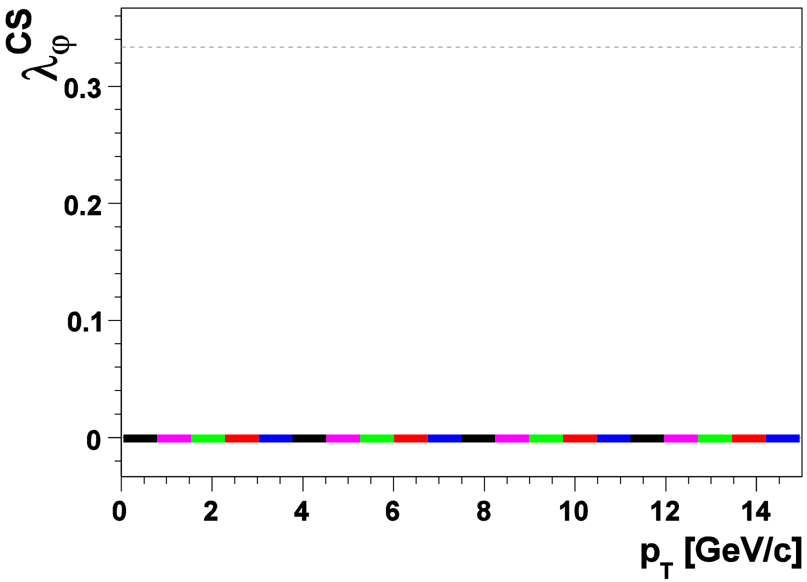
We consider Υ decay. For simplicity of illustration we assume that each experiment has a flat acceptance in its nominal rapidity range:

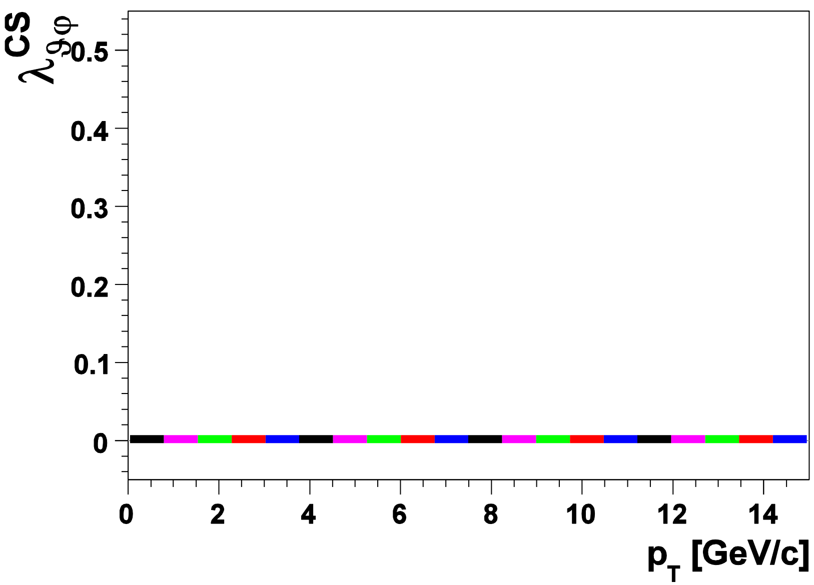
CDF	$\lvert y \rvert < 0.6$
D0	$\lvert y \rvert < 1.8$
ATLAS & CMS	$\lvert y \rvert < 2.5$
ALICE e^+e^-	$\lvert y \rvert < 0.9$
ALICE $\mu^+\mu^-$	$2.5 < \lvert y \rvert < 4$
LHCb	$2 < \lvert y \rvert < 5$

The lucky frame choice

(CS in this case)







