Testing SUSY

G.G.Ross, Planck 2010

Testing ^Low energy^ SUSY

The Hierarchy problem: $M_{Higgs}, M_{W,Z} \ll M_{Planck}, M_{GUT}, ..$ ✔

$\ll M_{SUSY}$??

Fine tuning $\Longrightarrow$ SUSY accessible to the LHC

Testing Low energy SUSY

MSSM: 105 +(19) Parameters !

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- CMSSM $\mu_0, m_0, m_{1/2}, A_0, B_0$

Focus point…non-degenerate scalars more fine-tuned

Testing Low energy SUSY

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105 +(19) Parameters !

- CMSSM $\mu_0, m_0, m_{1/2}, A_0, B_0$
- Gauge mediation – no focus point

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- Gauge mediation – no focus point
- Non universal gaugino masses - new focus point
- New heavy states – higher dimension operators

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- CMSSM $\mu_0, m_0, m_{1/2}, A_0, B_0$
- Gauge mediation – no focus point
- Non universal gaugino masses - new focus point
- New heavy states – higher dimension operators
- LHC – discovery potential in 1st run

- # Fine tuning in the CMSSM

$\mu_0, m_0, m_{1/2}, A_0, B_0$

$M_S^2 = m_{q_3} m_{U_3} \geq (500 GeV)^2$

$$M_{h^0}^2 = M_Z^2 \cos^2 2\beta + \frac{3 M_t^2 h_t^2}{4\pi^2}\left(\ln\left(\frac{M_S^2}{M_t^2}\right) + \delta_t\right) + \cdots \quad \geq 114 GeV (SM)$$

$$M_Z^2 = a_0 m_0^2 + a_{1/2} m_{1/2}^2 + a_\mu \mu^2 + .. \ll \tilde{m}_{q_i}^2, M_i^2$$

$$0.6 m_{q_3}^2(M_X) + 0.6 m_{U_3}^2(M_X) + 3M_3^2(M_X) + 0.2 A_t^2(M_X) - 2\mu^2(M_X) + \ldots$$

SPS1a

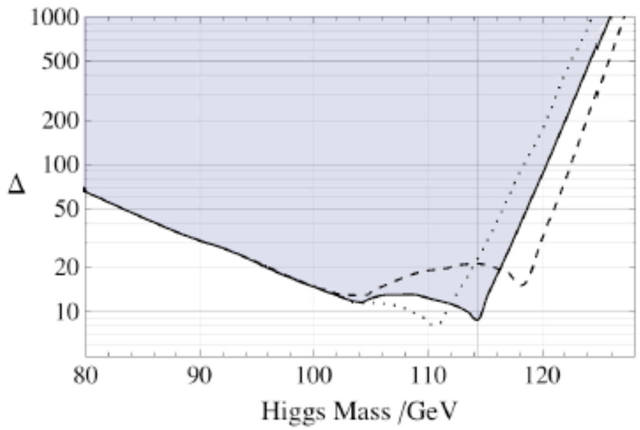
Quantitative measure of fine tuning:

$$\Delta(a_i) = \left| \frac{(a_i \; or \; \delta a_i)}{M_Z} \frac{\partial M_Z}{\partial a_i} \right|$$

$$\Delta_{\max} = Max_{a_i} \Delta(a_i)$$

Ellis, Enquist, Nanopoulos, Zwirner
Barbieri, Giudice
Ciafaloni, Strumia, Romanino

Fine tuning in the CMSSM



Constraints

SUSY particle masses

$$3.20 < 10^4\ \mathrm{Br}(b \to s\gamma) < 3.84$$

$$\mathrm{Br}(b \to \mu\mu) < 1.8 \times 10^{-8}$$

$$\delta a_\mu < 292 \times 10^{-11}$$

$$-0.0007 < \delta\rho < 0.0012$$

Radiative EW breaking

Relic density unrestricted

$$\Delta \equiv \max \left|\Delta_p\right|_{p=\{\mu_0^2, m_0^2, m_{1/2}^2, A_0^2, B_0^2, h_t ..\}}, \qquad \Delta_p \equiv \frac{\partial \ln v^2}{\partial \ln p}$$

Sample technique: Mathematica code search, then SoftSUSY †

Couplings and masses evaluated to two loop (leading log) order
…enhanced sensitivity due to small tree-level quartic coupling $\lambda = \frac{1}{8}\left(g_1^2 + g_2^2\right)\cos^2 2\beta$

Cassel, Ghilencea, GGR

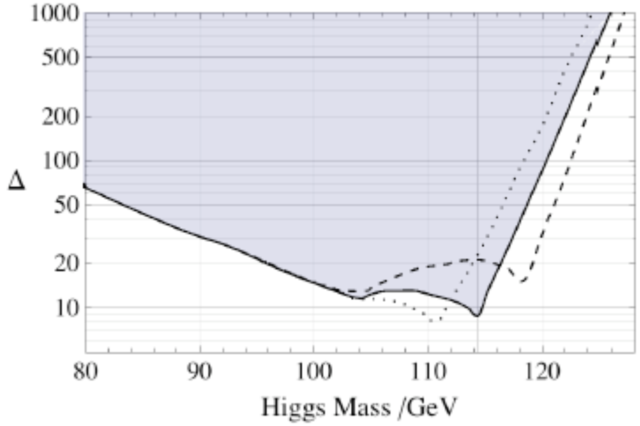
c.f. earlier work : Dimopoulos, Giudice
Chankowski, Ellis, Olechowski, Pokorski

$$\Delta_{Min} = 9, \quad m_h = 114 \pm 2GeV$$

(No Higgs bound applied)

