Quark and lepton flavor mixing in S_4 flavor symmetry

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arXiv:1004.5004

Mixing matrices of leptons and quarks

Lepton mixing is close to tri-bimaximal mixing:

$\theta_{12} \simeq 35.3^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$ Harrison, Perkins, Scott 2002

$$V_{\rm MNS} \sim \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

($\theta_{12} \sim 34^\circ$, $\theta_{23} \sim 42^\circ$, $\theta_{13} \sim 6.8^\circ$
M.C. G-Garcia, M. Maltoni, J. Salvado, arXiv:1001.4524)

CKM matrix:

$$|V_{CKM}| = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}$$

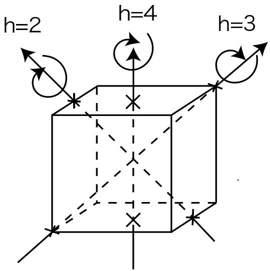
($\theta_{12} \simeq 13^\circ$, $\theta_{23} \simeq 2.4^\circ$, $\theta_{13} \simeq 0.21^\circ$ PDG 2008)

Basic structure of S_4

S_4 is the permutation of four elements or rotations of cube

Fundamental representations are $\mathbf{1_1}$, $\mathbf{1_2}$, $\mathbf{2}$, $\mathbf{3_1}$, and $\mathbf{3_2}$.

	h	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{2}$	$\mathbf{3}_1$	$\mathbf{3}_2$
C_1	1	1	1	2	3	3
C_3	2	1	1	2	-1	-1
C_6	2	1	-1	0	1	-1
$C_{6'}$	4	1	-1	0	-1	1
C_8	3	1	1	-1	0	0



- Multiplication rules with triplet

$$\mathbf{3_1}\times\mathbf{3_1}=\mathbf{1}_1+\mathbf{2}+\mathbf{3}_1+\mathbf{3}_2,\quad \mathbf{3_2}\times\mathbf{3_2}=\mathbf{1}_1+\mathbf{2}+\mathbf{3}_1+\mathbf{3}_2,$$

$$\mathbf{3_1}\times\mathbf{3_2}=\mathbf{1}_2+\mathbf{2}+\mathbf{3}_1+\mathbf{3}_2,\quad \mathbf{3_1}\times\mathbf{2}=\mathbf{3_1}+\mathbf{3_2},$$

$S_4 \times Z_4$ SUSY GUT model

quark and lepton / right handed neutrino

	(F_1, F_2, F_3)	(T_1, T_2)	T_3	(N_e^c, N_μ^c)	N_τ^c
$SU(5)$	$\bar{5}$	10	10	1	1
S_4	$\mathbf{3_1}$	$\mathbf{2}$	$\mathbf{1_1}$	$\mathbf{2}$	$\mathbf{1_2}$
Z_4	i	$-i$	-1	1	1
$U(1)_F$	0	1	0	1	0

Higgs / scalar / FN field

	$H_{5,\bar{5}}$	H_{45}	χ_u	χ_N	χ_D	χ_ℓ	χ'_ℓ	χ	Φ
$SU(5)$	$5, \bar{5}$	45	1	1	1	1	1	1	1
S_4	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3_2}$	$\mathbf{3_1}$	$\mathbf{3_1}$	$\mathbf{1_1}$	$\mathbf{1_1}$
Z_4	1	-1	$-i$	1	$-i$	-1	i	i	1
$U(1)_F$	0	0	-1	-2	0	0	0	-1	-1

cf. B. Dutta, Y. Mimura, R. N. Mohapatra, arXiv:0911.2242
C. Hagedorn, S. F. King, C. Luhn, arXiv:1003.4249
R.d.A. Toorop, F. Bazzocchi, L. Merlo, arXiv:1003.4502

