Spontaneous CP Violation vs. Collective Symmetry Breaking

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Summary

- Spontanous CPV requires both spontaneous and explicit breaking of global symmetry.
- This condition might interfere with Collective Symmetry Breaking.
- Example 1: SU(5)/SO(5) variant can yield $\mathcal{O}(1)$ phase.
- Example 2: SU(6)/SO(6) model yields two $\mathcal{O}(1)$ phases.

Spontaneous CP Violation: Basics

A model is explicitly CP conserving

iff

there exists a "real basis" (in which all the couplings are real).

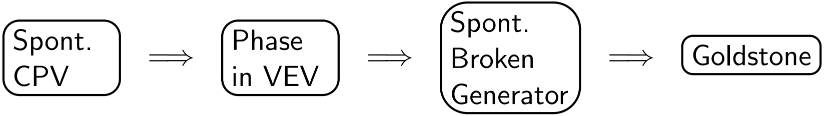
An explicitly CP conserving model is

spontaneously CP violating

iff

There is no real basis in which all the VEVs are real.

Conditions for Spontaneous CP Violation



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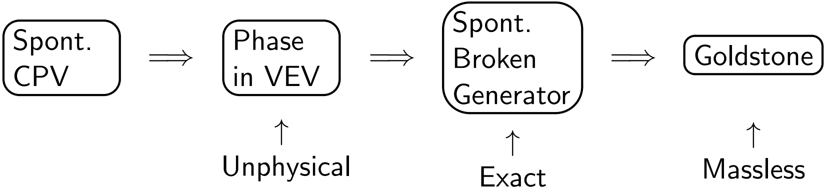

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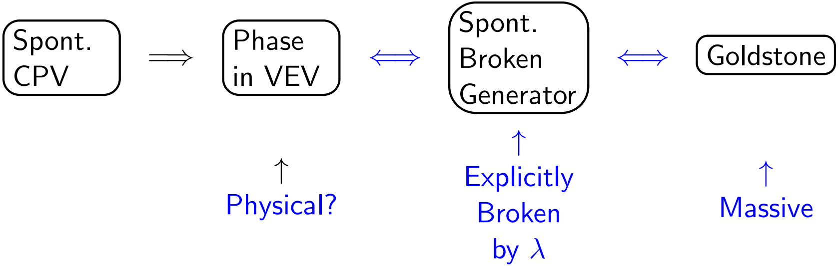


Conditions for Spontaneous CP Violation

Spont. CPV $\Longleftrightarrow$ Phase in VEV $\Longleftrightarrow$ Spont. Broken Generator $\Longleftrightarrow$ Goldstone

Phase in VEV: $\uparrow$ Physical

Spont. Broken Generator: $\uparrow$ Explicitly Broken by λ $\uparrow$ λ transforms as complex irrep.

Goldstone: $\uparrow$ Massive

Example (Simplest SCPV?)

$$V = -m_+^2 |\phi_+|^2 - m_-^2 |\phi_-|^2 + \lambda_+ |\phi_+|^4 + \lambda_- |\phi_-|^4 + g \left[(\phi_+ \phi_-)^2 + \mathrm{c.c.} \right]$$

- Exact Symmetry: U(1)
- for much of parameter space, global minimum at:

$$\langle \phi_+ \rangle = v_+ \left(e^{i\theta} \right), \qquad \langle \phi_- \rangle = i v_- \left(e^{i\theta} \right)$$

$$\Downarrow$$

Spontaneous CP Violation: $\mathrm{Arg} \frac{\langle \phi_+ \rangle}{\langle \phi_- \rangle} = \pi/2$

Example (Simplest SCPV?) - cont.

In terms of the above conditions:

- In the Limit $g \to 0$, the symmetry is U(1)$\times$U(1).
- Both U(1)'s are broken spontaneously.
- U$(1)_A$ broken explicitly by $g \neq 0$.

$\Longrightarrow$ Spontaneous CP Violation

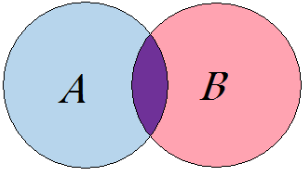
Little Higgs: Basics

- Higgs is pseudo-Goldstone - gets mass through loops

  $\Longrightarrow$ Naively $\delta m_H^2 \sim (\Lambda/4\pi)^2 \Longrightarrow \Lambda \gtrsim 10\mathrm{TeV}$

- Explicit Breaking is *Collective*:
- Two terms, each breaking either A or B
- Only diagrams involving both terms give $\delta m_H \neq 0$
- more vertexes $\Rightarrow$ No quadratic divergence at one-loop

  $\Longrightarrow \quad m_H \sim f/4\pi \sim \Lambda/(4\pi)^2 \quad \Longrightarrow \quad \Lambda \gtrsim 1\mathrm{TeV}$



