Intersecting Brane Worlds and Gauge Couplings at 1-Loop

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based on **NPB829** (2010) 225 (`arXiv:0910.0843[hep-th]`) with Florian Gmeiner

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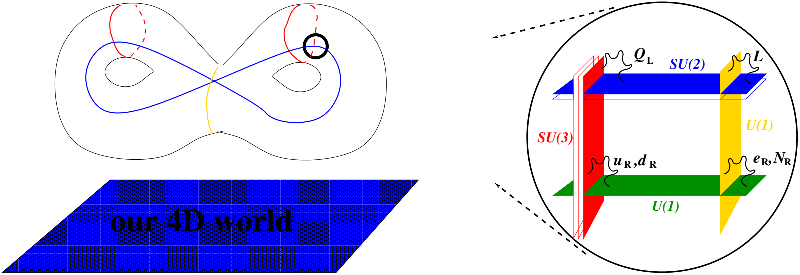
Motivation

Basic concept for the Standard Model on D6-branes:



- all **globally consistent** (SUSY) SM & GUT examples in IIA orientifolds with D6-branes are on **toroidal orbifolds**
- in particular *fractional D6-branes* on $T^6/\mathbb{Z}_6$ and $T^6/\mathbb{Z}'_6$ have proven fertile
- we expect improved models on *rigid D6-branes* on orbifolds $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6$ and $T^6/\mathbb{Z}_2 \times \mathbb{Z}'_6$ with discrete torsion

Motivation cont'd

- the computation of **massless spectrum** (i.e. gauge groups and particles) is well understood
- what about the **4D field theory**?
  - dimensional reduction of 10D type IIA SUGRA to 4D provides a few *tree level* terms *(without matter fields)*
  - on orbifolds, one can compute interactions using CFT techniques for string perturbation theory (at tree and 1-loop level)
  - first step: stringy **1-loop** correction to the **gauge couplings**
    - moduli dependence in the string frame ⋆
    - numerical values ⋆
    - these corrections reappear as exponential prefactors of instantonic couplings

Outline

- What are gauge thresholds?
- How to compute them?
- The generic result for D6-branes on orbifolds $T^6/\mathbb{Z}_N$
- $T^6/\mathbb{Z}_6$ example
- Conclusions & Outlook

What is a gauge threshold? Why is it important?

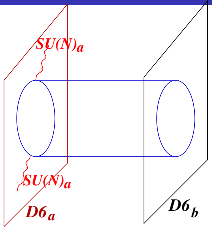
- 4D $\mathcal{N}=1$ SUSY gauge coupling at 1-loop:

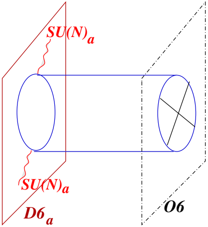
$$\boxed{\frac{8\pi^2}{g_a^2(\mu)} = \frac{8\pi^2}{g_{a,\mathrm{string}}^2} + \frac{b_a}{2}\ln\left(\frac{M_{\mathrm{string}}^2}{\mu^2}\right) + \frac{\Delta_a}{2}}$$

- with the tree-level D6-branes $\frac{8\pi^2}{g_{a,\mathrm{string}}^2} = \frac{M_{\mathrm{Planck}}}{M_{\mathrm{string}}}\overline{\mathrm{Vol}(\mathrm{D6_a})}$
- the **1-loop field theory** running of massless particles $b_a = -3\,C_2(\mathrm{Vector}) + \sum C_2(\mathrm{Matter})$
- the **1-loop gauge threshold** correction Δ_a from the complete tower of massive strings
- not only for $SU(N_a)$ but also $SO(2M_c)$, $Sp(2M_c)$ and $U(1)_X$
- the *holomorphic* gauge kinetic function f_a is not renormalised beyond 1-loop $\boxed{8\pi^2\Re(f_a) + \Delta_0(\mathcal{K}) + \sum_r \Delta_r(\mathcal{K}, K_r)}$
- the *non-holomorphic* Kähler potential $\mathcal{K}$ is unprotected

The gauge threshold computation for $SU(N_a)$

- only open strings charged under the gauge group $SU(N_a)$ contribute
- **Annulus:** contributes to all matter in the adjoint, bifundamental, symmetric & antisymmetric rep. $\mathcal{T}^A(D6_a, D6_b) = \int_0^\infty dl\, l^\varepsilon \left[\partial^2_{B_{\rm mag}} \langle D6_a(B_{\rm mag})| e^{-\pi l H_{\rm closed}} |D6_b\rangle\right]_{B_{\rm mag}=0}$
- **Möbius strip:** contributes to matter in the (anti)symmetric rep. $\mathcal{T}^M(D6_a, O6) = \int_0^\infty dl\, l^\varepsilon \left[\partial^2_{B_{\rm mag}} \langle D6_a(B_{\rm mag})| e^{-\pi l H_{\rm closed}} |O6\rangle\right]_{B_{\rm mag}=0}$





