The Flavor of Heterotic Gauge/Gravity Duality

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Based on

JHEP 0910:004, 2009
+ *unpublished work*

Hermitian YM Instantons on CY Cones

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Outline

- Motivation
- Hermitian YM instantons on Calabi-Yau cones
- Comments on the gauge theory duals

Gauge/Gravity Duality

Starting with Maldacena-Nuñez solution, there's been great deal of interest on constructing gravity duals of N=1 SQCD using 5-branes:

- Basic Type II construction describes 5-branes wrapped on compact 2-cycles of M^4 x (deformed conifold).
- The geometry has non-zero NS flux H_3 *and* position dependent dilaton ;
- The dual gauge theory has N=1 susy and at low-energies is a 4d SYM theory.

Gauge/Gravity Duality

- Adding flavor (fundamental matter) proceeds now by wrapping extra 5-branes (or 7-branes) on non-compact 2-cycles extending to the UV.

- There are some technical issues regarding the singular nature of these new sources, usually tackled by smearing along isometries...

Here, I shall be interested on similar constructions with heterotic NS 5 branes,

with an important twist

Heterotic Gauge/Gravity Duality

- Instead of using *smeared* 5-branes to flavor the theory, consider a *necessary* ingredient of heterotic theories:

Hermitian YM instantons

- Crucial is that these act as sources for 3-form flux

$$dH_3 \sim \mathrm{tr}\mathcal{F}^2 - \mathrm{tr}\mathcal{R}^2$$

pretty much like 5-branes would do, but they are *naturally smeared.*

We will thus proceed to

- construct HYM instantons on 6d CY cones

and check if

- these have interesting field theory duals

For simplicity (just for this talk), I shall consider the backreaction on the geometry/string coupling only towards the end of the talk...

6D Calabi-Yau Cones

- Any 6D CY cone is a cone over a 5D Einstein-Sasaki

$$ds_B^2 = d\rho^2 + \rho^2 ds_{Sasaki}^2$$

where the latter is a fibration over an Einstein-Kähler 2-fold:

$$ds_{Sasaki}^2 = \eta^2 + 2\mathcal{K}_{w\bar{w}} dw d\bar{w}$$

- *Here,* η *is given as*

$$\eta = d\phi - i(\mathcal{K}_w dw - \mathcal{K}_{\bar{w}} d\bar{w})$$

while $\mathcal{K}$ *is the 4d Kähler potential.*

➡ The Sasaki space has at least one isometry, $\partial/\partial\phi$, which is inherited by the CY cone.

HYM Instantons on CY$_3$ cones

An *hermitian Yang-Mills* instanton is a rather special solution of the Yang-Mills equations *DF*=0, *D*F*=0 in that:

- It has the lowest value of the action (within its topological class);
- It has a non-vanishing instanton charge;
- It satisfies 1st order equations on the gauge connection *A*

$$\mathcal{F}_{ij}(\mathcal{A}) = 0, \qquad g^{i\bar{j}}\mathcal{F}_{i\bar{j}}(A) = 0$$

- In 4d CY the condition of *HYMness* reduces to the familiar condition of (anti-)self-duality $F=\pm{}^*F$.

Ansatz

On a CY_3 cone, there is a natural *Ansatz* for the YM gauge connection A: Take $\mathcal{A}$ to be the (1,0)-part of A, and

$$\mathcal{A} = \Phi\epsilon + \mathcal{B}$$

where

$$\Phi = \Phi(\rho, w, \bar{w}), \quad \mathcal{B} = \mathcal{B}_a(\rho, w, \bar{w}) dw^a$$
$$\epsilon = d\ln\rho + i\eta$$

Remarks:

- The w^a are holomorphic coordinates on the 4d Einstein-Kahler base space.
- As the CY, **the connection is invariant under the isommetry** $\phi \to \phi + c$.
- We will use some of the gauge freedom to set $\Phi = \Phi^\dagger$.

- Φ is adjoint valued Higgs, $\mathcal{B}$ gauge field from 4d point of view.

