From Hidden Symmetry to Extra Dimensions

A 5-dimensional formulation of the D-BESS model and its connection with RS1

Francesco Coradeschi

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based on

R. Casalbuoni, F. Coradeschi, S. De Curtis, D. Dominici, PRD77 (2008) 095005
F. Coradeschi, S. De Curtis, D. Dominici, arXiv:1001.2716 (hep-ph)

Outline

On EW breaking

- $SU(2)_L \otimes U(1)_Y$ symmetry breaking: an experimental certainty.
- Exact nature of the symmetry breaking sector is unknown
- In the SM: broken by the VEV of a scalar doublet. Single neutral scalar left in the spectrum
- Suffers the so-called Hierarchy problem. In short: why is $M_W \ll M_{Pl}$?

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- Many possible solutions. Two "main roads": SUSY and Strong Interactions (variety of proposals: Higgsless, Goldstone bosons . . .)
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- In Strong Interactions: EW symmetry broken by strong interactions at the $\sim$ TeV scale. Higgs scalar is either absent or a bound state. Difficult to properly define a realistic theory!

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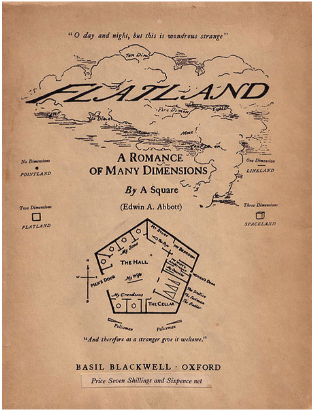
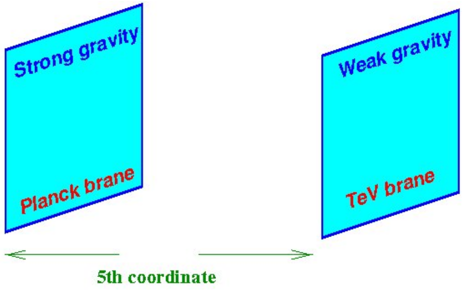
An alternative: Extra Dimensions!



The World is apparently 4 dimensional, but at distances shorter than those yet probed Nature could be described by a theory with extra dimensions

An alternative: Extra Dimensions!

- RS model: only one, strongly warped compact extra dimension.
- Gravity is concentrated on one brane, the Planck (UV) brane and we live on the TeV (IR) brane, a little apart. Gravity on our second brane appears to be weak.
- The Higgs field feels an effective scale much lower than the Planck scale: $\Lambda \simeq M_{Pl} e^{-k\pi R}$



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- BESS: effective theory - low-energy limit of a strongly-interacting EW breaking sector (Casalbuoni et al. '95)
- Based on chiral Lagrangians and the Hidden Symmetry approach, both "borrowed" from low-energy QCD
- Starting point is the (experimentally confirmed) $SU(2) \otimes SU(2)$ global custodial symmetry of the EW breaking sector, partly gauged to $SU(2) \otimes U(1)$ then spontaneously broken
- An extra "hidden" $SU(2) \otimes SU(2)$ gauge symmetry is added; the associated vector fields are interpreted as bound states of the strong sector

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The BESS model

- Set-up: let $G = \left(SU(2)_L \otimes SU(2)_R\right)_{glob.}$ and $H = \left(SU(2)_L \otimes SU(2)_R\right)_{loc.}$
- Put in 3 $SU(2)$-valued fields, L, R and M such that

$$L(x) \to g_L L(x) h_L(x), \quad R(x) \to g_R R(x) h_R(x),$$
$$M(x) \to h_R^\dagger(x) M h_L(x) \qquad (g_L \otimes g_R \in G, \quad h_L \otimes h_R \in H')$$

- Write down the most general Lagrangian with these d.o.f. invariant under the assumed symmetry
- Add weak interactions by gauging a proper subgroup of G
- Scalar bound states can also be introduced

(Casalbuoni, et al. '97)