Dirac neutrino

- $(N_e^c, N_\mu^c) L \chi_D H_u/\Lambda$,

	(N_e^c, N_μ^c)	L	χ_D
S_4	$\mathbf{2}$	$\mathbf{3}_1$	$\mathbf{3}_2$
Z_4	1	i	$-i$

- $N_\tau^c L \chi_D H_u/\Lambda$,

	N_τ^c	L	χ_D
S_4	$\mathbf{1}_2$	$\mathbf{3}_1$	$\mathbf{3}_2$
Z_4	1	i	$-i$

Majorana neutrino

- $(N_e, N_\mu)^2 \Phi^2/\Lambda\bar{\Lambda}^2,\ N_\tau^2$

	(N_e, N_μ)	N_τ
S_4	$\mathbf{2}$	$\mathbf{1}_2$
Z_4	1	1

- $(N_e, N_\mu)^2 \chi_N$

	(N_e, N_μ)	χ_N
S_4	$\mathbf{2}$	$\mathbf{2}$
Z_4	1	1

VEVs are written as $\langle \chi_D \rangle = (\alpha_{D_1}, \alpha_{D_2}, \alpha_{D_3})\Lambda$ and $\langle \chi_N \rangle = (\alpha_{N_1}, \alpha_{N_2})\Lambda$,

$$M_D = v_u \begin{pmatrix} 2y_1^D \lambda \alpha_{D_1}/\sqrt{6} & -y_1^D \lambda \alpha_{D_2}/\sqrt{6} & -y_1^D \lambda \alpha_{D_3}/\sqrt{6} \\ 0 & y_1^D \lambda \alpha_{D_2}/\sqrt{2} & -y_1^D \lambda \alpha_{D_3}/\sqrt{2} \\ y_2^D \alpha_{D_1} & y_2^D \alpha_{D_2} & y_2^D \alpha_{D_3} \end{pmatrix}$$

$$M_N = \begin{pmatrix} y_1^N \lambda^2 \bar{\Lambda} + y_2^N \alpha_{N_2} \Lambda & y_2^N \alpha_{N_1} \Lambda & 0 \\ y_2^N \alpha_{N_1} \Lambda & y_1^N \lambda^2 \bar{\Lambda} - y_2^N \alpha_{N_2} \Lambda & 0 \\ 0 & 0 & M \end{pmatrix}$$

Neutrino

$\langle\chi_D\rangle = \alpha_D(1,1,1)\Lambda, \quad \langle\chi_N\rangle = (0,\alpha_N)\Lambda.$

$$M_D = v_u\alpha_D \begin{pmatrix} 2y_1^D\lambda/\sqrt{6} & -y_1^D\lambda/\sqrt{6} & -y_1^D\lambda/\sqrt{6} \\ 0 & y_1^D\lambda/\sqrt{2} & -y_1^D\lambda/\sqrt{2} \\ y_2^D & y_2^D & y_2^D \end{pmatrix},$$

$$M_N = \begin{pmatrix} y_1^N\lambda^2\bar\Lambda + y_2^N\alpha_N\Lambda & 0 & 0 \\ 0 & y_1^N\lambda^2\bar\Lambda - y_2^N\alpha_N\Lambda & 0 \\ 0 & 0 & M \end{pmatrix}$$

With the see-saw mechanism

$$M_\nu = \frac{b+c}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{3a-b}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{b-c}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

This matrix leads tri-bimaximal mixing

$$a = \frac{(y_2^D\alpha_D v_u)^2}{M}, \quad b = \frac{(y_1^D\lambda\alpha_D v_u)^2}{y_1^N\lambda^2\bar\Lambda + y_2^N\alpha_N\Lambda}, \quad c = \frac{(y_1^D\lambda\alpha_D v_u)^2}{y_1^N\lambda^2\bar\Lambda - y_2^N\alpha_N\Lambda}.$$

Dirac and Majorana neutrinos including next-to-leading terms

For Dirac neutrinos

$$M_D = v_u \begin{pmatrix} 2y_1^D \lambda \alpha_D/\sqrt{6} & -y_1^D \lambda \alpha_D/\sqrt{6} & -y_1^D \lambda \alpha_D/\sqrt{6} \\ y_\Delta^D \lambda \alpha_\ell \alpha_{\ell'} & y_1^D \lambda \alpha_D/\sqrt{2} & -y_1^D \lambda \alpha_D/\sqrt{2} \\ y_2^D \alpha_D & y_2^D \alpha_D & y_2^D \alpha_D \end{pmatrix}$$

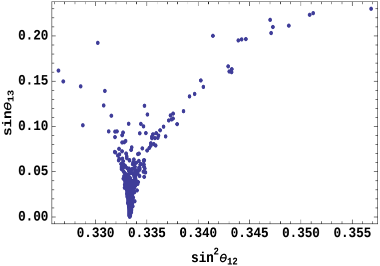
$$U_{e3} \sim -\frac{\sqrt{6} y_\Delta^D \alpha_\ell \alpha_{\ell'}}{3 y_1^D \alpha_D} \sim \mathcal{O}(10^{-2})$$

