GUTs and Flavour: $SU(5) \times S_4$ as Example

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Outline

- Observations: fermion masses and mixings
- Basics of $SU(5) \times S_4$ model
- Main results of the model
- Different aspects of the model
- Conclusions & Outlook

Observations: Fermion Masses and Mixings

	Mass at M_Z	in units of $m_t(M_Z)$
u	(1.7 ± 0.4) MeV	λ^8
c	(0.62 ± 0.03) GeV	λ^4
t	(171 ± 3) GeV	1

$$\lambda \equiv \theta_C \approx 0.22$$

	Mass at M_Z	in units of $m_b(M_Z)$
d	(3.0 ± 0.6) MeV	λ^4
s	(54 ± 8) MeV	λ^2
b	(2.87 ± 0.03) GeV	1

	Mass at M_Z	in units of $m_\tau(M_Z)$
e	$(0.486570161 \pm 0.000000042)$ MeV	$\lambda^{4\div5}$
μ	$(102.7181359 \pm 0.0000092)$ MeV	λ^2
τ	$1.74624^{+0.00020}_{-0.00019}$ GeV	1

Observations: Fermion Masses and Mixings

- Mild hierarchy among light neutrino masses
  - Two mass squared differences Δm_{21}^2 and $|\Delta m_{31}^2|$ are known ($2\,\sigma$)

$$\Delta m_{21}^2 = \left(7.59^{+0.44}_{-0.37}\right)\cdot 10^{-5}\ \text{eV}^2 \quad \text{and} \quad |\Delta m_{31}^2| = \left(2.40^{+0.24}_{-0.22}\right)\cdot 10^{-3}\ \text{eV}^2$$

  - Cosmological data give upper bound on m_0

$$\sum m_i \lesssim 0.7\ \text{eV} \qquad (2\,\sigma)$$

  - The bounds on m_β and $|m_{ee}|$ also constrain m_0

$$m_\beta \leq 2.2\,\text{eV} \quad \text{and} \quad |m_{ee}| \leq (0.2 ... 1)\,\text{eV}$$

- Normal (NH) & inverted hierarchy (IH) still allowed

Observations: Fermion Masses and Mixings

- The mixing pattern in the lepton sector is very peculiar

$\sin^2(\theta^l_{12}) = 0.318^{+0.042}_{-0.028}$, $\sin^2(\theta^l_{23}) = 0.50^{+0.13}_{-0.11}$ and $\sin^2(\theta^l_{13}) \leq 0.039$

$\theta^l_{12} = (34.3^{+2.5}_{-1.7})^\circ$, $\theta^l_{23} = (45.0^{+7.5}_{-6.4})^\circ$ and $\theta^l_{13} \leq 11.4^\circ$ $(2\,\sigma)$

compare to quark sector $\theta^q_{12} \approx 13^\circ$, $\theta^q_{23} \approx 2.4^\circ$ and $\theta^q_{13} \approx 0.21^\circ$

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- Special mixing patterns
  - $\mu\tau$ symmetry

$$\sin^2(\theta^l_{23}) = \tfrac{1}{2}\,, \quad \sin^2(\theta^l_{13}) = 0$$

$$\Rightarrow U_{MNS} = \begin{pmatrix} \cos(\theta^l_{12}) & \sin(\theta^l_{12}) & 0 \\ -\frac{\sin(\theta^l_{12})}{\sqrt{2}} & \frac{\cos(\theta^l_{12})}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin(\theta^l_{12})}{\sqrt{2}} & -\frac{\cos(\theta^l_{12})}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

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- Special mixing patterns
  - Tri-bimaximal (TB) mixing

$$\sin^2(\theta^l_{12}) = \frac{1}{3}\ , \quad \sin^2(\theta^l_{23}) = \frac{1}{2}\ , \quad \sin^2(\theta^l_{13}) = 0$$
$$\Rightarrow U_{MNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Basic Setup

- SUSY $SU(5)$ Grand Unified Theory in 4 dimensions
- Effective theory with cutoff M for flavour dynamics
- Flavour symmetry is S_4
- $F \sim (\overline{\mathbf{5}}, \mathbf{3})$ and $N \sim (\mathbf{1}, \mathbf{3})$ for TB mixing
- $T_3 \sim (\mathbf{10}, \mathbf{1})$ and $T \sim (\mathbf{10}, \mathbf{2})$ for top quark mass
- Flavons (gauge singlets) break S_4 spontaneously
- Additional (global) $U(1)$ symmetry for forbidding unwanted operators
- GUT Higgs fields are $H_5 \sim (\mathbf{5}, \mathbf{1})$, $H_{\overline{5}} \sim (\overline{\mathbf{5}}, \mathbf{1})$ and $H_{\overline{45}} \sim (\overline{\mathbf{45}}, \mathbf{1})$

