Algebraic Singularity Method for Mass Measurements with Missing Energy

Ian-Woo Kim

University of Wisconsin-Madison

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and work in progress

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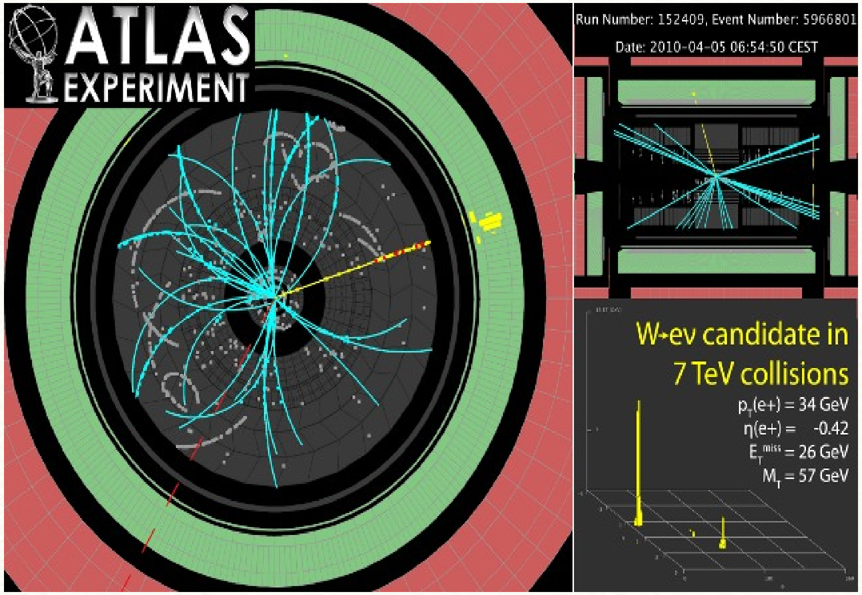
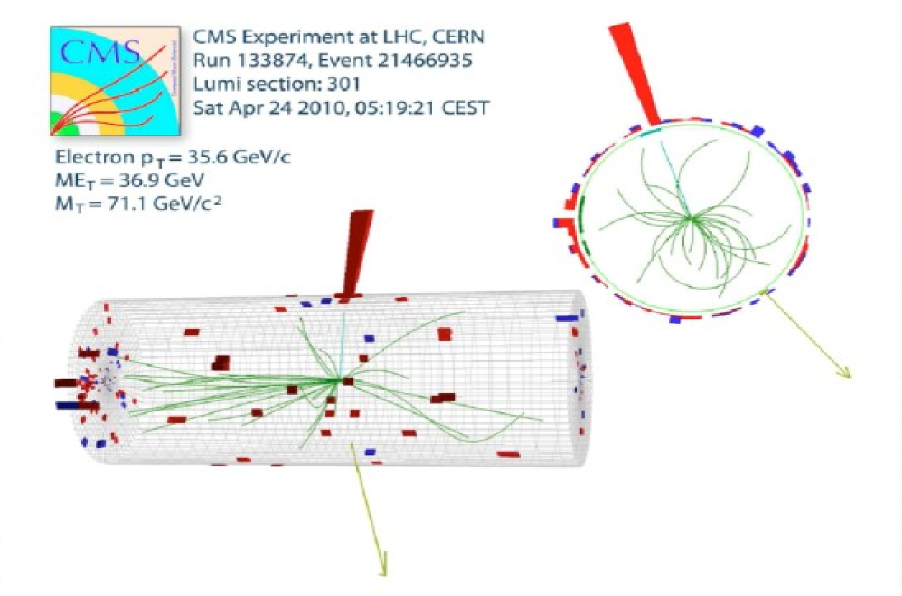
Introduction

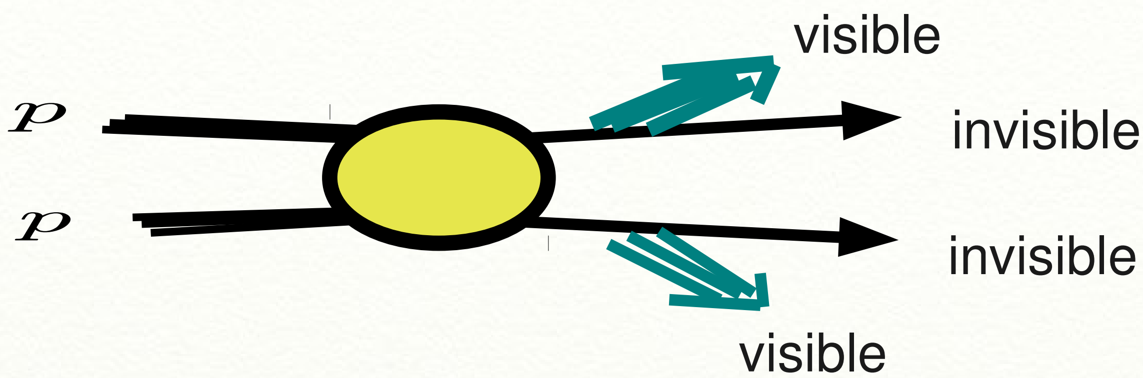
First W candidates in ATLAS and CMS at 7 TeV





- Prepare for discovery of Dark Matter particle!
- Search for missing energy signatures
  (“invisible” or “missing” particles)

Many efforts on Missing Mass Measurements...



I focus on nonreconstructable event :