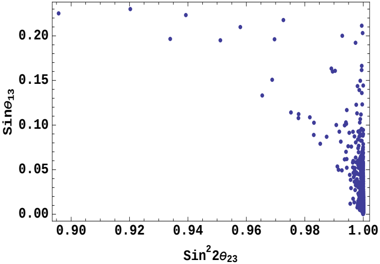
For Majorana neutrinos

$$M_N = \begin{pmatrix} y_1^N \lambda^2 \bar{\Lambda} + y_2^N \alpha_N \Lambda & 0 & 0 \\ 0 & y_1^N \lambda^2 \bar{\Lambda} - y_2^N \alpha_N \Lambda & 0 \\ 0 & 0 & M \end{pmatrix}$$
$$+\Lambda \begin{pmatrix} y_{\Delta_1}^N \alpha_u \alpha & y_{\Delta_1}^N \alpha_u \alpha & -\frac{\lambda}{\sqrt{6}} y_{\Delta_2}^N \alpha_D \alpha_{\ell'} + \frac{\lambda}{\sqrt{2}} y_{\Delta_3}^N \lambda \alpha_\ell^2 \\ y_{\Delta_1}^N \alpha_u \alpha & -y_{\Delta_1}^N \alpha_u \alpha & -\frac{\lambda}{\sqrt{2}} y_{\Delta_2}^N \alpha_D \alpha_{\ell'} + \frac{\lambda}{\sqrt{6}} y_{\Delta_3}^N \alpha_\ell^2 \\ -\frac{\lambda}{\sqrt{6}} y_{\Delta_2}^N \alpha_D \alpha_{\ell'} + \frac{\lambda}{\sqrt{2}} y_{\Delta_3}^N \alpha_\ell^2 & -\frac{\lambda}{\sqrt{2}} y_{\Delta_2}^N \alpha_D \alpha_{\ell'} + \frac{\lambda}{\sqrt{6}} y_{\Delta_3}^N \alpha_\ell^2 & y_{\Delta_4}^N \alpha_\ell^2 \end{pmatrix}.$$

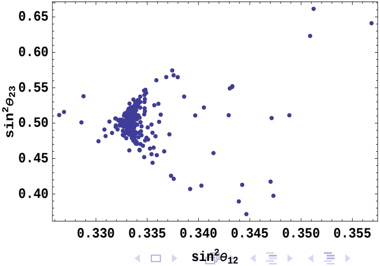
$$U_{e3} \sim \frac{y_{\Delta_1}^N \alpha_u \alpha}{y_2^N \alpha_N} \sim \mathcal{O}(10^{-2})$$

Numerical analysis of lepton mixing (Charged lepton is diagonal)





- When $\sin^2\theta_{12} \geq 0.335$, $\theta_{13} \gtrsim 0.05$
- When $\sin^2 2\theta_{23} \leq 0.99$, $\theta_{13} \gtrsim 0.05$



Charged lepton

- $(R^c_e, R^c_\mu) L \chi_\ell H_{45} \Phi^2/\Lambda\bar{\Lambda}^2$,

	(R^c_e, R^c_μ)	L	χ_ℓ	H_{45}
S_4	$\mathbf{2}$	$\mathbf{3}_1$	$\mathbf{3}_1$	$\mathbf{1}_1$
Z_4	$-i$	i	-1	-1

- $R^c_\tau L \chi'_\ell H_d/\Lambda$,

	R^c_τ	L	χ'_ℓ
S_4	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{3}_1$
Z_4	-1	i	i

VEVs are $\langle \chi_\ell \rangle = (\alpha_{\ell_1}, \alpha_{\ell_2}, \alpha_{\ell_3})\Lambda$, $\langle \chi_{\ell'} \rangle = (\alpha_{\ell'_1}, \alpha_{\ell'_2}, \alpha_{\ell'_3})\Lambda$

$$M_\ell = -3y_1\lambda v_{45} \begin{pmatrix} 0 & \alpha_{\ell_2}/\sqrt{2} & -\alpha_{\ell_3}/\sqrt{2} \\ -2\alpha_{\ell_1}/\sqrt{6} & \alpha_{\ell_2}/\sqrt{6} & \alpha_{\ell_3}/\sqrt{6} \\ 0 & 0 & 0 \end{pmatrix} + y_2 v_d \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{\ell'_1} & \alpha_{\ell'_2} & \alpha_{\ell'_3} \end{pmatrix}$$