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The BESS model

- QCD-scaled TC theories are generally strongly constrained (excluded) by electroweak precision observables
- Possible parametrizations: S, T, U (Peskin & Takeuchi '90) or $\epsilon_{1,2,3}$ parameters (Altarelli & Barbieri '90)
- In BESS, custodial symmetry guarantees $\epsilon_1 \sim \epsilon_2 \sim 0$ at L.O., while

$$\epsilon_3 \simeq \frac{g^2}{g''^2} \sim 0.4/(g''^2)$$

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The BESS model: D-BESS

- There exists one choice of parameters - for which total global symmetry is enhanced to a maximal $(SU(2) \otimes SU(2))^3$ - that leads to a vanishing L.O. contribution: D-BESS model
- N.L.O. contribution is suppressed by a factor $\frac{m_Z^2}{M^2}$
- For reference, the Lagrangian

$$\mathcal{L} = \frac{v^2}{4} \left\{ a_1 \mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + a \left(\mathrm{Tr}\left[D_\mu L^\dagger D^\mu L\right] + \mathrm{Tr}\left[D_\mu R^\dagger D^\mu R\right] \right) \right\} + \mathcal{L}_{kin}.$$

- ϵ parameters are proportional to: $-\frac{g^2}{g''^2}\frac{m_Z^2}{M^2}$

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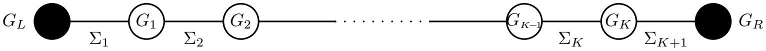
Generalized D-BESS

- Goal: connect D-BESS to Extra Dimension. Need to use the tool of Dimensional Deconstruction
(Arkani-Hamed et al. '01) (Cheng et al. '01)
- Extend the hidden symmetry: use a "moose" Lagrangian (Casalbuoni et al. '04)

$$\mathcal{L} = -\frac{1}{2}\sum_{i=1}^{K}\frac{1}{g_i^2}\mathrm{Tr}\left[\mathbf{F}^i_{\mu\nu})\right]^2 + \sum_{i=1}^{K+1} f_i^2 \mathrm{Tr}\left[D_\mu {\Sigma^i}^\dagger D^\mu \Sigma^i\right]$$

$$\left(D_\mu\Sigma^i = \partial_\mu\Sigma^i + i\mathbf{A}^{i-1}_\mu\Sigma^i - i\Sigma^i\mathbf{A}^i_\mu, \quad \mathbf{A}^0 = \mathbf{A}^{K+1} = 0\right)$$



Generalized D-BESS

- All G_i are (independent) $SU(2)$. Similarly to BESS: weak interactions initially turned off. Symmetry groups $G_L \equiv SU(2)_L,\ G_R \equiv SU(2)_R$ are global
- Generic leading order contribution to ϵ_3 parameter:

$$\epsilon_3 \sim \frac{g^2}{g_c^2}, \qquad (g_i \sim g_c, \quad f_i \sim f_c)$$

- Challenge: is it possible to get a vanishing L.O. contribution (generalizing D-BESS) ?

Generalized D-BESS

$\Rightarrow$ Yes if and only if 1+ "link couplings" (f_i) vanishes independently $\boxed{\rightarrow \text{cutting}}$

- But: 3 Goldstones are lost; cannot give masses to all of the vectors!
- Necessary to add a new field U

$$U \rightarrow g_L U g_R^\dagger, \quad g_{L(R)} \in SU(2)_{L(R)}$$

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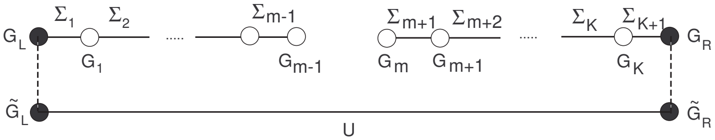
Generalized D-BESS

- "Cut" model defined by:



$$\mathcal{L} = f_0^2 \mathbf{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + \sum_{i=1}^{m-1} f_i^2 \mathbf{Tr}\left[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i\right]$$

$$+ \sum_{i=m+1}^{K+1} f_i^2 \mathbf{Tr}\left[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i\right] - \frac{1}{2g_i^2} \sum_{i=1}^{K} \mathbf{Tr}\left[(F_{\mu\nu}^i)^2\right].$$

Generalized D-BESS

- Contributions to ϵ parameters are next-to-leading,

$$\epsilon_i \sim \frac{g^2}{g_c^2} \frac{m_Z^2}{\bar{M}^2}$$

- Introducing - for ease of calculation - a reflection symmetry on the moose (K → $2N$) (Casalbuoni et al. '07):

$$\epsilon_1 = -\frac{(c_\theta^4 + s_\theta^4)}{c_\theta^2} \overline{X}, \quad \epsilon_2 = -c_\theta^2 \overline{X}, \quad \epsilon_3 = -\overline{X}$$

$$\overline{X} = \frac{g^2}{\bar{G}^2} \frac{m_Z^2}{\bar{M}^2}$$

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$$\frac{1}{\bar{G}^2} = \sum_{i=1}^{N}\frac{1}{g_i^2}, \qquad \frac{1}{\bar{M}^2} = \bar{G}^2\sum_{i=1}^{N}\frac{1}{g_i^2}\sum_{j=1}^{i}\frac{1}{f_j^2}\sum_{k=j}^{N}\frac{1}{g_k^2}$$