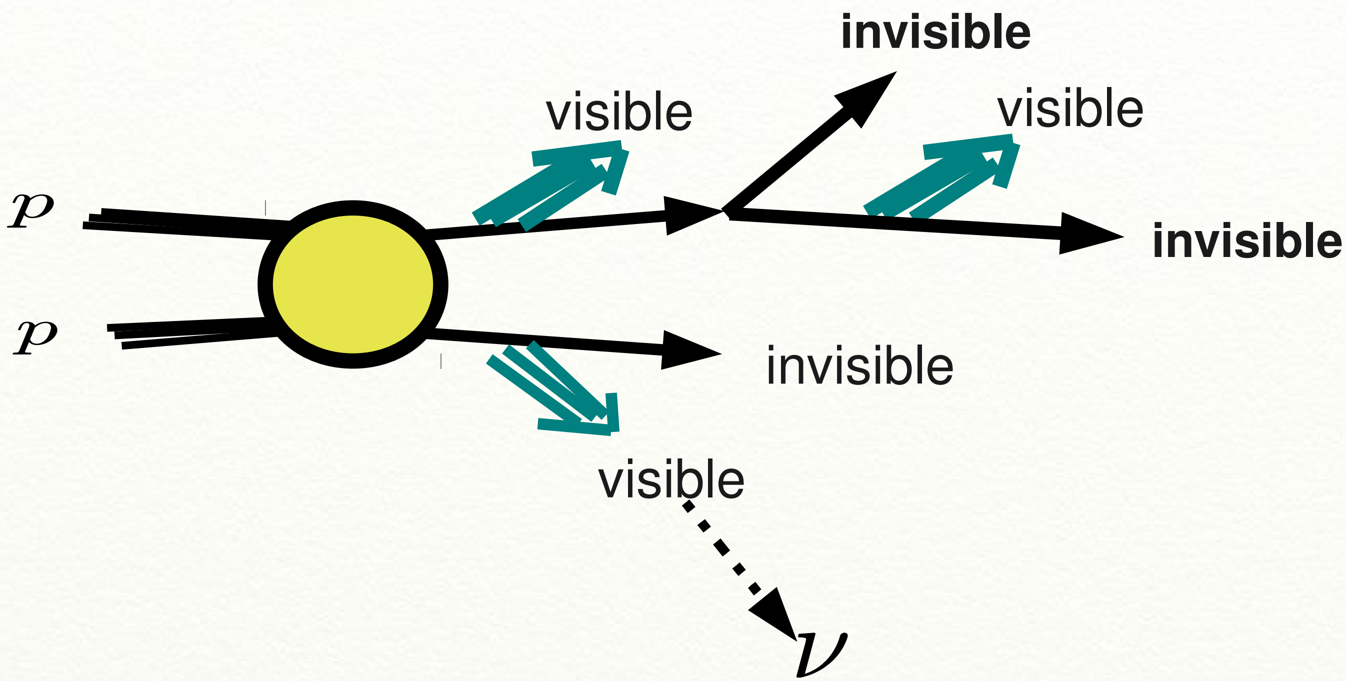
Popular techniques:

- Using end point or cusps in various kinematic distribution
- Transverse mass variables mT2 : End-point curve shows a kink at the true mass value

For summary, refer to Choi's talk (Monday)

Event topology may be even more complex.

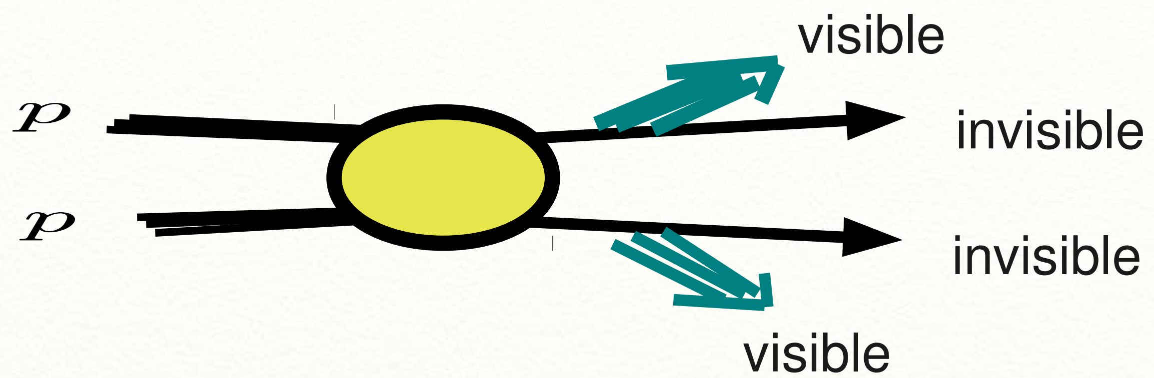
(more missing particle including neutrino, asymmetric chain)



Do we have a generalized mass measurement method?

What is the essence of missing mass measurements?

Many efforts on Missing Mass Measurements...



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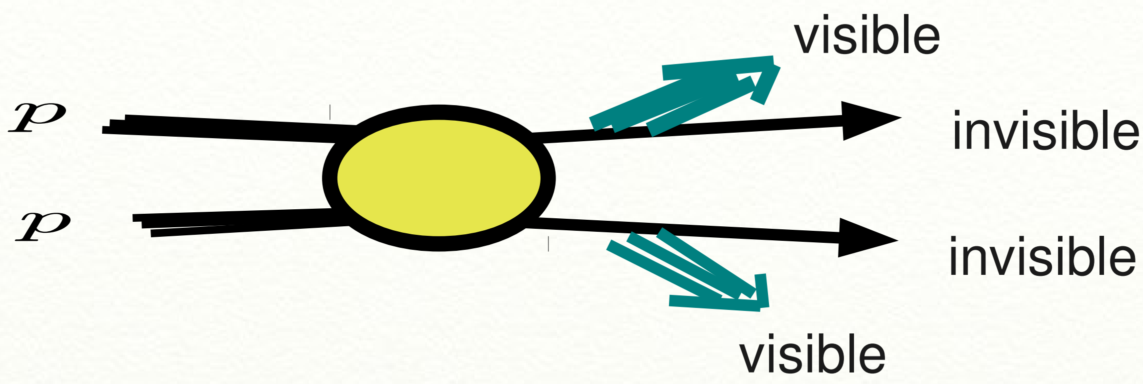

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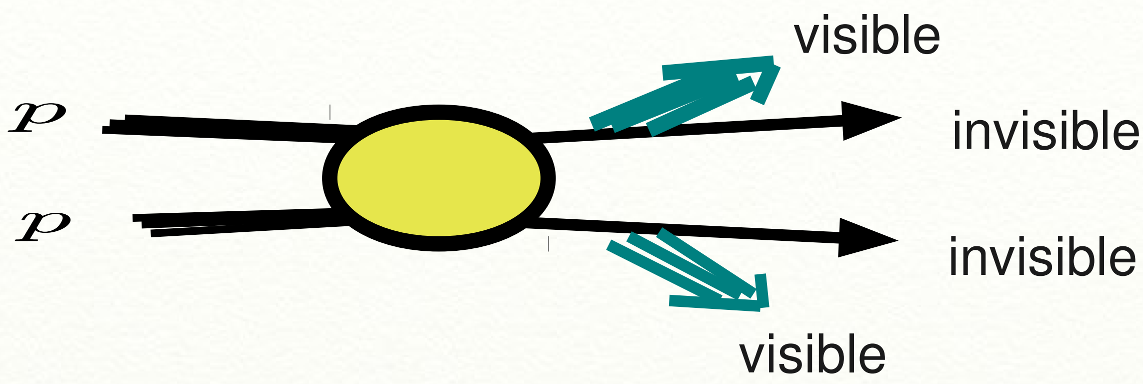