Down-type quarks are related to charged letptons due to $SU(5)$

Down-type quarks

- $(Q_1, Q_2)D^c\chi_\ell H_{45}\Phi^2/\Lambda\bar{\Lambda}^2$,

	(Q_1, Q_2)	D^c	χ_ℓ	H_{45}
S_4	$\mathbf{2}$	$\mathbf{3}_1$	$\mathbf{3}_1$	$\mathbf{1}_1$
Z_4	$-i$	i	-1	-1

- $Q_3 D^c \chi'_\ell H_d/\Lambda$,

	Q_3	D^c	χ'_ℓ
S_4	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{3}_1$
Z_4	-1	i	i

VEVs are $\langle\chi_\ell\rangle = (\alpha_{\ell_1}, \alpha_{\ell_2}, \alpha_{\ell_3})\Lambda$, $\langle\chi_{\ell'}\rangle = (\alpha_{\ell'_1}, \alpha_{\ell'_2}, \alpha_{\ell'_3})\Lambda$

$$M_d = y_1\lambda v_{45}\begin{pmatrix} 0 & -2\alpha_{\ell_1}/\sqrt{6} & 0 \\ \alpha_{\ell_2}/\sqrt{2} & \alpha_{\ell_2}/\sqrt{6} & 0 \\ -\alpha_{\ell_3}/\sqrt{2} & \alpha_{\ell_3}/\sqrt{6} & 0 \end{pmatrix} + y_2 v_d \begin{pmatrix} 0 & 0 & \alpha_{\ell'_1} \\ 0 & 0 & \alpha_{\ell'_2} \\ 0 & 0 & \alpha_{\ell'_3} \end{pmatrix}$$

Charged leptons and down-type quarks

We take the alignment of VEVs as follows

$\langle\chi_\ell\rangle = (0, \alpha_\ell, 0)\Lambda, \quad \langle\chi'_\ell\rangle = (0, 0, \alpha_{\ell'})\Lambda,$

$$M_l = \begin{pmatrix} 0 & -\frac{3y_1\lambda\alpha_\ell v_{45}}{\sqrt{2}} & 0 \\ 0 & -\frac{3y_1\lambda\alpha_\ell v_{45}}{\sqrt{6}} & 0 \\ 0 & 0 & y_2\alpha_{\ell'}v_d \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & 0 & 0 \\ \frac{y_1\lambda\alpha_\ell v_{45}}{\sqrt{2}} & \frac{y_1\lambda\alpha_\ell v_{45}}{\sqrt{6}} & 0 \\ 0 & 0 & y_2\alpha_{\ell'}v_d \end{pmatrix}$$

$$M_l^\dagger M_l = v_d^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 6|\bar{y}_1\lambda\alpha_\ell|^2 & 0 \\ 0 & 0 & |y_2\alpha_{\ell'}|^2 \end{pmatrix}, \quad M_d^\dagger M_d = v_d^2|\bar{y}_1\lambda\alpha_\ell|^2 \begin{pmatrix} \frac{1}{2} & \frac{1}{2\sqrt{3}} & 0 \\ \frac{1}{2\sqrt{3}} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{|y_2\alpha_{\ell'}|^2}{|\bar{y}_1\lambda\alpha_\ell|^2} \end{pmatrix}$$

$m_e^2 = 0,\ m_\mu^2 = 6|\bar{y}_1\lambda\alpha_\ell v_d|^2,\ m_\tau^2 = |y_2\alpha_{\ell'}v_d|^2$

$m_d^2 = 0,\ m_s^2 = \frac{2}{3}|\bar{y}_1\lambda\alpha_\ell v_d|^2,\ m_b^2 = |y_2\alpha_{\ell'}v_d|^2 \qquad (\bar{y}_1 = y_1 v_{45}/v_d)$