CDF

D0

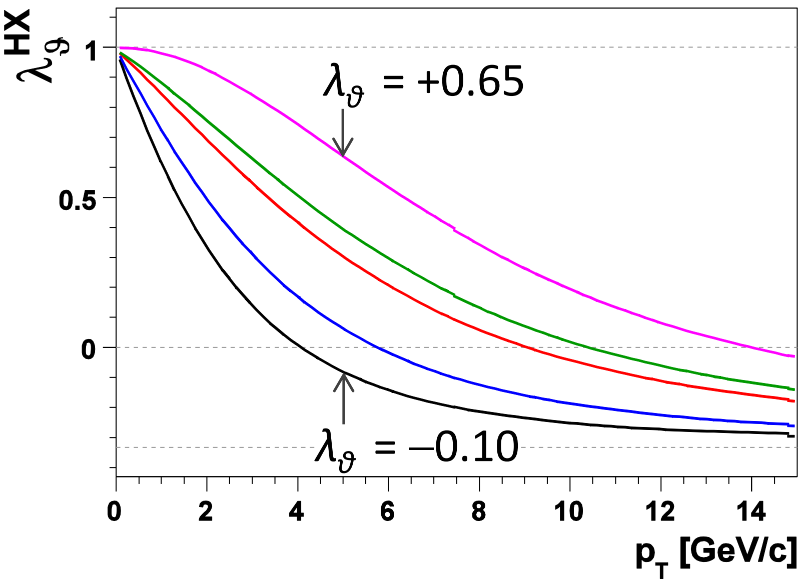
ATLAS / CMS

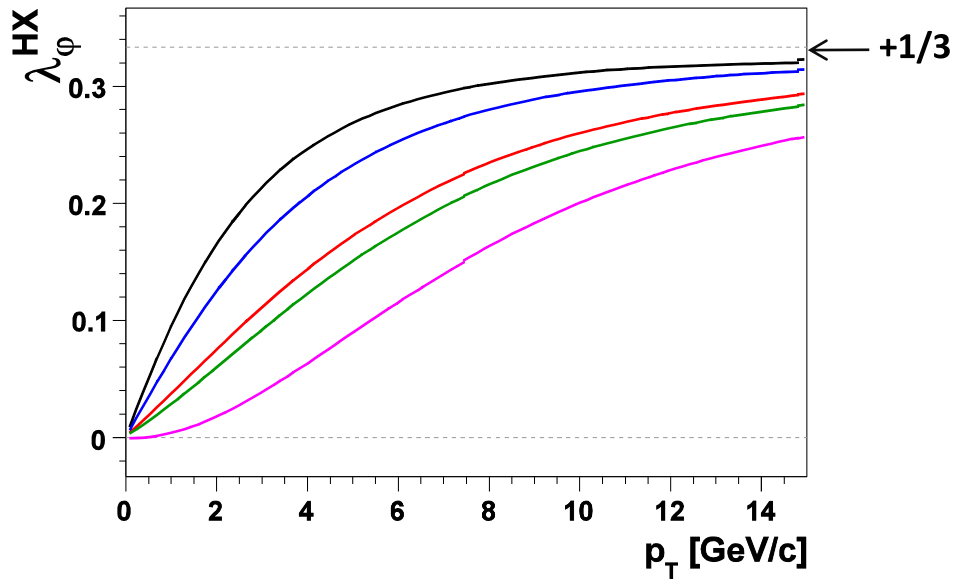
ALICE e^+e^-

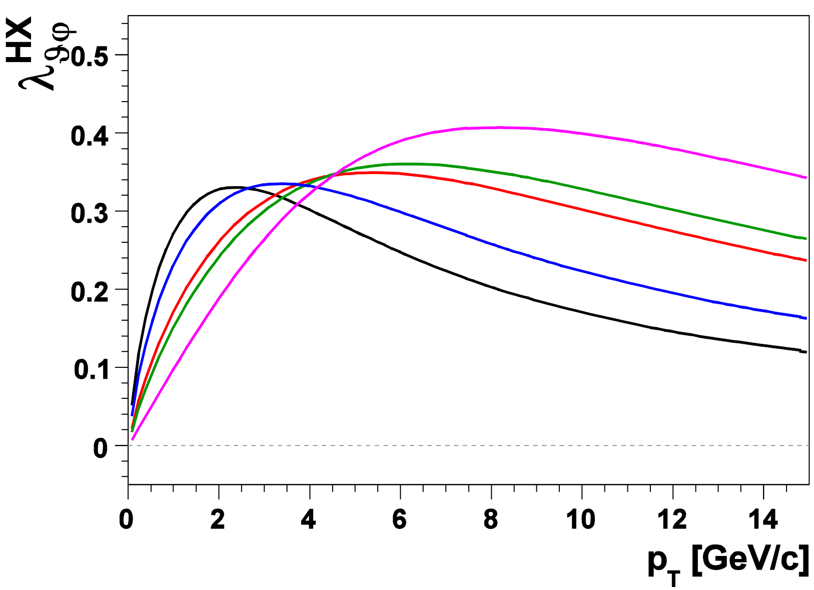
ALICE $\mu^+\mu^-$ / LHCb

Less lucky choice

(HX in this case)







CDF

D0

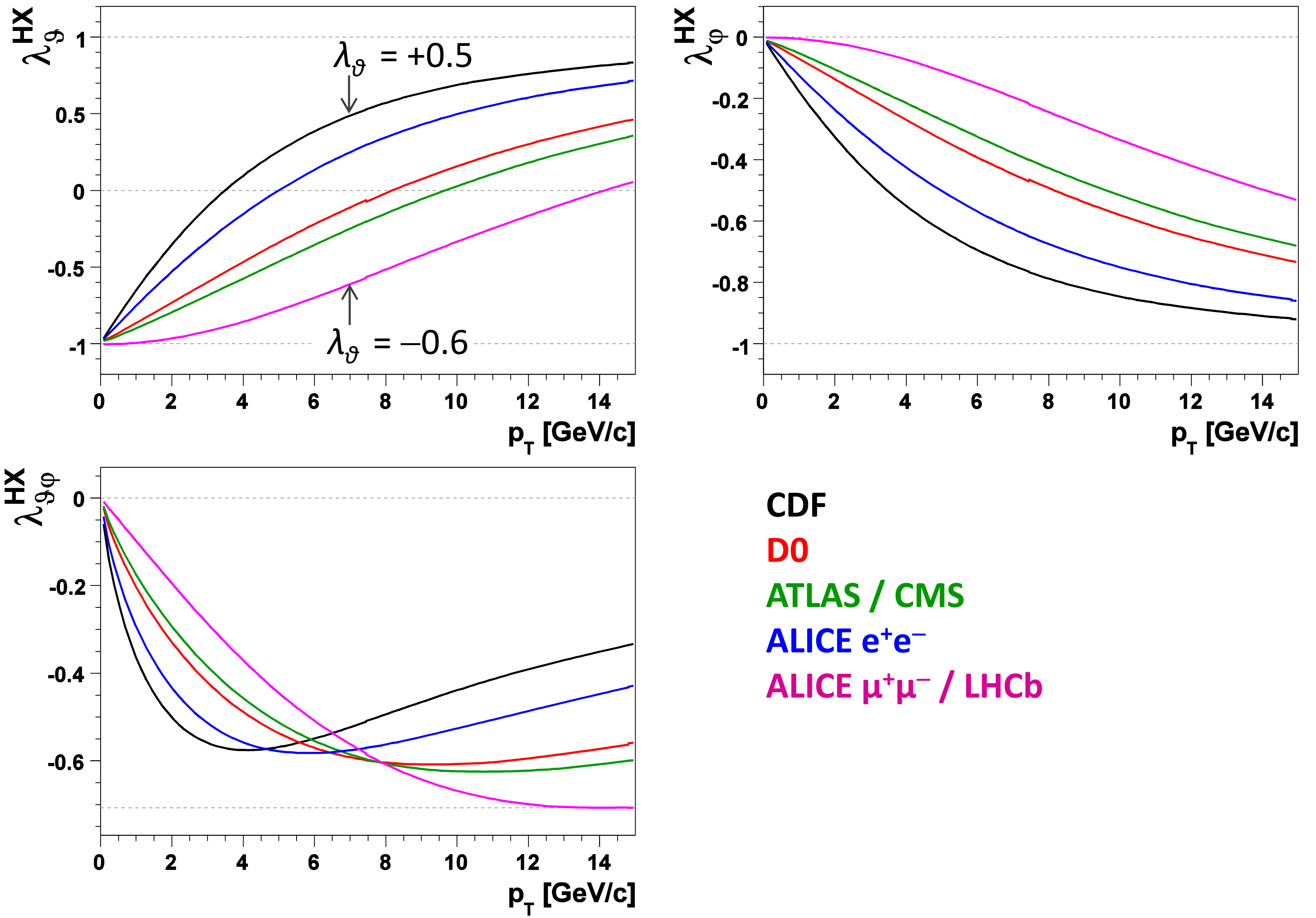
ATLAS / CMS

ALICE e^+e^-

ALICE $\mu^+\mu^-$ / LHCb

One more example

"natural" polarization $\boldsymbol{\lambda_\vartheta = -1}$ in the CS frame, as seen in the HX frame



Message nº2

When observed in an arbitrarily chosen frame, the simplest possible pattern of a constant natural polarization may be seen as a complex decay distribution rapidly changing with p_{T} and rapidity. This is not wrong, but gives a misleading view of the phenomenon, even inducing an artificial dependence of the measurement on the specific kinematic window of the experiment.