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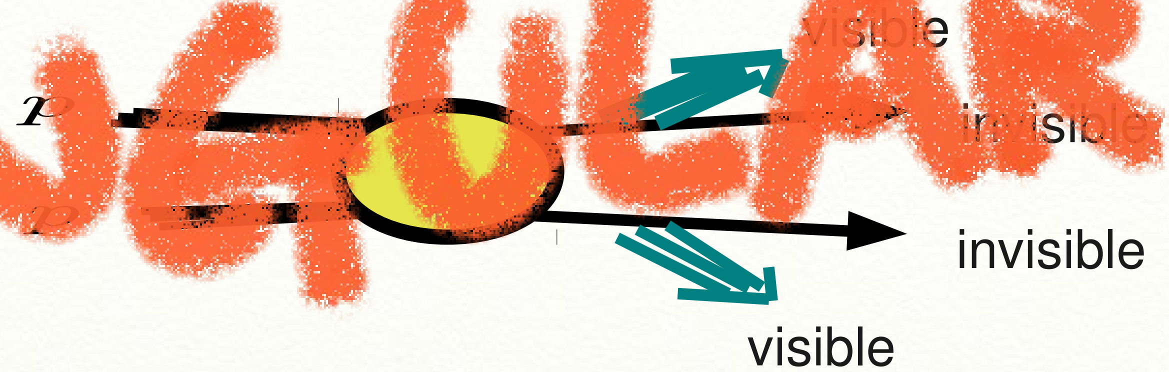
SINGULARITY



I focus on nonreconstructable events

= NONSMOOTH POINT IN DISTRIBUTION

Popular techniques:

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Algebraic Singularity Method

- Mass Measurements in a disciplined and generalized way

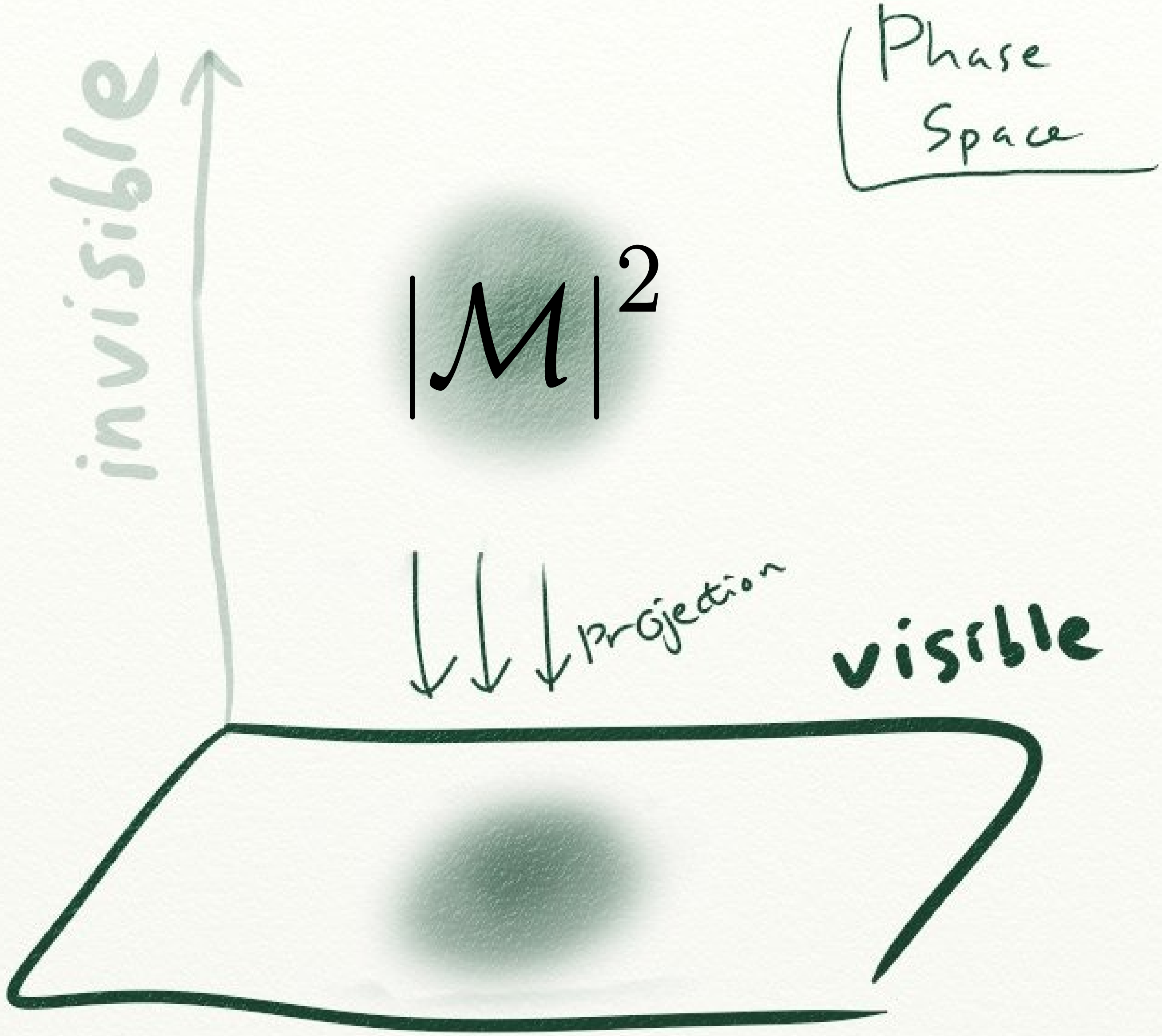
- Based on the observation that mass measurements rely on singularity structures in PS

