Dirac-Born-Infeld actions and the dilaton in N=2 Supersymmetry

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Based on: NA, Antoniadis, Derendinger, Tziveloglou, "Nonlinear Supersymmetry, Brane-bulk Interactions and Super-Higgs without Gravity" (arXiv:0911.5212) and "The Hypermultiplet with Heisenberg Isometry in N=2 Global and Local Supersymmetry" (arXiv:1005.0323)

Motivation

- Effective theory approach to string compactifications
- Type II on Calabi-Yau give N=2 supergravity in 4D → concentrate on global N=2 supersymmetry
- D-branes break half of supersymmetry which is then nonlinearly realised on their worldvolume
- Universal Hypermultiplet has Heisenberg symmetry preserved in string perturbation theory

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Aim

Description of D-branes coupled to Universal Hypermultiplet (UH) with off-shell global $N = 2$ supersymmetry: one linearly realised and one nonlinearly.

Results

- Global $N = 2$ supersymmetry and Heisenberg symmetry uniquely determine the form of the theory
- Generalized the derivation of the DBI from constrained superfields to the coupling to the dilaton (single-tensor) with one linear and one nonlinear supersymmetry
- Nonlinear $N = 2$ vector multiplet coupled to full single-tensor: no orientifold truncation
- New super-Higgs without gravity
- Low energy effective theory description of string theory compactified on rigid Calabi-Yau including perturbative corrections

D-brane effective action

- D-branes have a gauge field living on their worldvolume (open strings).
- The effective theory is the Born-Infeld. [Fradkin-Tseytlin 1985]
- The coupling of the worldvolume fields to bulk fields (closed strings) is (for a space-filling D3-brane in a fixed, flat background)

$$
\begin{aligned}
S_{D3} &= -T_{D3} \int d^4\xi \, e^{-\phi} \sqrt{-\det\left(\eta_{\mu\nu} + B_{\mu\nu} + F_{\mu\nu}\right)} \\
&\quad + \int \left(\frac{1}{2} C_0 (B_2 + F)^2 + C_2 \wedge (B_2 + F) + C_4 \right)
\end{aligned}
$$

[Leigh 1989]

where the second line is the topological term with the coupling to RR fields.

Universal hypermutiplet in string compactifications

- At string tree-level the UH is described by a NLSM with target space $SU(2,1)/[SU(2)\times U(1)]$, which has 8 isometries
  [Cecotti, Ferrara, Girardello, 1989]
- String loop corrections yield a one-parameter deformation proportional to the Euler characteristic $\chi_E = 2(h_{1,1} - h_{2,1})$, breaking $SU(2,1)$ down to Heisenberg (3 isometries)

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Metric of target manifold (quaternion-Kähler) with Heisenberg symmetry:

$$ds^2_{local} = \frac{V-\chi}{(V+\chi)^2}\left(\frac{dV^2}{4V} + d\eta^2 + d\varphi^2\right) + \frac{4V}{(V+\chi)^2(V-\chi)}\Big(d\tau + \eta\, d\varphi\Big)^2 ,$$

with $V+\chi = e^{-2\phi_4}$, $\varphi \leftrightarrow C_{\mu\nu}$, $\tau \leftrightarrow B_{\mu\nu}$, $\eta = C_0$ and $\chi \propto \chi_E$.
[Antoniadis, Minasian, Theisen, Vanhove 2003]

Heisenberg algebra

In IIB we have two anti-symmetric tensors $B_{\mu\nu}$ (NS-NS) and $C_{\mu\nu}$ (R-R) and a R-R scalar C_0 with gauge and shift symmetries

$$\delta B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}\,, \quad \delta C_{\mu\nu} = 2\partial_{[\mu}\tilde{\Lambda}_{\nu]} + \lambda B_{\mu\nu}\,, \quad \delta C_0 = \lambda,$$

which realise the Heisenberg algebra

$$[\delta_1, \delta_2] C_{\mu\nu} = 2\partial_{[\mu}\lambda_2\Lambda_{1\nu]} - 2\partial_{[\mu}\lambda_1\Lambda_{2\nu]}\,.$$

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- Can easily prove that there is a unique theory of a hypermultiplet in global $N = 2$ with Heisenberg symmetry
- Precise relation to the string-loop corrected UH in the gravity-decoupling limit determines the identification of fields [AADT, 1005.0323]

The constrained vector multiplet

Usual $N=2$ vector superfield is

$$\mathcal{W} = X + \sqrt{2} i \widetilde{\theta}^{\alpha} W_{\alpha} - \frac{1}{4} \widetilde{\theta}\widetilde{\theta} \overline{DDX}$$

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We impose a constraint on the $N = 2$ vector superfield $\mathcal{W}$