Measure in more than one frame.

Message nº3

In more complex cases (quarkonium not produced as a pure state; superposition of different processes...) the observed distribution has an *intrinsic* kinematic dependence.

For the comparison between experiments and with theory it is necessary to take into account the *role of the experimental acceptance*. The experiments measure the net polarization of the specific cocktail of quarkonium events accepted by detector, trigger and analysis cuts. If the polarization depends on the kinematics ("intrinsically" and/or "extrinsically"), the average polarization depends on the *effective population of the collected events* in the considered kinematic interval.

Two experiments may find different average polarizations in the *same* kinematic range if they have very different *acceptance shapes* in that range. The problem can be solved by presenting the result in a fine scan of the kinematic phase space.

The *theoretical calculations* for the *average* polarization in a certain kinematic range should in principle take into account how the momentum distribution is distorted by the acceptance of the specific experiment. As a better alternative, for each experiment theorists may provide several curves as a function of p_T, one for each rapidity value, rather than integrating over rapidity as is currently done.

Avoid (as much as possible) kinematic averages.

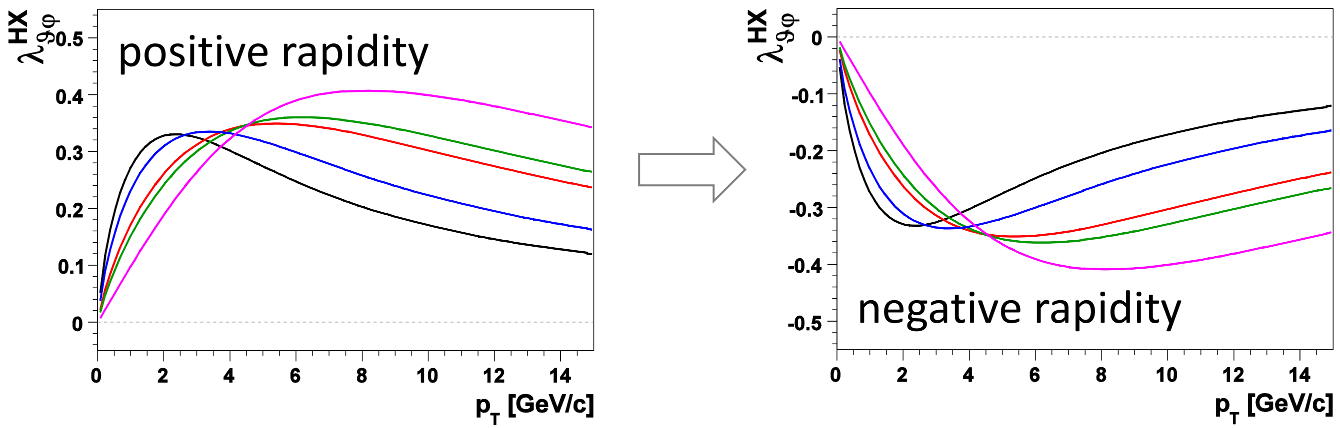
Mind the sign

1. always state the exact definition of the *y* axis: the sign of $\lambda_{\vartheta\varphi}$ depends on it!

Moreover, stay away from artificially "parity violating" definitions of the *y* axis direction, like

$$\hat{y} = \frac{\vec{P}'_{\mathrm{beam1}} \times \vec{P}'_{\mathrm{beam2}}}{\left|\vec{P}'_{\mathrm{beam1}} \times \vec{P}'_{\mathrm{beam2}}\right|}, \qquad \hat{y} = \frac{\vec{p}_{J/\psi} \times \vec{P}'_{\mathrm{beam1}}}{\left|\vec{p}_{J/\psi} \times \vec{P}'_{\mathrm{beam1}}\right|}, \qquad \ldots$$

They cause a change in sign of the measured $\lambda_{\vartheta\varphi}$ passing from positive to negative rapidity, e.g. in the example shown before:



As a consequence, $\lambda_{\vartheta\varphi}$ *would always be measured to be zero* over any rapidity range symmetric wrt zero (if the detection efficiency is symmetric wrt zero)

2. avoid averaging positive- with negative-rapidity results OR change the sign of *y* axis definitions like the above ones when passing from positive to negative rapidity

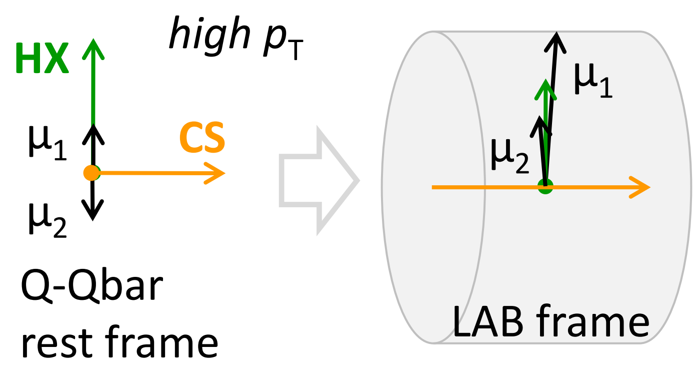
Polarization dependence of the dilepton acceptance