Main Results

- TB mixing encoded in right-handed neutrino Majorana mass terms
- Large top quark mass through $T + T_3 \sim \mathbf{2} + \mathbf{1}$ under S_4
- Quark mixings arise from down quark sector
- GST and GJ relations are achieved through specific high energy completion
- Corrections from charged lepton sector relevant for lepton mixings, lead to sum rules

Group Theory of S_4

- S_4 is the permutation group of four distinct objects, isomorphic to the symmetry group O of a regular octahedron, with order 24
- Irred. reps. are $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$ and $\mathbf{3}'$
- Generators S, T and U fulfill

$$S^2 = \mathbb{1} \; , \;\; T^3 = \mathbb{1} \; , \;\; U^2 = \mathbb{1} \; ,$$
$$(ST)^3 = \mathbb{1} \; , \;\; (SU)^2 = \mathbb{1} \; , \;\; (TU)^2 = \mathbb{1} \; , \;\; (STU)^4 = \mathbb{1}$$

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- Irred. reps. are $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$ and $\mathbf{3}'$
- Choice of S, T and U $(\omega = e^{2\pi i/3})$

$$\mathbf{1}: \quad S = 1\,, \qquad T = 1\,, \qquad U = 1\,,$$

$$\mathbf{1}': \quad S = 1\,, \qquad T = 1\,, \qquad U = -1\,,$$

$$\mathbf{2}: \quad S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \qquad U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\mathbf{3}: \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

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TB Mixing in Neutrino Sector

Field	F	N	H_5	$\Phi^\nu_{3'}$	Φ^ν_2	Φ^ν_1
$SU(5)$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3'}$	$\mathbf{2}$	$\mathbf{1}$
$U(1)$	y	$-y$	0	$2y$	$2y$	$2y$
$U(1)$	4	-4	0	8	8	8

Yukawas at leading order

$$y_D F N H_5 + \alpha N N \Phi^\nu_1 + \beta N N \Phi^\nu_2 + \gamma N N \Phi^\nu_{3'}$$

For vacuum

$$\langle \Phi^\nu_{3'} \rangle \propto (1,1,1)^t \ , \qquad \langle \Phi^\nu_2 \rangle \propto (1,1)^t \ , \qquad \langle \Phi^\nu_1 \rangle \neq 0$$

which leaves invariant the group generated by S and U in the neutrino sector ...

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Yukawas at leading order

$$y_D F N H_5 + \alpha N N \Phi^\nu_1 + \beta N N \Phi^\nu_2 + \gamma N N \Phi^\nu_{3'}$$

... we get

$$M_D = y_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_u \quad \text{and} \quad M_R = \begin{pmatrix} \alpha\varphi^\nu_1 + 2\gamma\varphi^\nu_{3'} & \beta\varphi^\nu_2 - \gamma\varphi^\nu_{3'} & \beta\varphi^\nu_2 - \gamma\varphi^\nu_{3'} \\ \beta\varphi^\nu_2 - \gamma\varphi^\nu_{3'} & \beta\varphi^\nu_2 + 2\gamma\varphi^\nu_{3'} & \alpha\varphi^\nu_1 - \gamma\varphi^\nu_{3'} \\ \beta\varphi^\nu_2 - \gamma\varphi^\nu_{3'} & \alpha\varphi^\nu_1 - \gamma\varphi^\nu_{3'} & \beta\varphi^\nu_2 + 2\gamma\varphi^\nu_{3'} \end{pmatrix}$$

Up Quarks

Field	T_3	T	H_5	Φ_2^u	$\widetilde{\Phi}_2^u$
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$U(1)$	0	x	0	$-2x$	0
$U(1)$	0	5	0	-10	0

Yukawas at leading order

$$T_3 T_3 H_5 + \frac{1}{M} T T \Phi_2^u H_5 + \frac{1}{M^2} T T \Phi_2^u \widetilde{\Phi}_2^u H_5$$

