Non-Abelian Discrete Symmetry in SUSY Flavor Model

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Plan of my talk

1. Introduction

2. S_4 Flavor Model in Quarks and Leptons

3. S_4 Flavor Symmetry in Sleptons

4. Summary

1. Introduction

Three Flavor global analysis strongly suggests

Tri-bimaximal Mixing Harrison, Perkins, Scott (2002)

M.C. G-Garcia, M. Maltoni, J. Salvado, arXiv:1001.4524

parameter	best fit	1σ	3σ	tri-bi
θ_{12}	34.4°	$33.4^\circ - 35.4^\circ$	$31.5^\circ - 37.6^\circ$	35.3°
θ_{23}	42.3°	$39.5^\circ - 47.6^\circ$	$35.2^\circ - 53.7^\circ$	45°
θ_{13}	6.8°	$3.2^\circ - 9.4^\circ$	$< 13.2^\circ$	0°
$\Delta m^2_{\rm sol}$ $[10^{-5}{\rm eV}^2]$	7.59	7.39-7.79	6.90-8.20	*
$\Delta m^2_{\rm atm}$ $[10^{-3}{\rm eV}^2]_N$	2.51	2.39-2.63	2.15-2.90	*

$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$

$$U_{\rm tri-bimaximal} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Consider the structure of Neutrino Mass Matrix, which gives Tri-bi maximal mixing

$$M_\nu^{\rm exp} = \frac{m_1+m_3}{2}\begin{pmatrix}1 & & \\ & 1 & \\ & & 1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1 & & \\ & & 1\\ & 1 & \end{pmatrix}$$

A_4 structure is hidden :

- The 3rd term is A_4 symmetric
- A 3-dim higgs gives the general A_4-symmetric Majorana mass term:

$$M_\nu^{A_4} = \begin{pmatrix}a & & \\ & b & \\ & & c\end{pmatrix} - \frac{1}{3}\begin{pmatrix}a & c & b\\ c & b & a\\ b & a & c\end{pmatrix} + x\begin{pmatrix}1 & & \\ & & 1\\ & 1 & \end{pmatrix}$$

$$a = b = c \iff V_{\rm tri-bi}$$

T', S_4 , Δ(54) flavor models also give Tri-bi maximal mixing !

Flavor Symmetries of Neutrinos connect with Other Physical Phenomena.

● Ue3=0 in Tri-bimaximal mixing!

There are hints Non-zero U_{e3} in experiments.
How can one predict U_{e3} ?

● CKM mixing in Quarks ? Cabibbo angle?

We need Quark-lepton unification in a GUT.

● SUSY Flavor Sector, SUSY FCNC

We discuss the case of S_4 symmetry.

2 S_4 Flavor Model in Quarks and Leptons

H. Ishimori, K. Saga, Y. Shimizu, M. Tanimoto, arXiv:1004.5004

$S_4 \times Z_4$ with SUSY SU(5) GUT

S_4 group is the symmetry group of octahedron or permutation of four elements. Number of elements is 24.

- Irreducible representations of S_4 are 3_1, 3_2, 2, 1_1, and 1_2.
- Multiplication rules are

$$3_1 \times 3_1 = 1_1 + 2 + 3_1 + 3_2$$
$$3_2 \times 3_2 = 1_1 + 2 + 3_1 + 3_2$$
$$3_1 \times 3_2 = 1_2 + 2 + 3_1 + 3_2$$
$$2 \times 3_1 = 3_1 + 3_2$$
$$2 \times 3_2 = 3_1 + 3_2$$
$$2 \times 2 = 1_1 + 1_2 + 2$$
$$\vdots$$

etc.

- S_4 invariant representation is 1_1.

3_1, 3_2, 2, 1_1, 1_2

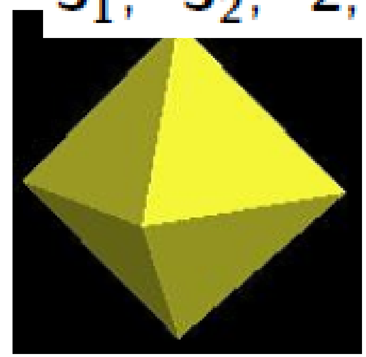

Figure: S_4 symmetry: Octahedron

B.Dutta, Y. Mimura, R.N. Mohapatra, arXiv:0911.2242. SO(10)
C.Hagedorn, S. F. King, C. Luhn, arXiv:1003.4249. SU(5)
R.d.A. Toorop, F. Bazzocchi, L. Merlo, arXiv: 1003.4502. Pati-Salam

