MSSM forecast for the LHC

Collab. with

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JHEP 0903:075,2009

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and with Roberto Trotta

(to appear)

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Start of LHC

Which kind of SUSY is more likely to be there?

Idea:

maybe prejudices

Use all the exp. and theor. wisdom to detemine

Relative probability of different regions of the parameter space

MSSM Forecast

Some previous literature on this subject

[R. R. de Austri, R. Trotta and L. Roszkowski] [D. E. Lopez-Fogliani, L. Roszkowski, R. R. de Austri and T. A. Varley] [B. C. Allanach, K. Cranmer, C. G. Lester and A. M. Weber] [S. S. AbdusSalam, B. C. Allanach, M. J. Dolan, F. Feroz and M. P. Hobson] [S. S. AbdusSalam, B. C. Allanach, F. Quevedo, F. Feroz and M. Hobson] [...] [Strumia]

[O. Buchmueller, R. Cavanaugh, A. De Roeck, J.R. Ellis, H. Flacher, S. Heinemeyer, G. Isidori, K.A. Olive, F.J. Ronga and G. Weiglein]

Appropriate framework:

Bayesian approach

which allows to identify and separate in a neat way the objective and subjective pieces and information

Bayesian approach

(in two-minutes !)

- ★ Model: θ_i
- ★ Exp. data

$$p(\theta_i|\text{data})$$

Bayesian approach

Likelihood ($\mathcal{L}$)

Posterior (pdf)

prior

$$p(\theta_i|\text{data}) = \frac{p(\text{data}|\theta_i)\ p(\theta_i)}{p(\text{data})}$$

parameters of the model

norm. constant

Posterior: our state of knowledge about θ_i after we have seen the data

Likelihood: probability of obtaining the data if θ_i are true

Prior: what we know about θ_i before seeing the data

Ignoring the prior and identifying

$$p(\theta_i|\text{data}) \;\equiv\; p(\text{data}|\theta_i)$$

implicitely amounts to

$$p(\theta_i) \;=\; \text{const.} \;\equiv\; \textit{“flat”}$$

But, e.g. $\theta_i \longrightarrow \theta_i^2$

⇨ $\textit{“flat”} \longrightarrow \textit{“non-flat”}$

Besides:

Statistical analysis requires a choice for the allowed range of θ_i

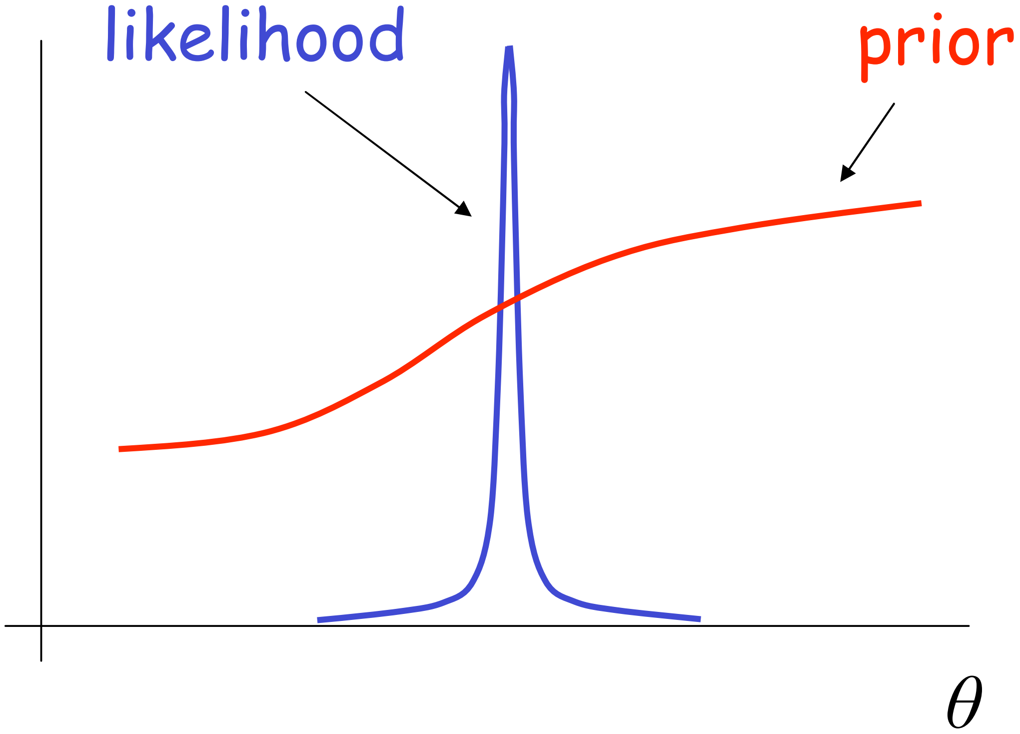
Suppose we are interested in $p(\theta_1|\mathrm{data})$

$$p(\theta_1|\mathrm{data}) = \underbrace{\int d\theta_2 \cdots d\theta_N \; p(\theta_i|\mathrm{data})}_{\text{Marginalization procedure}}$$

We need the range of the definite integration

Marginalization procedure

If data are good enough to select a small region of {θ}, then the prior $p(\theta)$ becomes irrelevant



$$p(\theta_i|\text{data}) \equiv p(\text{data}|\theta_i)$$

Usual priors:

- *Flat:* $p(\theta) = \text{const.}$

  All values of θ equally probable

- *Logarithmic:*

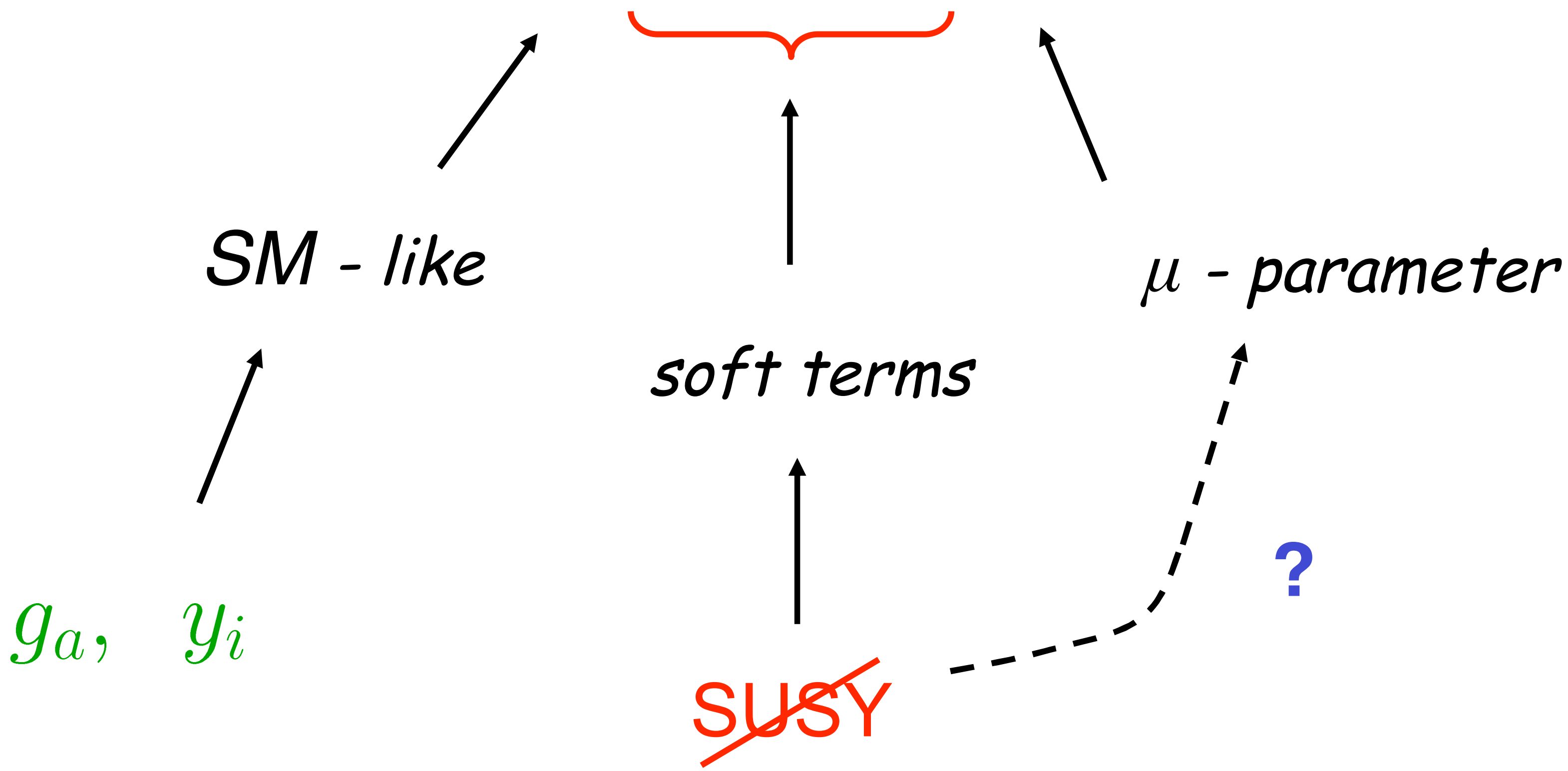
  $$p(\ln\theta) = \text{const.}$$

  $$p(\theta) \propto \frac{1}{\theta}$$

  All magnitudes of θ equally probable

...back to the **MSSM**

$$\{\theta_i\} \equiv \{s, m, M, A, B, \mu\}$$



GOAL:

Scan MSSM parameter space

$$\{\theta_i\} = \{s, m, M, A, B, \mu\}$$

evaluating

$$p(\theta_i|\mathrm{data}) \propto p(\mathrm{data}|\theta_i)\; p(\theta_i)$$

Forecast map for LHC

We have improved previous studies of this kind in several aspects.