Imposing the HYM conditions $\mathcal{F}_{ij} = 0 = g^{i\bar{j}}\mathcal{F}_{i\bar{j}}$ we then find

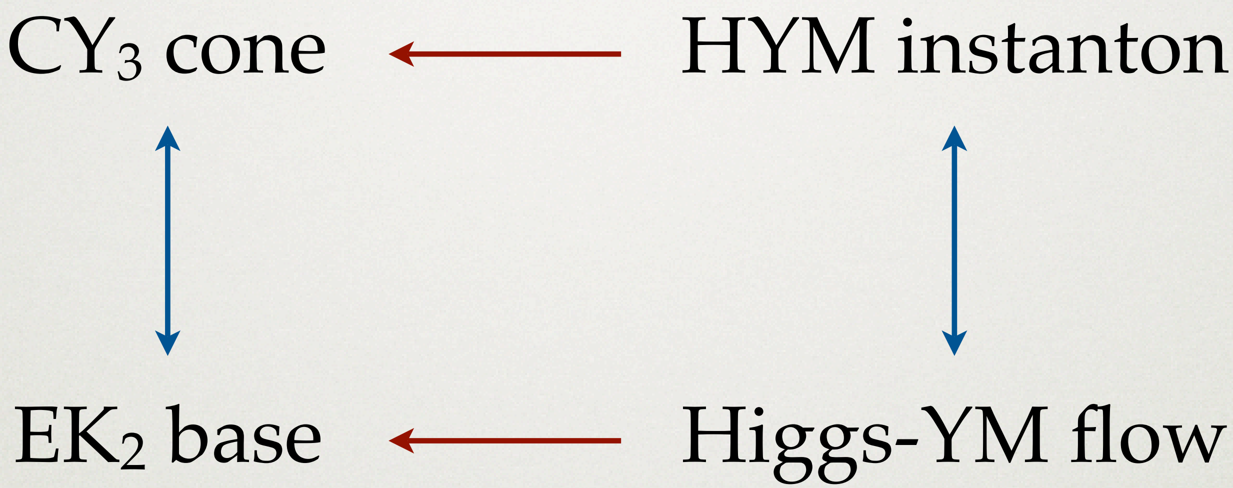
$$\mathcal{F}_{ab}(\mathcal{B}) = 0$$

$$\partial_t \mathcal{B} = D_{\mathcal{B}}\Phi$$

$$\partial_t \Phi = -2\Phi + \frac{1}{4} g^{a\bar{b}} \mathcal{F}_{a\bar{b}}(B)$$

where $t=2\,ln\varrho$ is a new radial coordinate.

The above equations can be seen as ***flow equations***, with flow parameter t, for the adjoint Higgs Φ and the Yang-Mills connection $\mathcal{B}$ on the 4d Einstein-Kahler space.


CY_3 cone
HYM instanton
EK_2 base
Higgs-YM flow

The Higgs-YM-Flow

We list a few *properties* of the flow:

(1) instantons solutions correspond to flows between 2 fix-points of the flow equations with

$$\mathcal{F}_{ab}(\mathcal{B}) = 0 \qquad D_{\mathcal{B}}(g^{a\bar{b}}\mathcal{F}_{a\bar{b}}) = 0$$

That is, the flow interpolates between 2 solutions of the YM equations on the 4D EK space;

(2) there is an entropy functional

$$N(t) = (\partial_t + 2)\int_{EK} \mathrm{tr}\Phi^2$$

satisfying $\partial_t N(t) \geq 0$.

Solutions of the Flow

- Since a CY_3 has SU(3) holonomy, it is natural to search for SU(3) instantons. The simplest ansatz is now to take the adjoint Higgs Φ to be *constant* on the EK base:

$$\Phi(t) = \varphi(t) \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2}\mathbf{1_2} \end{pmatrix}$$

- Then, the flow equations imply:

$$\mathcal{B}(t) = \begin{pmatrix} \partial\mathcal{K} & 0 \\ E(t) & \mathcal{C} + \mathbf{1_2}(\partial\mathcal{K}) \end{pmatrix} , \quad E_a(t) = -\sqrt{2} e^{\frac{3}{2}\int \varphi dt} e_a$$

where $\mathcal{C}$ is the $U(2)$ spin connection on the EK_2 base, and e_a the zweibein.