$S_4 \times Z_4 \times U(1)_{FN}$ with SUSY SU(5) GUT

	(T_1, T_2)	T_3	(F_1, F_2, F_3)	(N_e^c, N_μ^c)	N_τ^c	H_5	$H_{\bar{5}}$	H_{45}	Θ
$SU(5)$	10	10	$\bar{5}$	1	1	5	$\bar{5}$	45	1
S_4	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_4	$-i$	-1	i	1	1	1	1	-1	1
$U(1)_{FN}$	ℓ	0	0	m	0	0	0	0	-1

	(χ_1, χ_2)	(χ_3, χ_4)	(χ_5, χ_6, χ_7)	$(\chi_8, \chi_9, \chi_{10})$	$(\chi_{11}, \chi_{12}, \chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	$-\ell$	$-n$	0	0	0	$-\ell$

Up quarks   **M_R**   **Dirac Neutrinos**   **Charged leptons Down quarks**

$10\ (q_1, u^c, e^c)$ $\bar{5}\ (d^c, l_e)$

We take l=m=1, n=2.

Right-handed neutrinos are $SU(5)$ gauge singlets

$S_4 \times Z4 \times U(1)FN \times SU(5)$ invariant superpotential

$$
\begin{aligned}
w = {} & y_1^u (T_1, T_2) \otimes T_3 \otimes (\chi_1, \chi_2) \otimes H_5/\Lambda + y_2^u T_3 \otimes T_3 \otimes H_5 \\
& + y_1^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes \Theta^{2m}/\bar{\Lambda}^{2m-1} \\
& + y_2^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes (\chi_3, \chi_4) \otimes \Theta^{2m-n}/\bar{\Lambda}^{2m-n} + M N_\tau^c \otimes N_\tau^c \\
& + y_1^D (N_e^c, N_\mu^c) \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5 \otimes \Theta^m/(\Lambda \bar{\Lambda}^m) \\
& + y_2^D N_\tau^c \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5/\Lambda \\
& + y_1 (F_1, F_2, F_3) \otimes (T_1, T_2) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_{45} \otimes \Theta^\ell/(\Lambda \bar{\Lambda}^\ell) \\
& + y_2 (F_1, F_2, F_3) \otimes T_3 \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{\bar{5}}/\Lambda,
\end{aligned}
$$

Define VEVs : $\langle \chi_i \rangle \equiv u_i$ and $\alpha_i \equiv u_i/\Lambda$

Vacuum alignment

take vacuum alignment $(u_8, u_9, u_{10}) = (0, u_9, 0)$ and $(u_{11}, u_{12}, u_{13}) = (0, 0, u_{13})$

$$M_l = \begin{pmatrix} 0 & -3y_1\lambda^\ell \alpha_9 v_{45}/\sqrt{2} & 0 \\ 0 & -3y_1\lambda^\ell \alpha_9 v_{45}/\sqrt{6} & 0 \\ 0 & 0 & y_2\alpha_{13}v_d \end{pmatrix}$$

$$M_l^\dagger M_l = v_d^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 6|\bar{y}_1\lambda^\ell\alpha_9|^2 & 0 \\ 0 & 0 & |y_2|^2\alpha_{13}^2 \end{pmatrix}$$

$$m_e^2 = 0 \ , \quad m_\mu^2 = 6|\bar{y}_1\lambda^\ell\alpha_9|^2 v_d^2 \ , \quad m_\tau^2 = |y_2|^2\alpha_{13}^2 v_d^2$$

No mixing in the left-hand !

$\theta_{12}=60^\circ$ in the right-hand !

Taking vacuum alignment $(u_3, u_4) = (0, u_4)$ and $(u_5, u_6, u_7) = (u_5, u_5, u_5)$

$$M_N = \begin{pmatrix} \lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} + y_2^N \alpha_4 \Lambda) & 0 & 0 \\ 0 & \lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} - y_2^N \alpha_4 \Lambda) & 0 \\ 0 & 0 & M \end{pmatrix}$$

$$M_D = y_1^D \lambda^m v_u \begin{pmatrix} 2\alpha_5/\sqrt{6} & -\alpha_5/\sqrt{6} & -\alpha_5/\sqrt{6} \\ 0 & \alpha_5/\sqrt{2} & -\alpha_5/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} + y_2^D v_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_5 & \alpha_5 & \alpha_5 \end{pmatrix}$$