In particular:

★ We have not made any ad hoc assumption to penalize fine-tuned regions of the parameter space.

The penalization arises from the Bayesian analysis itself.

What does this mean?

Recall a usual assumption:

$$\underbrace{m, M, A, B, \mu}_{(\equiv M_{\rm soft})} \quad \text{should be} \quad \lesssim \mathcal{O}(TeV)$$

in order to get a

Natural Electroweak Breaking

(with **no fine-tunings**)

Conventional Measure of Fine-Tuning:

$$\frac{\Delta M_Z^2}{M_Z^2} \simeq c_i \frac{\Delta \theta_i}{\theta_i}$$

Ellis, Enqvist, Nanopoulos & Zwirner

Barbieri-Giudice

$$\equiv \qquad c_i = \frac{\partial \ \ln M_Z^2}{\partial \ \ln \theta_i}, \qquad c = \mathrm{Max}\{c_i\}$$

$c^{-1} \sim$ probability of cancellation between the various contributions to get $M_Z \sim \mathcal{O}(90\mathrm{GeV})$

(Strumia; JAC, Espinosa & Hidalgo)

One typically requires $c \lesssim 10$

Two practices:

- ★ Incorporate the fine-tuning measure into the prior:

$$p(m, M, A, B, \mu) \longrightarrow \frac{1}{c}\ p(m, M, A, B, \mu)$$

- ★ Restrict the soft parameters to the $\lesssim \mathcal{O}(TeV)$ range

But all this is arbitrary.
E.g. the results are strongly dependent on the ranges adopted.

However, since naturalness arguments are deep down statistical arguments, one might expect that an effective penalization of fine-tunings arises from the Bayesian analysis itself.

...and this is really what happens.

Cabrera, Ruiz de Austri, J.A.C. 09

Instead solving μ^2 in terms of M_Z and the other soft terms and, treat M_Z as another exp. data

☆ Approximate the likelihood as

$$\mathcal{L} = N_Z \, e^{-\frac{1}{2}\left(\frac{M_Z - M_Z^{\exp}}{\sigma_Z}\right)^2} \mathcal{L}_{\rm rest}$$

$$\simeq \delta(M_Z - M_Z^{\exp}) \, \mathcal{L}_{\rm rest}$$

Likelihood associated to the other observables

★ Use M_Z to marginalize μ

$$
\begin{aligned}
p(s,m,M,A,B|\ \text{data}) &= \int d\mu\; p(s,m,M,A,B,\mu|\text{data}) \\
&\simeq \mathcal{L}_{\text{rest}} \left[\frac{d\mu}{dM_Z}\right]_{\mu_Z} p(s,m,M,A,B,\mu_Z)\ .
\end{aligned}
$$

$$
p(s,m,M,A,B|\ \text{data}) = 2\,\mathcal{L}_{\text{rest}}\, \frac{\mu_Z}{M_Z}\, \frac{1}{c_\mu}\, p(s,m,M,A,B,\mu_Z)
$$

fine-tuning penalization !

In practice you pick up a Jacobian factor:

$$\{\mu, y_t, B\} \xrightarrow{\boldsymbol{J}} \{M_Z, m_t, \tan\beta\}$$

$$J = \left[\frac{E}{R_\mu^2}\right] \frac{y}{y_{\rm low}} \frac{t^2-1}{t(1+t^2)} \frac{B_{\rm low}}{\mu_Z}$$

model-independent part !

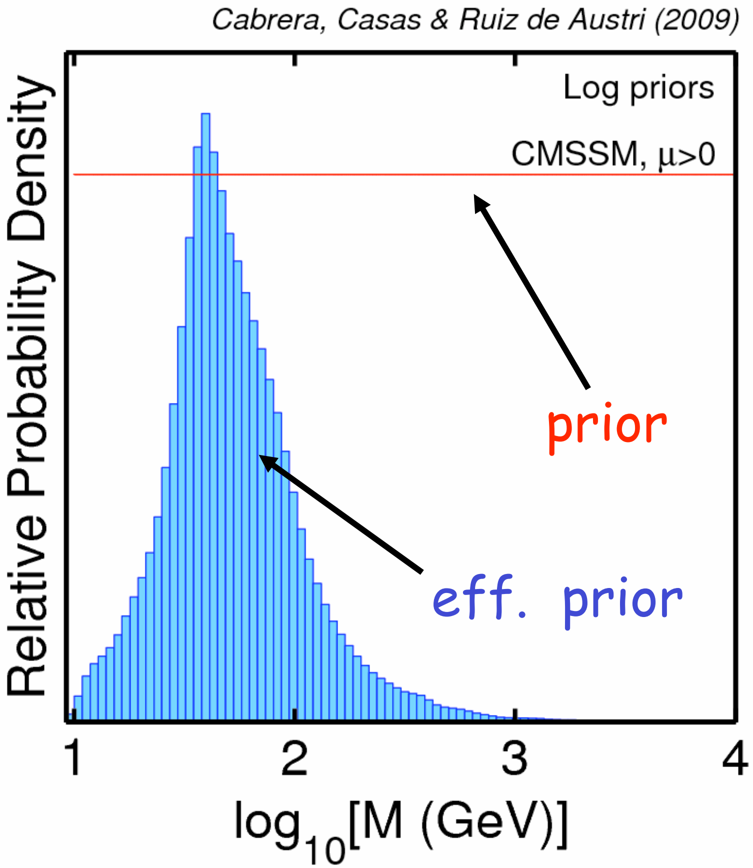
- It contains the fine-tuning penalization
- It penalizes large tan β
- It applies to any MSSM (not just CMSSM)

Finally, for the prior

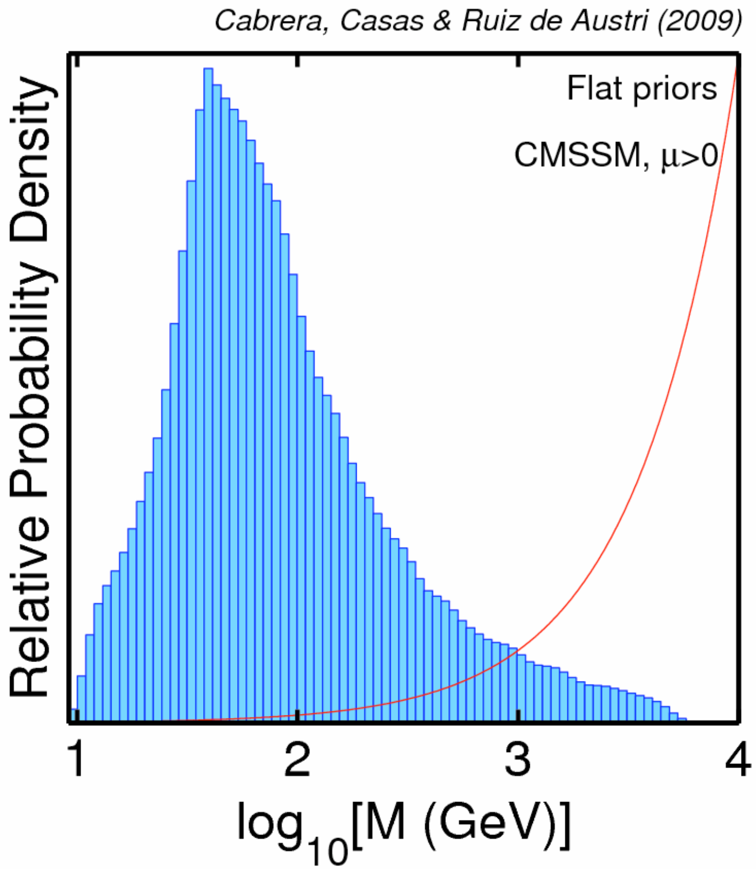
$$p(m, M, A, B, \mu)$$

we have taken the two basic possibilities:

- flat
- logarithmic



Log prior



Flat prior

★ $M_Z^{\rm exp}$ brings SUSY to the LHC region

★ We may vary $M_{\rm soft}$ up to M_X

The results do not depend on the range chosen

★ This suggests that large $M_{\rm soft}$ is disfavoured, even in a Landscape context