→ 5.5 (improved minimisation of potential) Horton

Fine tuning in the CMSSM



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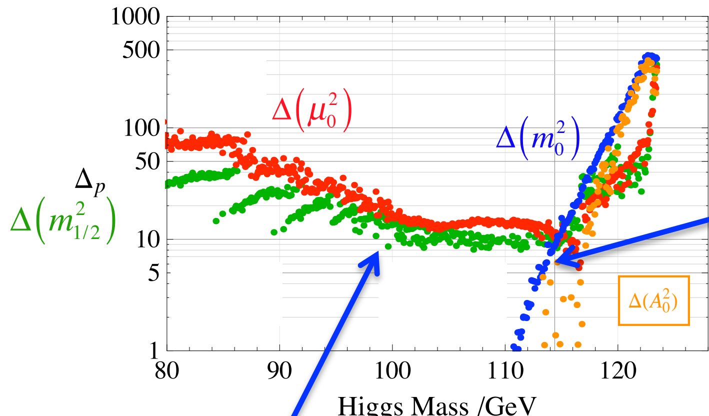
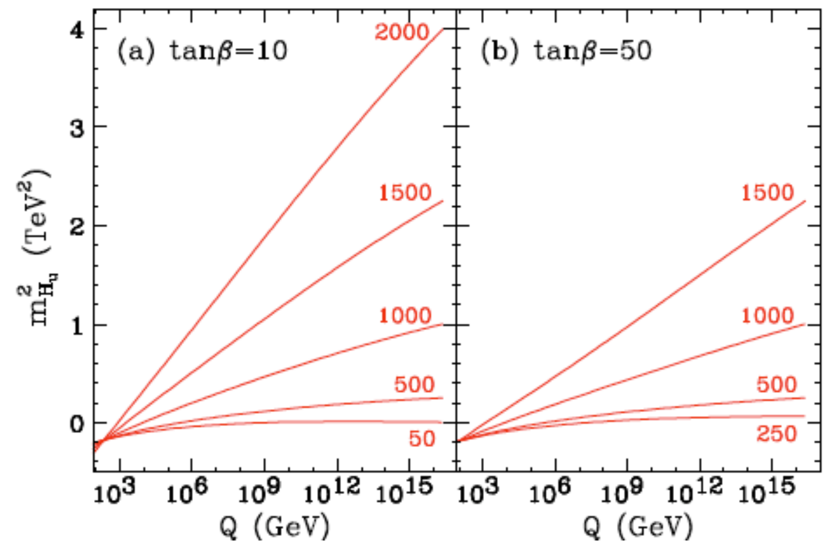


Limit of focus point

λ decrease

$$v^2 = -\frac{m^2}{\lambda}$$

Focus Point



$$2|y_t|^2\left(m_{H_u}^2+m_{Q_3}^2+m_{\bar{u}_3}^2\right)+2|a_t|^2$$

$$16\pi^2\frac{d}{dt}m_{H_u}^2=3X_t-6g_2^2|M_2|^2-\frac{6}{5}g_1^2|M_1|^2$$

$$16\pi^2\frac{d}{dt}m_{Q_3}^2=X_t+X_b-\frac{32}{3}g_3^2|M_3|^2-6g_2^2|M_2|^2-\frac{2}{15}g_1^2|M_1|^2$$

$$16\pi^2\frac{d}{dt}m_{\bar{u}_3}^2=2X_t-\frac{32}{3}g_3^2|M_3|^2-\frac{32}{15}g_1^2|M_1|^2$$

$$m_{H_u}^2\left(Q^2\right)=m_{H_u}^2\left(M_P^2\right)+\frac{1}{2}\left(m_{H_u}^2\left(M_P^2\right)+m_{Q_3}^2\left(M_P^2\right)+m_{\bar{u}_3}^2\left(M_P^2\right)\right)\left[\left(\frac{Q^2}{M_P^2}\right)^{\frac{3y_t^2}{4\pi^2}}-1\right]$$

m_0^2 $\quad$ $3m_0^2$ $\quad$ $\simeq -\frac{2}{3},\ Q^2\simeq M_Z^2$

"Focus point": $m_{H_u}^2(0)=m_{Q_3}^2(0)=m_{\bar{u}_3}^2(0)\equiv m^2$

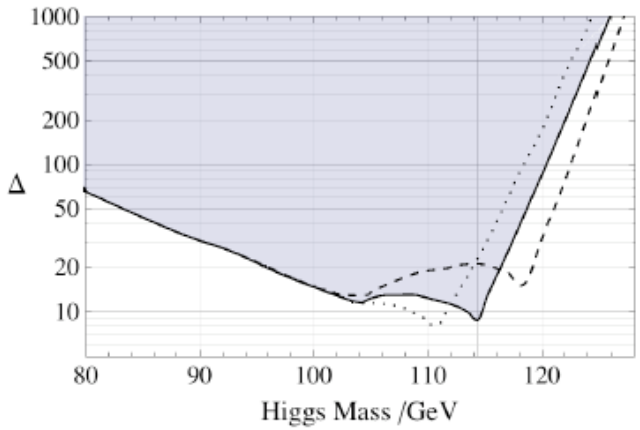
$$m_{H_u}^2(t_0)=a_0\, m^2+..,\ a_0\le 0.1$$

$i.e.\, m_{Q_3}^3, m_{\bar{u}_3}^2 \gg M_Z^2$ possible

"Natural" choice

Feng, Matchev, Moroi
Chan, Chattopadhay, Nath
Barbieri, Giudice

Fine tuning in the CMSSM



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Relic density unrestricted

$$\Delta_{Min} = 9(5.5), \quad m_h = 114 \pm 2 GeV$$

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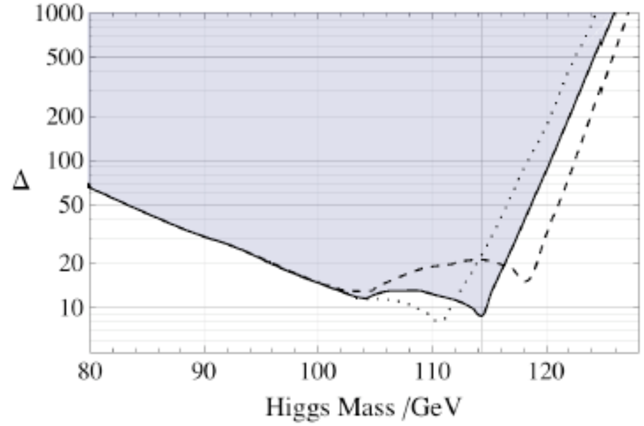

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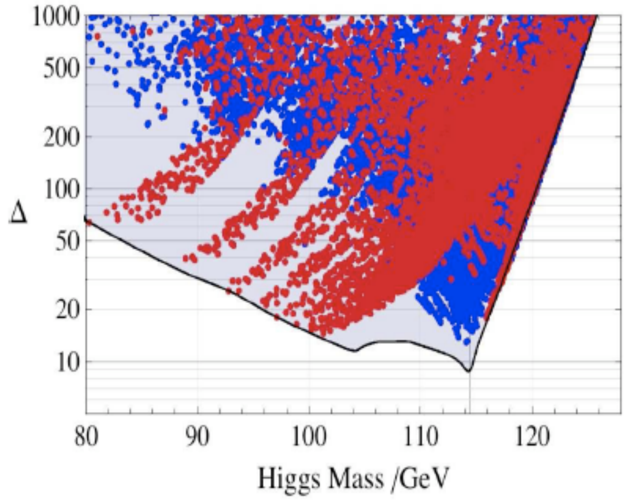