For

$$\langle \Phi_2^u \rangle \ , \ \langle \widetilde{\Phi}_2^u \rangle \ \propto (0,1)^t \ \text{ and } \ \langle \Phi_2^u \rangle / M \ , \ \langle \widetilde{\Phi}_2^u \rangle / M \approx \lambda^4$$

we get

$$M_u \approx \text{diag}(\varphi_2^u \widetilde{\varphi}_2^u / M^2, \varphi_2^u / M, 1) v_u \approx \text{diag}(\lambda^8, \lambda^4, 1) v_u$$

Down Quarks & Charged Leptons

Field	T_3	T	F	$H_{\overline{5}}$	$H_{\overline{45}}$	Φ_3^d	$\widetilde{\Phi}_3^d$	Φ_2^d
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$
$U(1)$	0	x	y	0	z	$-y$	$-x-y-2z$	z
$U(1)$	0	5	4	0	1	-4	-11	1

Yukawas at leading order

$$\frac{1}{M}FT_3\Phi_3^dH_{\overline{5}}+\frac{1}{M^2}(F\widetilde{\Phi}_3^d)_1(T\Phi_2^d)_1H_{\overline{45}}+\frac{1}{M^3}(F\Phi_2^d\Phi_2^d)_3(T\widetilde{\Phi}_3^d)_3H_{\overline{5}}$$

For vacuum alignment

$$\langle\Phi_3^d\rangle\propto(0,1,0)^t\ ,\ \ \langle\widetilde{\Phi}_3^d\rangle\propto(0,-1,1)^t\ ,\ \ \langle\Phi_2^d\rangle\propto(1,0)^t$$

and

$$\langle\Phi_3^d\rangle/M\approx\lambda^2\ ,\ \langle\widetilde{\Phi}_3^d\rangle/M\approx\lambda^3\ ,\ \ \langle\Phi_2^d\rangle/M\approx\lambda$$

GJ and GST Relations

... we find GJ relations

$$m_d \approx 3\, m_e \quad \text{and} \quad m_s \approx m_\mu/3 \quad \text{and} \quad m_b \approx m_\tau$$

and hierarchies

$$m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1 \quad \text{with} \quad m_b \approx m_\tau \approx \lambda^2\, v_d$$

Since $M_u \propto$ diag we get also the GST relation

$$\tan\theta_{12}^q \approx \sqrt{m_d/m_s}$$

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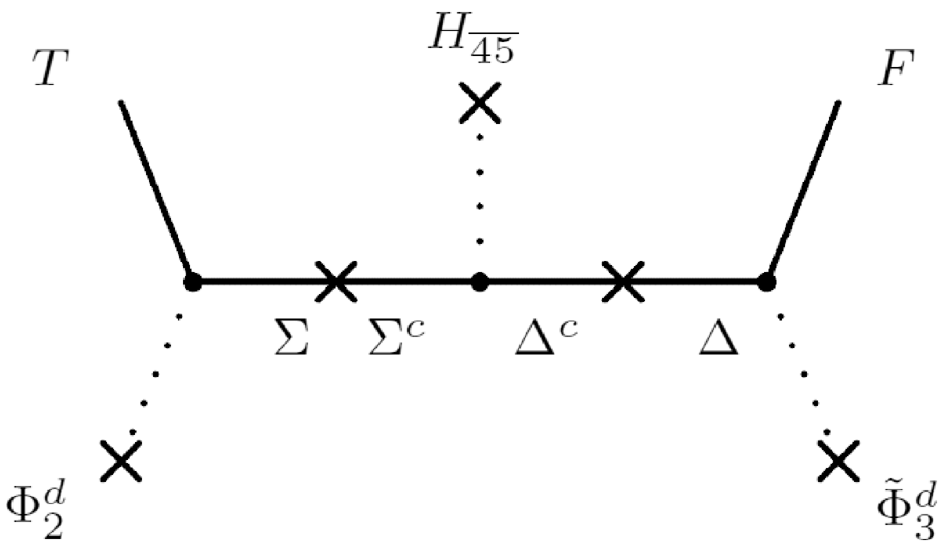
Note for $\frac{1}{M^2}(F\widetilde{\Phi}^d_3)_1(T\Phi^d_2)_1 H_{\overline{45}}$ specified high energy completion is needed



