Leptogenesis and Lepton Flavour Violation in See-Saw Models with A_4 Symmetry

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C. Hagedorn, E.M., S. T. Petcov, JHEP **0909** (2009) 115
C. Hagedorn, E.M., S. T. Petcov, JHEP **1002** (2010) 047

Tri-bimaximal mixing

- Mixing pattern in the lepton sector revealed by neutrino experiments:

$$\sin^2\theta_{12} = 0.304^{+0.066}_{-0.054} \quad \text{and} \quad \sin^2\theta_{23} = 0.50^{+0.17}_{-0.14} \quad (3\sigma)$$

$$\sin^2\theta_{13} < 0.056 \quad (3\sigma)$$

- Tri-bimaximal mixing

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}$$

- TB mixing can arise at LO in a class of models in which the alternating (tetrahedral) group A_4 serves as flavour symmetry group
- A_4 models have a very simple structure and a rich phenomenology: *light ν mass spectrum with normal and inverted ordering, constraints on Majorana phases, thermal leptogenesis, ...*

See-saw models with A_4 flavour symmetry

- Left-handed lepton doublets l and right-handed neutrinos ν^c transform as triplets under A_4
- Right-handed charged leptons e, μ and τ in one-dimensional unitary representations of A_4
- Neutrino Yukawa couplings at LO in powers of the cut-off Λ

$$y_\nu(\nu^c l)h_u \; + \; \ldots$$

- Majorana mass matrix of right-handed neutrinos generated through the coupling:

$$a\xi(\nu^c\nu^c) + b(\nu^c\nu^c\varphi_S) \; + \; \ldots$$

$$\varphi_S \sim \mathbf{3} \qquad \xi \sim \mathbf{1}$$

$$\langle\varphi_S\rangle \; = \; v_S\,\epsilon\,\Lambda(1,1,1)^t \qquad \langle\xi\rangle \; = \; u\epsilon\Lambda$$

- $\epsilon \sim 0.007 \div 0.05$ is given by the ratio of the VEV of the flavon field and the cut-off scale Λ of the theory
- $2 \lesssim \tan\beta \lesssim 15$ related to the value of ϵ

See-saw models with A_4 flavour symmetry

$$m_D = y_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_u \qquad m_M = \begin{pmatrix} X+2Z & -Z & -Z \\ -Z & 2Z & X-Z \\ -Z & X-Z & 2Z \end{pmatrix}$$

$$U_{TB}{}^T m_M U_{TB} = \text{diag}(M_1 e^{i\phi_1}, M_2 e^{i\phi_2}, M_3 e^{i\phi_3})$$

- The model at LO is determined by three parameters:

$$\alpha = |3Z/X| \,, \qquad \phi = \arg(Z) - \arg(X) \,, \qquad |X| \sim 10^{14\div 15}\,\text{GeV}$$

- Light neutrino masses from type I see-saw mechanism: $m_i = y_\nu^2 v_u^2 / M_i$
- Neutrino mixing matrix TB at LO:

$$U_{\text{PMNS}} \approx U_{TB}\,\text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$$

$$\alpha_{21} \equiv \phi_2 - \phi_1 \qquad \alpha_{31} \equiv \phi_3 - \phi_1$$

- Majorana phases depend at LO only on α and ϕ:

$$\tan\alpha_{21} = -\frac{\alpha\sin\phi}{1+\alpha\cos\phi} \qquad \tan\alpha_{31} = 2\,\frac{\alpha\sin\phi}{\alpha^2-1}$$

See-saw models with A_4 flavour symmetry

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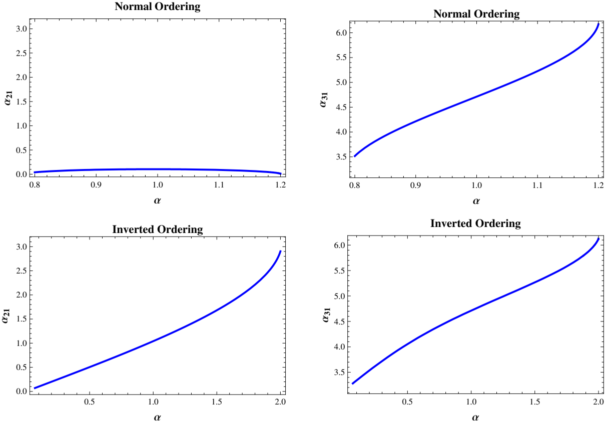
- Low energy constraints on the parameter space:

$$\frac{\Delta m^2_\odot}{|\Delta m^2_{\mathrm{A}}|} = \frac{(1+\alpha^2 - 2\,\alpha\,\cos\phi)(\alpha + 2\cos\phi)}{4\,|\cos\phi|} = 0.032 \pm 0.006$$