- Systematic and mathematical formulation

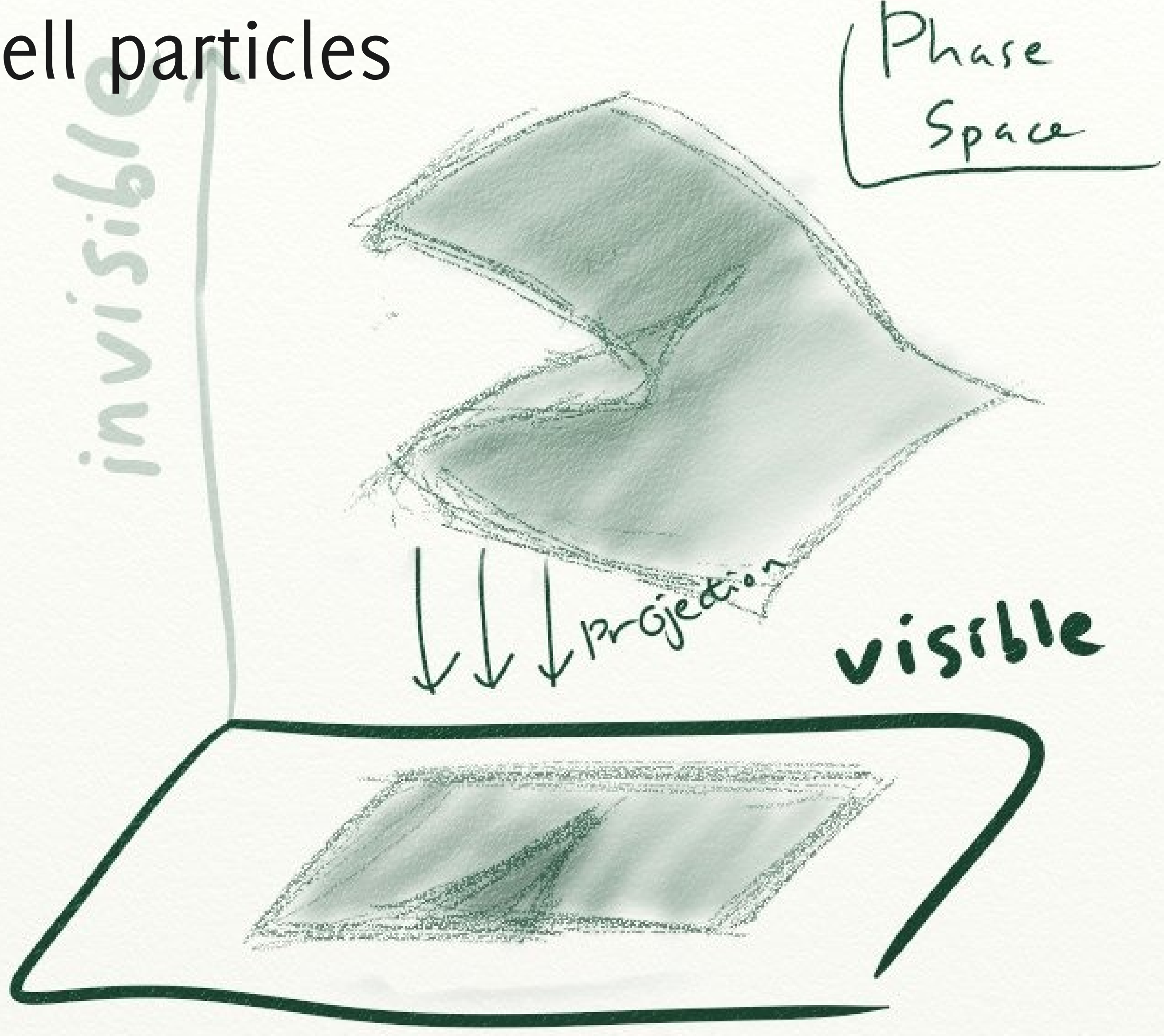
- Provide an optimized kinematic variable:
  "singularity coordinate"

- Subsumes the previous known techniques and gives a better understanding.

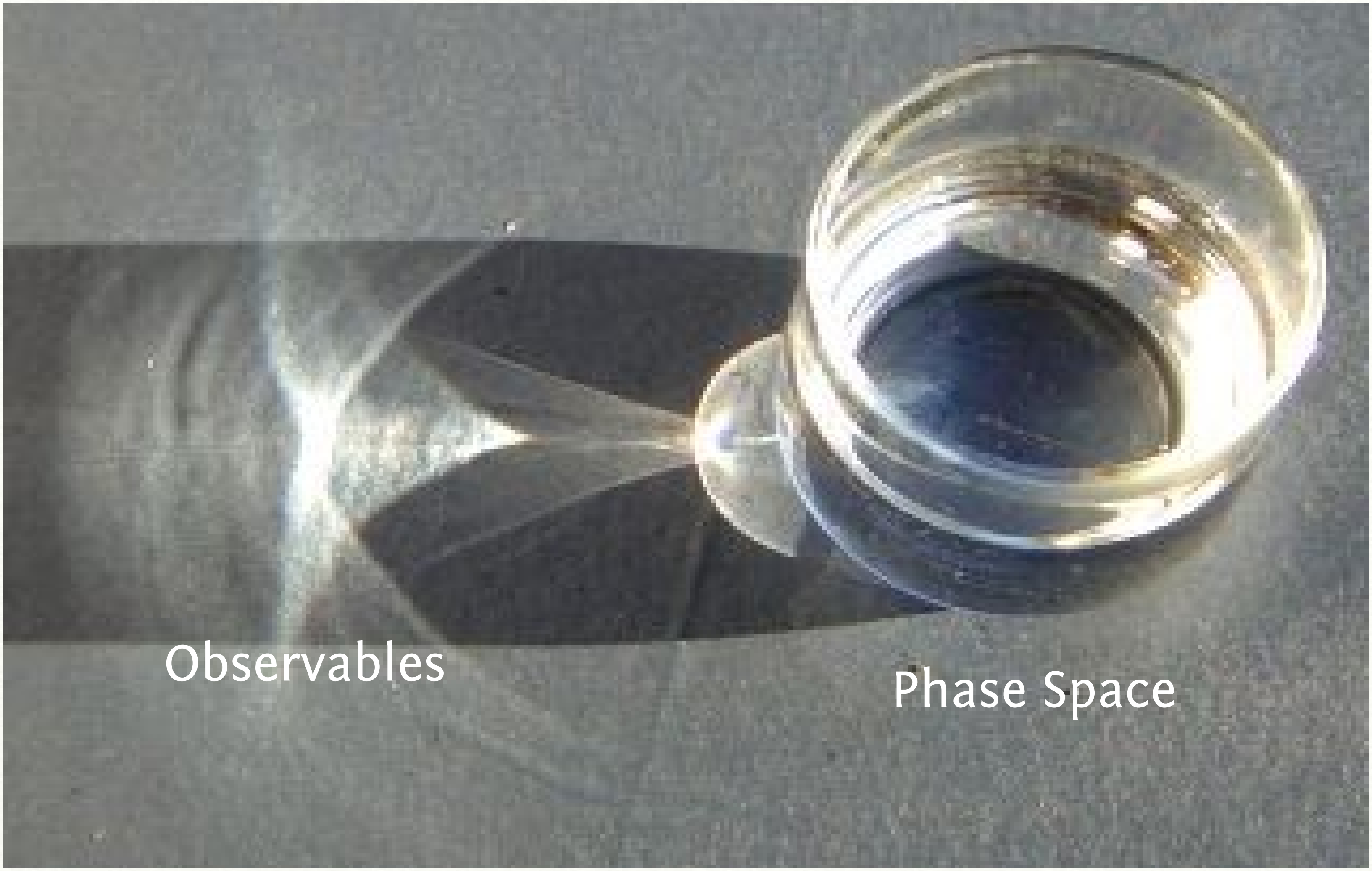
Why do Singularities Appear?