- Finally, φ(*t*) satisfies the 2nd order equation

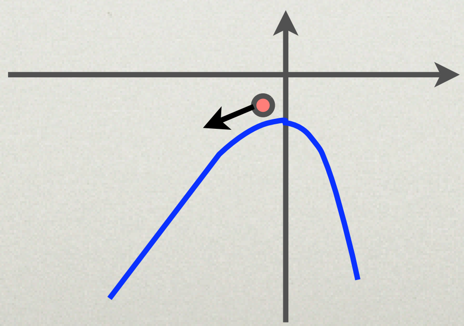
$$\partial_t \varphi + 2\varphi + 1 - e^{3\int^t \varphi} = 0$$

This has a 1-parameter (non-singular) solution with monotically changing φ(*t*) , interpolating between $-\frac{1}{2}$ at the *UV* and 0 at the *IR*:

In fact, introducing $X = \int^t \varphi dt'$ *, this Eq. becomes*

$$\partial_t^2 X + 2\partial_t X = -\frac{d}{dX}(X - \tfrac{1}{3}e^{3X})$$

which describes the motion of a damped particle in the potential $V(X) = X - \frac{1}{3}e^{3X}$



Remarks:

- For every CY$_d$ cone we can find a 1-parameter family of SU(d) HYM instantons. This parameter is the instanton's size.
- For specific CY$_3$ cones, e.g. the conifold, it is possible to find *also* SU(2) instantons (or flows) satisfying similar equations;
- The instanton charge, $N_3 = \int_{CY_3} ch_3(\mathcal{F})$, is finite and depends only on the topology of the 5d Sasaki-Einstein space and of the gauge bundle:

$$N_3 = -\frac{1}{4\pi}\int_{SE_5} \eta \wedge p_1(EK_2) + \frac{5}{8\pi^3} Vol(SE_5)$$

Comments on Gauge Theory duals

I will comment on the conifold case:

Let us recall that:

- the conifold is a CY_3 cone for which the Einstein-Kahler base is a direct product $S^2 \times S^2$ of 2 round spheres;
- following Aharony et al. '98, heterotic conifolds have gauge theory duals only in the decoupling limit defined by imposing that at the UV the string coupling vanishes as

$$g_s = e^{\phi} \rightarrow \frac{c}{\rho^2}$$

- In turn, this implies we have to look also at the 3-form *NS* flux H_3 induced by the gauge instantons, and its backreaction on the string coupling and the conifold geometry (including α′-corrections);

Doing this we find:

$$H_3 = c_1 N(t)\ \eta \wedge J$$

and the conifold geometry becomes

$$e^{\phi}\left[\frac{d\rho^2}{f^2} + \rho^2 f^2 \eta^2 + \rho^2 (ds^2_{S^2} + ds^2_{S^2})\right]$$

where the string coupling and f^2 obey flow equations

$$f^2 \partial_\rho e^{\phi} = -c_2 \frac{N(\rho)}{\rho^3} \qquad \partial_\rho f^2 = f^2 \partial_\rho \phi - \frac{6}{\rho}(f^2 - 1)$$

Remarks:

- The only instanton dependent quantity entering these equations is $N(t)$, the sum of the "entropies" of the SU(3) and SU(2) instantons;

- For a **slow varying** $N(t)$, we expect f^2 to be slow varying too, and the string coupling to be well approximated by

$$e^{\phi} \sim \frac{N(\rho)}{\rho^2}$$

The Dual

The dual is the (4d) gauge theory obtained by wrapping *NS 5*-branes on holomorphic 2-cycles on the base of the conifold. (That is, little string theory compactified on those 2-cycles.)

We know the following properties:

- The gauge theory has N=1 supersymmetry
- Global symmetries are the isometries of conifold times the SO(32)/G where G=SU(3)xSU(2)^{k}xU(1)$^{k'}$ depends on number and type of instantons breaking the SO(32);

- Gauge symmetry, resulting from compactification of the 6D Sp($2N_c$) theory has number of colors given by the NS-flux H_3 . Since $H_3 \sim N(t)$ decreases from the UV to the IR, gauge symmetry is broken every time we pass by an instanton; the same happens with N_f , in fact we have $N_f \sim N_c$

- The gauge coupling is proportional to the size of the wrapped S^2. That is

$$\frac{1}{g^2} \sim e^{\phi}\rho^2 \sim N(\rho)$$

- Again, since N is monotonically decreasing towards the *IR*, where it vanishes, we find that this gauge theory is *strongly coupled* in the *IR*, *weak* in the *UV.*

- The instanton moduli space should match the gauge theory moduli space.

Outlook

For an isommetry preserving ansatz:

- We've shown that the HYM equation on CY_d cones is equivalent to an Higgs-YM flow on the EK_{d-1} base.
- Several properties of the flow (entropy $N(t)$, instanton numbers) could be derived, and
- 1-parameter families of SU(d) instanton solutions were explicitely presented.

Open issues:

- computing metrics on moduli space
- modifying UV b. c.'s for further gauge/gravity duals