Multi-Higgs Models and Minimal Flavour Violation

talk given at Planck 2010

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based on work done in collaboration with

F. J. Botella and M. N. Rebelo

and earlier work, with

W. Grimus and L. Lavoura

For the relationship with the Minimal Flavour Violation principle, see G. Isidori's talk and

A. Buras, M. Carlucci, S. Gori, G. Isidori 1005.53v1 (2010)

- Neutral Currents have played a crucial rôle in the construction and experimental tests of unified gauge theories

EPS Prize to Gargamelle

Important features of Flavour-Changing neutral currents (FCNC):

- Forbidden at tree level both in the gauge and scalar sectors, in the Standard Model and most of its extensions.

- At loop level, FCNC are generated and they have played a crucial rôle in testing the SM and putting bounds on New Physics beyond the SM:

$K^0 - \bar{K}^0$ mixing

$D^0 - \bar{D}^0$ mixing

$B_d^0 - \bar{B}_d^0$ mixing

$B_s^0 - \bar{B}_s^0$ mixing

rare Kaon decays

rare b-meson decays

CP violation

Two dogmas:

- No Z-mediated FCNC at tree level

- No FCNC in the scalar sector at tree level

Glashow and Weinberg (Phys. Rev. D 15, 1958 (1977))
E. A. Paschos, P.R.D. (1977)
derived necessary and sufficient conditions for this:

(i) "All quarks of fixed charge and helicity must transform according to the same irreducible representation of $SU(2)$ and correspond to the same eigenvalue of T_3"

(ii) "All quarks should receive their contributions in the quark mass matrix from a single neutral Higgs vev.

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Natural conservation laws for neutral currents*

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(Received 20 August 1976)

We explore the consequences of the assumption that the direct and induced weak neutral currents in an SU(2)⊗U(1) gauge theory conserve all quark flavors *naturally*, i.e., for all values of the parameters of the theory. This requires that all quarks of a given charge and helicity must have the same values of weak T_3 and $\vec{T}^2$. If all quarks have charge $+2/3$ or $-1/3$ the only acceptable theories are the "standard" and "pure vector" models, or their generalizations to six or more quarks. In addition, there are severe constraints on the couplings of Higgs bosons, which apparently cannot be satisfied in pure vector models. We also consider the possibility that neutral currents conserve strangeness but not charm. A natural seven-quark model of this sort is described. The experimental consequences of charm nonconservation in direct or induced neutral currents are found to be quite dramatic.

I. INTRODUCTION

It has been known for many years that there are no strangeness-changing neutral-current weak interactions, or none with anything like the strength of the familiar charged-current weak interactions. We see this from the slowness of such decays as $K_L^0 \to \mu^+\mu^-$ and $K^+ \to \pi^+\nu\bar{\nu}$, and even more strongly (and independently of the nature of the lepton couplings) from the size of the K_1^0-K_2^0 mass difference. an effective strangeness-changing neutral current of order αG_F. In still other theories, the exchange of Higgs bosons can produce a strangeness-changing neutral current of roughly the same order. In principle these effects may perhaps be eliminated by a retuning of the parameters of the theory (including in the last case the parameters of the Higgs-boson interactions), but we would find a theory much more attractive if the neutral currents conserved strangeness naturally.

Can one violate these two Dogmas in "reasonable" extensions of the SM ?

Yes !

"reasonable" means that FCNC should be naturally suppressed, without fine-tuning.

In the gauge sector, the dogma can be violated through the introduction of $Q = -1/3$ and/or $Q = 2/3$ vector like quarks.

$\Downarrow$

Naturally small violations of 3×3 unitarity of V_{CKM}

$\Downarrow$

Naturally suppressed FCNC at tree level!

Example : Addition of one $Q = -1/3$ vector-like quark.

D_L , D_R → singlets under $SU(2)_L$.

(Large number of references ...)