Majorana phases and leptogenesis



C. Hagedorn, E.M., S. T. Petcov, JHEP **0909** (2009) 115

- Majorana phases α_{21} and α_{31} *provide necessary CP violation for leptogenesis*
- $\sin\phi < 0$ is derived by the requirement that the predicted baryon asymmetry in the thermal leptogenesis scenario has correct sign
- Deviations from TB mixing pattern *are important* in order to have a CP asymmetry different from zero and successful leptogenesis $\rightarrow$ model dependent NLO corrections

Specific SUSY A_4 models

We study LFV in two specific models in the mSUGRA framework:

- G. Altarelli and F. Feruglio, Nucl. Phys. B **741** (2006) 215

Field	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	φ_T	φ_S	$\xi,\ \tilde{\xi}$
A_4	3	1	$1''$	$1'$	3	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	ω^2	ω^2

$$\text{Flavour Group}: \ = A_4 \times Z_3 \times U(1)_{FN}$$

- G. Altarelli and D. Meloni, J. Phys. G **36** (2009) 085005

Field	l	e^c	μ^c	τ^c	ν^c	h_d	h_u	φ_T	ξ'	φ_S	ξ
A_4	3	1	1	1	3	1	$1''$	3	$1'$	3	1
Z_4	i	1	i	-1	-1	1	1	i	i	1	1

$$\text{Flavour Group}: \ = A_4 \times Z_4$$

The form of the NLO corrections of the neutrino Yukawa couplings, charged lepton and RH neutrino mass matrix provides different predictions on the charged lepton flavour violating radiative decays in the mSUGRA scenario

$B(\ell_i \to \ell_j + \gamma)$: leading order contribution

- We consider flavour universal boundary conditions at the GUT scale (mSUGRA): m_0, $m_{1/2}$, $A_0 = a_0 m_0$, $\tan\beta$
- LFV in Y_ν $\to$ LFV in the slepton mass matrix at low energy due to RGE

$$\frac{B(\ell_i \to \ell_j + \gamma)}{B(\ell_i \to \ell_j + \nu_i + \bar{\nu}_j)} \approx B_0(m_0, m_{1/2}) \left| \sum_k (\hat{Y}_\nu^\dagger)_{ik} \log\left(\frac{M_X}{M_k}\right) (\hat{Y}_\nu)_{kj} \right|^2 \tan^2\beta$$

$$\hat{Y}_\nu = V_R^T \, \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) \, U_{TB}^T \, Y_\nu \, V_{eL}$$

Borzumati and Masiero, Phys. Rev. Lett. **57** (1986) 961
Hisano et al., Phys. Lett. B **357** (1995) 579

...

$$B_0(m_0, m_{1/2}) \approx \frac{\alpha_{em}^3}{G_F^2 m_S^8} \left| \frac{(3 + a_0^2) m_0^2}{8\pi^2} \right|^2$$

$$m_S^8 \approx 0.5 \, m_0^2 \, m_{1/2}^2 \, (m_0^2 + 0.6 \, m_{1/2}^2)^2$$

S. T. Petcov et al., Nucl. Phys. B **676** (2004) 453

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- LFV in $Y_\nu \to$ LFV in the slepton mass matrix at low energy due to RGE

$$B(\ell_i \to \ell_j + \gamma) \;\propto$$
$$\left|(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ij} \log\left(\frac{m_1}{m_*}\right) + (\hat{Y}_\nu^\dagger)_{i2}(\hat{Y}_\nu)_{2j} \log\left(\frac{m_2}{m_1}\right) + (\hat{Y}_\nu^\dagger)_{i3}(\hat{Y}_\nu)_{3j} \log\left(\frac{m_3}{m_1}\right)\right|^2$$

$$m_* \;=\; \frac{v_u^2 y_\nu^2}{M_X} \cong (1.5 \times 10^{-3}\,\mathrm{eV})\, y_\nu^2 \sin^2\beta \approx 1.5 \times 10^{-3}\,\mathrm{eV}$$

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General Features of $B(\ell_i \to \ell_j + \gamma)$