Phase Space is condensed to restricted submanifold in presence of on-shell particles

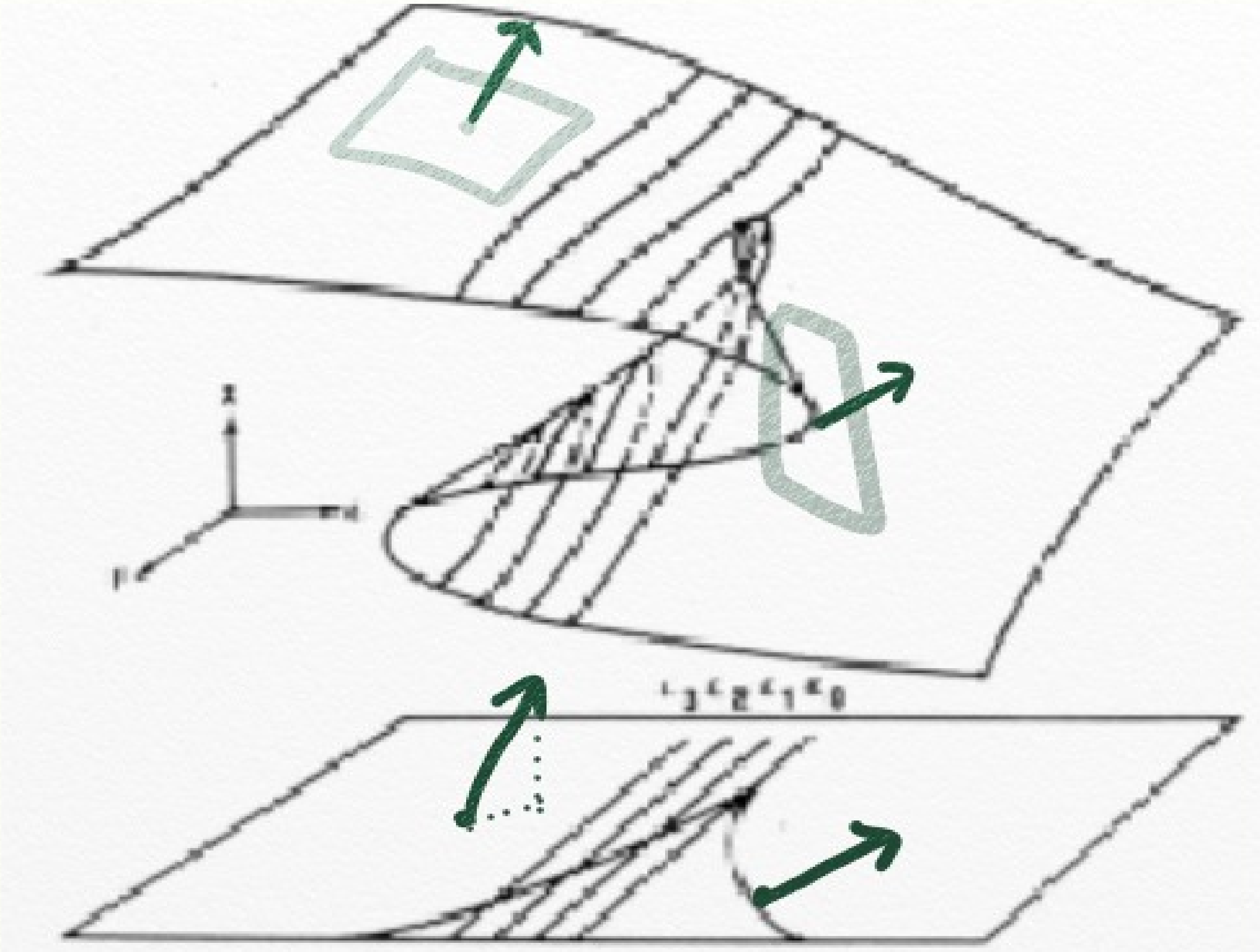


We read singularity patterns in observables and reconstruct the shape of PS.



Algebraic Singularity Method

IWK (2009)



Singularities happen when the tangent space is aligned along invisible momentum direction.

- PS = Solution Space of Coupled Polynomial Eqs.

$$\begin{aligned} g_1(x,q) &= 0 \\ g_2(x,q) &= 0 \\ g_3(x,q) &= 0 \\ &\vdots \end{aligned}$$

x : invisible momentum q : visible momentum

- At a singularity point in event observable space,

Jacobian $\left(\frac{\partial g_i}{\partial x_j}\right)$ has a reduced rank.