Charged currents :

$$\begin{bmatrix} \bar{u} & \bar{c} & \bar{t} \end{bmatrix}_L \gamma^\mu \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & V_{uD} \\ V_{cd} & V_{cs} & V_{cb} & V_{cD} \\ V_{td} & V_{ts} & V_{tb} & V_{tD} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} W_\mu + \text{h.c.}$$

Non-orthogonality of columns leads to terms like :

$$\frac{g}{2\cos\theta_W} Z_{bd} \, \bar{b}_L \gamma_\mu d_L Z^\mu$$

$$Z_{bd} = V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^*$$

Z_{bd} suppressed by $\left(\frac{m}{M}\right)^2$

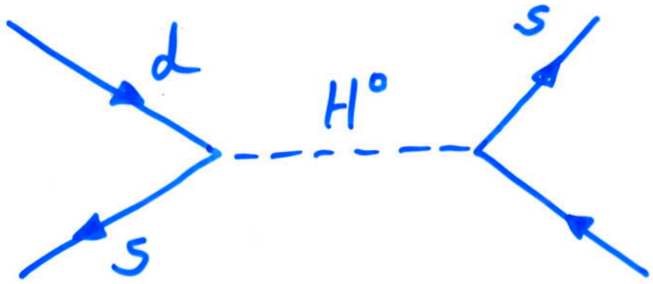
where m → mass of standard quarks

M → mass of D-quark

Scalar Sector

Can one have scalar-mediated FCNC at tree level, but somehow suppressed by "small V^{CKM} elements"?

Most-dangerous couplings :



$K_L - K_S$ mass diff.

$\Downarrow$

$m_H \gtrsim 1$ TeV

CP violation ε_K

$m_H \gtrsim 30$ TeV

The idea that FCNC could in principle be suppressed by small V^{CKM} elements was suggested by various authors :

- L. Hall, S. Weinberg
- A. Antaramian, L. Hall, A. Rasin
- Yoshipura, S.D. Rindani

Interesting, but ad-hoc assumptions not based on an exact (or softly broken) symmetry of the Lagrangian

Question : Can one have a multi-Higgs extension of the SM where, as the result of a family symmetry there are FCNC at tree level, but with all the couplings controlled by V^{CKM}, with no other flavour parameters ?

- First we show that this looks like a "Mission Impossible"

- Then will show that there are models which fulfill the above condition but the number of models is severely restricted

BGL → G.C. Branco, W. Grimus, L. Lavoura Phys. Lett 1996

F. Botella, M. N. Rebelo, GCB

arXIV 0911.1753 (Nov. 2009)

Phys. Lett B (2010)

The requirement of rephasing Invariance

Let us consider a FCNC transition connecting a quark d_j to another quark d_k. The transition could be mediated by a scalar or by a vector boson.

$$\mathcal{L}_{scalar} = \bar{d}_{L_j} \Gamma^{S}_{jk} d_{R_k} S$$

$$\mathcal{L}_{vector} = \bar{d}_{L_j} \Gamma^{V}_{jk} \gamma_\mu d_{L_k} V^\mu$$

Γ^S, Γ^V may arise at tree level or in higher orders. Assume that d_j denote quark mass eigenstates. Under rephasing of quark fields:

$$d_j \to d'_j = \exp(-i\beta_j)\, d_j$$

Γ^S, Γ^V have to transform in such a way that the above interactions remain invariant. This implies that under rephasing

$$\Gamma_{jk} \to \Gamma'_{jk} = \exp\left[i(\beta_k - \beta_j)\right] \Gamma_{jk}$$

If we require that the flavour dependence of Γ_{jk} is completely controlled by V^{CKM}, this severely restricts the functional dependence of Γ_{jk} on V_{CKM}.

The simplest allowed forms allowed by rephasing invariance are :

$$\Gamma_{jk} = \sum_{\alpha} c_{\alpha} V_{\alpha j} V^{*}_{\alpha k}$$

Note that the form of Z_{bd} found previously, satisfies this requirement (as it had to!), with

$$c_{\alpha} = 1 \text{ for all } c_{\alpha}$$

$$Z_{bd} = V_{ud} V^{*}_{ub} + V_{cd} V^{*}_{cb} + V_{td} V^{*}_{tb}$$

The case of Two Higgs doublets Models

Yukawa Interactions

$$\mathcal{L}_Y = -\overline{Q}_L^0 \Gamma_1 \Phi_1 d_R^0 - \overline{Q}_L^0 \Gamma_2 \phi_2 d_R^0 - \overline{Q}_L^0 \Delta_1 \tilde{\Phi}_1 u_R^0 - \overline{Q}_L^0 \Delta_2 \tilde{\Phi}_2 u_R^0 + \text{h.c.}$$

$Q_L^0 \rightarrow$ left-handed doublets

Γ_i, Δ_j Yukawa couplings

So this is a two-Higgs doublet model of type III.