Relic density restricted

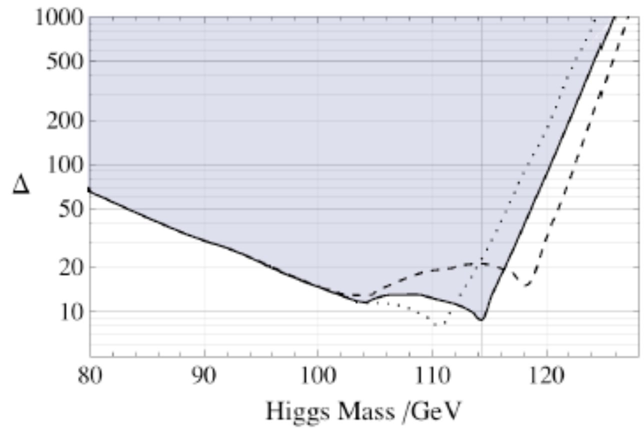
within 3σ WMAP

< 3σ WMAP

$$\Delta_{Min} = 15, \quad m_h = 114.7 \pm 2GeV$$

$$\Delta_{Min} = 18, \quad m_h = 115.9 \pm 2GeV$$

Fine tuning in the CMSSM



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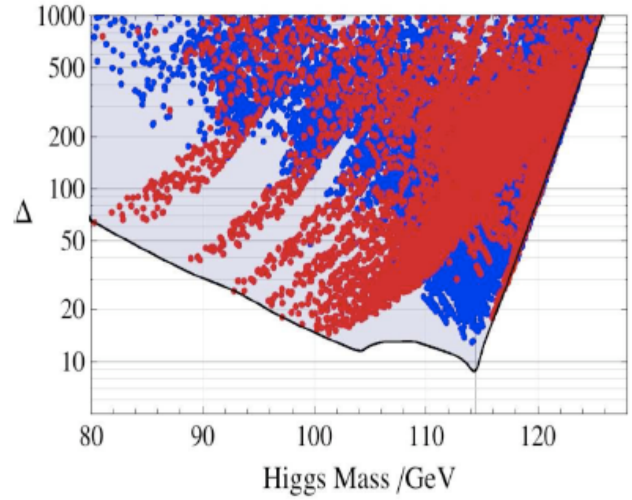
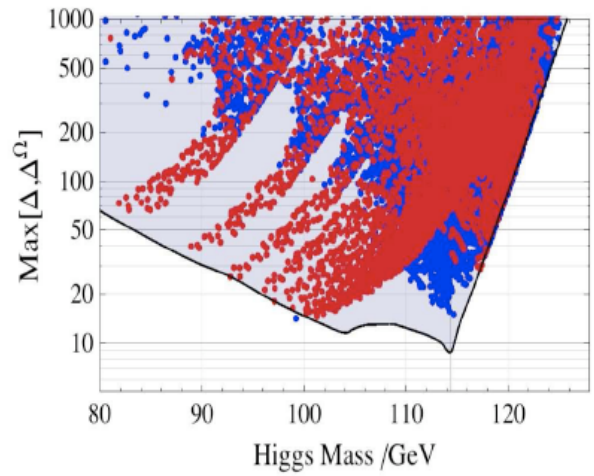
Relic density unrestricted

$$\Delta^{\Omega} = \max\left|\frac{\partial \ln \Omega h^2}{\partial \ln q}\right|_{q=m_0,m_{1/2},A_0,B_0}$$

Ellis, Olive





$$\Delta_{Min} = 29, \quad m_h = 117 \pm 2 GeV$$

Direct dark matter searches: CDMS (spin independent)

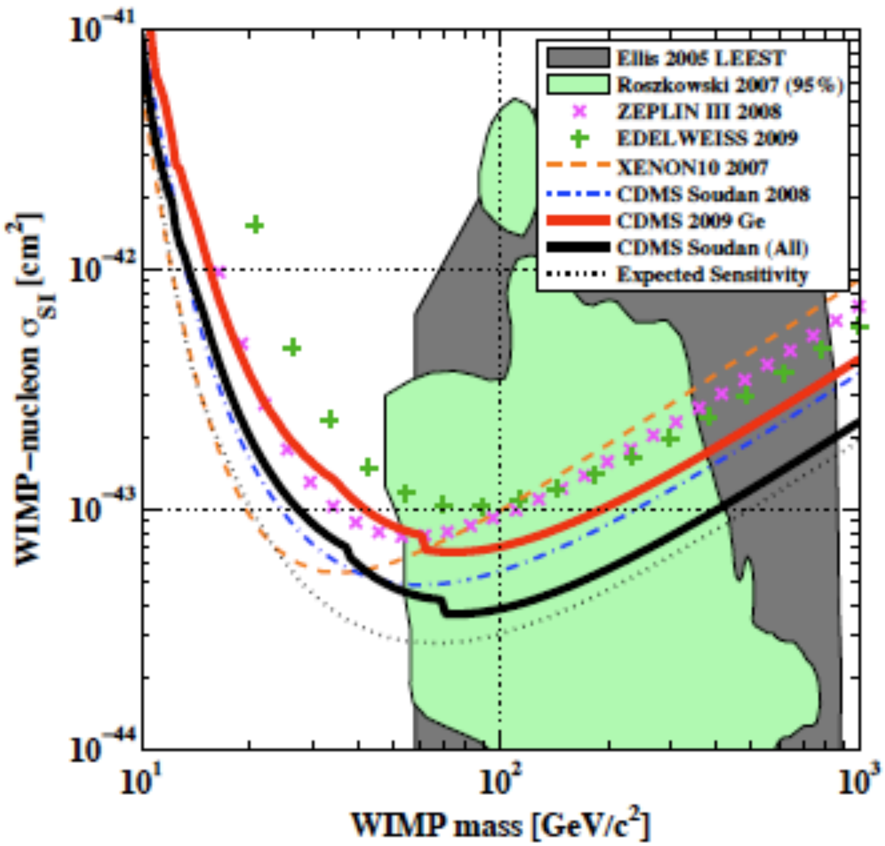
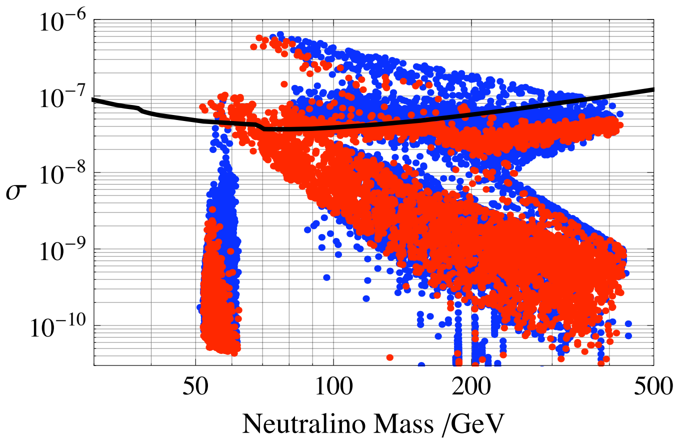
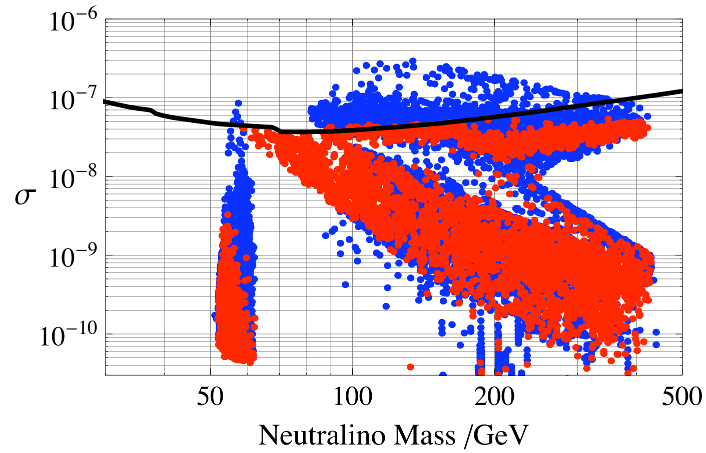