GJ and GST Relations

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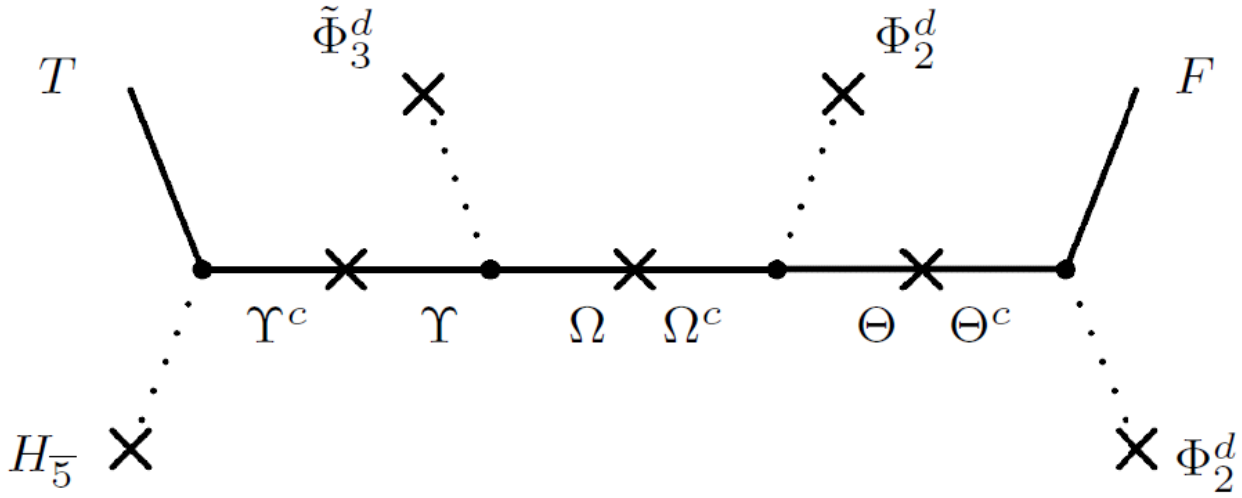
Note for $\frac{1}{M^3}(F\Phi_2^d\Phi_2^d)_3(T\widetilde{\Phi}_3^d)_3 H_{\overline{5}}$ specified high energy completion is needed



Lepton Mixings: Sum Rules

Mixing in neutrino sector (up to corrections of order λ^4)

$$\sin^2(\theta^\nu_{12}) \approx \frac{1}{3}\,, \quad \sin^2(\theta^\nu_{23}) \approx \frac{1}{2}\,, \quad \sin^2(\theta^\nu_{13}) \approx 0$$

Mixing in charged lepton sector

$$\sin(\theta^e_{13}) \approx \lambda^4\,, \quad \tan(\theta^e_{12}) \approx \tan(\theta^q_{12})/3\,, \quad \tan(\theta^e_{23}) \approx \lambda^4$$

$$\Downarrow$$

Lepton mixings obey sum rules

$$\sin^2(\theta^l_{12}) \approx \frac{1}{3}\left(1 + 2\sqrt{2}\sin(\theta^l_{13})\cos(\delta^l)\right)\,, \quad \sin(\theta^l_{13}) \approx \tan(\theta^q_{12})/(3\sqrt{2})$$

and for $\sin(\theta^l_{13}) \equiv \frac{r}{\sqrt{2}}$ and $\sin(\theta^l_{23}) \equiv \frac{1}{\sqrt{2}}(1+a)$ holds: $\quad a \approx -r^2/4$.

Conclusions & Outlook

- $SU(5) \times S_4$ - a minimal model for TB mixing in GUT context
- Nearly TB mixing in neutrino sector through non-trivial S_4 breaking
- Large top quark mass due to $T + T_3 \sim \mathbf{2} + \mathbf{1}$
- Incorporation of GST and GJ relations in the model
- Particular choice of $U(1)$ charges x, y and z allows vacuum alignment to be stable under NLO corrections and very small contributions to fermion masses and mixings from NLO terms

Not discussed

- GUT Higgs (super-)potential
- RG and threshold effects on fermion masses and mixings
- Anomaly constraints from $U(1)$
- Phenomenology related to sfermions

Thank you.

Back up

Messengers

Field	Σ	Σ^c	Δ	Δ^c	Υ	Υ^c	Ω	Ω^c	Θ	Θ^c
$SU(5)$	$\overline{\mathbf{10}}$	$\mathbf{10}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{5}$
S_4	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
$U(1)$	$-x-z$	$x+z$	$x+2z$	$-x-2z$	x	$-x$	$y+2z$	$-y-2z$	$y+z$	$-y-z$
$U(1)$	-6	6	7	-7	5	-5	6	-6	5	-5