Results

Using MultiNest sampler

F. Feroz, M.P. Hobson and M. Bridges

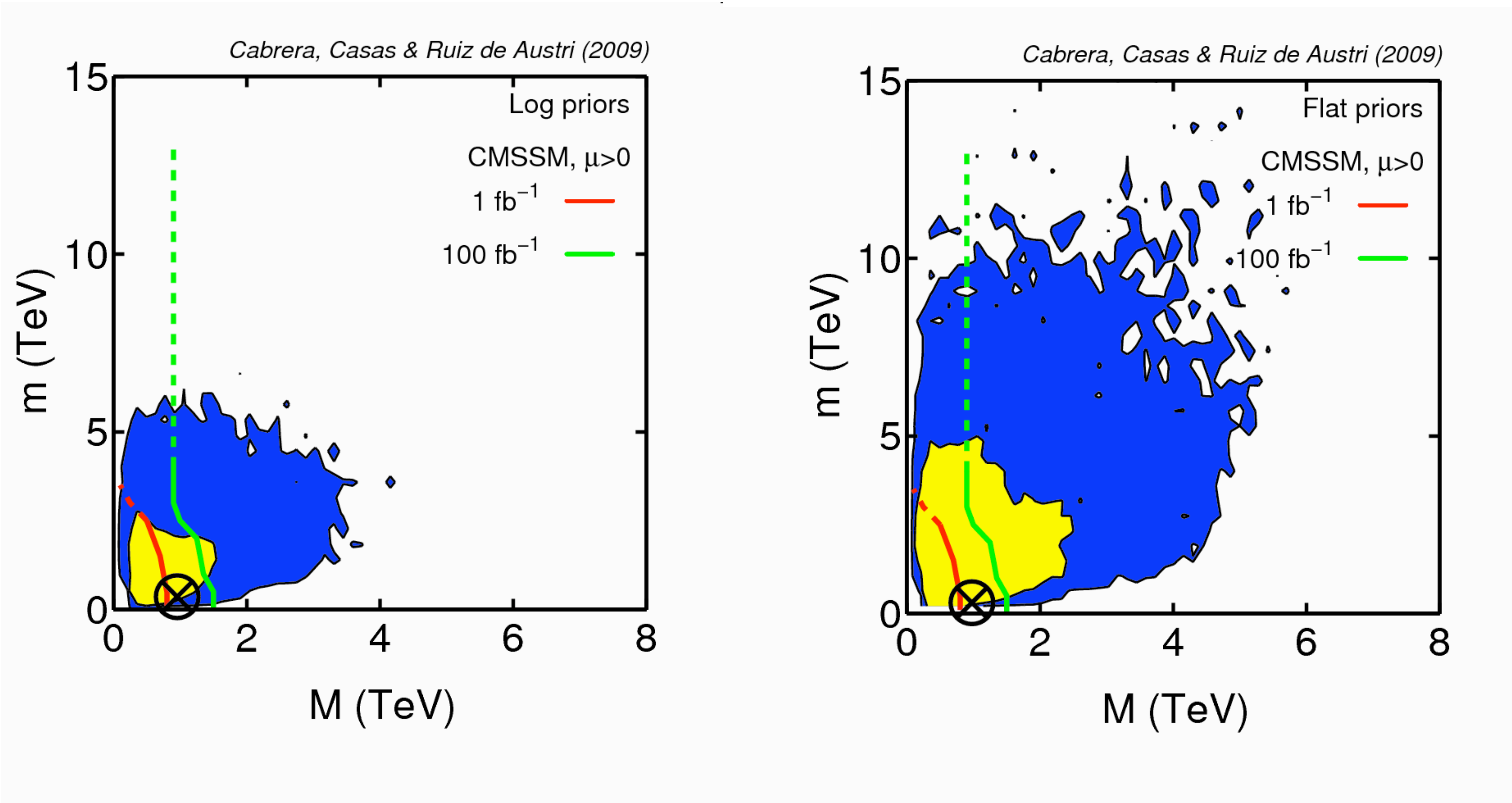
Experimental Constraints

We have considered 3 groups of exp. constraints:

E.W. and B-physics observabes, and limits on particle masses

Constraints from $(g\text{-}2)_\mu$

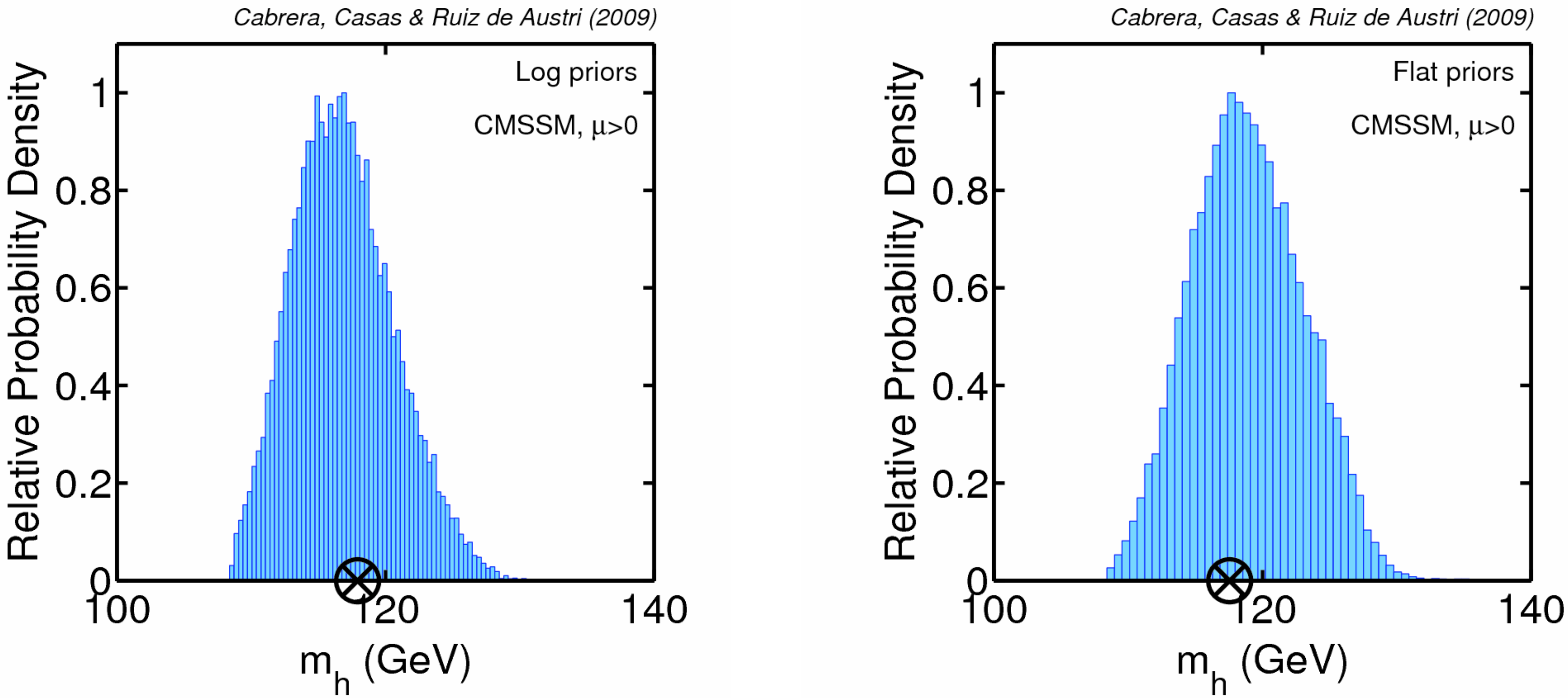
Constraints from Dark Matter abundance



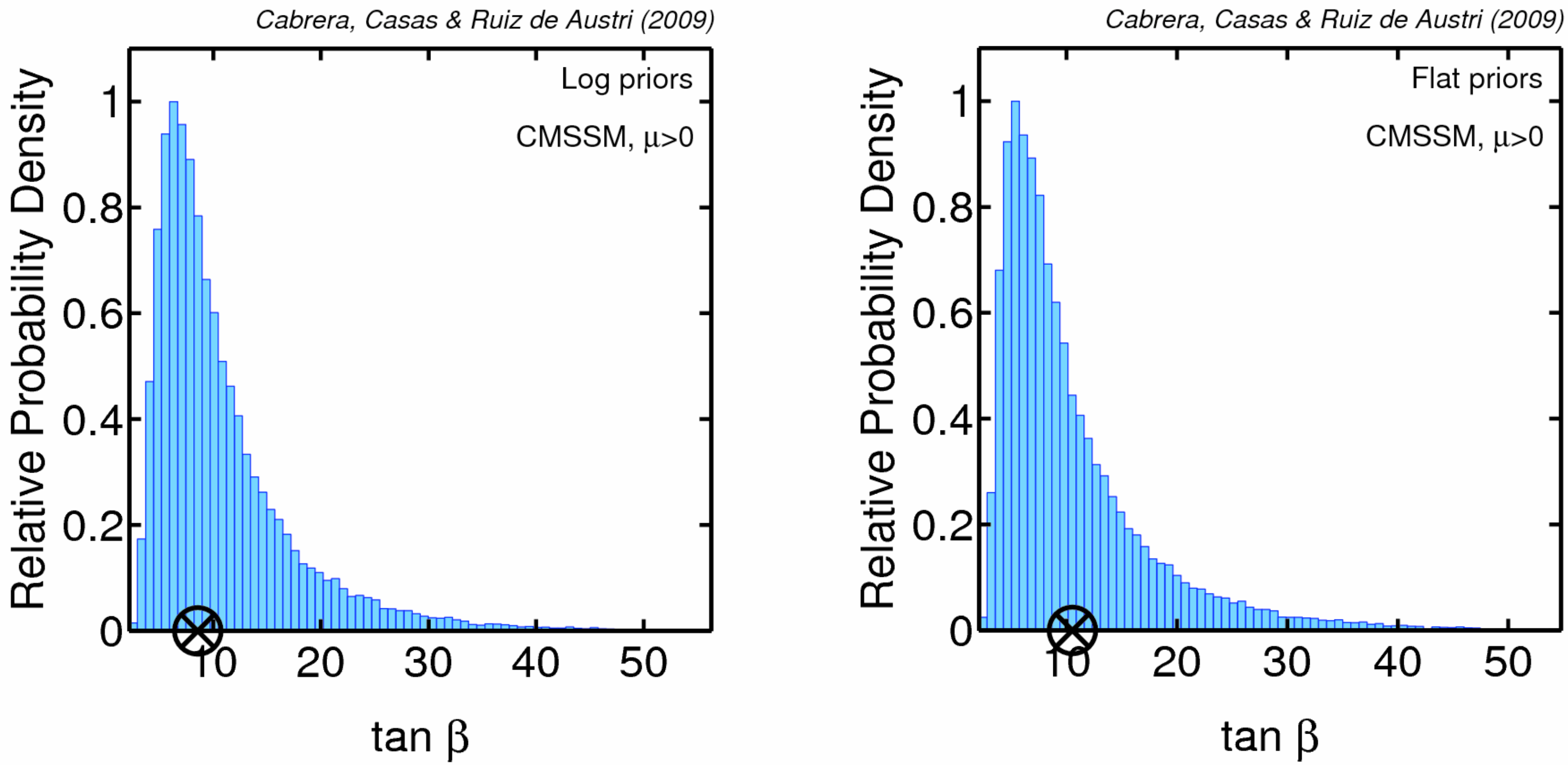
68% c.l.