[Roček-Tseytlin 1998]

$$\mathcal{W}_{nl}^2 = \left(\mathcal{W} - \frac{1}{2\kappa} \widetilde{\theta}\widetilde{\theta} \right)^2 = \mathcal{W}^2 - \frac{1}{\kappa} \widetilde{\theta}\widetilde{\theta} \mathcal{W} = 0$$

In terms of $N = 1$ superfields the $\tilde{\theta}\tilde{\theta}$ component is

$$WW - \frac{1}{2} X \overline{DDX} = \frac{1}{\kappa} X$$

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- Need to deform the 2nd SUSY transformation of W_{α}
- Gaugino transforms as a goldstino.
- Solution of the constraint $X(WW)$ contains the Born-Infeld [Bagger-Galperin 1996]

Hypermultiplet as a single-tensor

- Usual description of single-tensor in terms of L (linear) and Φ (chiral)
- L contains $\partial_{[\mu} B_{\nu\rho]}$, χ_α contains $B_{\mu\nu}$, with $L = D^\alpha \chi_\alpha - \overline{D}_{\dot\alpha} \overline{\chi}^{\dot\alpha}$

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- L contains $\partial_{[\mu} B_{\nu\rho]}$, χ_α contains $B_{\mu\nu}$, with $L = D^\alpha \chi_\alpha - \overline{D}_{\dot\alpha} \overline{\chi}^{\dot\alpha}$
- $N = 2$ supersymmetrisation of $\delta B_{\mu\nu} = \partial_{[\mu} \Lambda_{\nu]}$ requires the introduction of another chiral superfield Y containing a 4-form
- We can construct a chiral $N = 2$ superfield

$$\mathcal{Y} = Y + \sqrt{2}\widetilde{\theta}^\alpha \chi_\alpha - \widetilde{\theta}\widetilde{\theta}\left[\frac{i}{2}\Phi + \frac{1}{4}\overline{DDY}\right]$$

- $N = 2$ gauge variation is

$$\delta\mathcal{Y} = \widehat{\mathcal{W}}$$

Dilaton-Brane coupling and DBI

Take as Lagrangian [AADT, 0911.5215]

$$\mathcal{L} = \int d^2\theta d^2\widetilde{\theta}\, i\mathcal{Y}\mathcal{W}_{nl} + \mathrm{c.c.} + \mathcal{L}_{ST,kin}$$

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After integration of auxiliary field in W_α the bosonic Lagrangian is

$$\begin{aligned}\mathcal{L}_{DBI,bos.} \quad &= \quad \frac{\mathrm{Re}\,\phi}{4\kappa}\left[1 - \sqrt{1 + \frac{C^2}{2(\mathrm{Re}\,\phi)^2}}\sqrt{-\det(\eta_{\mu\nu} + 2\sqrt{2}\kappa F_{\mu\nu})}\right] \\ &\quad + \epsilon^{\mu\nu\rho\sigma}\left(\frac{\kappa}{4}\mathrm{Im}\,\phi\, F_{\mu\nu}F_{\rho\sigma} - \frac{1}{4}b_{\mu\nu}F_{\rho\sigma} + \frac{1}{24\kappa}C_{\mu\nu\rho\sigma}\right)\end{aligned}$$

We recovered the DBI action with

- field dependent coefficient in front of the square root generated by integration of auxiliary field
- Full topological term
- Invariance under non-linearly realized 2nd SUSY

Analysis of the vacuum

There is a scalar potential

$$V(C, \mathrm{Re}\,\phi) = \frac{\mathrm{Re}\,\phi}{4\kappa}\left[\sqrt{1 + \frac{C^2}{2(\mathrm{Re}\,\phi)^2}} - 1\right]$$

- $\mathrm{Im}\,\phi$ is a flat direction.
- SUSY vacuum at $\langle C \rangle = 0$, for any value of $\mathrm{Re}\,\phi$.

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Properties of SUSY vacuum:

- ϕ is massless
- C is massive $m_C^2 = \frac{1}{4\kappa\langle \mathrm{Re}\,\phi \rangle}$, same mass as the vector
- Partial breaking of supersymmetry $N = 2$ to $N = 1$

Analysis of the vacuum: super-Higgs without gravity

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- ϕ is massless
- C is massive $m_C^2 = \frac{1}{4\kappa\langle \mathrm{Re}\,\phi\rangle}$, same mass as the vector
- Partial breaking of supersymmetry $N = 2$ to $N = 1$
- Gaugino transforms as a goldstino, but is not massless
- The mixing term $\chi\lambda$ gave a mass to the gaugino, but 2nd SUSY preserved since its variation is canceled by the variation of the 4-form.

Conclusions

- Global $N = 2$ supersymmetry and Heisenberg symmetry uniquely determine the form of the theory
- Generalized the derivation of the DBI from constrained superfields to the coupling to the dilaton (single-tensor) with one linear and one nonlinear supersymmetry
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