$\theta_{12}^d = 60^\circ$ and other angles are zero

Including next-to-leading terms,
charged leptons and down-type quarks become

$$M_l = \begin{pmatrix} \epsilon_{11} & -3y_1\lambda\alpha_\ell v_{45}/\sqrt{2} + \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & -3y_1\lambda\alpha_\ell v_{45}/\sqrt{6} + \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & y_2\alpha_{\ell'} v_d + \epsilon_{33} \end{pmatrix},$$

$$M_d = \begin{pmatrix} \bar\epsilon_{11} & \bar\epsilon_{12} & \bar\epsilon_{13} \\ \frac{\sqrt{3}m_s}{2} + \bar\epsilon_{21} & \frac{m_s}{2} + \bar\epsilon_{22} & \bar\epsilon_{23} \\ \bar\epsilon_{31} & \bar\epsilon_{32} & y_2\alpha_{\ell'} v_d + \bar\epsilon_{33} \end{pmatrix}$$

$\epsilon_{mn} \sim \mathcal{O}(\alpha_i\alpha_j)v_d,\ \alpha_i \sim \mathcal{O}(10^{-2})$

$\epsilon_{11} = (y_{\Delta_b}\alpha_D\alpha - 3\bar y_{\Delta_{c_2}}\alpha_u\alpha_D)v_d, \quad \epsilon_{21} = -3\bar y_{\Delta_{c_1}}\alpha_u\alpha_D v_d,$

$\bar\epsilon_{11} = (y_{\Delta_b}\alpha_D\alpha - 3\bar y_{\Delta_{c_2}}\alpha_u\alpha_D)v_d, \quad \bar\epsilon_{21} = \bar y_{\Delta_{c_1}}\alpha_u\alpha_D v_d, \quad \cdots$

Since $m_e^2 = \frac{3}{2}(\frac{1}{6}\epsilon_{11}^2 - \frac{1}{\sqrt{3}}\epsilon_{11}\epsilon_{21} + \frac{1}{2}\epsilon_{21}^2)$, $m_d^2 = \frac{3}{2}(\frac{1}{6}\bar\epsilon_{11}^2 - \frac{1}{\sqrt{3}}\bar\epsilon_{11}\bar\epsilon_{21} + \frac{1}{2}\bar\epsilon_{21}^2)$,

$m_e^2 : m_d^2 = 1 : 9$ when $\alpha = -\frac{5(\sqrt{3}\bar y_{\Delta_{e_1}} - \bar y_{\Delta_{e_2}})}{y_{\Delta_d}}\alpha_u$ or $\alpha = -\frac{2(\sqrt{3}\bar y_{\Delta_{e_1}} - \bar y_{\Delta_{e_2}})}{y_{\Delta_d}}\alpha_u$

$\theta_{12}^l \sim (m_e/m_\mu),\ \theta_{13}^l \sim (m_e/m_\tau),\ \theta_{23}^l \sim (m_e m_\mu/m_\tau^2)$

$\theta_{12}^d \sim (m_d/m_s),\ \theta_{13}^d \sim (m_d/m_b),\ \theta_{23}^d \sim (m_d/m_b)$

Up-type quarks

- $(Q_1, Q_2) T^c \chi_u H_u / \Lambda$,

	(Q_1, Q_2)	T^c	χ_u
S_4	$\mathbf{2}$	$\mathbf{1}_1$	$\mathbf{2}$
Z_4	$-i$	-1	$-i$

- $Q_3 (U^c, C^c) \chi_u H_u / \Lambda$,

	Q_3	(U^c, C^c)	χ_u
S_4	$\mathbf{1}_1$	$\mathbf{2}$	$\mathbf{2}$
Z_4	-1	$-i$	$-i$

- $Q_3 T^c H_u$

	Q_3	T^c
S_4	$\mathbf{1}_1$	$\mathbf{1}_1$
Z_4	-1	-1

- $(Q_1, Q_2)(U^c, C^c) \chi_u^2 H_u / \Lambda^2$,
  $(Q_1, Q_2)(U^c, C^c) \chi^2 H_u / \Lambda^2$

	(Q_1, Q_2)	(U^c, C^c)	χ_u	χ
S_4	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}_1$
Z_4	$-i$	$-i$	$-i$	$-i$