Using Algebraic Technique for Reduced Rank condition

- Groebner basis : sequentially solvable form

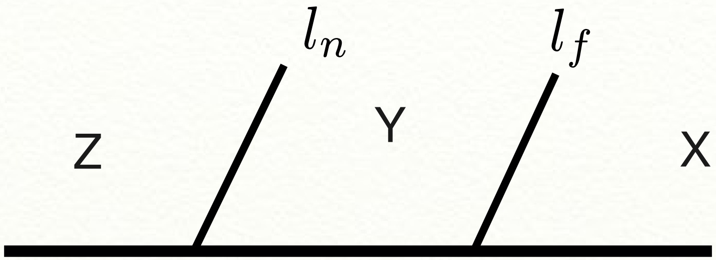
$$\begin{aligned} g_1(x_1, x_2, x_3, x_4, \dots) &= 0 \\ g_2(x_2, x_3, x_4, \dots) &= 0 \\ g_3(x_3, x_4, \dots) &= 0 \\ g_4(x_4, \dots) &= 0 \end{aligned}$$

-
- Jacobian matrix has upper triangular form.

$$\left(\frac{\partial g_i}{\partial x_j}\right) = \begin{pmatrix} X & \dots & \dots & \\ & X & X & \dots \\ & & X & \dots \end{pmatrix}$$

- Vanishing diagonal component is necessary in this basis for a reduced rank of Jacobian.

Ex) Simple Cascade Decay



Kinematic constraint equations:

$$X^2 = m_X^2$$
$$(X + l_f)^2 = m_Y^2$$
$$(X + l_f + l_n)^2 = m_Z^2$$

Event unknowns :

$$X^\mu = (X_0, X_1, X_2, X_3)$$

Event Observable :

$$l_n^\mu \qquad l_f^\mu$$

ASM guides the Best Choice of Kinematic Variable

- Jacobian matrix for Groebner basis:

$$\left(\frac{\partial g_i}{\partial X'_j}\right) = \begin{pmatrix} 2m_{ll} & 0 & 0 & 0 \\ & 2m_{ll} & 0 & 0 \\ & & m_{ll}^2 X'_1 & m_{ll}^2 X'_2 \end{pmatrix}$$

- reduced rank Jacobian arise when 3rd row vanishes

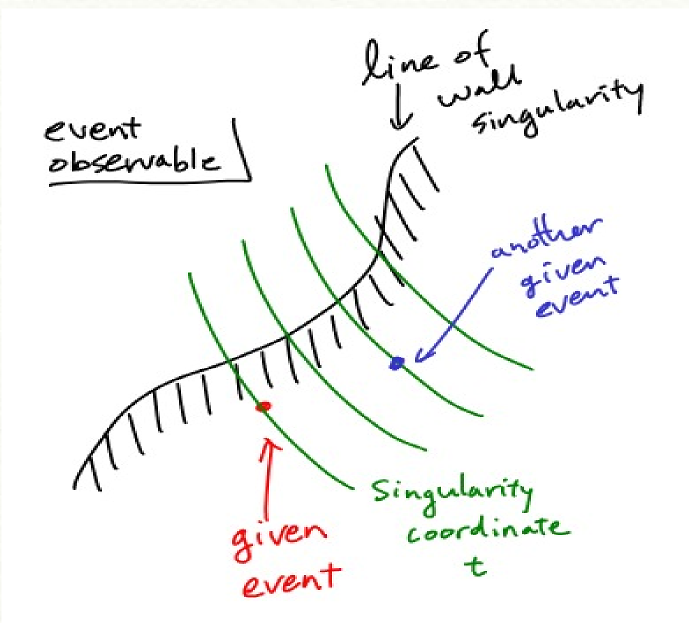
$$m_{ll}^2 = \frac{(m_Z^2 - m_Y^2)(m_Y^2 - m_X^2)}{m_Y^2} \equiv m_{ll}^{(\mathrm{end})\,2}$$

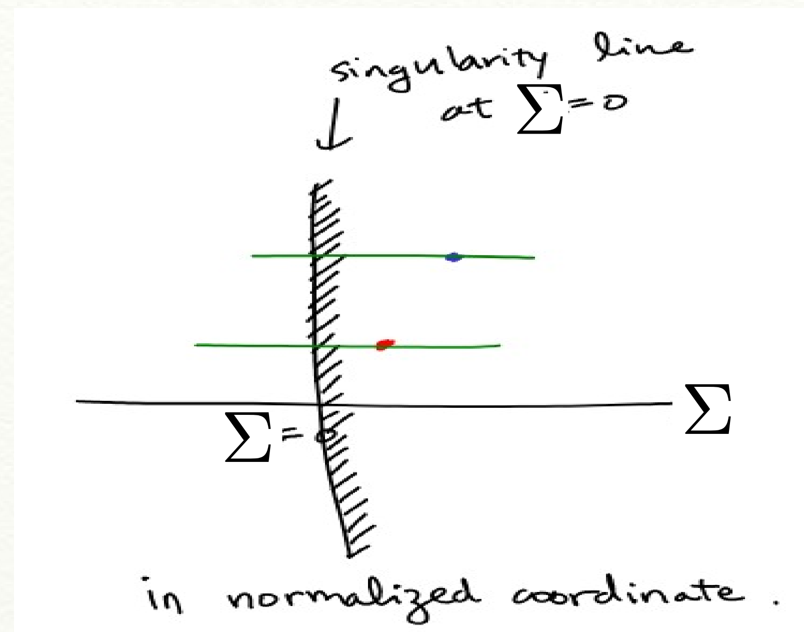
- Note that we did not start from the invariant mass. Algebraic Singularity Method shows that the invariant mass has singularities.

Singularity Coordinate

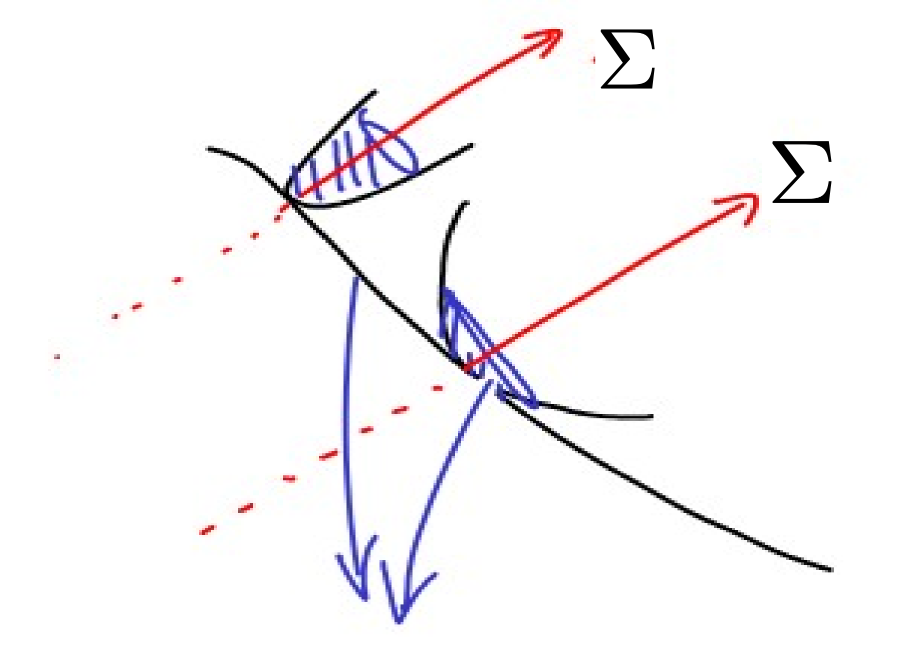
1. Singularities must collectively appear on the same point in the singularity coordinate.