FIG. 4: 90% C.L. upper limits on the WIMP-nucleon spin-independent cross section as a function of WIMP mass. The red (upper) solid line shows the limit obtained from the exposure analyzed in this work. The solid black line shows the combined limit for the full data set recorded at Soudan.

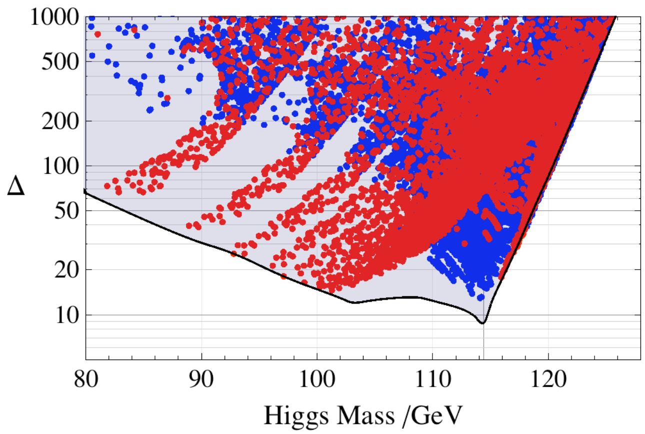
CDMS sensitivity to CMSSM

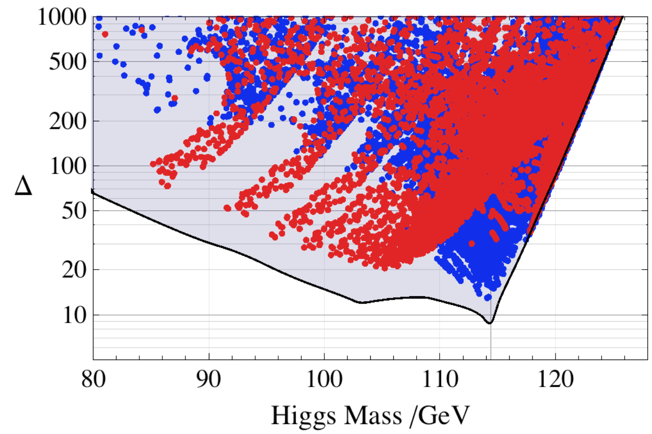




Unconstrained

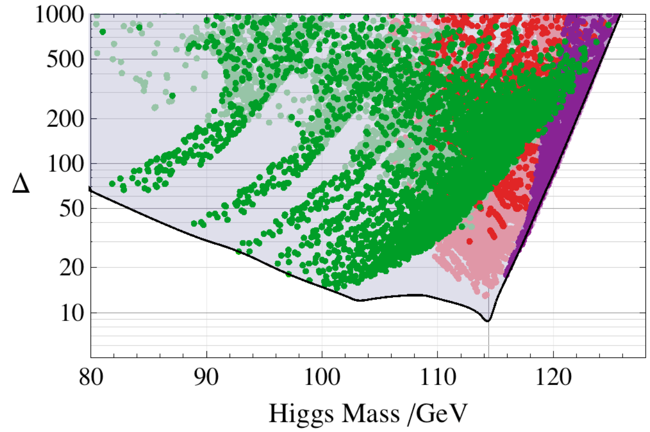
… with CDMS constraint

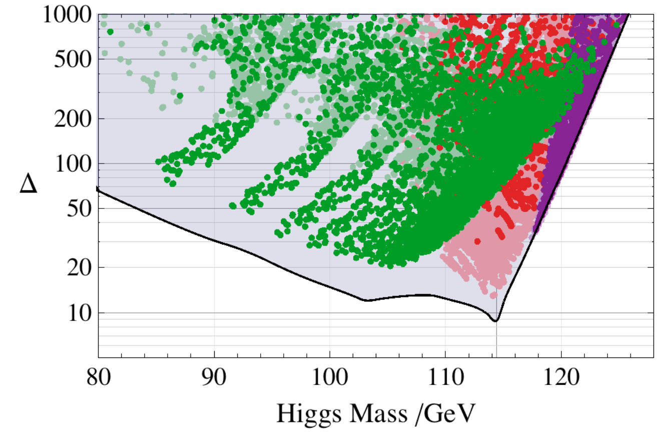


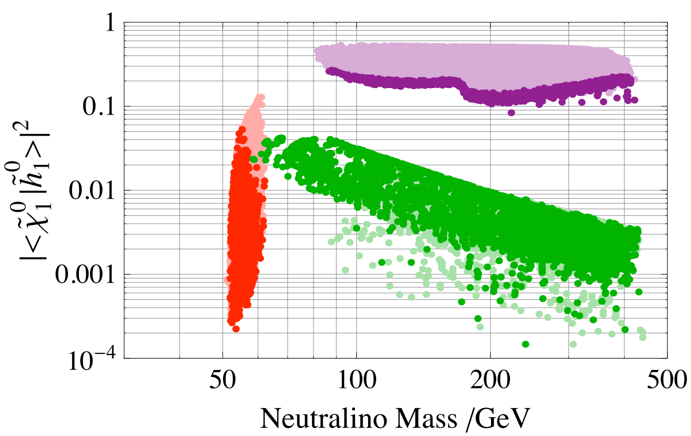


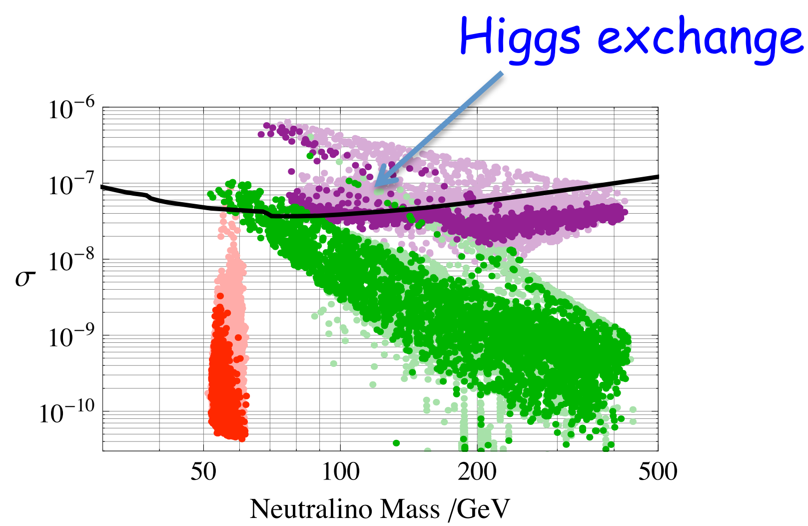
Cassel
microOMEGA

LSP composition





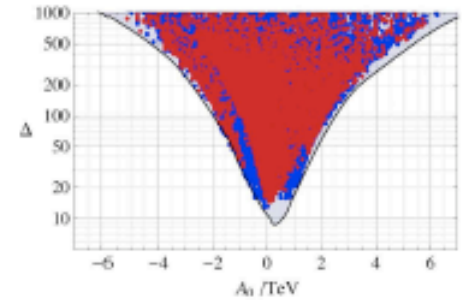




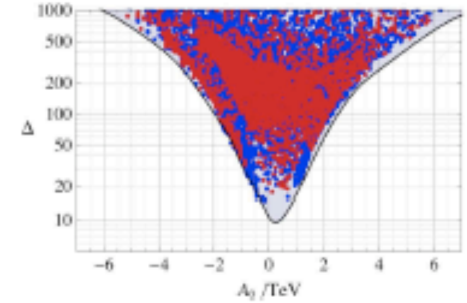
SUSY parameters

$\Delta < 100$