VEVs are $\langle \chi_u \rangle = (\alpha_{u_1}, \alpha_{u_2})\Lambda$, $\langle \chi \rangle = \alpha\Lambda$,

$$M_u = v_u \begin{pmatrix} A+B & y^u_{\Delta_{a_2}} \alpha_{u_1}\alpha_{u_2} & y^u_1 \alpha_{u_1} \\ y^u_{\Delta_{a_2}} \alpha_{u_1}\alpha_{u_2} & A-B & y^u_1 \alpha_{u_2} \\ y^u_1 \alpha_{u_1} & y^u_1 \alpha_{u_2} & y^u_2 \end{pmatrix},$$

where $A = y^u_{\Delta_{a_1}}(\alpha^2_{u_1} + \alpha^2_{u_2}) + y^u_{\Delta_b}\alpha^2$, $B = y^u_{\Delta_{a_2}}(\alpha^2_{u_1} - \alpha^2_{u_2})$

Up-type quarks

$\langle \chi_u \rangle = \alpha_u(1,1)\Lambda, \quad \langle \chi \rangle = \alpha\Lambda$

$$M_u = v_u \begin{pmatrix} 2y^u_{\Delta_{a_1}}\alpha_u^2 + y^u_{\Delta_b}\alpha^2 & y^u_{\Delta_{a_2}}\alpha_u^2 & y^u_1\alpha_u \\ y^u_{\Delta_{a_2}}\alpha_u^2 & 2y^u_{\Delta_{a_1}}\alpha_u^2 + y^u_{\Delta_b}\alpha^2 & y^u_1\alpha_u \\ y^u_1\alpha_u & y^u_1\alpha_u & y^u_2 \end{pmatrix}$$

$m_u = ((2y^u_{\Delta_{a_1}} - y^u_{\Delta_{a_2}})\alpha_u^2 + y_{\Delta_b}\alpha^2)v_u,$
$m_c = ((2y^u_{\Delta_{a_1}} + y^u_{\Delta_{a_2}})\alpha_u^2 + y_{\Delta_b}\alpha^2)v_u - 2\frac{y_1^{u2}}{y_2^u}\alpha_u^2 v_u, \; m_t = |y^u_2|v_u$
$\theta^u_{12} = 45^\circ$, $\theta^u_{13} = 0^\circ$, and $\theta^u_{23} \simeq \sqrt{m_c/m_t}$

CKM matrix

$$U_u \simeq \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix},$$

$$U_d \simeq \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_{12}^d & \theta_{13}^d \\ -\theta_{12}^d - \theta_{13}^d\theta_{23}^d & 1 & \theta_{23}^d \\ -\theta_{13}^d + \theta_{12}^d\theta_{23}^d & -\theta_{23}^d - \theta_{12}^d\theta_{13}^d & 1 \end{pmatrix}.$$

where $r_t = \sqrt{m_t/(m_c + m_t)}$ and $r_c = \sqrt{m_c/(m_c + m_t)}$

Then CKM matrix is

$$V^0 = U_u^\dagger U_d \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & -r_c \\ 0 & r_c & r_t \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 15^\circ & \sin 15^\circ & 0 \\ -\sin 15^\circ & \cos 15^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & \theta_{12}^d & \theta_{13}^d \\ -\theta_{12}^d - \theta_{13}^d\theta_{23}^d & 1 & \theta_{23}^d \\ -\theta_{13}^d + \theta_{12}^d\theta_{23}^d & -\theta_{23}^d - \theta_{12}^d\theta_{13}^d & 1 \end{pmatrix}.$$

$V_{us} \sim \sin 15^\circ,\ V_{ub} \sim 0,\ V_{cb} \sim r_c \simeq 0.048$

$$V^0_{us} \simeq \theta^d_{12}\cos 15^\circ + \sin 15^\circ,$$
$$V^0_{ub} \simeq \theta^d_{13}\cos 15^\circ + \theta^d_{23}\sin 15^\circ,$$
$$+r_c(\theta^d_{23} + \theta^d_{12}\theta^d_{13}) \ ,$$
$$V^0_{cb} \simeq -r_t\theta^d_{13}e^{i\rho}\sin 15^\circ$$
$$+r_t\theta^d_{23}e^{i\rho}\cos 15^\circ - r_c \ ,$$