- For NO light ν mass spectrum *all three branching ratios scale as* ϵ^0 and

$$B(\tau \to \mu + \gamma) \approx 10 B(\tau, \mu \to e + \gamma)$$

  suppression of the branching ratios given by off-diagonal elements of slepton mass matrix being generated through RGE

- For IO light ν mass spectrum

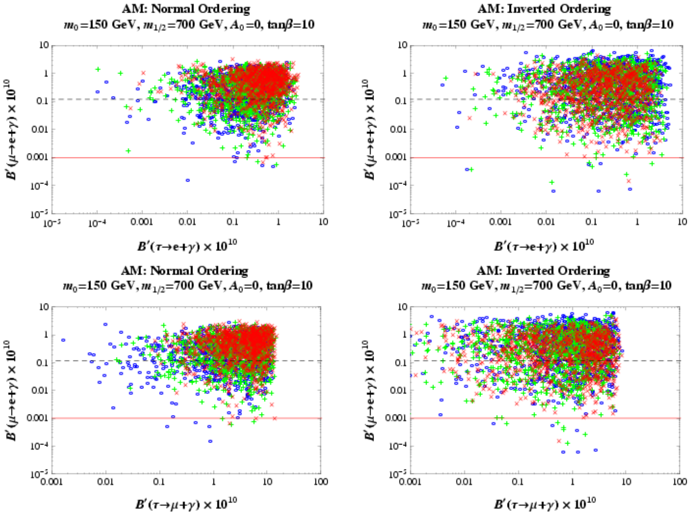
$$\frac{B(\tau \to e + \gamma)}{B(\tau \to e\nu\bar{\nu})} \approx \frac{B(\mu \to e + \gamma)}{B(\mu \to e\nu\bar{\nu})} \propto \epsilon^2$$

  for $\epsilon \approx 0.04$ *and "generic" NLO correction* $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ij} \approx \mathcal{O}(\epsilon)$
  $\to B(\mu \to e + \gamma)$ *and* $B(\tau \to e + \gamma)$ *scale as few* $\times\, \epsilon$
  $B(\tau \to \mu + \gamma)$ receives a contribution of order ϵ^0 like in NO

- For both NO and IO neutrino mass spectra $B(\tau \to e + \gamma)$ and $B(\tau \to \mu + \gamma)$ are in general below the sensitivity of present and future experiments

- Stronger constraints on the parameter space of the models come from present and future bounds on $\mu \to e + \gamma$

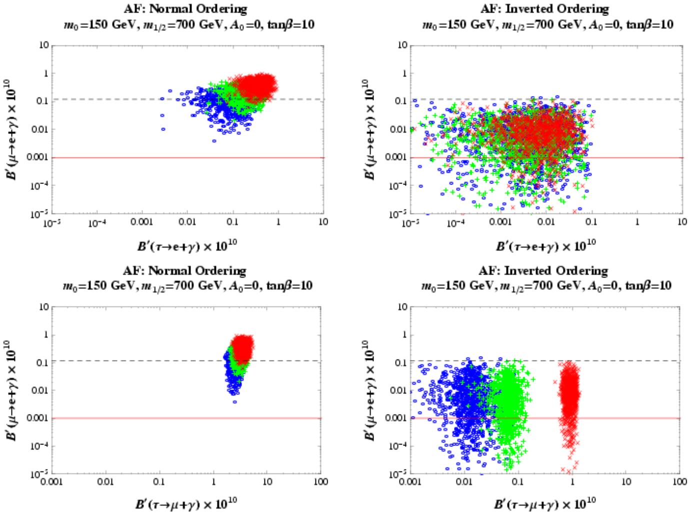
$A_4 \times Z_4$: $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ij} \approx \mathcal{O}(\epsilon)$ $\epsilon = 0.04$



NO : $m_1 = 3.8\,10^{-3}$ eV ($\times$), $5\,10^{-3}$ eV ($+$), $7\,10^{-3}$ eV ($\circ$)

IO : $m_3 = 0.02$ eV ($\times$), 0.06 eV ($+$), 0.1 eV ($\circ$)

$A_4 \times Z_3 \times U(1)_{FN}$: $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ij} \approx \mathcal{O}(\epsilon^2)$ $\epsilon = 0.04$



NO : $m_1 = 3.8\,10^{-3}$ eV (×), $5\,10^{-3}$ eV (+), $7\,10^{-3}$ eV (○)