Outline

Continuum limit of GD-BESS

(F.C. et al. '10)

GD-BESS describes a discretized 5^{th} dimension. I want the continuum limit. But two nontrivial questions:

- How to interpret the cut link?
- How to treat the apparent nonlocality of the U field?

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- How to interpret the cut link?
- How to treat the apparent nonlocality of the U field?

$\Rightarrow$ "Flip" one side of the moose!

Continuum limit of GD-BESS

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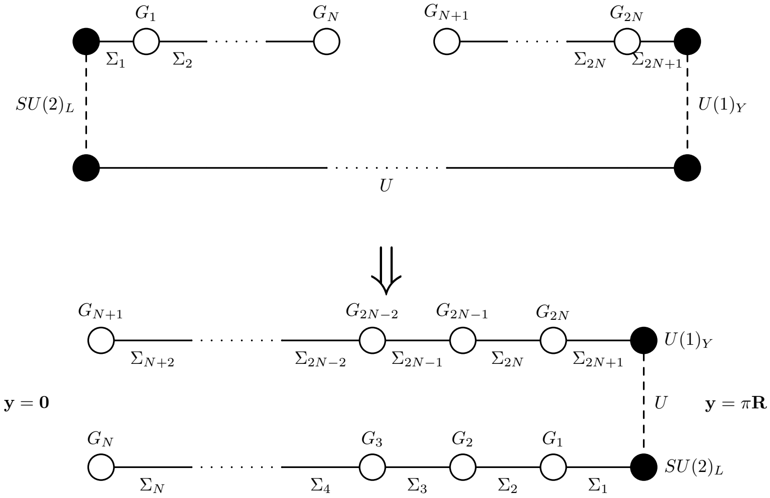
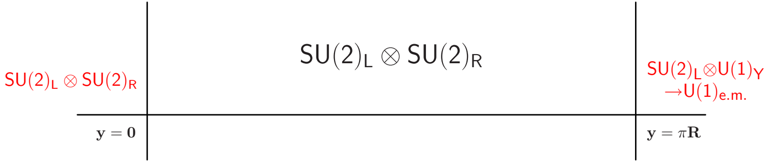


Continuum limit of GD-BESS

- The model can be seen as an $SU(2)_L \otimes SU(2)_R$ theory discretized on N sites
- The set-up of deconstructed model additionally prescribes:
  1. Neumann BCs on the $y = 0$ Brane
  2. The symmetry breaking pattern on the $y = \pi R$ Brane
  3. Localized kinetic terms at $y = \pi R$

Continuum limit of GD-BESS



Symmetry scheme of the 5D D-BESS

Continuum limit of GD-BESS

$$\frac{g_i^2}{N} \to \frac{g_{5L,R}^2}{\pi R}, \qquad f_i^2 \to b(y)\frac{N}{\pi R g_5^2}$$

$\Rightarrow$ f_i on the two side of the moose must be identified

$$ds^2 = b(y)\eta_{\mu\nu}\, dx^\mu dx^\nu + dy^2$$

$$S = \int d^4x \int_0^{\pi R} \sqrt{-g}\, dy \Big[-\frac{1}{4g_{5L}^2} L^a_{MN} L^{a\,MN} - \frac{1}{4g_{5R}^2} R^a_{MN} R^{a\,MN} + \delta(y-\pi R) \left(-\frac{1}{4\tilde{g}^2} L^a_{\mu\nu} L^{a\,\mu\nu} - \frac{1}{4\tilde{g}'^2} R^3_{\mu\nu} R^{3\,\mu\nu} - \frac{\tilde{v}^2}{4} (D_\mu U)^\dagger D^\mu U + fermions \right) \Big]$$