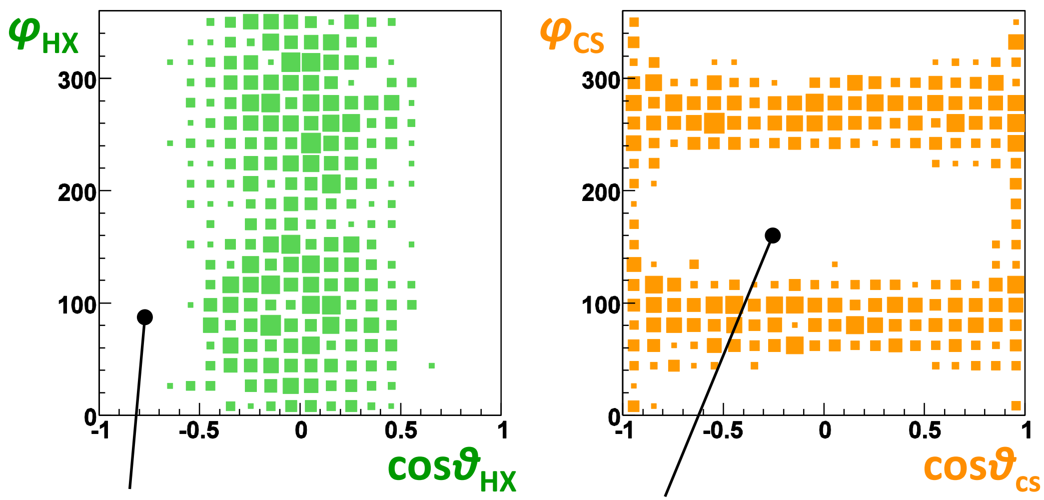
Absolute cross section measurements depend on the knowledge (or lack of knowledge) of the polarization. In fact, the probability that quarkonium dileptons are accepted and reconstructed is very sensitive to the *quarkonium decay kinematics*.

In experiments like CMS, for example, the minimum detector sensitivity to the muon momenta and the trigger cuts strongly suppress events where the two muons are emitted perpendicularly to the beam line, because one of the two has small lab p_T. At *high quarkonium* p_T, this configuration corresponds

to $\cos\vartheta_{HX} \approx \pm 1$

or to $\cos\vartheta_{CS} \approx 0$ and $\varphi_{CS} \approx 0^\circ, 180^\circ, 360^\circ$





$\Upsilon(1S)$, CMS-like MC with $p_T(\mu) > 3$ GeV/c for both muons

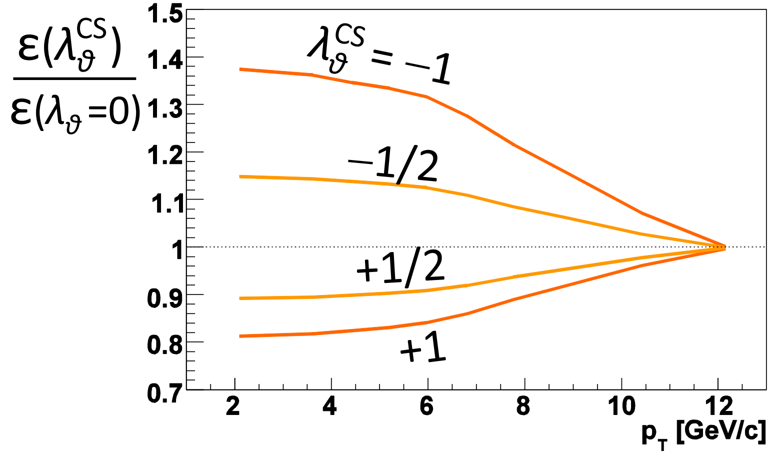
$p_T(\Upsilon) > 10$ GeV/c, $|y(\Upsilon)| < 1$,

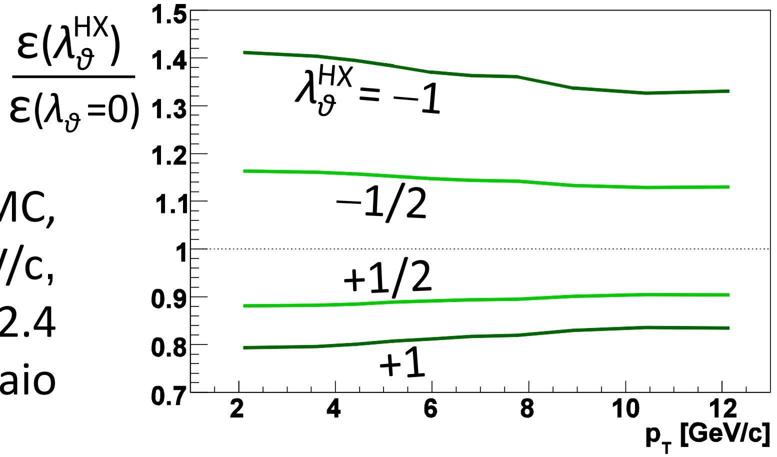
The efficiency determination in the zero-acceptance domains will be 100% dependent on the polarization information fed into the Monte Carlo simulation.

Not a one-dimensional problem either

The knowledge of the polar anisotropy alone in one arbitrarily chosen frame (or a range of hypotheses about it) is not, a priori, a sufficient input to an accurate determination of the experimental kinematic acceptance for the dileptons from quarkonium.
Ideally, the acceptance normalization of cross-section measurements (and the associated systematic uncertainty) should take into account a range of hypotheses for the *full* dilepton decay distribution.

In the absence of a complete measurement of such a distribution (early LHC analyses), one possible procedure is to *determine the frame which maximizes the dependence of the acceptance on* λ_ϑ and use that frame for the determination of the corresponding maximum systematic variation.





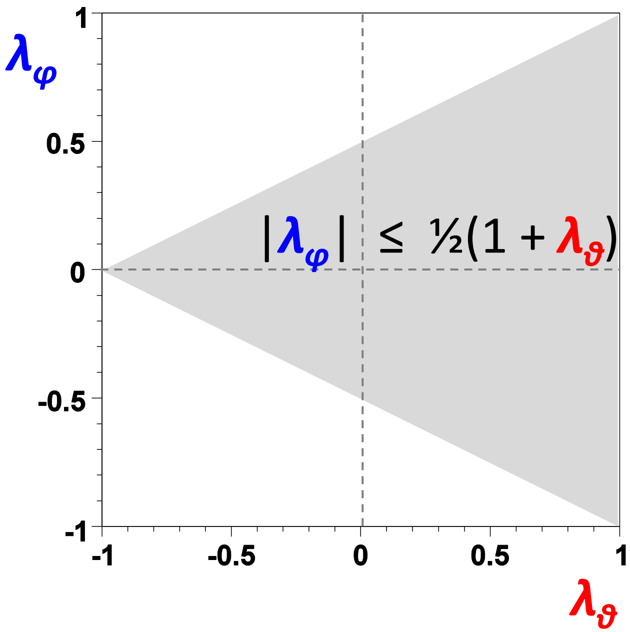
Υ(1S), CMS-like MC,
$p_T(\mu) > 3$ GeV/c,
$|y(\Upsilon)| < 2.4$
work by Sérgio Sampaio