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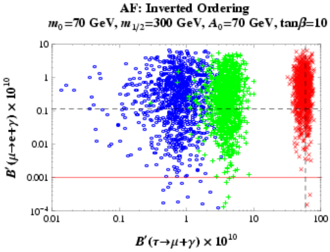
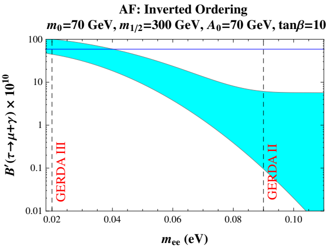
$A_4 \times Z_3 \times U(1)_{FN}$





$$\textit{large } B(\mu \to e + \gamma) > 10^{-13} \textit{ and } B(\tau \to \mu + \gamma) \approx 10^{-9}$$

$$\text{for } m_3 \approx 0.02 \text{ eV} \to m_{ee} \approx \sqrt{m_3^2 + |\Delta m_{\rm A}^2|}|2 + e^{i\alpha_{21}/2}|/3 \approx (0.018 \div 0.054) \text{ eV}$$

Summary

- A_4 models predict at LO in the symmetry breaking parameter $\epsilon \approx 0.007 \div 0.05$ exact TB mixing in the neutrino sector
- Parameter space at LO is highly constrained by low energy neutrino phenomenology and *e.g.* by the requirement of successful thermal leptogenesis ($\sin\phi < 0$)
- LO results are corrected by higher order terms, suppressed in general by a factor ϵ, which give deviations from TB mixing and can play an important role in the phenomenology of the models (*e.g.* viable thermal leptogenesis)
- Large $\mu \to e + \gamma$ decay rate can be realized in this scenario for a SUSY mass scale observable at LHC and put strong constraints on the parameter space of specific models
- $\tau \to e + \gamma$ and $\tau \to \mu + \gamma$ are in general below the sensitivity of present and future experiments

Generic NLO corrections to the superpotential

$\mu \to e + \gamma$	
$(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}$ $(\hat{Y}_\nu^\dagger)_{22}(\hat{Y}_\nu)_{21}$ $(\hat{Y}_\nu^\dagger)_{23}(\hat{Y}_\nu)_{31}$	$y_\nu\,(\,w''\overline{y}^\nu_{1'} + w'y^\nu_{1''} - x_2(y^\nu_A + y^\nu_S) - x_3(\overline{y}^\nu_A + \overline{y}^\nu_S)\,)\,\epsilon + \mathcal{O}(\epsilon^2)$ $\frac{1}{3}\,y_\nu^2 \;+\; \mathcal{O}(\epsilon)$ $\mathcal{O}(\epsilon)$
$\tau \to e + \gamma$	
$(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{31}$ $(\hat{Y}_\nu^\dagger)_{32}(\hat{Y}_\nu)_{21}$ $(\hat{Y}_\nu^\dagger)_{33}(\hat{Y}_\nu)_{31}$	$y_\nu\,(\,w''y^\nu_{1'} + w'\overline{y}^\nu_{1''} + x_2(\overline{y}^\nu_A - \overline{y}^\nu_S) + x_3(y^\nu_A - y^\nu_S)\,)\,\epsilon + \mathcal{O}(\epsilon^2)$ $\frac{1}{3}\,y_\nu^2 \;+\; \mathcal{O}(\epsilon)$ $\mathcal{O}(\epsilon)$
$\tau \to \mu + \gamma$	
$(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{32}$ $(\hat{Y}_\nu^\dagger)_{32}(\hat{Y}_\nu)_{22}$ $(\hat{Y}_\nu^\dagger)_{33}(\hat{Y}_\nu)_{32}$	$y_\nu\,(\,w''\overline{y}^\nu_{1'} + w'y^\nu_{1''} + 2\,x_2\,y^\nu_S + 2\,x_3\,\overline{y}^\nu_S\,)\,\epsilon + \mathcal{O}(\epsilon^2)$ $\frac{1}{3}\,y_\nu^2 \;+\; \mathcal{O}(\epsilon)$ $-\frac{1}{2}\,y_\nu^2 \;+\; \mathcal{O}(\epsilon)$

$$\mathrm{AF}: \quad w' = w'' = 0 \quad (x_1, x_2, x_3) \propto (1, 0, 0)$$
$$\mathrm{AM}: \quad w' = w'' = 0 \quad (x_1, x_2, x_3) \propto (1, 1, 1)$$