Quark mass matrices:

$$M_d = \frac{1}{\sqrt{2}}\left(v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2\right)$$

$$M_u = \frac{1}{\sqrt{2}}\left(v_1 \Delta_1 + v_2 e^{-i\alpha} \Delta_2\right)$$

Diagonalized by:

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{diag.}(m_d\ m_s\ m_b)$$

$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{diag.}(m_u, m_c, m_t)$$

Expand Φ_j :

$$\Phi_j = e^{i\alpha_j} \begin{bmatrix} \phi_j^+ \\ 1/\sqrt{2}\,(v_j + \rho_j + i\eta_j) \end{bmatrix} \qquad j = 1,2$$

It is convenient to define new fields
G^+, G^0, H^+, I, H^0, R :

$$\begin{bmatrix} G^+ \\ H^+ \end{bmatrix} = O \begin{bmatrix} \phi_1^+ \\ \phi_2^+ \end{bmatrix} ; \begin{bmatrix} G^0 \\ I \end{bmatrix} = O \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} ; \begin{bmatrix} H^0 \\ R \end{bmatrix} = O \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$

$$O = \frac{1}{v} \begin{bmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{bmatrix} \quad ; \quad v = \sqrt{v_1^2 + v_2^2} = \left(\sqrt{2}\, G_F\right)^{-1/2} \approx 246\,\text{GeV}$$

$G^+, G^0 \rightarrow$ Goldstone bosons

$H^0, R, I \rightarrow$ neutral Higgs

$H^+ \rightarrow$ Charged Higgs

It is convenient to write the
Yukawa couplings in terms of quark mass eigenstates and the new fields
$H^\pm, H^0, R, I$

$$\mathcal{L}_Y = \frac{\sqrt{2}\, H^+}{v}\, \bar{u}\left[V N_d \gamma_R + N_u^{\dagger} V \gamma_L\right] d_L + h.c. -$$

$$- \frac{H^0}{v}\left[\bar{u}\, D_u\, u + \bar{d}\, D_d\, d\right] -$$

$$- \frac{R}{v}\left[\bar{u}\left(N_u \gamma_R + N_u^{\dagger}\gamma_L\right) u + \bar{d}\left(N_d \gamma_R + N_d^{\dagger}\gamma_L\right) d\right]$$

$$+ i\, \frac{I}{v}\left[\bar{u}\left[N_u \gamma_R - N_u^{\dagger}\gamma_L\right] u - d\left[N_d \gamma_R - N_d^{\dagger}\gamma_L\right] d\right]$$

$u, d \rightarrow$ quark mass eigenstates

$\gamma_L = (1-\gamma_5)/2 \quad ; \quad \gamma_R = (1+\gamma_5)/2$

The matrices N_d, N_u are given by:

$$N_d = \frac{1}{\sqrt{2}}\, U_{dL}^{\dagger}\left[v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2\right] U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}}\, U_{uL}^{\dagger}\left[v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2\right] U_{uR}$$

Flavour Changing Neutral Couplings are controlled by the matrices N_d, N_u

For generic 2 Higgs doub. models N_d is arbitrary!!

It is convenient to write N_d in the following way:

$$N_d = \underbrace{\frac{v_2}{v_1} D_d}_{\text{Conserves flavour}} - \underbrace{\frac{v_2}{\sqrt{2}}\left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) U_{dL}^{\dagger}\, e^{i\alpha}\, \Gamma_2\, U_{dR}}_{\text{leads to FCNC}}$$

From the above expression for N_d, one concludes that there are Two Major Obstacles which one has to surmount in order for N_d to be entirely controlled by V^{CKM}, with no free parameters.

- (i) It is U_{dL} rather than the combination $U_{uL}^{\dagger} U_{dL} \equiv V^{CKM}$ which appears in N_d

- (ii) How to get rid of the dependence on U_{dR}?

The first difficulty can be solved by means of a flavour symmetry constraining U_{uL} to have mixing only among two generations, for example:

$$U_{uL} = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In this case:

$$V^{CKM} = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ U_{d_{31}} & U_{d_{32}} & U_{d_{33}} \end{bmatrix} = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ U_{d_{31}} & U_{d_{32}} & U_{d_{33}} \end{bmatrix}$$

So one has

$$(V_{CKM})_{3j} = (U_{dL})_{3j}$$

In order to surmount difficulty (i) one has to further require that the flavour dependence of N_d on U_{dL} is only on the 3rd row of U_{dL}.