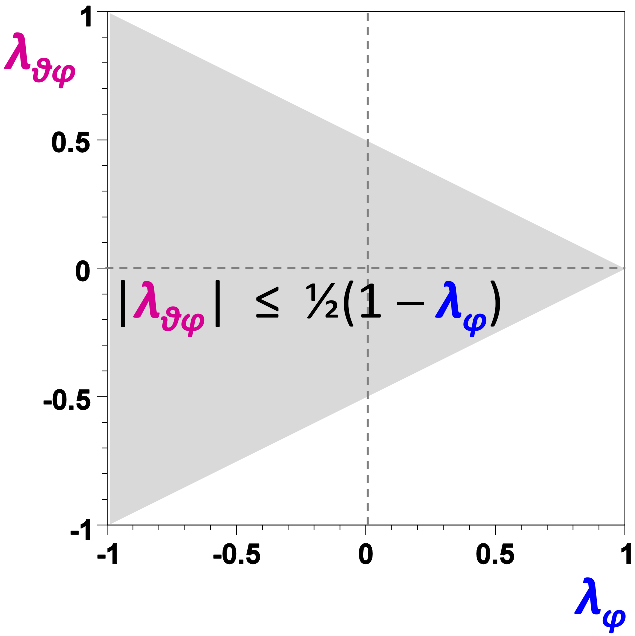
The parameter space

1. The ranges of allowed values for the three parameters of the angular distribution are correlated.
2. Simple relations constrain the way polar and azimuthal anisotropies can be observed in different frames.

Triangles

$|\lambda_\vartheta| = |(1\text{-}3P_0)/(1+P_0)| \leq 1$. By imposing that rotations of the polarization axis in the production plane do not violate this bound, the anisotropy parameters are found to satisfy the following triangle conditions:





Note: with the definition adopted here, $|\lambda_\varphi| \leq 1$ and $|\lambda_{\vartheta\varphi}| \leq 1$.
The widespread convention uses "ν" = $2\lambda_\varphi$ → $|\nu| \leq 2$.

Lines

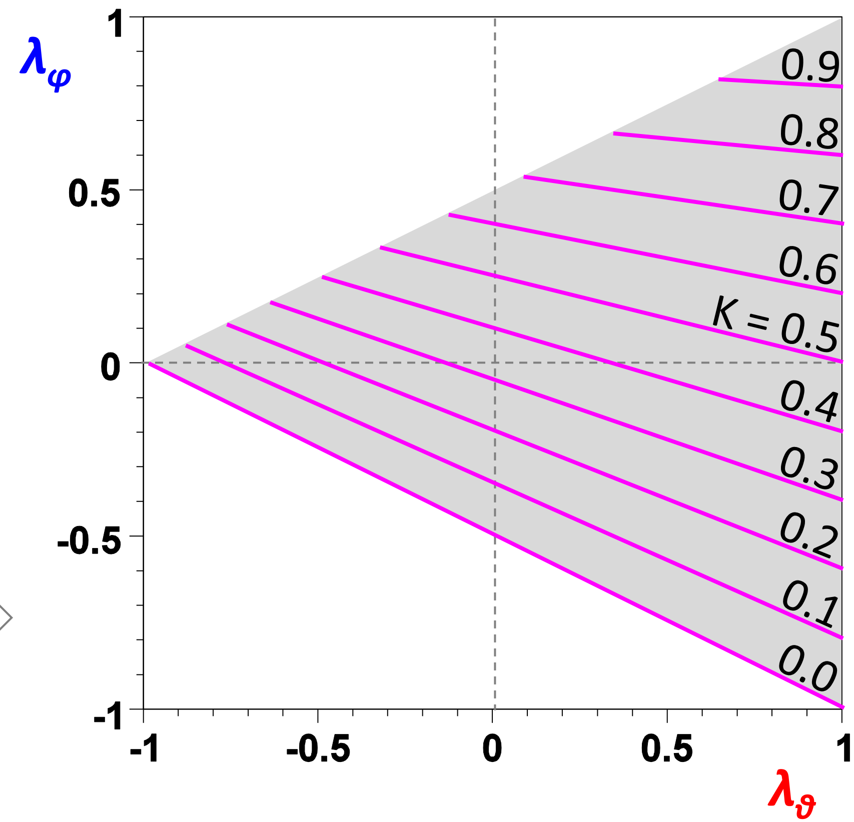
It is a completely general result that any combination of anisotropy parameters of the following kind is frame-independent:

$$K_{\{c_i\}} = \frac{c_1\,(3 + \lambda_\vartheta) \;+ c_2\;(1 - \lambda_\varphi)}{c_3\,(3 + \lambda_\vartheta) \;+ c_4\;(1 - \lambda_\varphi)}$$

Once a definition is chosen, each value of *K* defines one line in the (λ_ϑ , λ_φ) triangle.
Each line contains all (λ_ϑ , λ_φ) combinations corresponding to the *same decay distribution as observed in all possible frames*.

Example $K = \dfrac{1 + \lambda_\vartheta + 2\,\lambda_\varphi}{3 + \lambda_\vartheta}$ ⇨

[c_1=1, c_2=–2, c_3=1, c_4=0]



Basic meaning

Let us suppose that in the collected events quarkonium (*J* = 1) was produced through *n* different elementary subprocesses yielding angular momentum states of the kind

$$\left|\psi^{(i)}\right\rangle = A_0^{(i)}\,|0\rangle + A_{+1}^{(i)}\,|{+1}\rangle + A_{-1}^{(i)}\,|{-1}\rangle, \qquad i = 1, 2, \ldots n$$

(wrt a given quantization axis), each one with probability $f^{(i)}$ $\left(\sum f^{(i)} = 1\right)$.