After seesaw, we get the tri-bimaximal mixing

$$M_\nu = \frac{b+c}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{3a-b}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{b-c}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$a = \frac{(y_2^D \alpha_5 v_u)^2}{M}, \qquad b = \frac{(y_1^D \alpha_5 v_u \lambda^m)^2}{\lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} + y_2^N \alpha_4 \Lambda)}, \qquad c = \frac{(y_1^D \alpha_5 v_u \lambda^m)^2}{\lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} - y_2^N \alpha_4 \Lambda)}.$$

$$m_1 = b\ , \qquad m_2 = 3a\ , \qquad m_3 = c\ .$$

Determination of magnitudes α_i

Desired Vacuum Alignments **FN charges l=m=1, n=2**

$$(\chi_1, \chi_2) = (1, 1), \quad (\chi_3, \chi_4) = (0, 1),$$
$$(\chi_5, \chi_6, \chi_7) = (1, 1, 1), \quad (\chi_8, \chi_9, \chi_{10}) = (0, 1, 0), \quad (\chi_{11}, \chi_{12}, \chi_{13}) = (0, 0, 1),$$

$$\alpha_3 = \alpha_8 = \alpha_{10} = \alpha_{11} = \alpha_{12} = 0,$$
$$\alpha_1 = \alpha_2 \simeq \sqrt{\left|\frac{y_2^u m_c}{2 y_1^{u2} v_u}\right|},$$
$$\alpha_4 = \frac{(y_1^D \lambda)^2 (m_3 - m_1) m_2 M}{6 y_2^N y_2^{D^2} m_1 m_3 \Lambda}, \qquad \alpha_5 = \alpha_6 = \alpha_7 = \frac{\sqrt{m_2 M}}{\sqrt{3} y_2^D v_u},$$
$$\alpha_9 = \frac{m_\mu}{\sqrt{6} |\bar{y_1}| \lambda v_d}, \qquad \alpha_{13} = \frac{m_\tau}{y_2 v_d} \ .$$

Putting observed masses and $M = 10^{12}$ GeV, we get

$$\alpha_1 \sim 3.0 \times 10^{-2}, \qquad \alpha_4 \sim 10^{-2},$$
$$\alpha_5 \sim 10^{-2}, \qquad \alpha_9 \sim 5.1 \times 10^{-3}, \qquad \alpha_{13} \sim 2.1 \times 10^{-2}.$$

The charged lepton mass matrix including the next-to-leading terms

next talk (Ishimori)

$$M_l^\dagger M_l \simeq \begin{pmatrix} |\epsilon_{11}|^2+|\epsilon_{21}|^2+|\epsilon_{31}|^2 & \frac{1}{2}(\sqrt{3}\epsilon_{11}^*+\epsilon_{21}^*)m_\mu & \epsilon_{31}^* m_\tau \\ \frac{1}{2}(\sqrt{3}\epsilon_{11}+\epsilon_{21})m_\mu & m_\mu^2 & \frac{1}{2}(\sqrt{3}\epsilon_{13}+\epsilon_{23})m_\mu \\ \epsilon_{31}m_\tau & \frac{1}{2}(\sqrt{3}\epsilon_{13}^*+\epsilon_{23}^*)m_\mu & m_\tau^2 \end{pmatrix}$$

$$\epsilon_{ij} = \mathcal{O}(\alpha_i\alpha_j v_d) = \mathcal{O}(m_e)$$

$$U_E = \begin{pmatrix} 1 & \mathcal{O}\left(\frac{m_e}{m_\mu}\right) & \mathcal{O}\left(\frac{m_e}{m_\tau}\right) \\ \mathcal{O}\left(\frac{m_e}{m_\mu}\right) & 1 & \mathcal{O}\left(\frac{m_e m_\mu}{m_\tau^2}\right) \\ \mathcal{O}\left(\frac{m_e}{m_\tau}\right) & \mathcal{O}\left(\frac{m_e m_\mu}{m_\tau^2}\right) & 1 \end{pmatrix}$$

Since the lepton mixing is given as $U = U_E^\dagger U_{\text{tri-bi}}$

we have non-zero U_{e3}

$$|U_{e3}| \sim \frac{1}{\sqrt{2}}\left(\mathcal{O}\left(\frac{m_e}{m_\mu}\right)\right), \quad |U_{e2}| \sim \frac{1}{\sqrt{3}}\left(1+\mathcal{O}\left(\frac{m_e}{m_\mu}\right)\right)$$

Quark Sector is predictable. Next talk (Ishimori)