Littlest Higgs: SU(5)/SO(5)

- SU(5) Global Symmetry
- Explicit breaking: by gauging $[\mathrm{SU}(2)\times \mathrm{U}(1)]^2 \supset \mathrm{SU}(5)$:

$$V = \begin{pmatrix} \mathrm{SU}(2)_1 & & \\ & 1 & \\ & & \mathrm{SU}(2)_2 \end{pmatrix}$$

$$Y_1 = \mathrm{diag}(3,3,-2,-2,-2),\ Y_2 = \mathrm{diag}(2,2,2,-3,-3)$$

- Spont. Breaking: $\Sigma_0 = \begin{pmatrix} & & \mathbb{1} \\ & 1 & \\ \mathbb{1} & & \end{pmatrix}$
- SU(5) →SO(5), $[\mathrm{SU}(2)\times \mathrm{U}(1)]^2 \to [\mathrm{SU}(2)\times \mathrm{U}(1)]_{\mathrm{diag}}$

Littlest Higgs (cont.)

- Goldstone bosons: $$\pi = \begin{pmatrix} \text{eaten} & H & \phi \\ H^\dagger & \text{eaten} & H^T \\ \phi^\dagger & H^* & \text{eaten} \end{pmatrix}$$
- H shifts under: $A =$SU(3)$_1$, $B =$SU(3)$_2$
- gauge interactions break A, B *collectively*
- $\delta m_H^2 \sim \left(\frac{gf}{4\pi}\right)^2 \log \Lambda$ at 1-loop
- Collective breaking also with Yukawa terms...

Mass Scales

$\uparrow$ UV completion

$\Lambda = 4\pi f$ (or lower)

$\updownarrow$ NLΣM

$\sim \frac{\Lambda}{4\pi}$ ϕ, W', Z', t' get masses

Standard Model

$\frac{f}{4\pi} \sim \frac{\Lambda}{(4\pi)^2}$ m_H

Standard Model

Spontaneous CPV vs. Collective Symmetry Breaking

Spontaneous CPV $\Longleftrightarrow$ There exists a generator X which is:

- Spontaneously Broken
- Explicitly Broken

But if X is spanned by both A+gauge and B+gauge,

$\Longrightarrow$ It must spoil Collective Symmetry Breaking.

SU(5)/SO(5) variant

- In Littlest Higgs: no SU(5)-related phase: $\Sigma_0 = \begin{pmatrix} & & \mathbb{1} \\ & 1 & \\ \mathbb{1} & & \end{pmatrix}$
- Variant by Csáki et al., Perelstein et al.:

  Don't gauge $Y' \equiv Y_1 - Y_2 \sim \mathrm{diag}(1,1,-4,1,1)$

  $\Longrightarrow$ CPV Phase: $\Sigma_0 = \begin{pmatrix} & & e^{i\delta}\mathbb{1} \\ & e^{-4i\delta} & \\ e^{i\delta}\mathbb{1} & & \end{pmatrix}$
- $\Longrightarrow$ ...but also a new Goldstone

SU(5)/SO(5) variant - cont.

Must break Y' explicitly (to avoid exact Goldstone)

- Problem: Explicit breaking of Y' breaks both $\mathrm{SU}(3)_{1,2}$:

$$Y' \sim \lambda_8^{\mathrm{SU}(3)_1} + Y \sim \lambda_8^{\mathrm{SU}(3)_2} + \# Y$$

- Solution: Introduce *small* non-collective breaking

$\Longrightarrow$ *large* phase

$\Longrightarrow$ Pseudo-Goldstone of Y' acquires weak scale mass.

- Spontaneous CPV phase generically $\mathcal{O}(1)$

SU(6)/SO(6) Model

- Explicit SU(6) breaking by gauging:

$$\left(\begin{array}{c|cc|c} SU(2)_1 & & & \\ \hline & 1 & & \\ & & 1 & \\ \hline & & & SU(2)_2 \end{array}\right)$$

- $Y_1 \sim \mathrm{diag}\{-2, -2, 1, 1, 1, 1\}$
- $Y_2 \sim \mathrm{diag}\{-1, -1, -1, -1, 2, 2\}$
- Global sym.: $SU(2)_M$ (must break!)

SU(6)/SO(6) - cont.