95% c.l.

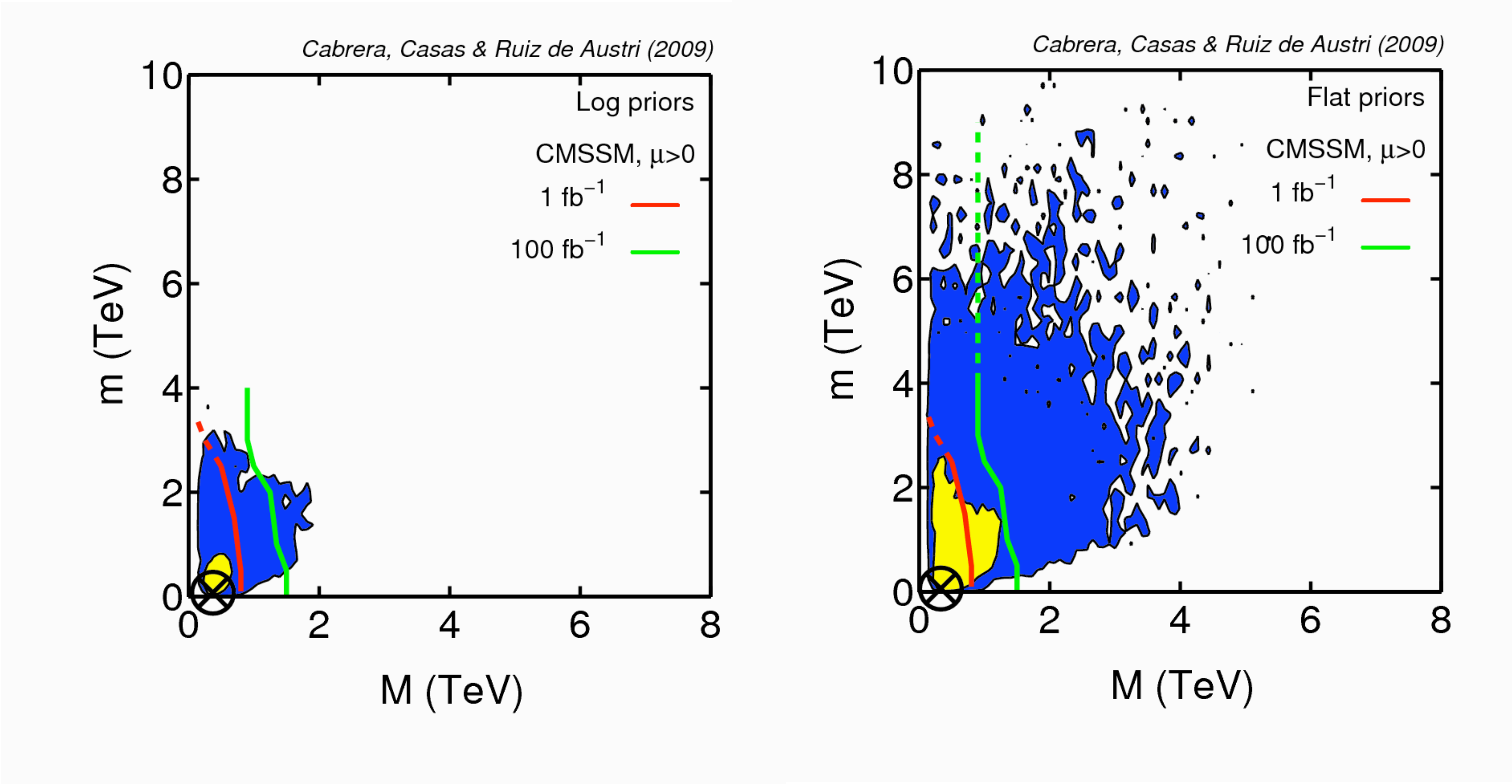
(LHC contours at 14 TeV C.M.)



Cabrera, Casas & Ruiz de Austri (2009)
Log priors
CMSSM, μ>0
Relative Probability Density
m_h (GeV)
Cabrera, Casas & Ruiz de Austri (2009)
Flat priors
CMSSM, μ>0
Relative Probability Density
m_h (GeV)

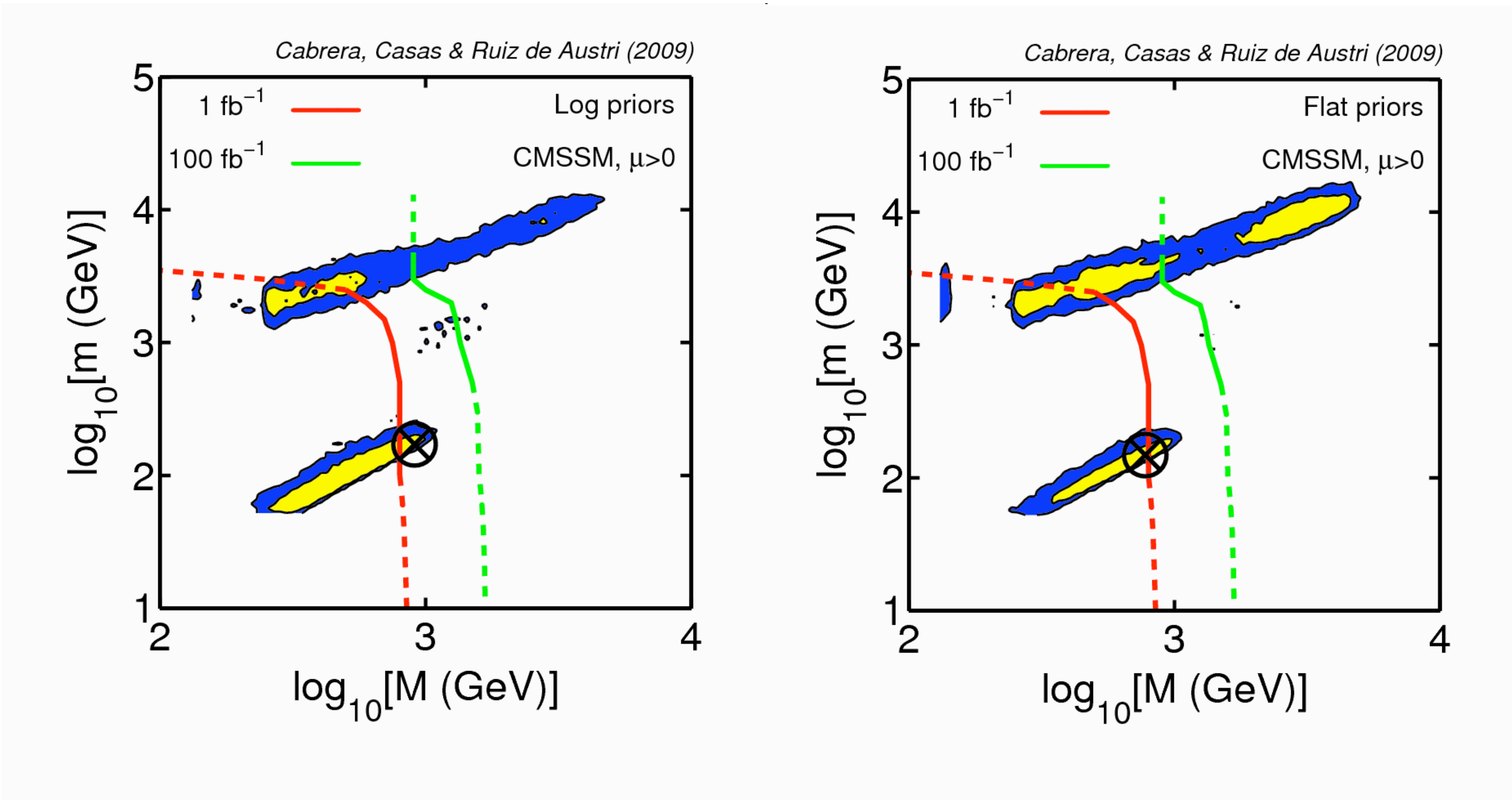

Cabrera, Casas & Ruiz de Austri (2009)
Log priors
CMSSM, μ>0
Relative Probability Density
0
0.2
0.4
0.6
0.8
1
10
20
30
40
50
tan β
Cabrera, Casas & Ruiz de Austri (2009)
Flat priors
CMSSM, μ>0
Relative Probability Density
0
0.2
0.4
0.6
0.8
1
10
20
30
40
50
tan β