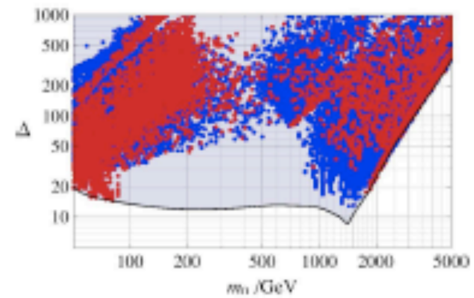
$$m_h < 121 \text{ GeV}$$
$$\mu < 680 \text{ GeV}$$
$$m_0 < 3.2 \text{ TeV}$$
$$5.5 < \tan\beta < 55$$
$$120 \text{ GeV} < m_{1/2} < 720 \text{ GeV}$$
$$-2.0 \text{ TeV} < A_0 < 2.5 \text{ TeV}$$



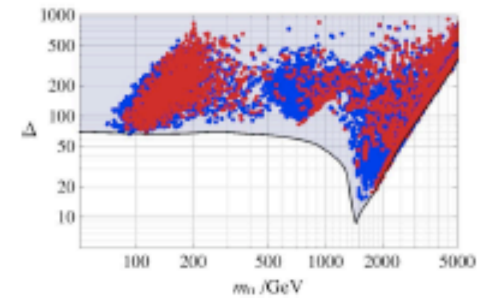
(a) Fine tuning vs A_0



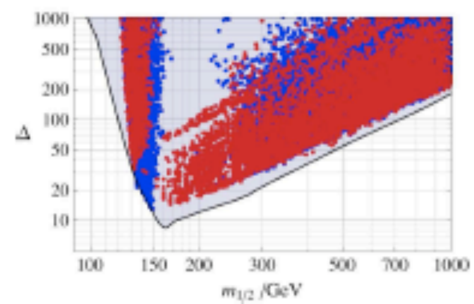
(b) Fine tuning vs A_0, $m_h > 114.4$GeV



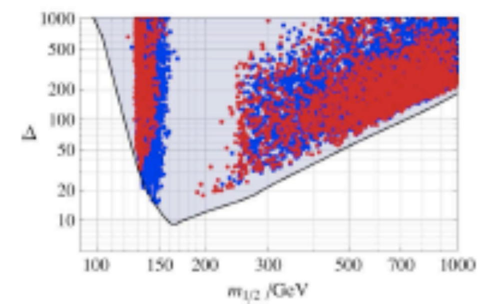
(c) Fine tuning vs m_0



(d) Fine tuning vs m_0, $m_h > 114.4$ GeV



(e) Fine tuning vs $m_{1/2}$



(f) Fine tuning vs $m_{1/2}$, $m_h > 114.4$ GeV

Sparticle upper bounds

$\tilde{g}$	χ_1^0	χ_2^0	χ_3^0	χ_4^0
1720	305	550	660	665

$\chi_1^\pm$	$\chi_2^\pm$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{b}_1$	$\tilde{b}_2$
550	670	2080	2660	2660	3140