Continuum limit of GD-BESS

- ϵ parameters calculation: matches the deconstructed one (again in L-R symmetric limit $g_{5L} = g_{5R} = g_5$ for simplicity)

$$\frac{1}{\bar{G}^2} \equiv \sum_i \frac{1}{g_i^2} \rightarrow \frac{\pi R}{g_5^2}$$

$$\frac{1}{\bar{M}^2} = \frac{1}{\pi R} \int_0^{\pi R} dy \int_y^{\pi R} dz\, z\, b^{-1}(z)$$

$$\left(\epsilon_1 = -\frac{(c_\theta^4 + s_\theta^4)}{c_\theta^2} \overline{X}, \quad \epsilon_2 = -c_\theta^2 \overline{X}, \quad \epsilon_3 = -\overline{X} \right)$$

Unitarity and EW fits

- Effective model: what is the breakdown scale Λ?
- May estimate Λ looking for unitarity violation in tree-level partial wave amplitudes
- Two sources of unitarity violation:
  1. Theory is in 5D $\to \infty$ resonances
  2. W and Z are coupled to the chiral field U
- Second source is dominant: unitarity violation at a low scale ~ 1.7 TeV

Unitarity and EW fits

- Solution: insert a (Higgs-like) scalar bound state by promoting:

$$U \to M \equiv \frac{\rho}{\sqrt{2}} U.$$

mimicking the matrix formulation of the SM Higgs sector

- If scalr mass $m_H < 1$ TeV, unitarity violation is postponed to

$$\Lambda = \frac{16\pi^2}{g_5^2}\sqrt{b(\pi R)} \qquad \text{(dimensional estimate)}$$

Outline

Spectrum and EW constraints: AdS background

- Most interesting case: a slice of AdS_5 space, $b(y) = e^{-2ky}$
- The spectrum has to be determined numerically
- $\bar{M}$ given by

$$\frac{1}{\bar{M}^2} \simeq \frac{1}{2k^2} e^{2k\pi R} k\pi R$$

Comparison with RS1

- Dropping left-right symmetry, the general result for the ϵ parameters reads

$$\epsilon_1 = -\frac{m_Z^2}{\bar{M}^2}\left(c_\theta^2 \pi R \frac{g^2}{g_{5L}^2} + s_\theta^2 \pi R \frac{g'^2}{g_{5R}^2}\right)$$

$$\epsilon_2 = -\frac{m_Z^2}{\bar{M}^2} c_\theta^2 \pi R \frac{g^2}{g_{5L}^2}, \quad \epsilon_3 = -\frac{m_Z^2}{\bar{M}^2} c_\theta^2 \left(\pi R \frac{g^2}{g_{5L}^2} + \pi R \frac{g'^2}{g_{5R}^2}\right)$$

- It is possible to decouple brane kinetic terms by sending the corresponding couplings to ∞

Comparison with RS1

- One gets

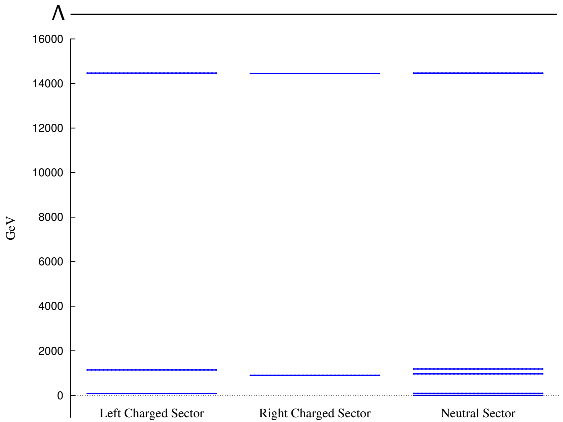
$$\pi R \frac{g^2}{g_{5L}^2} \to 1, \quad \pi R \frac{g'^2}{g_{5R}^2} \to 1$$

or

$$\epsilon_1 = -\frac{m_Z^2}{\bar{M}^2}, \quad \epsilon_2 = -\frac{m_Z^2}{\bar{M}^2} c_\theta^2, \quad \epsilon_3 = -2\frac{m_Z^2}{\bar{M}^2} c_\theta^2$$