Generic form of the amplitude for $\frac{1}{\varepsilon} \rightsquigarrow \ln\left(\frac{M_{\rm string}}{\mu}\right)^2$

$$\mathcal{T} \sim \underbrace{c_1 \cdot VII}_{\text{tadpole}} \int_0^\infty dl + \underbrace{c_2 \cdot III}_{\beta-\text{function}} \ln\left(\frac{M_{\rm string}}{\mu}\right)^2 + \underbrace{\text{finite(moduli)}}_{\text{gauge threshold } \Delta}$$

Technical Interlude: Jacobi ϑ functions in string amplitudes

- **vacuum amplitudes** in the tree channel

$$\mathcal{A} = \int dl \sum_{\alpha,\beta} (-1)^{2(\alpha+\beta)} \frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}{\eta^3}(2il) \times A^{\alpha\beta}_{\mathrm{comp}}(l), \quad \mathcal{M} = \int dl \sum_{\alpha,\beta} (-1)^{2(\alpha+\beta)} \frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}{\eta^3}(2il - \frac{1}{2}) \times M^{\alpha\beta}_{\mathrm{comp}}(l)$$

- **gauging** the partition function, i.e. $\mathcal{A} \to \mathcal{T}^A$ and $\mathcal{M} \to \mathcal{T}^M$ results in replacing $\frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}{\eta^3}(\tau) \longrightarrow \frac{\vartheta''\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}{\eta^3}(\tau)$
- For toroidal orbifolds, per compact two-torus **Jacobi ϑ functions** occur *(multiplied by lattice sums for parallel branes)*
- Jacobi ϑ function identities for **SUSY angles**

Angles	untwisted sector	$\mathbb{Z}_2$ twisted sector
$(0,0,0)$	$\frac{\vartheta_3''(\vartheta_3)^3 - \vartheta_2''(\vartheta_2)^3 - \vartheta_4''(\vartheta_4)^3}{\eta^{12}}(0,\tau) = 0$	$\frac{\vartheta_3''\vartheta_3(\vartheta_2)^2 - \vartheta_2''\vartheta_2(\vartheta_3)^2}{\eta^6(\tau)(\vartheta_4)^2}(0,\tau) = 4\pi^2$
$(\phi,-\phi,0)$	$4\pi^2$	$\pi\left(\frac{\vartheta_4'}{\vartheta_4}(\phi,\tau) + \frac{\vartheta_1'}{\vartheta_1}(-\phi,\tau)\right)$
$(\phi,0,-\phi)$	$4\pi^2$	$4\pi^2$
$(\phi^{(1)},\phi^{(2)},\phi^{(3)})$	$\pi\sum_{i=1}^3 \frac{\vartheta_1'}{\vartheta_1}(\phi^i,\tau)$	$\pi\left(\sum_{i=1,3}\frac{\vartheta_4'}{\vartheta_4}(\phi^i,\tau) + \frac{\vartheta_1'}{\vartheta_1}(\phi^2,\tau)\right)$

- these are multiplied by intersection numbers I_{ab} and $I_{ab}^{\mathbb{Z}_2}$ *(or (length)^2 V_{ab} for parallel branes)*

Matching of numerical prefactors

- **Chiral matter** is known from intersection numbers on $T^6/\mathbb{Z}_{2N}$

$$\Pi_a \circ \Pi_b = -\sum_{k=0}^{N-1} \frac{I_{a(\theta^k b)} + I^{\mathbb{Z}_2}_{a(\theta^k b)}}{2} \quad \Rightarrow \quad \varphi^{ab}_{\text{chiral}} = \sum_{k=0}^{N-1} |\ldots|$$

reformulated *chiral* RR tad. contribution	$\longleftarrow$	**start:** 1-loop β function for *chiral* matter	$\longrightarrow$	gauge threshold contribution from *chiral* matter
$\downarrow$				
reformulated *non-chiral* RR contribution	$\longrightarrow$	1-loop β funct. for *non-chiral* matter	$\longrightarrow$	gauge threshold contribution from *non-chiral* matter

- cross-check with constraints on (maximal) **non-chiral matter** from Chan-Paton matrices
- no sign ambiguities, in particular for (anti)symmetrics
- economic way of obtaining the **full spectrum from intersection numbers only**!

$$\mathcal{T} = \underbrace{c_1 \cdot VII}_{\text{tadpole}} \int_0^\infty dl + \underbrace{c_2 \cdot III}_{\beta-\text{function}} \ln\left(\frac{M_{\text{string}}}{\mu}\right)^2 + \underbrace{\text{finite(moduli)}}_{\text{gauge threshold } \Delta}$$