The rotational properties of angular momentum eigenstates imply that each amplitude combination $K^{(i)} = ½\left|A_{+1}^{(i)} + A_{-1}^{(i)}\right|^2$ is independent of the choice of the quantization axis. The quantity

$$K = \sum_{i=1}^{n} f^{(i)} K^{(i)} = \frac{1}{2}\sum_{i=1}^{n} f^{(i)} \left|A_{+1}^{(i)} + A_{-1}^{(i)}\right|^2 \qquad (0 \leq K \leq 1)$$

is therefore frame-independent. It can be determined in any frame as

$$K = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta}$$

Advantages of reporting polarization results in terms of (this or another) *K*:

- the choice of the polarization frame is *really* arbitrary: the measurement must always yield the same value of *K*
- measurements and theoretical calculations are **free from the (acceptance-dependent!) “extrinsic” kinematic effect induced by frame transformations** → cleaner comparisons

Example

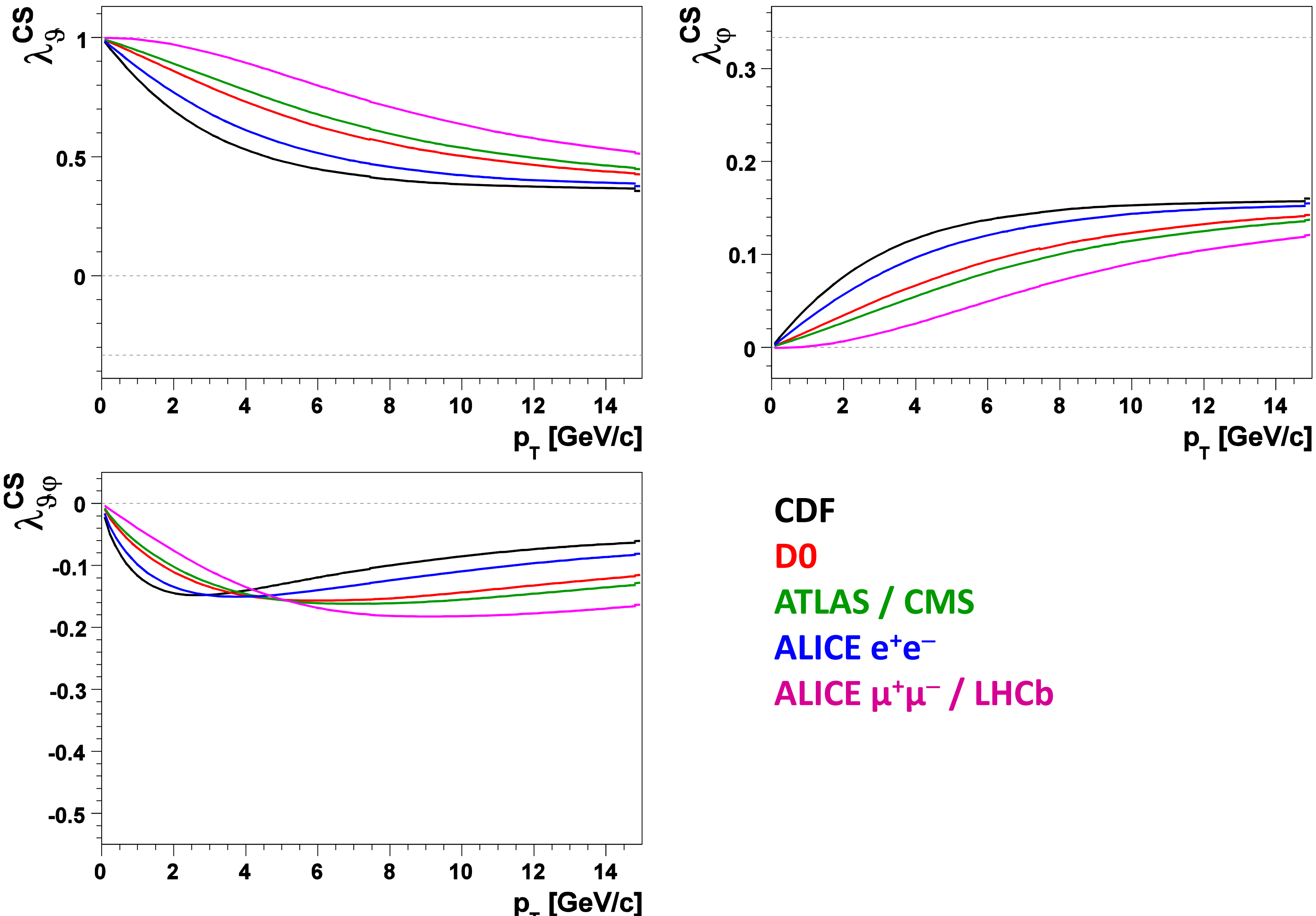
Let us consider, for illustrative purposes, the following (purely hypothetic) mixture of subprocesses for Υ production:

1) $\boldsymbol{f^{(1)}}$ **= 60%** of the events have a natural **transverse** polarization in the **CS** frame

2) $\boldsymbol{f^{(2)}}$ **= 40%** of the events have a natural **transverse** polarization in the **HX** frame