How to surmount obstacle (ii), i.e. how to avoid the dependence on U_{dR}?

Let us assume that Γ_2 is such that

$$\Gamma_2 \propto P M_d$$

where P is a fixed matrix. In this case:

$$U_{dL}^\dagger \Gamma_2 U_{dR} \propto U_{dL}^\dagger P M_d U_{dR}$$

$$= U_{dL}^\dagger P \underbrace{U_{dL} \overbrace{U_{dL}^\dagger M_d U_{dR}}^{D_d}}_{\mathbb{1}}$$

$$= U_{dL}^\dagger P U_{dL} D_d$$

The flavour structure of Γ_1, Γ_2 should be such that a fixed matrix P exists satisfying

$$\Gamma_2 \propto P M_d$$

One way of achieving this is by having:

$$P\,\Gamma_2 = k\,\Gamma_2 \qquad k \text{ is a constant}$$

$$P\,\Gamma_1 = 0$$

Recall that $M_d = 1/\sqrt{2}\left(v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2\right)$

It has been shown (Grimus, Lavoura, B.) that it is possible to find a family symmetry such that it leads to a flavour structure for Γ_i, Δ_i which imply FCNC at tree level, with strength completely controlled by V_{CKM}.

BGL have imposed the following symmetry on the Lagrangian:

$$\underbrace{\begin{aligned} Q^0_{L3} &\to \exp(i\alpha)\, Q^0_{L3} \\ u^0_{R3} &\to \exp(i2\alpha)\, u^0_{R3} \\ \phi_2 &\to \exp(i\alpha)\, \phi_2 \end{aligned}}$$

$\alpha \neq 0, \pi$

All other fields transform trivially

$$\Downarrow$$

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}$$

In this case:

$$P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad \frac{v_2}{\sqrt{2}} e^{i\alpha} \Gamma_2 = P M_d \; ; \; k=1$$

$$(N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \left(V_{CKM}^\dagger\right)_{i3} \left(V_{CKM}\right)_{3j} (D_d)_{jj}$$

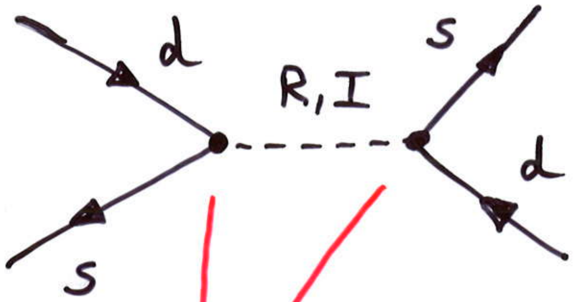
$$(N_u) = -\frac{v_1}{v_2} \operatorname{diag}(0, 0, m_t) + \frac{v_2}{v_1} \operatorname{diag.}(m_u, m_c, 0)$$

In this example the Higgs mediated FCNC are suppressed by the 3rd row of V^{CKM}. In this example, there are FCNC only in the down sector

The example given above corresponds to a class of six different models!!

An important feature of the Model that we have described :

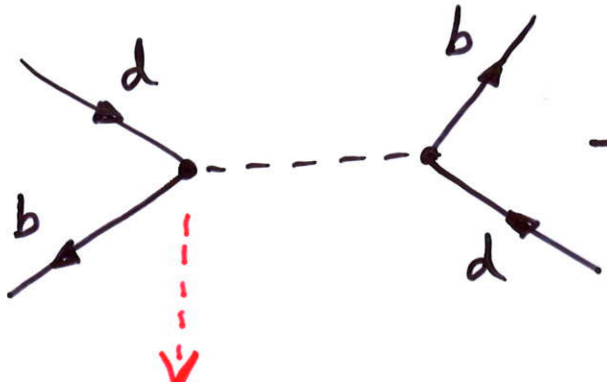
Strong and Natural suppression of the most "dangerous" processes :