Adding $(g-2)_\mu$ (using e+ e- data)

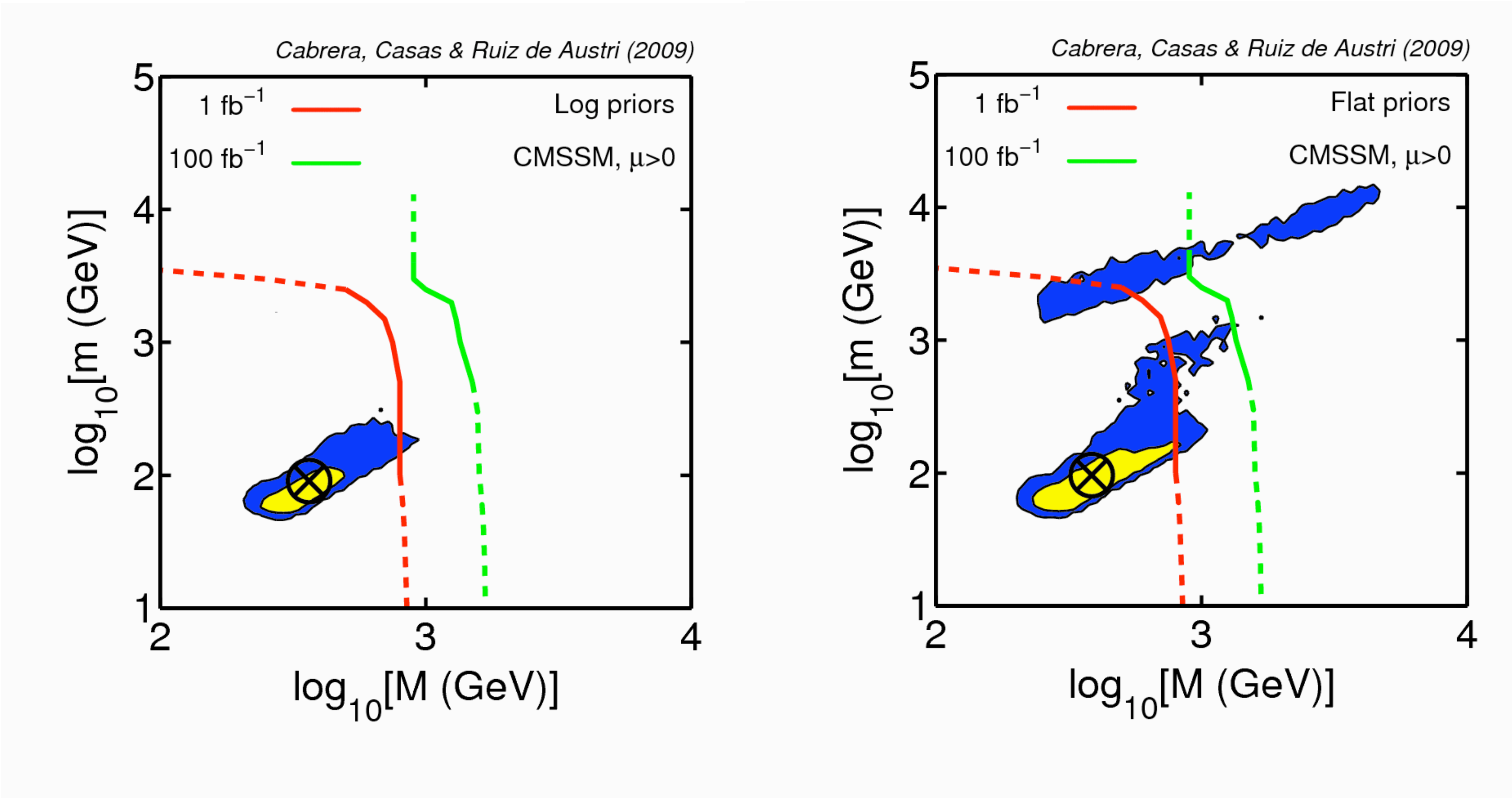


Preferred region clearly within the **LHC** *reach*

Adding Ω_{DM} [and not $(g-2)_\mu$]



Adding Ω_{DM} and $(g\text{-}2)_\mu$



Example of the usefulness of Bayesian techniques:

Cabrera, Ruiz de Austri, Trotta, J.A.C. (to appear)

Study of the tension (within the CMSSM) between

a_μ (using e+ e- data) ⟺ possibly large m_h

Intuitively

large m_h (say $m_h \gtrsim 135$ GeV)

$M_{\rm SUSY}$ large

SUSY decoupled

$\delta a_\mu^{\rm SUSY}$ negligible

3.3σ discrepancy with $a_\mu^{\rm exp}$

But, what is the tension if $m_h = 120\ \mathrm{GeV}, 125\ \mathrm{GeV}, \ \dots$?

Global likelihood of a model:

$$\sim \int d\theta\, p(\mathrm{data}|\theta)\, p(\theta) \equiv p(\mathrm{data})$$

Separate:

$$\{\mathrm{data}\} = \{\mathcal{D},\ D\}$$

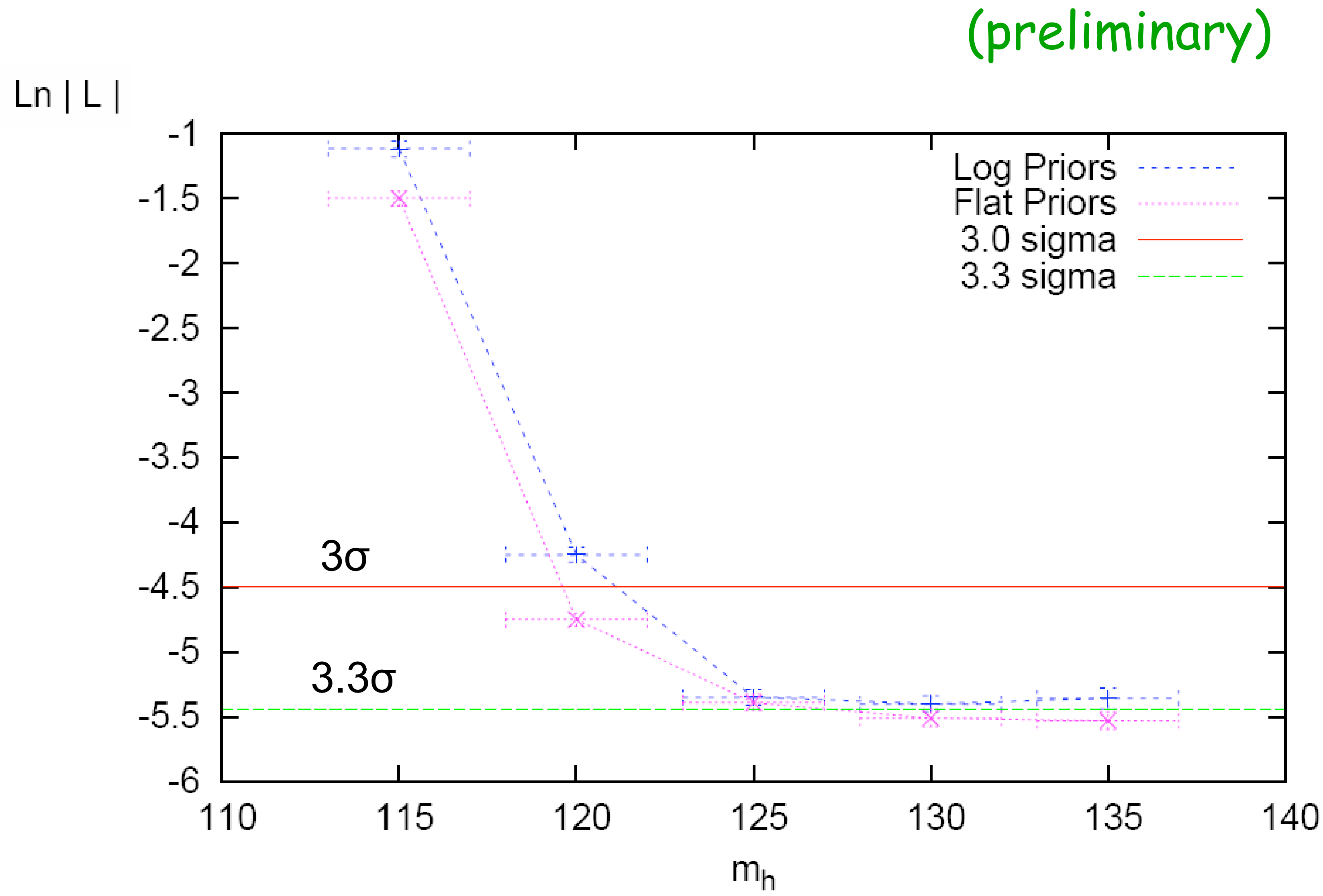
$\mathcal{D}$: to be tested

D: robust

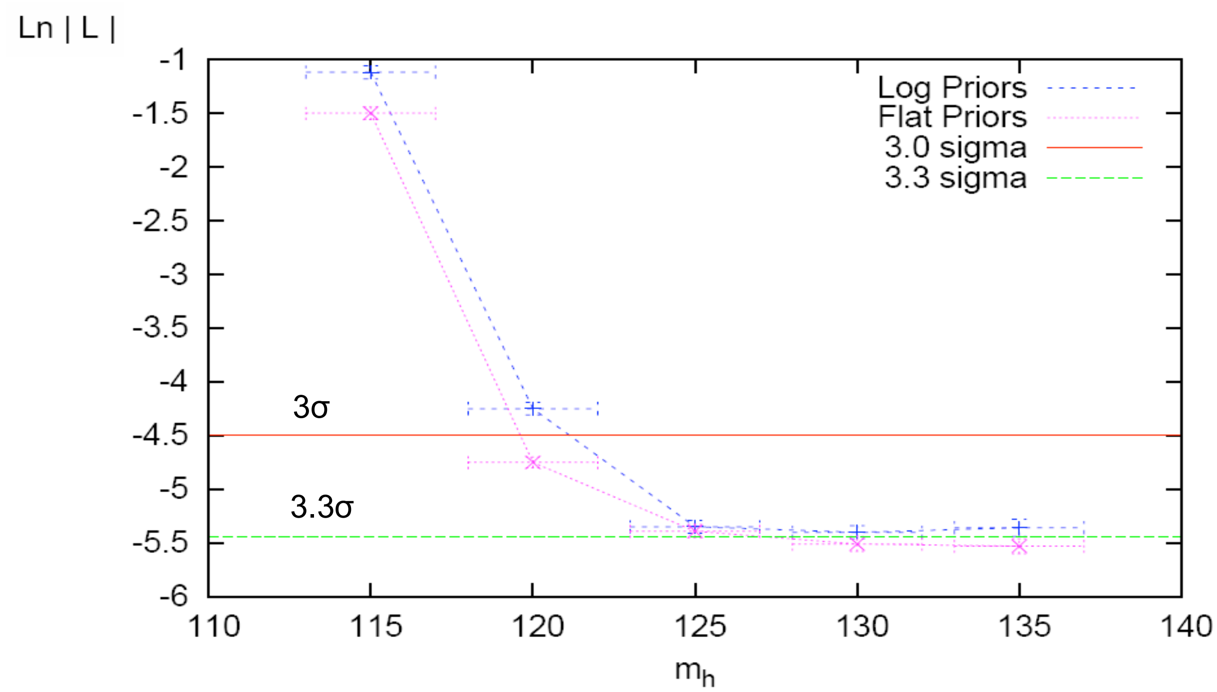
Global likelihood $\sim \dfrac{p(\mathcal{D}^{obs},\ D)}{p(\mathcal{D}^{max},\ D)} \equiv L$

close to a_μ^{SM}
if m_h large

You can evaluate L for any m_h



(preliminary)
Ln | L |
Log Priors
Flat Priors
3.0 sigma
3.3 sigma
3σ
3.3σ
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
110
115
120
125
130
135
140
m_h



If you require

- a_μ from e+ e- (with quoted uncertainties)
- discrepancy with exp < 3σ
- CMSSM

$m_h \lesssim 120$ GeV

CONCLUSIONS

- A rigorous study of the MSSM forecast for the LHC is possible.
- Bayesian analysis naturally penalizes fine-tunings. The exp. value of M_Z brings the relevant parameter space to the low-energy region (~accesible to LHC)
- The results are quite stable under changes of the initial prior (logarithmic or flat) or in the ranges of the parameters.
- Using E.W. + B-physics + Exp. limits on SUSY masses, the larger portion of the CMSSM is inside the LHC discovery potential.... but there is a substantial part outside

CONCLUSIONS (cont.)

- Things would change for worse if the Higgs mass is not close to the exp. limit

- Including (g-2) brings the preferred region of the par. space well inside the LHC scope.

- Demanding CDM abundance consistent with observations selects quite narrowed regions of the par. space.

with a grain of salt!

CONCLUSIONS (cont. 2)

- There is a (>3σ) tension between m_h and a_μ unless $m_h \lesssim 120\ \mathrm{GeV}$ (in the CMSSM)

Backup slides

We have already assumed a common prejudice about the parameters:

Universality

We will assume Univ. at High-Scale $(\sim M_X)$

CMSSM (or mSUGRA)

$$V(H_1, H_2) = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - 2B\mu H_1 H_2$$

$$+ \frac{1}{8}(g^2 + g'^2)\,(|H_1|^2 - |H_2|^2)^2$$

$$M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - 2\mu^2$$

$$\sin 2\beta = \frac{2\mu}{B}\,(m_{H_1}^2 + m_{H_2}^2 + 2\mu_{\rm low}^2)$$

Unnatural fine-tuning

unless $M_{\rm soft} \lesssim \mathcal{O}(\mathrm{TeV})$

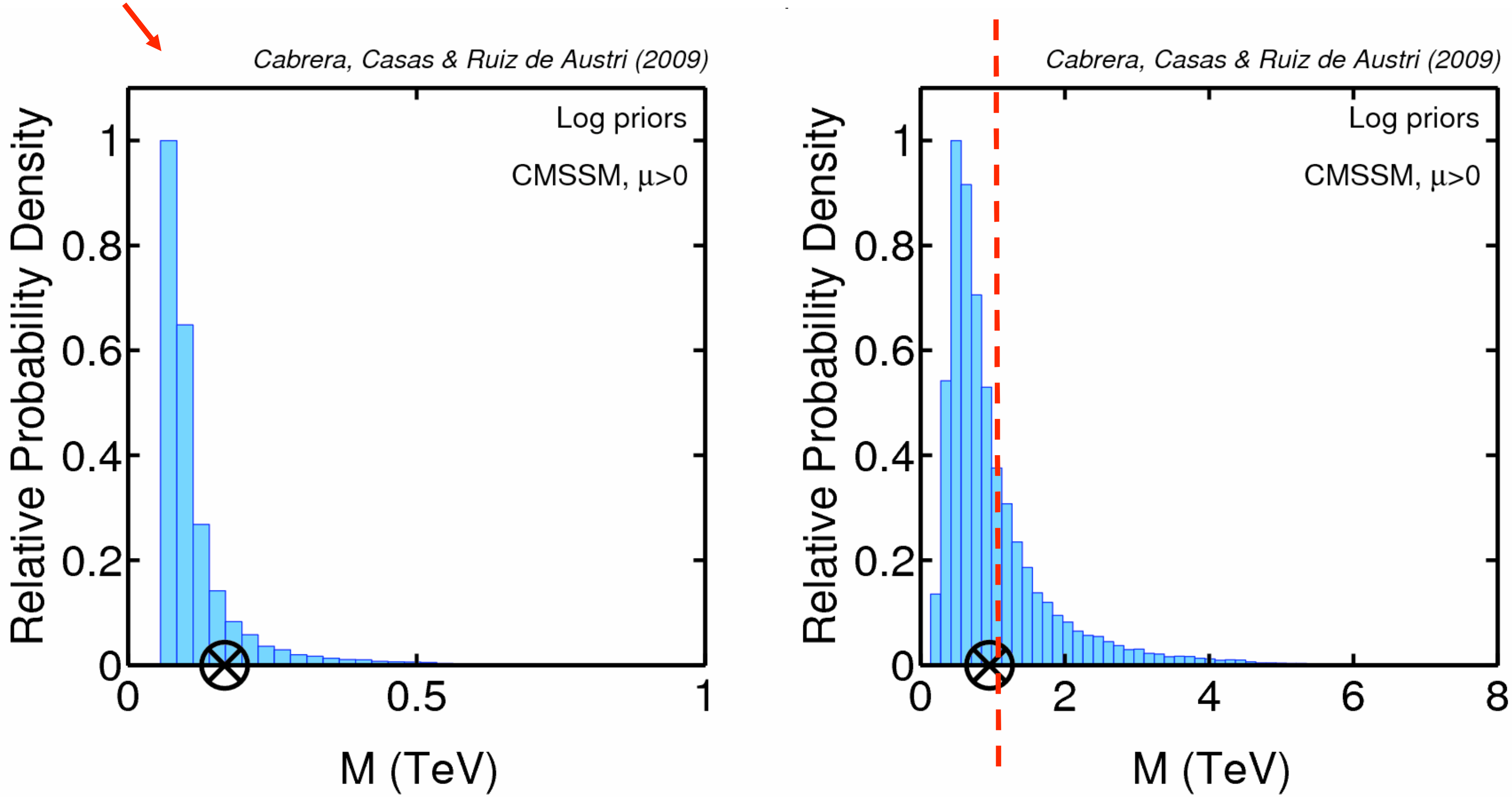
$$(\ \tan\beta \equiv \frac{v_2}{v_1}\ )$$

Observable	Mean value	Uncertainties	
	μ	σ (exper.)	τ (theor.)
M_W	80.398 GeV	27 MeV	15 MeV
$\sin^2\theta_{\rm eff}$	0.23149	17×10^{-5}	15×10^{-5}
$\delta a_\mu^{\rm SUSY} \times 10^{10} (e^+e^-)$	29.5	8.8	2.0
$\delta a_\mu^{\rm SUSY} \times 10^{10} (\tau)$	14.0	8.4	2.0
ΔM_{B_s}	17.77 $\rm ps^{-1}$	0.12 $\rm ps^{-1}$	2.4 $\rm ps^{-1}$
$BR(\overline{B} \to X_s\gamma) \times 10^4$	3.52	0.33	0.3
$\frac{BR(B_u\to\tau\nu)_{MSSM}}{BR(B_u\to\tau\nu)_{SM}}$	1.28	0.38	0
$\Delta_{0-} \times 10^2$	3.6	2.65	0
$\frac{BR(B\to D\tau\nu)}{BR(B\to De\nu)} \times 10^2$	41.6	12.8	3.5
R_{l23}	1.004	0.007	0
$BR(D_s \to \tau\nu) \times 10^2$	5.7	0.4	0.2
$BR(D_s \to \mu\nu) \times 10^3$	5.8	0.4	0.2
$\Omega_\chi h^2$	0.1099	0.0062	$0.1\, \Omega_\chi h^2$
	Limit (95% CL)		τ (theor.)
$BR(\overline{B}_s \to \mu^+\mu^-)$	$< 5.8 \times 10^{-8}$		14%
m_h	$>$ 114.4 GeV (SM-like Higgs)		3 GeV
ζ_h^2	$f(m_h)$ (see text)		negligible
$m_{\tilde q}$	$>$ 375 GeV		5%
$m_{\tilde g}$	$>$ 289 GeV		5%
other sparticle masses	As in table 4 of ref. [?].		