(General) Gauge mediation in the MSSM

$$M_{\tilde{\lambda}_i}(M_{mess}) = k_i \frac{\alpha_i(M_{mess})}{4\pi} \Lambda_G$$

$$m_{\tilde{f}}^2(M_{mess}) = 2 \sum_{i=1}^{3} C_i k_i \frac{\alpha_i^2(M_{mess})}{(4\pi)^2} \Lambda_S^2$$

$$k_i = (\frac{5}{3}, 1, 1)$$

$$k_i \alpha_i (M_{GUT}) = 1, \quad i = 1,2,3$$

(Ordinary gauge mediation $\Lambda_G = \Lambda_S$)

Meade, Seiberg, Shih

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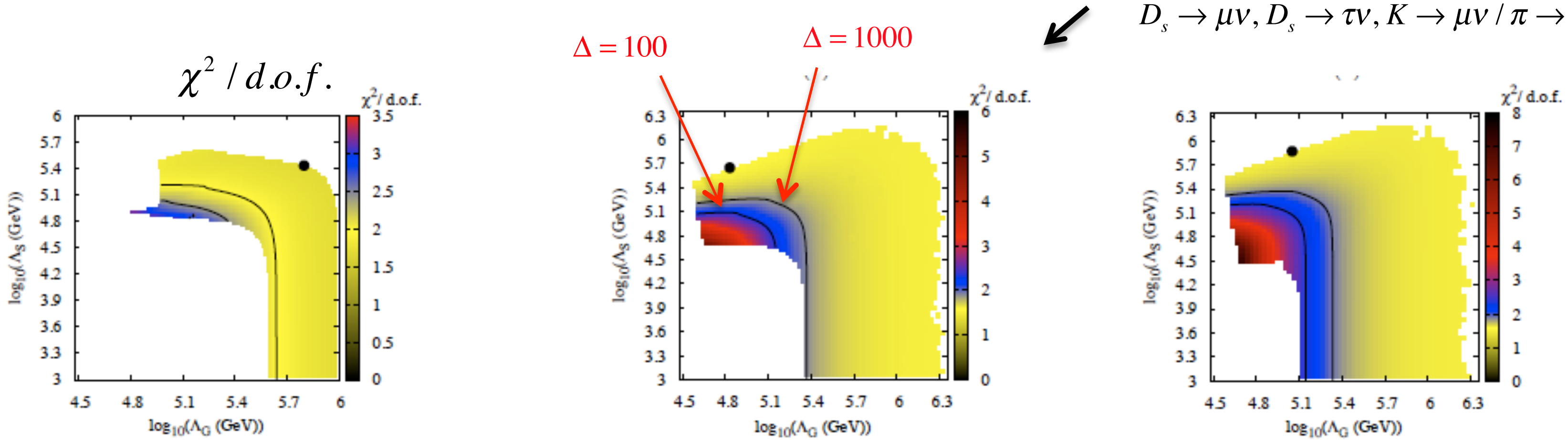
Meade, Seiberg, Shih

$B \to X_s \gamma, B \to \tau\mu, B \to \mu^+\mu^-, B \to D\tau\mu,$
$D_s \to \mu\nu, D_s \to \tau\nu, K \to \mu\nu / \pi \to \mu\nu, \Delta_{0-}$



$M_{Messenger}$ $10^6 GeV$ $10^{10} GeV$ $10^{14} GeV$

$$\Delta > 100$$

(no focus point)$\dagger$

Abel, Dolan, Jaeckel, Khoze
(Giusti, Romanino, Strumia)

Nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}3}^2) + 2|a_t|^2\right) - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

Natural ratios? e.g.:

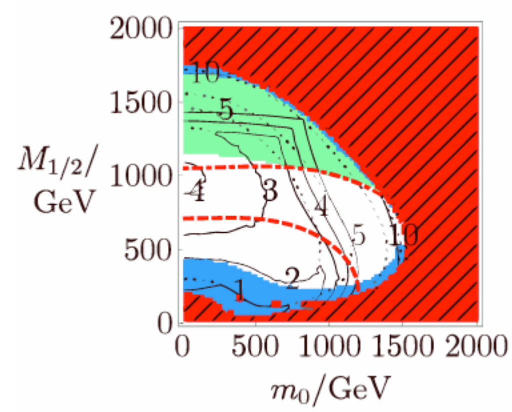
GUT: $SU(5):\ \Phi^N \subset (24\times 24)_{symm} = 1+24+75+200;\quad SO(10):\ (45\times 45)_{symm} = 1+54+210+770$

Representation	$M_3 : M_2 : M_1$ at M_{GUT} ($\eta_3 : 1 : \eta_1$)	$M_3 : M_2 : M_1$ at M_{EWSB} ($2.7\eta_3 : 1 : 0.5\eta_1$)
1	$1:1:1$	$5:2:1$
24	$2:(-3):(-1)$	$11:(-6):(-1)$
75	$1:3:(-5)$	$5:6:(-5)$
210	$5:15:(-1)$	$27:30:(-1)$
770	$(-10):15:101$	$(-54):30:101$

String: $(3+\delta_{GS}):(-1+\delta_{GS}):\left(-\frac{33}{5}+\delta_{GS}\right)$ $\quad(-11):(-12):12$ (OII, also mixed moduli anomaly)

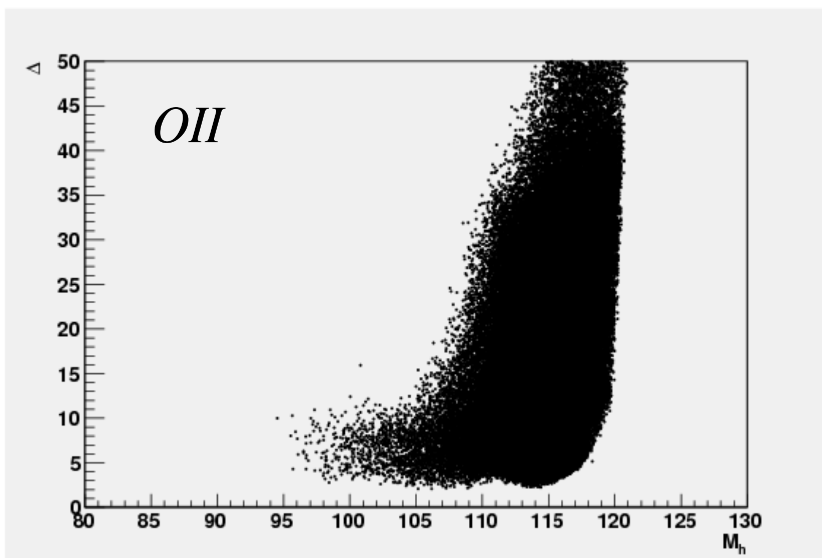
Horton,GGR
Choi,Jeong, Kobayashi,Okumura
Lebedev, Nilles, Ratz
Abe, Kobayashi, Okumura