$\Delta S = 2$ processes strongly suppressed

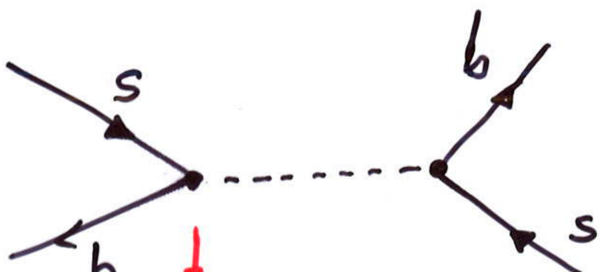
"light Higgs" are allowed

$V_{td} V_{ts}^{*} \sim \lambda^{5} \Rightarrow$ Altogether λ^{10} suppression



$\rightarrow$ may contribute significantly to $B_d - \bar{B}_d$ mixing

$V_{td} V_{tb}^{*} \sim \lambda^{3}$



Contribution to $B_s - \bar{B}_s$ mixing

$V_{ts} V_{tb}^{*} \sim \lambda^{2}$

So far, we have considered models which, due to the presence of a family symmetry, lead to FCNC completely controlled by V_{CKM}.

Can one make a "MFV-type" expansion of N_d^0, N_u^0?

It is clear that a necessary condition for N_d^0, N_u^0 to be of the "MFV-type" is that they should be functions of M_d, M_u and no other flavour dependent couplings

The terms entering in the expansion of N_d^0, N_u^0 should have the right transformation properties under weak basis (WB) transformations

Under a WB transformation, defined by:

$$Q^0_L \to W_L\, Q^0_L \;;\; d^0_R \to W^d_R\, d^0_R \;;\; u^0_R \to W^u_R\, u^0_R$$

The quark mass matrices M_u, M_d transform as:

$$M_d \to W_L^\dagger M_d W_R^d \;;\; M_u \to W_L^\dagger M_u W_R^u$$

The matrices U_{dL}, U_{dR}, U_{uL}, U_{uR} transform as:

$$U_{dL} \to W_L^\dagger U_{dL} \;;\; U_{uL} \to W_L^\dagger U_{uL}$$

$$U_{dR} \to W_R^{d\dagger} U_{dR} \quad U_{uR} \to W_R^{u\dagger} U_{uR}$$

while the Hermitian matrices $H_{d,u} \equiv M_{d,u} M^\dagger_{d,u}$ transform as:

$$H_d \to W_L^\dagger H_d W_L \;;\; H_u \to W_L^\dagger H_u W_L$$

It is convenient to write H_d, H_u in terms of projection operators (F. Botella, M. Nebot, O. Vives)

$$H_d = \sum_i m_{d_i}^2 \, P_i^{dL} \quad , \text{ where}$$

$$P_i^{dL} = U_{dL} P_i U_{dL}^\dagger \quad , \text{ with}$$

$$(P_i)_{jk} = \delta_{ij} \delta_{ik}$$

Obviously, under a WB transformation, N_d^0, N_u^0 should transform as M_d, M_u.

A MFV expansion for N_d^0, N_u^0 with proper transformation properties is:

$$N_d^0 = \lambda_1 M_d + \lambda_{2i} U_{dL} P_i U_{dL}^\dagger M_d + \lambda_{3i} U_{uL} P_i U_{uL}^\dagger \cdot M_d + \cdots$$

$$N_u^0 = \tau_1 M_u + \tau_{2i} U_{uL} P_i U_{uL}^\dagger + \tau_{3i} U_{dL} P_i U_{dL}^\dagger M_u + \cdots$$

In the mass eigenstate basis:

$$N_d^0 = \lambda_1 D_d + \lambda_{2i} P_i D_d + \lambda_{3i} V_{CKM}^\dagger P_i V_{CKM} D_d + \cdots$$

with analogous expression for N_u^0.

The BGL example considered before corresponds to the following truncation of our MFV expansion:

$$N^0_d = \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) U_{uL} P_3 U_{uL}^\dagger M_d$$

$$N^0_u = \frac{v_2}{v_1} M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) U_{uL} P_3 U_{uL}^\dagger M_u$$

Note that that the "truncation" corresponds to an exact symmetry of the Lagrangian.

There are 6 BGL type models but only one is compatible with the MFV principle

See : Gino Isidori talk and

A. Buras, M. Carlucci, S. Gori, G. Isidori 1005.5310V1

(appeared yesterday !!)

Conclusions

- Multi-Higgs models may play an important rôle in solving the flavour puzzle. For that it may be necessary to violate the Dogma of NFC in the scalar sector.
A theory of flavour may have its own mechanism of suppression of FCNC in the Higgs sector.

- LHC may bring some surprises in the scalar sector:
"Non-Standard Higgs"
with new hints to find a theory of flavour