E.W. and B-physics observabes, and limits on particle masses

Log pr

just with $M_Z^{\rm exp}$



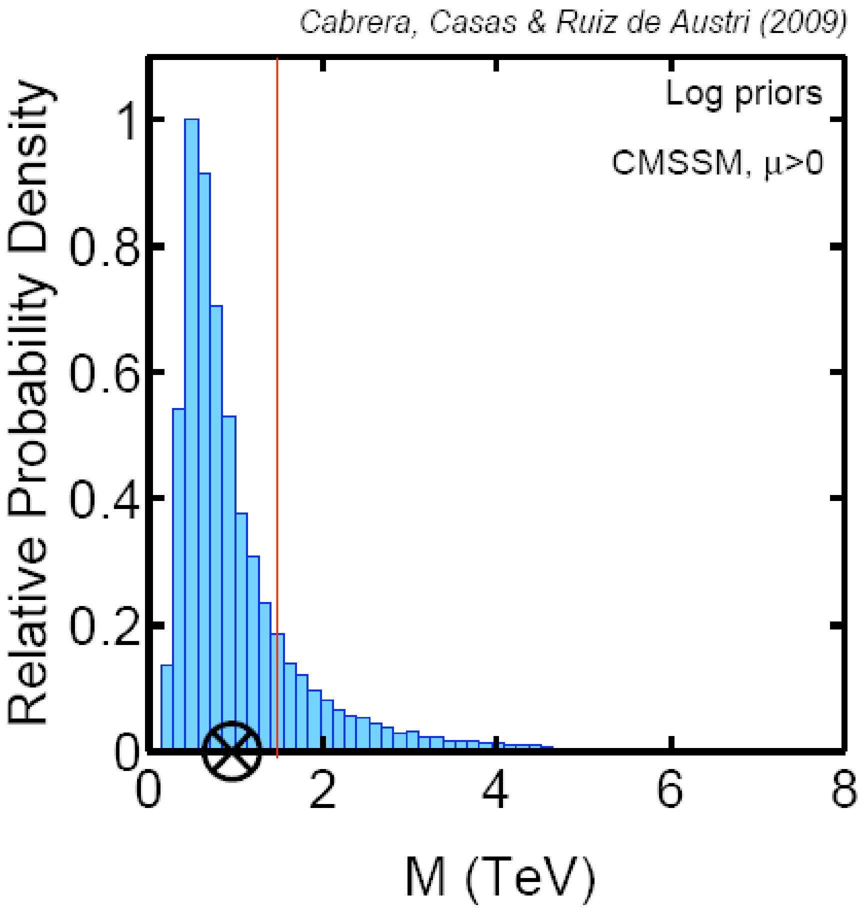
E.W. and B-physics observabes, and limits on particle masses

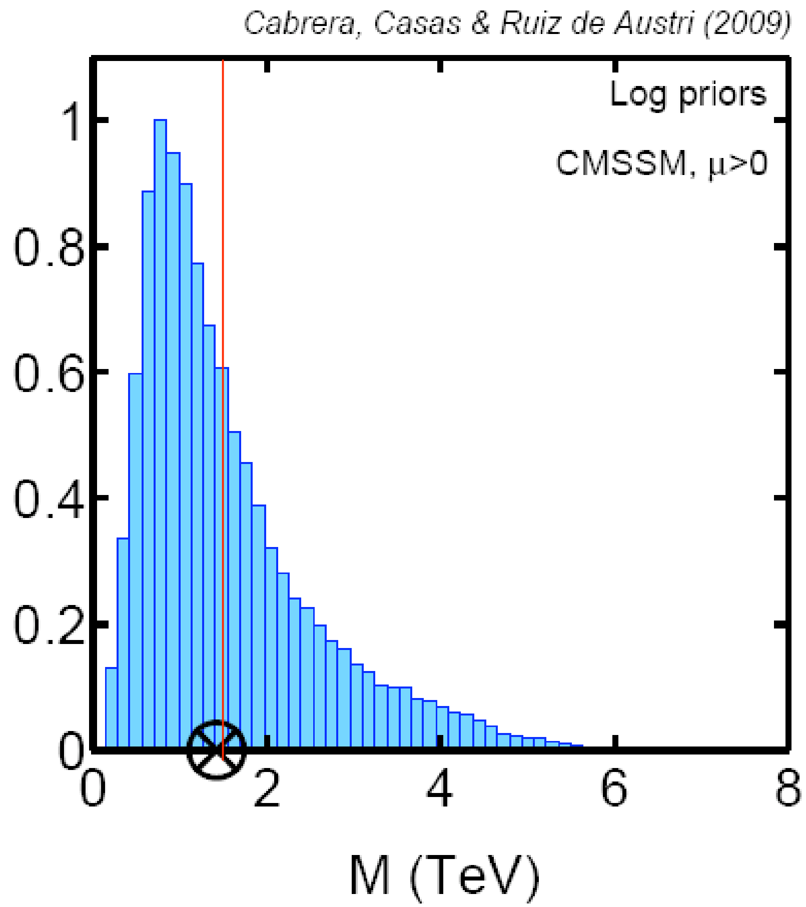
Shift (soft terms):

~100 GeV → ~1 TeV

- $B \to X_S\, \gamma$
- Higgs mass bound

$m_h > 114$ GeV → $m_h > 120$ GeV





A shift of a few GeV in the Higgs mass implies a shift of several hundred GeV in the soft masses

If the MSSM is true and we wish to detect it at the LHC, let us hope that the Higgs mass is close to the present exp. limit

For $\mu < 0$ the results are similar, with an important difference:

Now SUSY produces contributions to $(g\text{-}2)_\mu$ in the wrong direction.

So, adding $(g\text{-}2)_\mu$ (using e+ e- data) does not help in this case to bring the preferred region to the LHC reach.

And, of course, $\mu < 0$ gets strongly disfavoured with respect to $\mu > 0$.

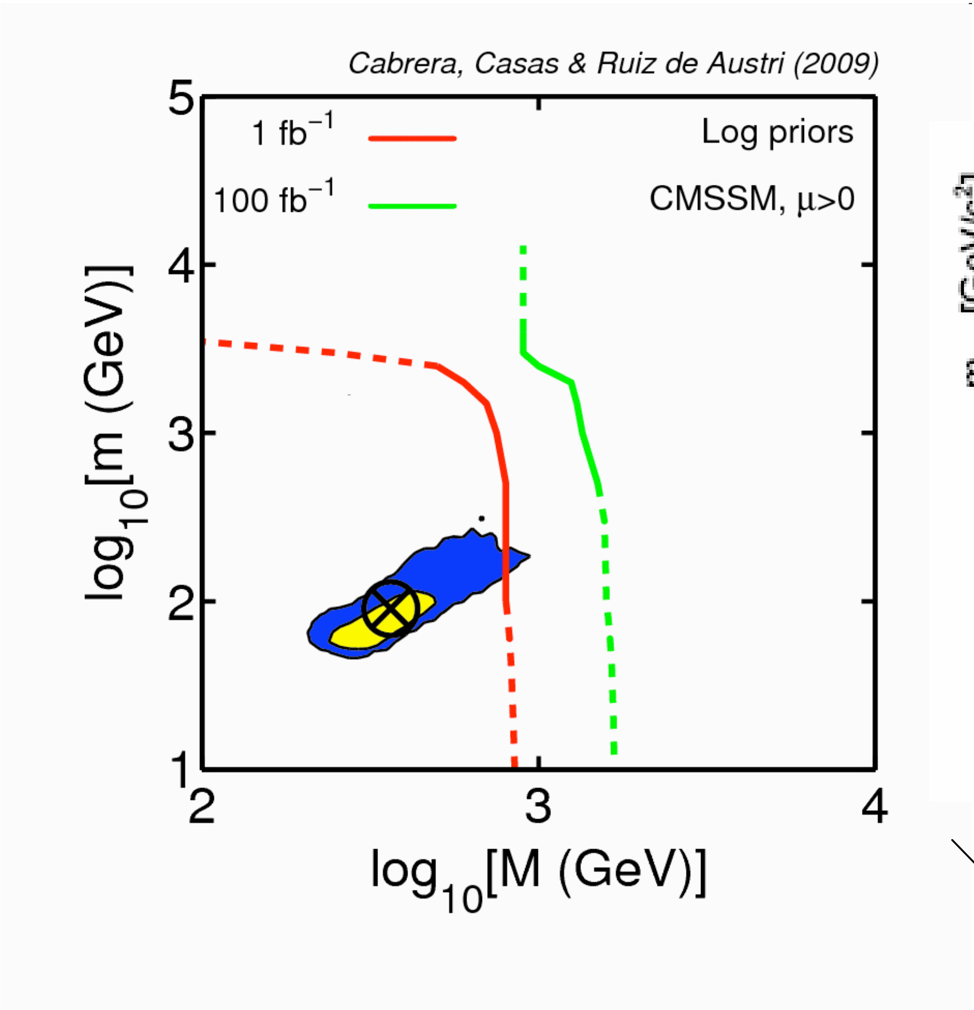
Besides, we have done a similar analysis for the fermion masses