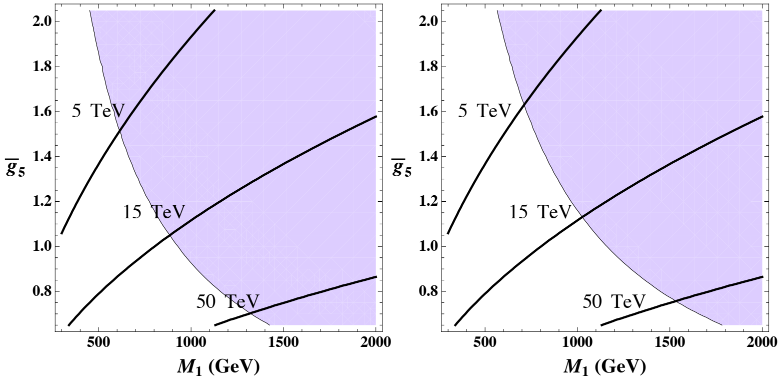
- This is equivalent to standard RS1 ($SU(2) \otimes U(1)$ in the Bulk, fermions and Higgs on IR Brane) (Csaki et al. '02)!
- Extra $SU(2)_R$ states do not contribute

AdS Spectrum



$g_5 = (\pi R)^{-\frac{1}{2}},\ \bar{M} = 900$	**0 mode(s)**	1^{st} **KK excit.(s)**	2^{nd} **KK excit.(s)**
Left charged	80.4	1180	14500
Right charged	-	900	14450
Neutral	0, 91.2	962, 1180	14450, 14500

AdS EW constraints

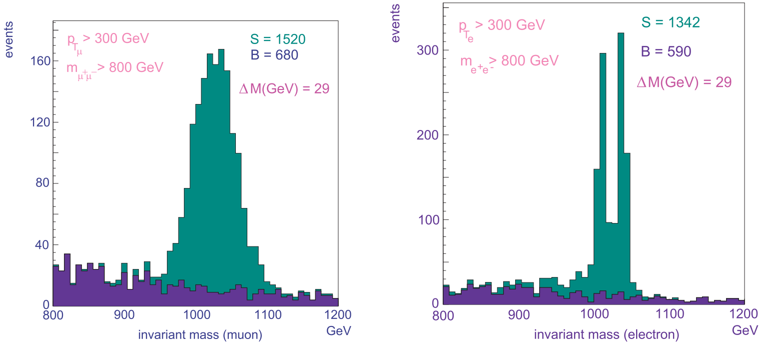


m_H = 300 GeV m_H = 114 GeV

Phenomenology at LHC

(Casalbuoni et al. '07)

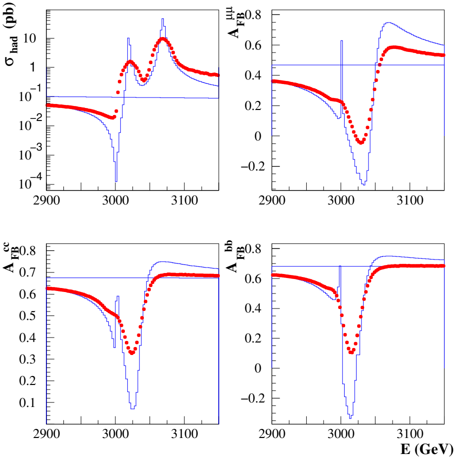
Only the lowest states of KK tower



Invariant mass distribution of the events ($\int \mathcal{L} = 100\,\text{fb}^{-1}$) in $pp \to L_3, R_3, Z, \gamma \to \mu^+\mu^-$ (left) and in $pp \to L_3, R_3, Z, \gamma \to e^+e^-$ (right), for $M = 1000\ GeV$ and $g/g'' = 0.2$, $g'' \equiv \sqrt{2}\bar{g}_5$. The splitting between the resonances is 29 GeV. The dashed line is the SM background. Pythia + CMSJET for fast CMS simulation.

Phenomenology at CLIC

(Battaglia et al. '02)



Hadronic cross section (upper left) and $\mu^+\mu^-$ (upper right), $c\bar{c}$(lower left) and $b\bar{b}$ (lower right) forward-backward asymmetries at energies around 3 TeV. Continuous lines represent the predictions for the D-BESS model with M = 3 TeV and $g/g'' = 0.15$, flat lines SM expectation and dots observable D-BESS signal after accounting for the CLIC.02 luminosity spectrum.

Summing up

- D-BESS - an effective TC theory - admits a direct generalization to a 5D theory
- In AdS background: result essentially equivalent to RS1 + kinetic terms on the IR Brane (see (Carena et al. '02))
- New resonances at a few TeV may be allowed
- Relatively low unitarity cut-off mostly $\sim$ 10-15 TeV in most parameter space for AdS background
- No time to talk about: flavour, FCNCs . . .

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