- Vacuum: $\Sigma_0 = \left(\begin{array}{c|cc|c} & & & \mathbb{1} \\ \hline & e^{i\alpha}\cos\theta & i\sin\theta & \\ & i\sin\theta & e^{-i\alpha}\cos\theta & \\ \hline \mathbb{1} & & & \end{array}\right)$
- Two phases α, θ
- Goldstone bosons: $\left(\begin{array}{c|cc|c} \mathbb{1} & H & K & \phi \\ \hline H^\dagger & \sigma & \rho & H^T \\ K^\dagger & \rho & -\sigma & K^T \\ \hline \phi^\dagger & H^* & K^* & \mathbb{1} \end{array}\right)$
- Collective Symmetry Breaking: $A =$SU$(4)_1$, $B =$SU$(4)_2$
- Yukawa can stabilize θ (giving mass to ρ):
  $A =$SU$(3)_1$, $B =$SU$(4)_2$
- -but not α: $T_\sigma \sim -2T^8_{\mathrm{SU}}(3)_1 + Y + Y'$, but is also SU$(4)_2$

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Backup Slides

NLSM (CWZ,CCWZ)

Goldstone Bosons:

- $\Sigma = e^{i\pi/f}\Sigma_0 e^{i\pi^T/f}, \quad \pi = \pi^a X^a$
- X^a belongs to

$$\mathrm{SU}(5)/\mathrm{SO}(5) \qquad \pi = \begin{pmatrix} \text{eaten} & H & \phi \\ H^\dagger & \text{eaten} & H^T \\ \phi^\dagger & H^* & \text{eaten} \end{pmatrix}$$

Condition Violation in Hypercharge Model

- $Y' \sim \lambda_8^{\mathrm{SU}(3)_1} + Y \sim \lambda_8^{\mathrm{SU}(3)_2} + \# Y$

- Explicit Breaking of $Y' \Longrightarrow$ expl. breaking of both $\mathrm{SU}(3)_{1,2}$
- But can still be fine if non-collective breaking is small, *e.g.* : $V \supset \frac{1}{16\pi^2}\Sigma_{33}$
  - keeps EW scale stable
  - Pseudo-Goldstone of Y' acquires weak scale mass
  - Spont. CPV phase generically $\mathcal{O}(1)$!

Yukawa Sector, SU(5)

- Add LH singlet ψ $\quad\Rightarrow\quad$ $Q^{i=1,2,3} \equiv \begin{pmatrix} Q^{a=1,2} \\ \psi \end{pmatrix}$
- Add RH singlet t'

$$\mathcal{L}_Y = \lambda_1 f \bar{Q}_i \Omega_1^i u + \lambda' f \bar{\psi} t', \qquad \Omega_1^i = \epsilon^{ijk} \epsilon^{xy} \Sigma_{jx} \Sigma_{ky}$$

- λ_1 : $\quad \mathrm{SU}(3)_1$ exact $\quad \mathrm{SU}(3)_2$ broken
- λ' : $\quad \mathrm{SU}(3)_1$ broken $\quad \mathrm{SU}(3)_2$ exact

Yukawas, SU(6)

- SU(2)$_M$ must be broken explicitly
  $\Longrightarrow$ break SU(4)$_{1,2}$ →SU(3)$_{1,2}$ explicitly:
- $\bar{Q}_i \left(\bar{\Sigma}^{i4}\Sigma_{44} + r\epsilon_{jk\ell}\epsilon_{xy}\bar{\Sigma}^{ij}\bar{\Sigma}^{kx}\bar{\Sigma}^{\ell y}\right) u + \lambda'\bar{\psi}t'$
  $\Longrightarrow$ breaks collectively SU(3)$_1$ and SU(4)$_2$
  $\Longrightarrow$ H is light. K, ρ are heavy

- diag$(0,0,1,-1,0,0)$ spanned by $\{T^8$ of SU(3)$_1$, Y, $Y'\}$ and also by {SU(4)$_2$}
  $\Longrightarrow$ can't be broken without spoiling collective SB

- Spoil collective SB slightly by $\varepsilon f^4 \bar{\Sigma}^{33}\Sigma_{44}$
  $\Longrightarrow$ σ at EW scale

- stabilizes $\theta \neq 0$ for generic r

- $\sqrt{2}e^{-i\alpha}\cos\theta\left[(2i - \sin\theta) + \frac{2i+\sin\theta}{3}H^\dagger H\right]\bar{Q}\tilde{H}u$

BSM quark-Higgs Couplings

Low energy implications:

$$\frac{Z^u_{\alpha\beta}}{f^2}\bar{Q}_\alpha \tilde{H} u_\beta H^\dagger H + \frac{Z^d_{\alpha\beta}}{f^2}\bar{Q}_\alpha H d_\beta H^\dagger H + \frac{Z^\ell_{\alpha\beta}}{f^2}\bar{L}_\alpha H \ell_\beta H^\dagger H$$

- Can get $Z_{\alpha\beta}$ from expanding Σ_{ij} in $\lambda \bar{Q}_i \Omega^i u$
- But... No relative phase in $H \leftrightarrow HH^\dagger H$ unless both $SU(3)_{1,2}$ are broken $\Longrightarrow$ phase in $Z_{\alpha\beta}$ is ε-suppressed