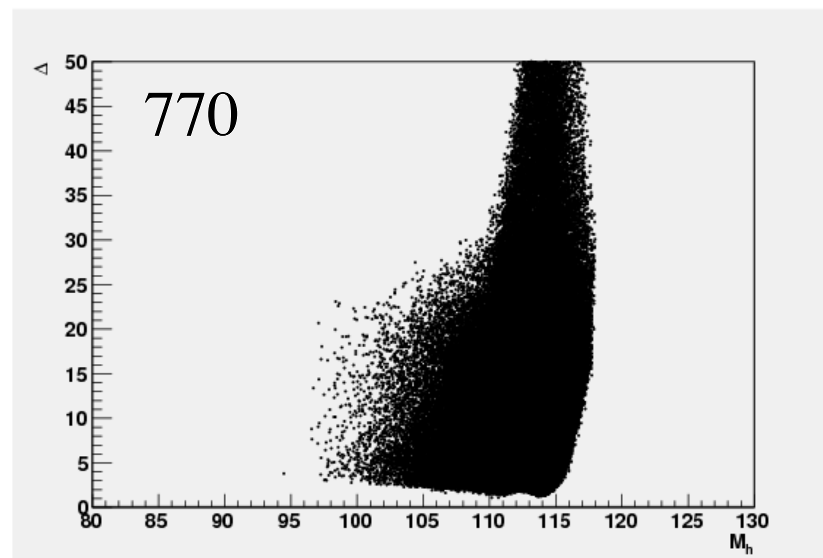
• Nonuniversal gaugino masses



(d) O-II

Δ





M_H M_H

D.Horton,GGR

Characteristic signals: $M_1 : M_2 : M_3 = 0.5\eta_1 : 1 : 2.7\eta_3$, light Higgsino $|\mu| \le 200 GeV \ll M_{1/2}$

…gauginos can be very heavy (Higgsino LSP subdominant cont to DM)

New heavy states – higher dimension operators

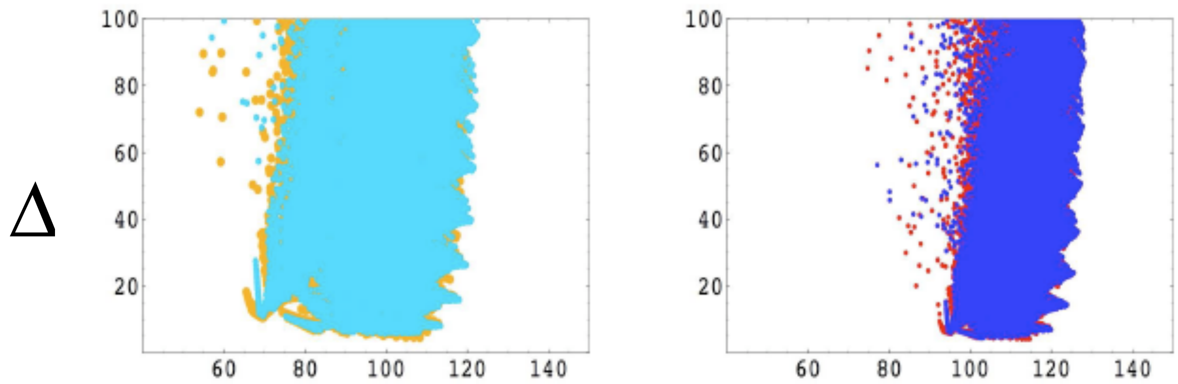
e.g. Massive gauge singlet (or triplet) coupled to Higgs fields:

$$\delta L = \int d^2\theta \frac{1}{M_*}\left(1 + c_0 P\right)\left(H_1 H_2\right)^2, \quad P = m_0\theta\theta$$

Dimension 5

$$\delta V = \varsigma_1 2\frac{\mu_0}{M_*}\left(\left|h_1\right|^2 + \left|h_2\right|^2\right)h_1 h_2 + \frac{1}{2}\varsigma_2\left(h_1 h_2\right)^2; \quad \varsigma_1 = 2\frac{\mu_0}{M_*}, \ \varsigma_2 = -2\frac{c_0 m_0}{M_*}$$

- New heavy states - higher dimension operators



m_h — MSSM $\qquad$ m_h — $\zeta_1 = \zeta_2 = 0.03$

$$v^2 = -\frac{m^2}{\lambda}$$ ⬆

$$1.5 < \tan\beta < 1,\ 50GeV < m_{0,1/2},\quad 130GeV < \mu_0 < 1TeV,\quad -10 < A_t < 10$$

Cassel, Ghilencea, GGR

Even for $M_* = 65\mu_0$ a significant shift of m_h for constant Δ
but … no lose … SUSY spectrum doesn't change much!

$$\delta V = \varsigma_1 2\frac{\mu_0}{M_*}\left(|h_1|^2 + |h_2|^2\right)h_1 h_2 + \frac{1}{2}\varsigma_2 (h_1 h_2)^2$$

↘ 0 in NMSSM (only small change in fine tuning)

≠0 with $M_* S^2$ term (For an explicit model see Delgado, Kolda, Olson, la Puente)

Summary

- Hierarchy problem $\Longrightarrow$ Low-energy SUSY

$$\Delta = Max_{a_i}\left|\frac{a_i \text{ or } \delta a_i}{M_Z}\frac{\partial M_Z}{\partial a_i}\right|,$$ measure of residual fine-tuning

- CMSSM

$$\Delta_{EW} = 9(18), \quad m_h = 114(116) \pm 2GeV$$

$$Max\left[\Delta_{EW}, \Delta_{\Omega}\right] = 15(29), \quad m_h = 114(116) \pm 2GeV$$