Down Quarks

$$M_d = v_d \begin{pmatrix} 0 & 0 & 0 \\ \bar{y}_1 \lambda^\ell \alpha_9/\sqrt{2} & \bar{y}_1 \lambda^\ell \alpha_9/\sqrt{6} & 0 \\ 0 & 0 & y_2 \alpha_{13} \end{pmatrix}$$

$$\bar{y}_1 v_d = y_1 v_{45}$$

$$M_d^\dagger M_d = v_d^2 \begin{pmatrix} \frac{1}{2}|\bar{y}_1 \lambda^\ell \alpha_9|^2 & \frac{1}{2\sqrt{3}}|\bar{y}_1 \lambda^\ell \alpha_9|^2 & 0 \\ \frac{1}{2\sqrt{3}}|\bar{y}_1 \lambda^\ell \alpha_9|^2 & \frac{1}{6}|\bar{y}_1 \lambda^\ell \alpha_9|^2 & 0 \\ 0 & 0 & |y_2|^2 \alpha_{13}^2 \end{pmatrix}$$

Left-handed mixing is given as

$$U_d^{(0)} = \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Up Quarks

We take alignment $\alpha_1 = \alpha_2$ **, we get**

$$M_u = v_u \begin{pmatrix} 2y^u_{\Delta_{a1}}\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & y^u_{\Delta_{a2}}\alpha_1^2 & y^u_1\alpha_1 \\ y^u_{\Delta_{a2}}\alpha_1^2 & 2y^u_{\Delta_{a1}}\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & y^u_1\alpha_1 \\ y^u_1\alpha_1 & y^u_1\alpha_1 & y^u_2 + y^u_{\Delta_c}\alpha_9^2 \end{pmatrix}$$

After rotating it by the orthogonal matrix,

$$U_u^{(0)} = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We obtain

$$\hat{M}_u = U_u^\dagger M_u U_u = v_u \begin{pmatrix} (2y^u_{\Delta_{a1}} - y^u_{\Delta_{a2}})\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & 0 & 0 \\ 0 & (2y^u_{\Delta_{a1}} + y^u_{\Delta_{a2}})\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & \sqrt{2}y^u_1\alpha_1 \\ 0 & \sqrt{2}y^u_1\alpha_1 & y^u_2 + y^u_{\Delta_c}\alpha_9^2 \end{pmatrix}$$

We obtain CKM matrix elements

$$V_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix}, \qquad r_c = \sqrt{\frac{m_c}{m_c + m_t}}, \qquad r_t = \sqrt{\frac{m_t}{m_c + m_t}},$$

$$U_u \simeq U_u^{(0)} P V_u = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix},$$

$$U_d \simeq \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_{12}^d & \theta_{13}^d \\ -\theta_{12}^d - \theta_{13}^d \theta_{23}^d & 1 & \theta_{23}^d \\ -\theta_{13}^d + \theta_{12}^d \theta_{23}^d & -\theta_{23}^d - \theta_{12}^d \theta_{13}^d & 1 \end{pmatrix}.$$

In the leading order, we predict

$$V_{us} \simeq \sin 15^\circ \simeq 0.26$$
$$V_{cb} \simeq \sqrt{m_c/m_t} \simeq 0.048$$
$$V_{ub} \simeq 0$$

3. S_4 Flavor Symmetry in Sleptons

Flavor symmetry constrains not only quark/lepton mass matrices,
but also mass matrices of their superpartner, i.e. squark/slepton

Specific patterns of squark/slepton mass matrices could be tested in future experiments.

In this talk, we concentrate on lepton FCNC.

Consider Soft SUSY Breaking Term in Supergravity.

we assume chiral superfields Φ_k to cause SUSY breaking

Flavor symmetry $S_4 \times Z_4$ requires

Second order Kähler potential of left-handed and right-handed leptons.

$$K = Z^{(L)}(\Phi) \sum_{i=e,\mu,\tau} |L_i|^2 + Z^{(R)}_{(1)}(\Phi) \sum_{i=e,\mu} |e_i|^2 + Z^{(R)}_{(2)}(\Phi)|e_\tau|^2$$

where $Z^{(L)}(\Phi)$, $Z^{(R)}_{(1)}(\Phi)$ and $Z^{(R)}_{(2)}(\Phi)$ are generic functions of moduli fields Φ.