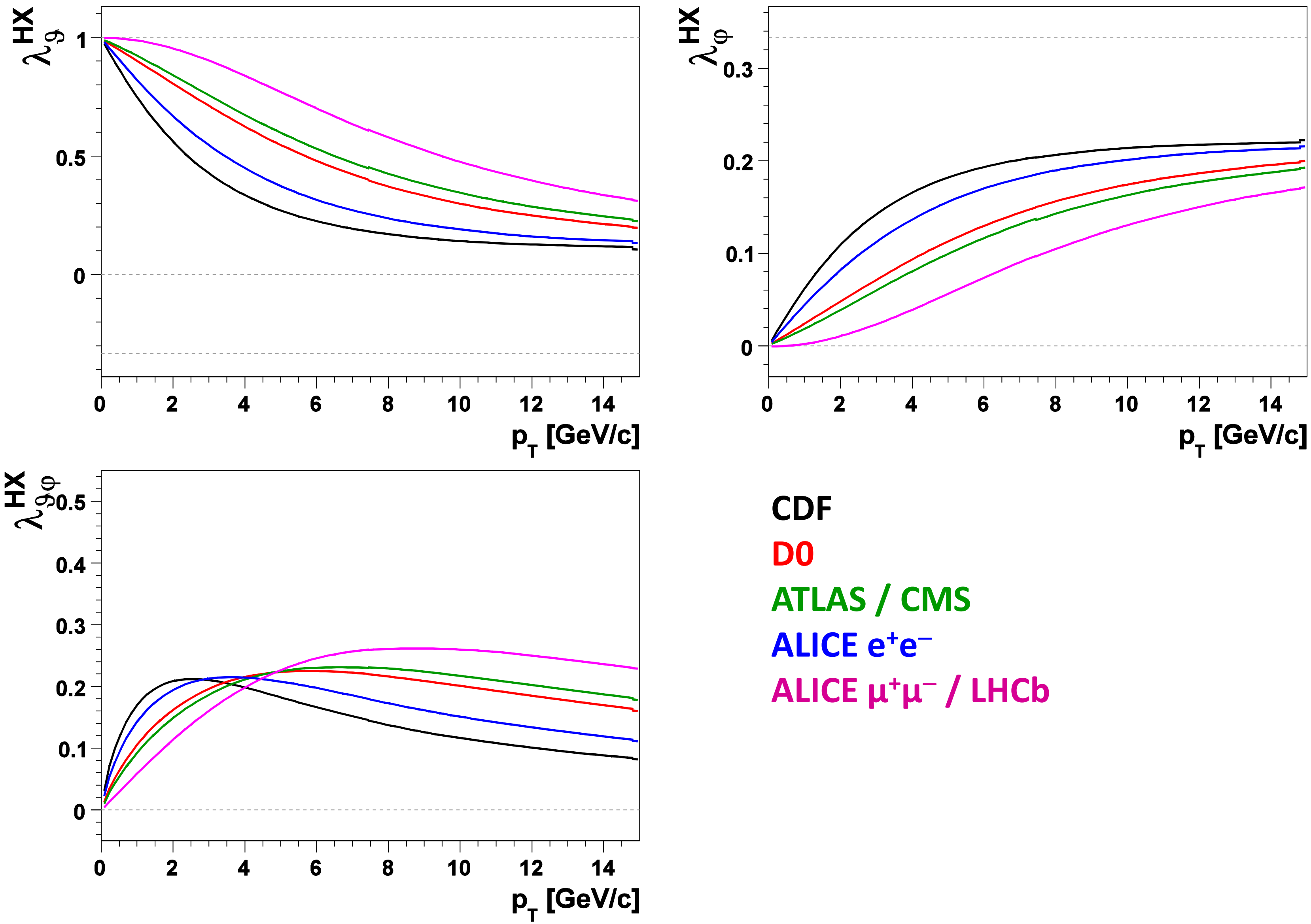
Frame choice 1

All experiments choose the CS frame



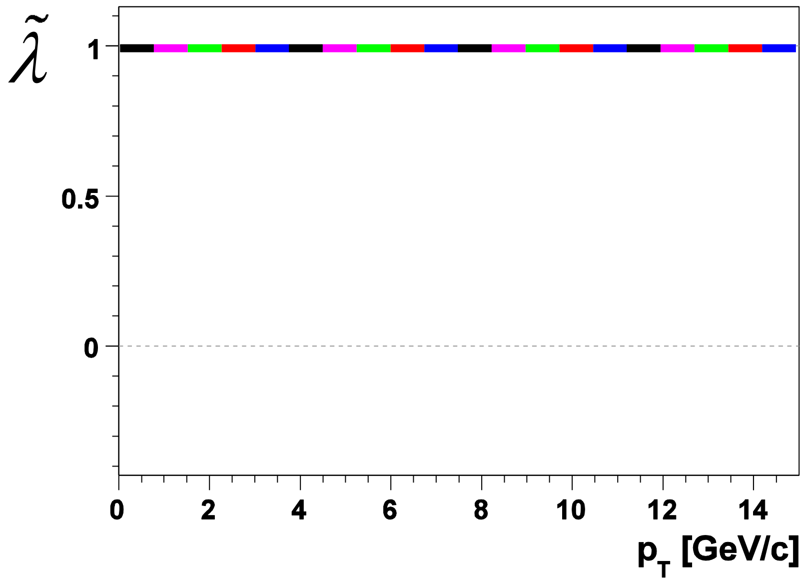
Frame choice 2

All experiments choose the HX frame



Any frame choice

The experiments measure an invariant quantity, for example $\tilde{\lambda} = \dfrac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi}$



[c_1=1, c_2=–3, c_3=0, c_4=1]

CDF
D0
ATLAS / CMS
ALICE e^+e^-
ALICE $\mu^+\mu^-$ / LHCb

When the observed distribution reflects the superposition of two or more "natural polarizations" $\lambda_\vartheta^{*(i)}$, the quantity $\tilde{\lambda}$ is equal to their weighted average, irrespectively of the directions of the corresponding axes:

$$\tilde{\lambda} = \frac{\displaystyle\sum_{i=1}^{n} \frac{f^{(i)}}{3 + \lambda_\vartheta^{*(i)}} \lambda_\vartheta^{*(i)}}{\displaystyle\sum_{i=1}^{n} \frac{f^{(i)}}{3 + \lambda_\vartheta^{*(i)}}}$$

The "Lam-Tung" limit

Another consequence of the rotational properties of angular momentum eigenstates is that for each single mixed state $\left|\psi^{(i)}\right\rangle = A_0^{(i)}\,|0\rangle + A_{+1}^{(i)}\,|{+1}\rangle + A_{-1}^{(i)}\,|{-1}\rangle$ there always exists a quantization axis *z'* with respect to which $A_0^{(i)\prime} = 0$.

Consequently, quarkonium produced in each single elementary subprocess is always characterized by a dilepton decay distribution of the type

$$\lambda_\vartheta^{(i)\prime} = +1, \quad \lambda_\varphi^{(i)\prime} = 2K^{(i)} - 1, \quad \lambda_{\vartheta\varphi}^{(i)\prime} = 0 \qquad \left(K^{(i)} = \tfrac{1}{2}\left|A_{+1}^{(i)} + A_{-1}^{(i)}\right|^2\right)$$

wrt its specific "$A_0^{(i)\prime} = 0$" axis.