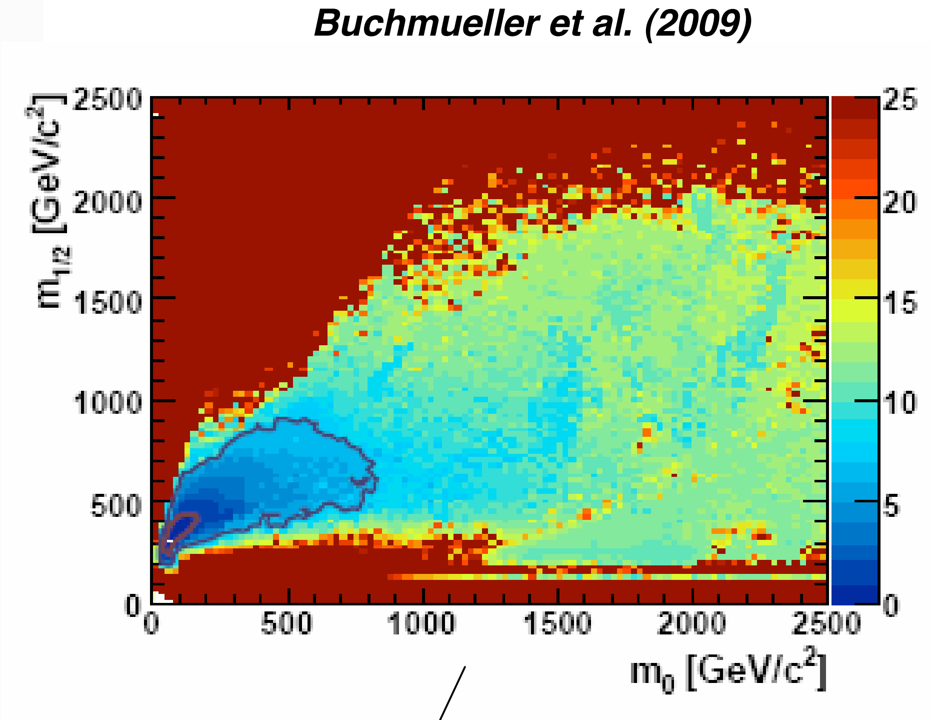
$$y_t \longrightarrow m_t$$

and traded

$$B \longrightarrow \tan\beta$$

Adding Ω_{DM} and $(g-2)_\mu$





similar

But in many cases (like SUSY), this is not the case

An assumption about the prior is necessary

What people usually do:

$$\{m, M, A, B, \mu\} \longrightarrow \{m, M, A, \tan\beta\} \qquad \& \ \text{sign}\ \mu$$

trade by tan β

eliminated by M_Z condition

Note that, in principle a H.E. point ($m, M, A, B = \text{large}$) is as probable as a L.E. one ($m, M, A, B = \mathcal{O}(\text{TeV})$)

The value of μ^2 is just tuned ...

Using

$$\mu_{\rm low}^2 = \frac{m_{H_1}^2 - m_{H_2}^2 t^2}{t^2 - 1} - \frac{M_Z^2}{2}$$

$$B_{\rm low} = \frac{s_{2\beta}}{2\mu_{\rm low}}(m_{H_1}^2 + m_{H_2}^2 + 2\mu_{\rm low}^2)$$

$$y_{\rm low} = \frac{m_t}{v\ s_\beta}\ .$$

and

$$\mu_{\rm low} = R_\mu(y)\mu, \qquad B_{\rm low} = B + \Delta_{RG}B(y), \qquad y_{\rm low} \simeq \frac{yE(Q_{\rm low})}{1 + 6yF(Q_{\rm low})}$$

SM-like parameters

$$s \equiv g_a\,, \quad y_i$$

Nuisance parameters

i.e. parameters we are not
very interested in

Usual technique to eliminate nuisance parameters:

marginalize them

$$p(m, M, A, B, \mu|\text{data}) \;=\; \int \, ds\; p(s, m, M, A, B, \mu|\text{data})$$

★ If nuisance parameter ⟷ high-quality exp. data

one-to-one

Then, the parameter is easily eliminated (without leaving any footprint)

E.g. g, g', g_3

(their prior is irrelevant)

★ This is not the case fo the Yukawa couplings, y_i , in the MSSM

E.g. $m_t = \frac{1}{\sqrt{2}} y_t \, v_2 = \frac{1}{\sqrt{2}} y_t \, v \sin\beta$

↑ Derived quantity

Two different points in the MSSM-par. space will have in general different y_t . Thus the relative probability depends upon $p(y_t)$.

(something ignored in previous literature)

★ Thus the marginalization of y_i leaves a footprint in the pdf

Likelihood ~ $\delta(m_t - m_t^{\exp})\ \delta(m_b - m_b^{\exp})\ \ldots.$

Take $m_t = \frac{1}{\sqrt{2}} y_t^{\rm low} v s_\beta, \quad m_b = \frac{1}{\sqrt{2}} y_b^{\rm low} v c_\beta, \ \cdots$

(with $y_i^{\rm low} = R_i y_i$)

$$\int [dy_t\ dy_b \cdots]\ p(y,m,M,A,B|\ \mathrm{data}) \ \sim\ p(y)\ \left|\frac{dy_t}{dm_t}\right| \left|\frac{dy_b}{dm_b}\right| \cdots \ = \ p(y)\ s_\beta^{-1}\ c_\beta^{-1}\ \cdots$$

Normally people just take y_i "as needed" to reproduce m_i and forget about.

This equivales to take

$$p(y_i) \propto \frac{1}{y_i} \qquad \text{(log prior)}$$

reasonable....

$$y_e \sim 10^{-6}, \quad y_t \sim 1$$

In order to write a sensible prior for $\{m, M, A, B, \mu\}$ one has to consider the dynamical origin of these parameters: ~~SUSY~~

They typically go like $\sim \dfrac{F}{\Lambda} \equiv M_S$

A particular soft term, say ***A***, receives several $\mathcal{O}\left(\dfrac{F}{\Lambda}\right)$ contributions (dep. on the details of ~~SUSY~~)

So, it is reasonable to expect

$$\begin{aligned}
-qM_S \leq & \;\; B \;\; \leq qM_S \\
-qM_S \leq & \;\; A \;\; \leq qM_S \\
0 \leq & \;\; m \;\; \leq qM_S \\
0 \leq & \;\; M \;\; \leq qM_S \\
0 \leq & \;\; \mu \;\; \leq qM_S
\end{aligned}$$

$q = \mathcal{O}(1)$

with *"flat"* probability

$$p(A) = \frac{1}{2M_S}, \qquad \text{etc.}$$

Recall: M_S is the scale of $\not{\text{SUSY}}$ in the observable

. sector

In principle M_S can have any value, say

$$M_S^0 \leq M_S \leq M_X \,, \qquad M_S^0 \sim 10 \text{ GeV}$$

with

flat: $p(M_S) = N_{M_S}$ or log: $p(M_S) = N_{M_S} \frac{1}{M_S}$

probability density

Log. prior:

$$p(m, M, A, B, \mu) \;=\; \int_{\max\{m,M,|A|,|B|,\mu,M_S^0\}}^{M_X} \;\; p(m, M, \mu, A, B)\, p(M_S)\, dM_S$$

$$\propto \; \frac{1}{[\max\{m, M, |A|, |B|, \mu, M_S^0\}]^5}$$

(neglecting $\frac{1}{M_X^5}$ terms)

For a particular parameter, say *M*:

$$\mathcal{P}(M) \propto \frac{1}{\max\{M, M_S^0\}}$$

Flat prior:

$$p(m, M, A, B, \mu) \;\propto\; \frac{1}{[\max\{m, M, |A|, |B|, \mu, M_S^0\}]^4}$$

(neglecting $\dfrac{1}{M_X^4}$ terms)

For a particular parameter, say *M*:

$$\mathcal{P}(M) \sim \ln \frac{M_X}{\max\{M, M_S^0\}} \;\sim\; \text{const.}$$

At first sight the log. prior implies a strong preference for the Low En. region of the par. space

But this is not so

$$\frac{\mathcal{P}(100\ \mathrm{GeV} \leq M \leq 2\ \mathrm{TeV})}{\mathcal{P}(2\mathrm{TeV} \leq M \leq M_X)} \simeq \frac{1}{11}$$

For flat prior:

$$\frac{\mathcal{P}(100\ \mathrm{GeV} \leq M \leq 2\ \mathrm{TeV})}{\mathcal{P}(2\mathrm{TeV} \leq M \leq M_X)} \simeq 6 \times 10^{-13}$$

But this was before including M_Z^{exp}

Including M_Z^{exp} amounts to use the effective prior. Then

log prior: $$\frac{\mathcal{P}(100\ \text{GeV} \leq M \leq 2\ \text{TeV})}{\mathcal{P}(2\text{TeV} \leq M \leq M_X)} \simeq 400$$

flat prior: $$\frac{\mathcal{P}(100\ \text{GeV} \leq M \leq 2\ \text{TeV})}{\mathcal{P}(2\text{TeV} \leq M \leq M_X)} \simeq 20$$

!