Slepton mass matrices are derived from

$$m^2_{\bar{I}J} K_{\bar{I}J} = m^2_{3/2} K_{\bar{I}J} + |F^{\Phi_k}|^2 \partial_{\Phi_k} \partial_{\bar{\Phi}_k} K_{\bar{I}J} - |F^{\Phi_k}|^2 \partial_{\bar{\Phi}_k} K_{\bar{I}L} \partial_{\Phi_k} K_{\bar{M}J} K^{L\bar{M}}$$

where $K_{IJ} = \partial_{\bar{I}} \partial_J K$ and K^{IJ} is its inverse.

$$(m^2_{\tilde{L}})_{ij} = \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_L^2 & 0 \\ 0 & 0 & m_L^2 \end{pmatrix}, \qquad (m^2_{\tilde{R}})_{ij} = \begin{pmatrix} m^2_{R(1)} & 0 & 0 \\ 0 & m^2_{R(1)} & 0 \\ 0 & 0 & m^2_{R(2)} \end{pmatrix}$$

For the left-handed sector, higher dimensional terms are given as

$$\Delta K_L = \sum_{i=1,3} Z^{(L)}_{\Delta_{a_i}}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L^c_e, L^c_\mu, L^c_\tau) \otimes (\chi_i, \chi_{i+1}) \otimes (\chi^c_i, \chi^c_{i+1})/\Lambda^2$$
$$+ \sum_{i=5,8,11} Z^{(L)}_{\Delta_{b_i}}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L^c_e, L^c_\mu, L^c_\tau) \otimes (\chi_i, \chi_{i+1}, \chi_{i+2}) \otimes (\chi^c_i, \chi^c_{i+1}, \chi^c_{i+2})/\Lambda^2$$
$$+ Z^{(L)}_{\Delta_c}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L^c_e, L^c_\mu, L^c_\tau) \otimes \chi_{14} \otimes \chi^c_{14}/\Lambda^2$$
$$+ Z^{(L)}_{\Delta_d}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L^c_e, L^c_\mu, L^c_\tau) \otimes \Theta \otimes \Theta^c/\bar{\Lambda}^2.$$

Left-handed Slepton mass matrix is

$$(m^2_{\tilde{L}})_{ij} = \begin{pmatrix} m^2_L + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \\ \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & m^2_L + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \\ \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & m^2_L + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \end{pmatrix}$$

$\tilde{\alpha}$ is a linear combination of α_i's.

Right-handed Slepton mass matrix is

$$
\begin{aligned}
\Delta K_R = & \sum_{i=1,3} Z^{(R)}_{\Delta_{a_i}}(\Phi)(R_e, R_\mu) \otimes (R^c_e, R^c_\mu) \otimes (\chi_i, \chi_{i+1}) \otimes (\chi^c_i, \chi^c_{i+1})/\Lambda^2 \\
& + \sum_{i=5,8,11} Z^{(R)}_{\Delta_{b_i}}(\Phi)(R_e, R_\mu) \otimes (R^c_e, R^c_\mu) \otimes (\chi_i, \chi_{i+1}, \chi_{i+2}) \otimes (\chi^c_i, \chi^c_{i+1}, \chi^c_{i+2})/\Lambda^2 \\
& + Z^{(R)}_{\Delta_c}(\Phi)(R_e, R_\mu) \otimes (R^c_e, R^c_\mu) \otimes \chi_{14} \otimes \chi^c_{14}/\Lambda^2 \\
& + Z^{(R)}_{\Delta_d}(\Phi)(R_e, R_\mu) \otimes R^c_\tau \otimes (\chi_1, \chi_2)/\Lambda^2 + Z^{(R)}_{\Delta_e}(\Phi)(R^c_e, R^c_\mu) \otimes R_\tau \otimes (\chi^c_1, \chi^c_2)/\Lambda^2 \\
& + \sum_{i=1,3} Z^{(R)}_{\Delta_{f_i}}(\Phi) R_\tau \otimes R^c_\tau \otimes (\chi_i, \chi_{i+1}) \otimes (\chi^c_i, \chi^c_{i+1})/\Lambda^2 \\
& + \sum_{i=5,8,11} Z^{(R)}_{\Delta_{g_i}}(\Phi) R_\tau \otimes R^c_\tau \otimes (\chi_i, \chi_{i+1}, \chi_{i+2}) \otimes (\chi^c_i, \chi^c_{i+1}, \chi^c_{i+2})/\Lambda^2 \\
& + Z^{(R)}_{\Delta_h}(\Phi) R_\tau \otimes R^c_\tau \otimes \chi_{14} \otimes \chi^c_{14}/\Lambda^2 \\
& + Z^{(R)}_{\Delta_i}(\Phi)(R_e, R_\mu) \otimes (R^c_e, R^c_\mu) \otimes \Theta \otimes \Theta^c/\bar{\Lambda}^2 \\
& + Z^{(R)}_{\Delta_j}(\Phi) R_\tau \otimes R^c_\tau \otimes \Theta \otimes \Theta^c/\bar{\Lambda}^2.
\end{aligned}
$$