2. Singularities must be perpendicular to the singularity coordinate direction.



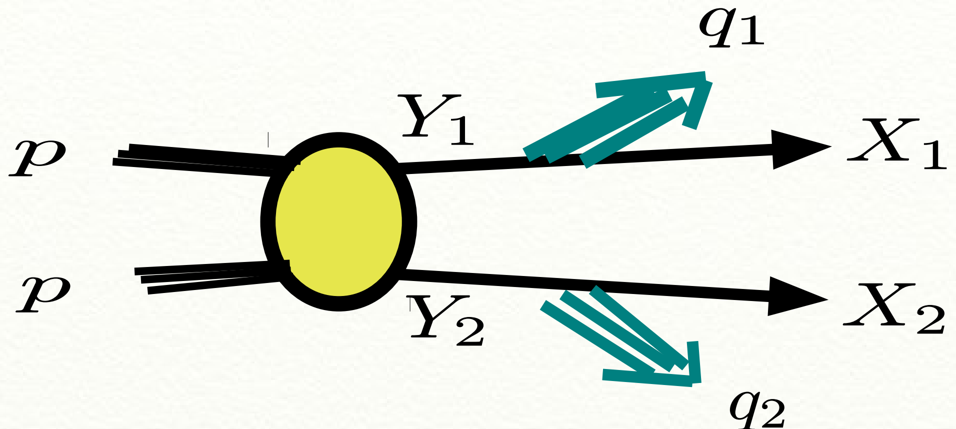


3. Each singularity must have the same statistical significance.



These conditions turn out nothing but 0^{th} , 1^{st} and 2^{nd} order boundary conditions at singular points.

Double Missing Particle Chain Topology



Kinematic Constraint Equations :

$$X_1^2 = m_X^2 \qquad X_2^2 = m_X^2$$

$$(X_1 + q_1)^2 = m_Y^2 \qquad (X_2 + q_2)^2 = m_Y^2$$

$$\vec{X}_{1T} + \vec{X}_{2T} = \vec{P}_T^{\text{miss}}$$

Event unknowns:

$$X_1^\mu = (X_{10}, X_{11}, X_{12}, X_{13}) \qquad X_2^\mu = (X_{20}, X_{21}, X_{22}, X_{23})$$

Event observables:

$$q_1^\mu = (q_{10}, q_{11}, q_{12}, q_{13}) \quad q_2^\mu = (q_{20}, q_{21}, q_{22}, q_{23}) \quad \vec{P}_T^{\text{miss}} = (P_{T1}, P_{T2})$$

Jacobian Matrix :

$$
\begin{array}{c}
g_6 \\ g_5 \\ g_4 \\ g_3 \\ g_2 \\ g_1
\end{array}
\begin{pmatrix}
X & X & & & & & X & X \\
& X & & & & & X & X \\
& & X & & & X & X & X \\
& & & 1 & & & 1 & \\
& & & & 1 & & & 1 \\
& & & & & X & X & X
\end{pmatrix}
$$

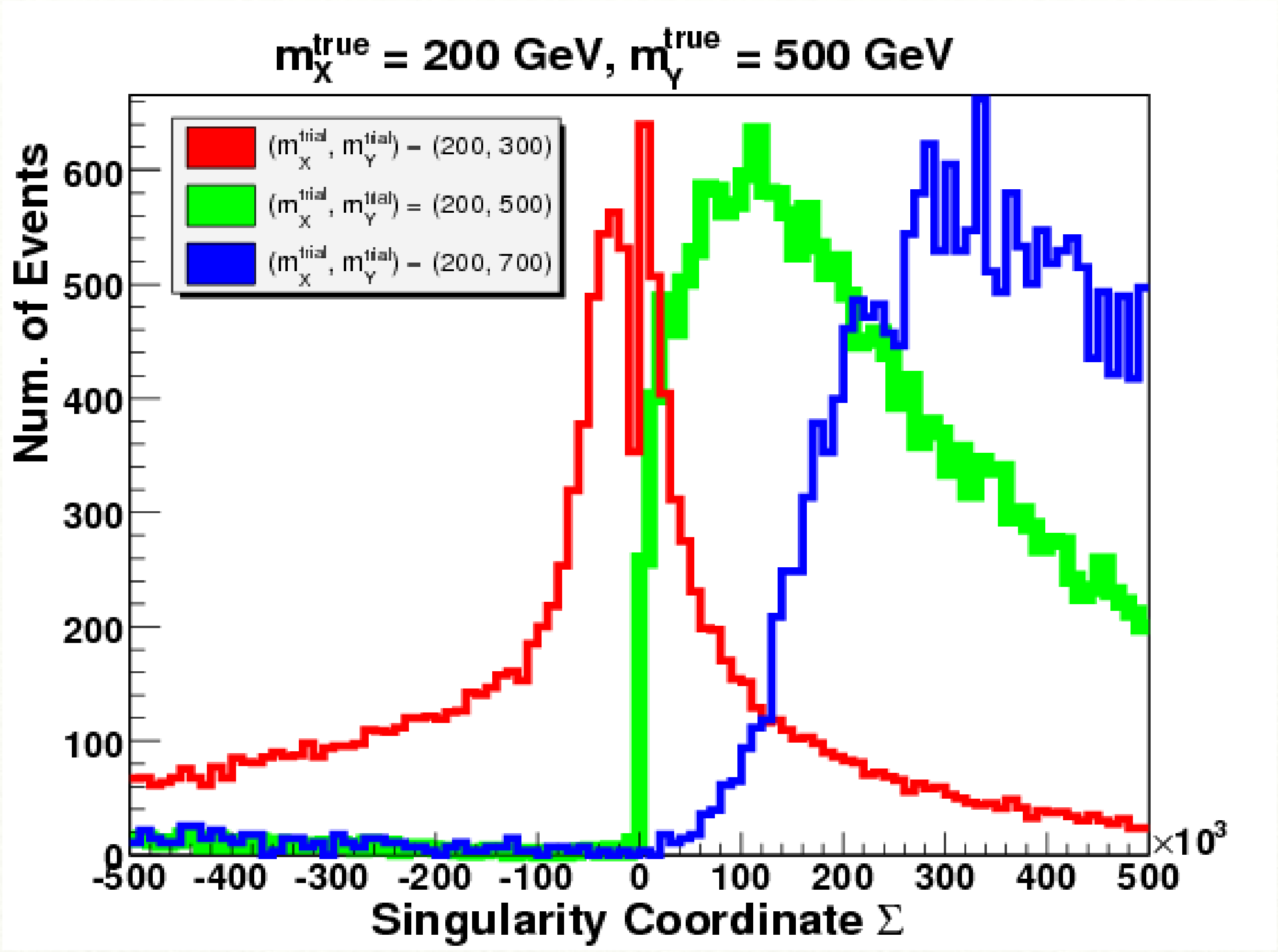
reduced rank condition :