$$r_t \simeq 1, \ r_c \simeq 0.048$$

CKM matrix at electroweak scale

$$V_{\rm CKM} \simeq \begin{pmatrix} V^0_{ud} & V^0_{us} & V^0_{ub}/h(t) \\ V^0_{cd} & V^0_{cs} & V^0_{cb}/h(t) \\ V^0_{td}/h(t) & V^0_{ts}/h(t) & V^0_{tb} \end{pmatrix}$$

$$h(t) \simeq 1.05$$

If we take $\rho = 123^\circ$, $\theta^d_{12} = -0.0340$, $\theta^d_{13} = 0.00626$, $\theta^d_{23} = -0.00880$, our calculated values

$$\alpha = 89.4^\circ, \quad \beta = 21.9^\circ, \quad \gamma = 68.7^\circ, \quad |J_{CP}| \simeq 3.06 \times 10^{-5}$$

are agreed with observed values

Summary

We proposed $S_4 \times Z_4$ flavor model with $SU(5)$ SUSY GUT

At the leading order

- lepton mixing is tri-bimaximal mixing
- Cabbibo angle is 15°

Including the next-to-leading terms

- CKM mixing angles are consistent with observed values
- U_{e3} is expected to be less than 0.1

	$(\chi_{15},\chi_{16},\chi_{17})$	χ_1^0	χ_2^0	χ_3^0	(χ_4^0,χ_5^0)
$SU(5)$	1	1	1	1	1
S_4	**3**	**1**	**1**	**1**	**2**
Z_4	-1	-1	i	-1	$-i$
$U(1)_{FN}$	$-z$	$2\ell+n$	0	2ℓ	z
$U(1)_R$	0	2	2	2	2

$$\begin{aligned} w' &= \kappa_1 (\chi_{u_1},\chi_{u_2}) \otimes (\chi_{u_1},\chi_{u_2}) \otimes (\chi_{N_1},\chi_{N_2}) \otimes \chi_1^0/\Lambda \\ &+ \eta_1 (\chi_{\ell_1},\chi_{\ell_2},\chi_{\ell_3}) \otimes \left(\chi_{\ell_1'},\chi_{\ell_2'},\chi_{\ell_3'}\right) \otimes \chi_2^0 \\ &+ \eta_2 (\chi_{u_1},\chi_{u_2}) \otimes (\chi_{u_1},\chi_{u_2}) \otimes \chi_3^0 + \eta_3 \chi \otimes \chi \otimes \chi_3^0 \\ &+ \eta_4 (\chi_{D_1},\chi_{D_2},\chi_{D_3}) \otimes (\chi_{15},\chi_{16},\chi_{17}) \otimes \left(\chi_4^0,\chi_5^0\right), \end{aligned}$$

With the conditions of potential minimum

$$
\begin{aligned}
\kappa_1 \left[2\chi_{u_1}\chi_{u_2}\chi_{N_1} + \left(\chi_{u_1}^2 - \chi_{u_2}^2\right)\chi_{N_2}\right]/\Lambda &= 0, \\
\eta_1 \left(\chi_{\ell_1}\chi_{\ell'_1} + \chi_{\ell_2}\chi_{\ell'_2} + \chi_{\ell_3}\chi_{\ell'_3}\right) &= 0, \\
\eta_2(\chi_{u_1}^2 + \chi_{u_2}^2) + \eta_3\chi^2 &= 0, \\
\frac{1}{\sqrt{2}}\eta_4\left(\chi_{D_2}\chi_{16} - \chi_{D_3}\chi_{17}\right) &= 0, \\
\frac{1}{\sqrt{6}}\eta_4\left(-2\chi_{D_1}\chi_{15} + \chi_{D_2}\chi_{16} + \chi_{D_3}\chi_{17}\right) &= 0\ ,
\end{aligned}
$$

Alignment of VEVs is consistent with above

$$
\chi_{u_1} = \chi_{u_2}, \quad \chi_{N_1} = 0, \quad \chi_{D_1} = \chi_{D_2} = \chi_{D_3}, \quad \chi_{\ell_1} = \chi_{\ell_3} = \chi_{\ell'_1} = \chi_{\ell'_2} = 0,
$$

$$
\chi^2 = -\frac{2\eta_2}{\eta_3}\chi_{u_1}^2, \quad \chi_{15} = \chi_{16} = \chi_{17}\ .
$$