$$
(m^2_{\tilde{R}})_{ij} = \begin{pmatrix} m^2_{R(1)} + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\alpha_1 m^2_{3/2}) \\ \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & m^2_{R(1)} + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\alpha_1 m^2_{3/2}) \\ \mathcal{O}(\alpha_1 m^2_{3/2}) & \mathcal{O}(\alpha_1 m^2_{3/2}) & m^2_{R(2)} + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \end{pmatrix}
$$

Move to Super-CKM basis (Diagonal Basis of Charged Lepton) in order to estimate magnitudes of FCNC.

Dominant term

$$(m^2_{L(R)})^{SCKM}_{12} \sim \theta^l_{L(R)12}(m^2_{L(R)11} - m^2_{L(R)22}) + m^2_{L(R)12}$$

where $\theta^\ell_{L12} \simeq \mathcal{O}(\frac{m_e}{m_\mu}), \quad \theta^\ell_{R12} \simeq \sin 60^\circ$

Mass Insertion Parameters

$$(\Delta^l_{LL(RR)})_{ij} \equiv \frac{(m^2_{L(R)})^{SCKM}_{ij}}{m^2_{SUSY}}$$

Our prediction is $(\Delta^l_{LL(RR)})_{12} = \mathcal{O}(\tilde{\alpha}^2) = \mathcal{O}(10^{-4})$.

Experimental Constraint

$$(\Delta^l_{LL(RR)})_{12} \leq \mathcal{O}(10^{-3}) \text{ when } m_{SUSY} = \mathcal{O}(100\text{GeV})$$

F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B477(1996) 321

The model is consistent with FCNC constraints !

A terms are obtained as

$$(m^2_{LR})^{SCKM}_{ij} = U^{\dagger}_E (m^2_{LR})_{ij} V_E \simeq m_{3/2} \begin{pmatrix} \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) \\ \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}(m_\mu) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) \\ \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}(m_\tau) \end{pmatrix}$$

$$\simeq m_{3/2} \begin{pmatrix} \mathcal{O}\left(m_e\right) & \mathcal{O}\left(m_e\right) & \mathcal{O}\left(m_e\right) \\ \mathcal{O}\left(m_e\right) & \mathcal{O}(m_\mu) & \mathcal{O}\left(m_e\right) \\ \mathcal{O}\left(m_e\right) & \mathcal{O}\left(m_e\right) & \mathcal{O}(m_\tau) \end{pmatrix}$$

$$(\Delta^l_{LR})_{12} = \mathcal{O}\left(\frac{m_e}{m_{SUSY}}\right) = \mathcal{O}(5 \times 10^{-6}).$$

Experimental Constraint

$$(\Delta^l_{LR})_{12} \le \mathcal{O}(10^{-6}) \text{ if } m_{SUSY} = \mathcal{O}(100\text{GeV})$$

$\mu \to e \gamma$ is expected to be observed soon !

4. Summary

☆ S_4 Flavor Symmetry can give realistic quark and lepton mixing matrices.

☆ S_4 discrete symmetry works to suppress FCNC in the framework of gravity mediation in SUSY breaking.

★ Squark sectors in S_4 Symmetry ?

★ Study of SUSY FCNC in other non-Abelian discrete symmetries.

Realization of Vacuum Alignment

Introduce driving fields with R parity 2

We can generate the vacuum alignment through F-terms by coupling flavons fields, which carry the R charge $+2$ under $U(1)_R$ symmetry.