What we have actually considered in the previous example is the "Drell-Yan-like case" $K^{(i)} = \tfrac{1}{2}$: each subprocess is characterized by a fully transversely polarized decay distribution ($\lambda_\vartheta^{(i)\prime} = +1,\ \lambda_\varphi^{(i)\prime} = \lambda_{\vartheta\varphi}^{(i)\prime} = 0$) *wrt a certain "natural" axis, which may be different from subprocess to subprocess.* Then $K = \sum f^{(i)} K^{(i)} = 1/2$:

$$K = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta} = \frac{1}{2} \quad \rightarrow \quad \lambda_\vartheta + 4\lambda_\varphi = 1 \qquad \text{(Lam-Tung relation)}$$

Actually, for the di-fermion decay of any *J* = 1 particle (even in parity-violating cases: *W*, *Z*) it is possible to calculate a frame-independent relation of the form

$$(1 - K)\lambda_\vartheta + 2\lambda_\varphi = 3K - 1 \qquad \text{with} \qquad K = \frac{1}{2}\sum_{i=1}^{n} f^{(i)}\left|A_{+1}^{(i)} + A_{-1}^{(i)}\right|^2 \qquad (0 \le K \le 1)$$

Why using two frames is important for the analysis

As already discussed, detector and data selection constraints strongly “polarize” the reconstructed dilepton events. Background processes may also affect the measured polarization if not well subtracted.

The “detector polarization frame” is naturally defined in the LAB frame, not in the quarkonium rest frame: there is no “rotation” correlating it with the physically interesting frames. Something similar may be expected for the “background polarization frame”. In general, therefore, the spurious “polarizations” do not follow the physical transformation rules from one quarkonium polarization frame to another.

If not well corrected and subtracted, these effects will affect the *shape* of the measured quarkonium distribution differently in different polarization frames. In particular, they will *violate the expected frame-independent relations* between the quarkonium angular parameters.

For this reason, checking whether the same value of an invariant quantity is obtained (within systematic errors) in two distinct polarization frames is a non-trivial test.

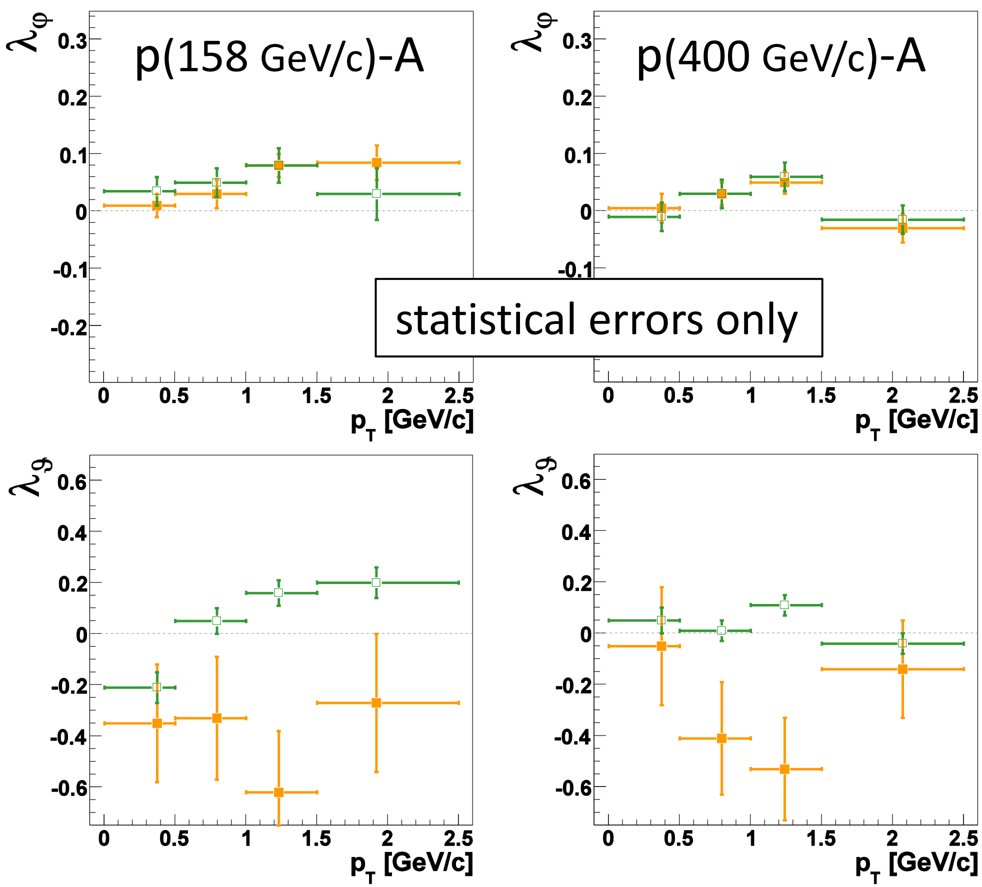
Any two physical polarization axes (defined in the quarkonium rest frame and belonging to the production plane) may be chosen. HX and CS frames are ideal choices at high p_T . At low p_T , where the difference between these two frames tends to vanish, any of the two and its exact orthogonal may be used to perform the test.

Example

Any of the invariant relations imply that, given two frames *A* and *B*,

$$\lambda_\vartheta^B = \lambda_\vartheta^A \Leftrightarrow \lambda_\varphi^B = \lambda_\varphi^A \Leftrightarrow \begin{cases} B = A \\ \text{or } \lambda_\vartheta = \lambda_\varphi = 0 \end{cases}$$

NA60 J/ψ prelim. (QM09) **HX** / **CS**



At first glance: λ_φ(CS) ≈ λ_φ(HX)
while λ_ϑ(CS) < λ_ϑ(HX)

→ check quantitatively by calculating the average "polarization" constant

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi}$$

$$\tilde{\lambda}(\text{HX}) - \tilde{\lambda}(\text{CS}) = \begin{cases} 0.49\ [\pm 0.13]\ \ 158\ \text{GeV/c} \\ 0.28\ [\pm 0.12]\ \ 400\ \text{GeV/c} \end{cases}$$

(errors not so relevant: CS and HX data are statistically correlated)

order of magnitude of the expected systematic error on the anisotropy parameters

Take-home messages

- Measure the full decay angular distribution
- Measure in no less than two reference frames
- Do not integrate out kinematic dependencies
- Report results also in terms of frame-invariant quantities