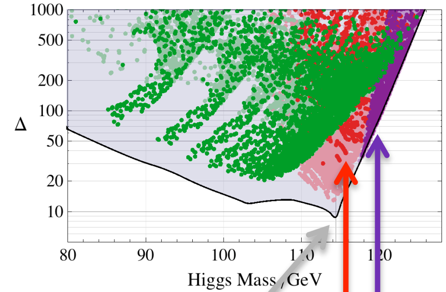
$$+\ CDMS \quad \rightarrow \quad 15(32)$$

LHC $\sqrt{s} = 10(14)TeV \quad \mathbf{L} = 10\, fb^{-1} \quad m_{\tilde{g}} \leq 1.9(2.4)TeV \quad \Delta_{EW} = 120(180)$

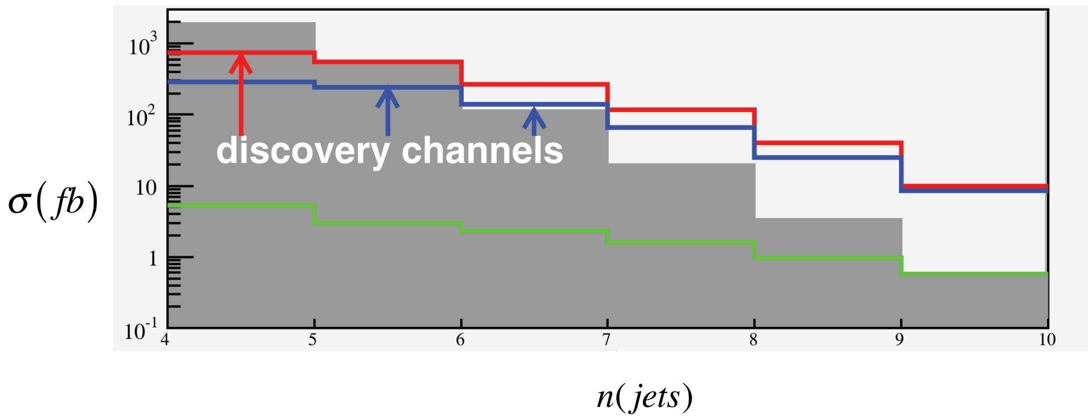
$\sqrt{s} = 7TeV \quad ?$

Baer et al

LHC - discovery potential in 1st run



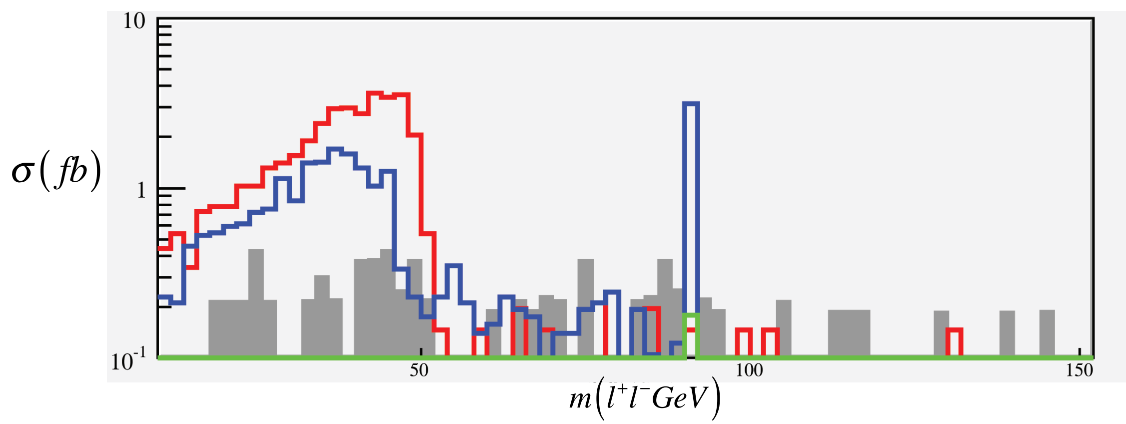
Cuts: $E_T^{miss} > 100GeV, n(j) \geq 4, p_T(j) > 50GeV, p_T(l) > 10GeV$



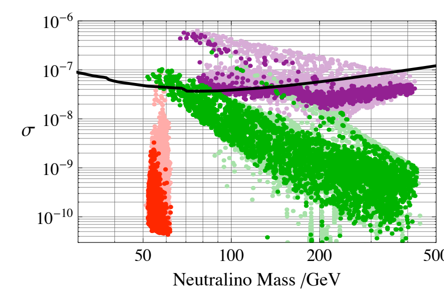
$\Delta = 9\ (5.5), m_{\tilde{g}} = 450GeV,$
$M_H \simeq 115GeV,$ no DM constraint

$\Delta = 30, m_{\tilde{g}} = 380GeV,$
$M_H \simeq 115GeV,$ DM within 3σ

$\Delta = 35, m_{\tilde{g}} = 860GeV,$
$M_H \simeq 118GeV,$ DM within σ



DM search complementarity:



SM background

Summary

- Hierarchy problem $\Longrightarrow$ Low-energy SUSY

$$\Delta = Max_{a_i} \left| \frac{a_i \text{ or } \delta a_i}{M_Z} \frac{\partial M_Z}{\partial a_i} \right|,$$ measure of residual fine-tuning

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$$\Delta_{EW} = 9(18), \quad m_h = 114(116) \pm 2GeV$$

$$Max\left[\Delta_{EW}, \Delta_{\Omega}\right] = 15(29), \quad m_h = 114(116) \pm 2GeV$$

$$+\ CDMS \quad \rightarrow \quad 15(32)$$

LHC $\sqrt{s} = 10(14)TeV$ $\mathbf{L} = 10\, fb^{-1}$ $m_{\tilde{g}} \leq 1.9(2.4)TeV$ $\Delta_{EW} = 120(180)$

$\sqrt{s} = 7TeV$ $L = 1\, fb^{-1}$ $m_{\tilde{g}} \leq 0.45 - 0.65\, TeV$ $\Delta_{EW} = 18(23)$ $\Delta_{TOT} = 35(40)$

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- MSSM...

  -Gauge mediation: $\Delta > 100$

  -Nondegenerate gauginos: $\Delta > 3, \ m_{\tilde{g}} \leq 4TeV$

  + D=5 operators: $m_h \rightarrow 130GeV$

...Higgs search ultimately the most sensitive