	(χ_1,χ_2)	(χ_3,χ_4)	(χ_5,χ_6,χ_7)	$(\chi_8,\chi_9,\chi_{10})$	$(\chi_{11},\chi_{12},\chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3'}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	$-\ell$	$-n$	0	0	0	$-\ell$
$U(1)_R$	0	0	0	0	0	0

	$(\chi_{15},\chi_{16},\chi_{17})$	χ_1^0	χ_2^0	χ_3^0	(χ_4^0,χ_5^0)
$SU(5)$	1	1	1	1	1
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$
Z_4	-1	-1	i	-1	$-i$
$U(1)_{FN}$	$-z$	$2\ell+n$	0	2ℓ	z
$U(1)_R$	0	2	2	2	2

$$
\begin{aligned}
w' = & \kappa_1 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes (\chi_3, \chi_4) \otimes \chi_1^0/\Lambda \\
& + \eta_1 (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_2^0 \\
& + \eta_2 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes \chi_3^0 + \eta_3 \chi_{14} \otimes \chi_{14} \otimes \chi_3^0 \\
& + \eta_4 (\chi_5, \chi_6, \chi_7) \otimes (\chi_{15}, \chi_{16}, \chi_{17}) \otimes (\chi_4^0, \chi_5^0) ,
\end{aligned}
$$

Scalar potential

$$
\begin{aligned}
V = & \left| \frac{\kappa_1}{\Lambda} \left[2\chi_1\chi_2\chi_3 + \left(\chi_1^2 - \chi_2^2\right) \chi_4 \right] \right|^2 + \left| \eta_1 \left(\chi_8\chi_{11} + \chi_9\chi_{12} + \chi_{10}\chi_{13} \right) \right|^2 \\
& + \left| \eta_2 (\chi_1^2 + \chi_2^2) + \eta_3 \chi_{14}^2 \right|^2 + \left| \frac{1}{\sqrt{2}} \eta_4 \left(\chi_6\chi_{16} - \chi_7\chi_{17} \right) \right|^2 \\
& + \left| \frac{1}{\sqrt{6}} \eta_4 \left(-2\chi_5\chi_{15} + \chi_6\chi_{16} + \chi_7\chi_{17} \right) \right|^2 .
\end{aligned}
$$

We obtain Desired Vacuum Alignment

$$
\chi_1 = \chi_2, \quad \chi_3 = 0, \quad \chi_5 = \chi_6 = \chi_7, \quad \chi_8 = \chi_{10} = \chi_{11} = \chi_{12} = 0,
$$

$$
\chi_{14}^2 = -\frac{2\eta_2}{\eta_3} \chi_1^2, \quad \chi_{15} = \chi_{16} = \chi_{17} .
$$

Our Multiplication Rule of S_4

$$
\begin{aligned}
(a_1, a_2)_2 \times (b_1, b_2)_2 &= (a_1b_1 + a_2b_2)_{1_1} + (-a_1b_2 + a_2b_1)_{1_2} \\
&+ (a_1b_2 + a_2b_1, a_1b_1 - a_2b_2)_2
\end{aligned}
$$

$$
\begin{aligned}
(a_1, a_2, a_3)_{3_1} \times (b_1, b_2, b_3)_{3_1} &= (a_1b_1 + a_2b_2 + a_3b_3)_{1_1} \\
&+ (\frac{1}{\sqrt{2}}(a_2b_2 - a_3b_3), \frac{1}{\sqrt{6}}(-2a_1b_1 + a_2b_2 + a_3b_3))_2 \\
&+ (a_2b_3 + a_3b_2, a_1b_3 + a_3b_1, a_1b_2 + a_2b_1)_{3_1} \\
&+ (a_3b_2 - a_2b_3, a_1b_3 - a_3b_1, a_2b_1 - a_1b_2)_{3_2}
\end{aligned}
$$

$$
\begin{aligned}
(a_1, a_2)_2 \times (b_1, b_2, b_3)_{3_1} &= (a_2b_1, -\frac{1}{2}(\sqrt{3}a_1b_2 + a_2b_2), \frac{1}{2}(\sqrt{3}a_1b_3 - a_2b_3))_{3_1} \\
&+ (a_1b_1, \frac{1}{2}(\sqrt{3}a_2b_2 - a_1b_2), -\frac{1}{2}(\sqrt{3}a_2b_3 + a_1b_3))_{3_2}
\end{aligned}
$$

Decomposition of S_4 invariant superpotential

$$w_l = -3y_1\left[\frac{e^c}{\sqrt{2}}(l_\mu\chi_9 - l_\tau\chi_{10}) + \frac{\mu^c}{\sqrt{6}}(-2l_e\chi_8 + l_\mu\chi_9 + l_\tau\chi_{10})\right]h_{45}\Theta^\ell/(\Lambda\bar{\Lambda}^\ell)$$
$$+ y_2\tau^c(l_e\chi_{11} + l_\mu\chi_{12} + l_\tau\chi_{13})h_d/\Lambda.$$