$$
\left.
\begin{aligned}
2(q_{10}^2 - q_{13}^2)X_{13} - 2A_1 q_{13} = 0 \\
2(q_{20}^2 - q_{23}^2)X_{23} - 2A_2 q_{23} = 0
\end{aligned}
\right\}
$$

$$
\det \begin{pmatrix}
\frac{\partial g_5}{\partial X_{11}} & \frac{\partial g_5}{\partial X_{12}} \\
\frac{\partial g_1}{\partial X_{11}} & \frac{\partial g_1}{\partial X_{12}}
\end{pmatrix} = 0
$$

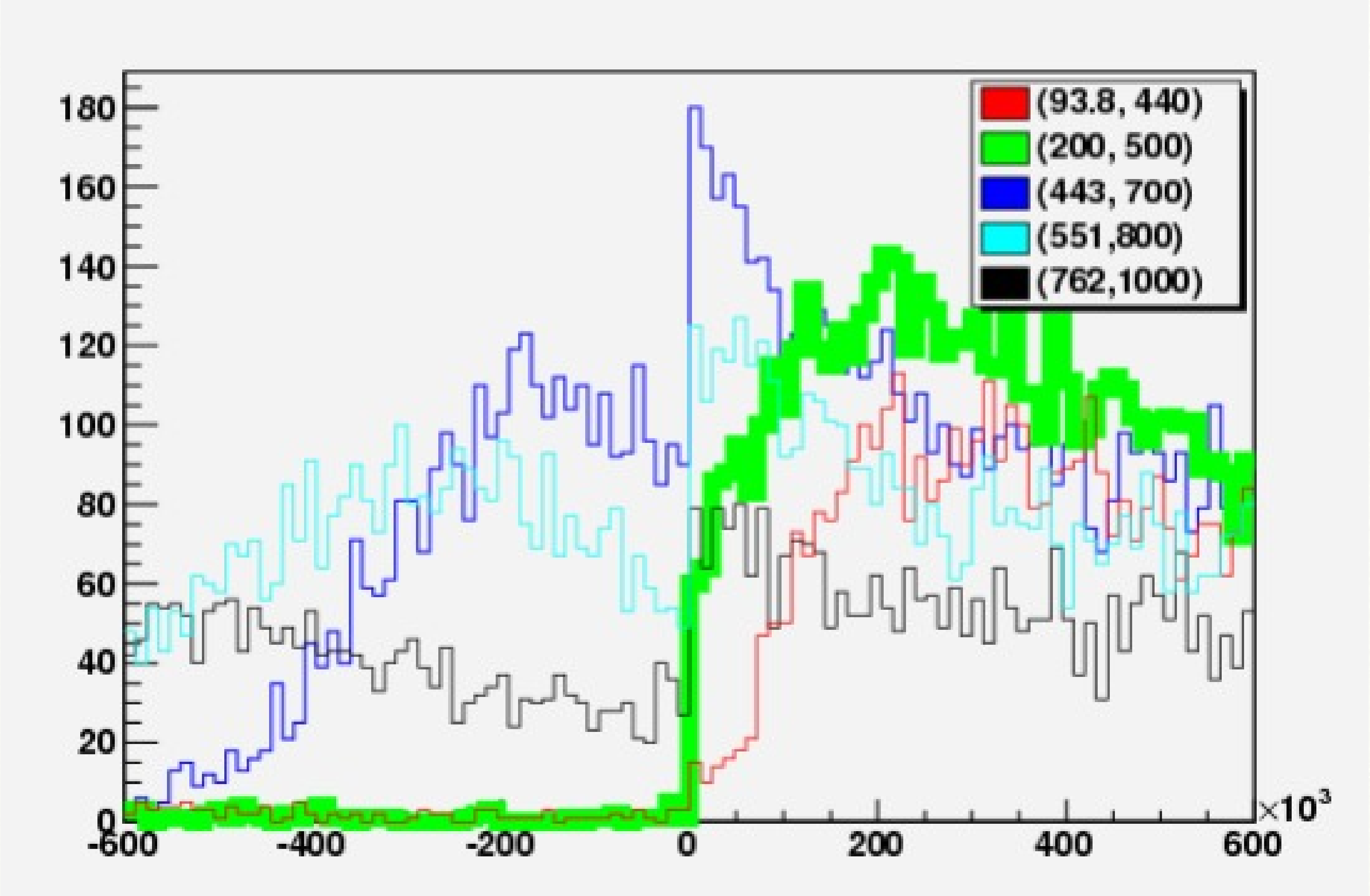
Yields the maximum MT2 condition.
(unbalanced or balanced)

Numerical Analysis



Singularity coordinate distribution along mT2 max curve.

IWK, work in progress



Conclusion

- Key Idea in mass measurement technique:
  PS folding $\longrightarrow$ Singularities

- A new method (Algebraic Singularity Method) developed for finding PS singularities and constructing optimized kinematic variable that exploits the singularities

- Naturally encompasses previous methods for non-reconstructable event topologies and can be generalized to more complicated event topologies.

Thank you!