$$w_N = y_1^N(N_e^cN_e^c + N_\mu^cN_\mu^c)\Theta^{2m}/\bar{\Lambda}^{2m-1}$$
$$+ y_2^N\left[(N_e^cN_\mu^c + N_\mu^cN_e^c)\chi_3 + (N_e^cN_e^c - N_\mu^cN_\mu^c)\chi_4\right]\Theta^{2m-n}/\bar{\Lambda}^{2m-n} + MN_\tau^cN_\tau^c,$$

$$w_D = y_1^D\left[\frac{N_e^c}{\sqrt{6}}(2l_e\chi_5 - l_\mu\chi_6 - l_\tau\chi_7) + \frac{N_\mu^c}{\sqrt{2}}(l_\mu\chi_6 - l_\tau\chi_7)\right]h_u\Theta^m/(\Lambda\bar{\Lambda}^m)$$
$$+ y_2^DN_\tau^c(l_e\chi_5 + l_\mu\chi_6 + l_\tau\chi_7)h_u/\Lambda.$$

We take VEV's

$$\langle h_u \rangle = v_u, \quad \langle h_d \rangle = v_d, \quad \langle h_{45} \rangle = v_{45}, \quad \langle \Theta \rangle = \theta,$$
$$\langle (\chi_3, \chi_4) \rangle = (u_3, u_4), \quad \langle (\chi_5, \chi_6, \chi_7) \rangle = (u_5, u_6, u_7),$$
$$\langle (\chi_8, \chi_9, \chi_{10}) \rangle = (u_8, u_9, u_{10}), \quad \langle (\chi_{11}, \chi_{12}, \chi_{13}) \rangle = (u_{11}, u_{12}, u_{13}),$$

$$\alpha_i \equiv u_i/\Lambda \text{ and } \lambda \equiv \theta/\bar{\Lambda}$$

We get **Lepton Mass Matrices**

$$M_l = -3y_1\lambda^\ell v_{45}\begin{pmatrix} 0 & \alpha_9/\sqrt{2} & -\alpha_{10}/\sqrt{2} \\ -2\alpha_8/\sqrt{6} & \alpha_9/\sqrt{6} & \alpha_{10}/\sqrt{6} \\ 0 & 0 & 0 \end{pmatrix} + y_2 v_d \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{11} & \alpha_{12} & \alpha_{13} \end{pmatrix}$$

$$M_N = \begin{pmatrix} \lambda^{2m-n}(y_1^N\lambda^n\bar{\Lambda} + y_2^N\alpha_4\Lambda) & y_2^N\lambda^{2m-n}\alpha_3\Lambda & 0 \\ y_2^N\lambda^{2m-n}\alpha_3\Lambda & \lambda^{2m-n}(y_1^N\lambda^n\bar{\Lambda} - y_2^N\alpha_4\Lambda) & 0 \\ 0 & 0 & M \end{pmatrix}$$

Due to m-n<0

$$M_D = y_1^D\lambda^m v_u \begin{pmatrix} 2\alpha_5/\sqrt{6} & -\alpha_6/\sqrt{6} & -\alpha_7/\sqrt{6} \\ 0 & \alpha_6/\sqrt{2} & -\alpha_7/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} + y_2^D v_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_5 & \alpha_6 & \alpha_7 \end{pmatrix}$$

Including next-to-leading order corrections, we get

$$V^0_{us} \simeq \theta^d_{12}\cos 15^\circ + \sin 15^\circ,$$
$$V^0_{ub} \simeq \theta^d_{13}\cos 15^\circ + \theta^d_{23}\sin 15^\circ,$$

$$V^0_{cb} \simeq -r_t\theta^d_{13}e^{i\rho}\sin 15^\circ + r_t\theta^d_{23}e^{i\rho}\cos 15^\circ - r_c\ ,$$
$$V^0_{td} \simeq -r_c\sin 15^\circ e^{i\rho} - r_c(\theta^d_{12} + \theta^d_{13}\theta^d_{23})e^{i\rho}\cos 15^\circ + r_t(-\theta^d_{13} + \theta^d_{12}\theta^d_{23})$$

The parameter set

$$\rho = 123^\circ, \quad \theta^d_{12} = -0.0340, \quad \theta^d_{13} = 0.00626, \quad \theta^d_{23} = -0.00880$$

reproduces observed values very well.

Values of parameters are consistent with our mass matrices.

$$\theta^d_{12} = \mathcal{O}\left(\frac{m_d}{m_s}\right) = \mathcal{O}\left(0.05\right), \quad \theta^d_{13} = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}\left(0.005\right), \quad \theta^d_{23} = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}\left(0.005\right)$$

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