- Use known spectrum for D6-branes at angles $(\phi^{(1)}, \phi^{(2)}, \phi^{(3)})$ or $(0,0,0)$:

$$b_{SU(N_a)} \supset \sum_b \frac{N_b}{2} \varphi^{ab} + \frac{N_a-2}{2}\varphi^{\text{Anti}_a} + \frac{N_a+2}{2}\varphi^{\text{Sym}_a} = \sum_b \frac{N_b}{4} |I_{ab} + I_{ab}^{\mathbb{Z}_2}| + \dots$$

- Untwisted: $\boxed{-\sum_b \frac{N_b}{2}(\kappa_{ab} + \kappa_{ab'}) + 2\kappa_{a,\Omega\mathcal{R}} = 0}$ with $\kappa_{ab} = \sum_{\text{cyclic}} V_{ab}^{(i)} I_{ab}^{(j\cdot k)}$ &

Twisted: $\boxed{\sum_b \frac{N_b}{2}(V_{ab}^{(2)} I_{ab}^{\mathbb{Z}_2,(1\cdot 3)} + V_{ab'}^{(2)} I_{ab'}^{\mathbb{Z}_2,(1\cdot 3)}) = 0} \Leftrightarrow \boxed{\Pi_a \star \left[\sum_b N_b(\Pi_b + \Pi'_b) - 4\,\Pi_{O6}\right] = 0}$

with $\star = -\frac{1}{2}[\text{diag}(V,I,I) + \text{diag}(I,V,I) + \text{diag}(I,I,V)]$

Annulus: $\frac{\text{Angle}}{\pi}$	sector	tadpole	$\ln\left(\frac{M_{\text{string}}}{\mu}\right)^2$	Δ_{ab} contributing to $\Delta_{SU(N_a)}$
$(0,0,0)$	$\mathbb{1}$	–	–	–
	$\mathbb{Z}_2$	$-\frac{N_b}{2} V_{ab}^{(2)} I_{ab}^{\mathbb{Z}_2,(1\cdot 3)}$	$-\frac{N_b}{2}\delta_{\sigma^2_{ab},0}\delta_{\tau^2_{ab},0} I_{ab}^{\mathbb{Z}_2,(1\cdot 3)}$	$\frac{N_b}{2} I_{ab}^{\mathbb{Z}_2,(1\cdot 3)} \Lambda(s^2_{ab}, t^2_{ab}, v_2; V_{ab}^{(2)})$
$(\phi^{(1)}, \phi^{(2)}, -\sum_{k=1}^2 \phi^{(k)})$	$\mathbb{1}$	$-\frac{N_b}{2}\sum_{i=1}^3 V_{ab}^{(i)} I_{ab}^{(j\cdot k)}$	$-\frac{N_b}{4} I_{ab} \sum_{i=1}^3 \text{sgn}(\phi^{(i)})$	$\frac{N_b}{2}\frac{I_{ab}}{2}\sum_{i=1}^3 \ln\left(\frac{\Gamma(\lvert\phi^{(i)}\rvert)}{\Gamma(1-\lvert\phi^{(i)}\rvert)}\right)^{\text{sgn}(\phi^{(i)})}$
	$\mathbb{Z}_2$	$-\frac{N_b}{2} V_{ab}^{(2)} I_{ab}^{\mathbb{Z}_2,(1\cdot 3)}$	$-\frac{N_b}{4} I_{ab}^{\mathbb{Z}_2} \sum_{i=1}^3 \text{sgn}(\phi^{(i)})$	$\frac{N_b}{2}\frac{I_{ab}^{\mathbb{Z}_2}}{2}\Big[\sum_{i=1}^3 \ln\left(\frac{\Gamma(\lvert\phi^{(i)}\rvert)}{\Gamma(1-\lvert\phi^{(i)}\rvert)}\right)^{\text{sgn}(\phi^{(i)})}$ $-2\ln(2)\sum_{j=1,3}\left(\text{sgn}(\phi^{(j)}) - 2\phi^{(j)}\right)\Big]$
Möbius: $(0,0,0)$	$\Omega\mathcal{R}\theta^{-m}$	–	–	–
$(\phi^{(1)}, \phi^{(2)}, -\sum_{l=1}^2 \phi^{(l)})$ $\lvert\phi^{(i)}\rvert, \lvert\phi^{(j)}\rvert \leq \lvert\phi^{(k)}\rvert < 1$ $\text{sgn}(\phi^{(i)}) = \text{sgn}(\phi^{(j)})$ $\neq \text{sgn}(\phi^{(k)})$	$\Omega\mathcal{R}\theta^{-m}$	$2\sum_{i=1}^3 \tilde{V}_{a,\Omega\mathcal{R}\theta^{-m}}^{(i)} \tilde{I}_a^{\Omega\mathcal{R}\theta^{-m},(j\cdot k)}$	$\tilde{I}_a^{\Omega\mathcal{R}\theta^{-m}}\left[H(\lvert\phi^{(k)}\rvert - \frac{1}{2}) - \frac{1}{2}\right]\cdot\text{sgn}(\phi^{(k)})$	$\frac{\tilde{I}_a^{\Omega\mathcal{R}\theta^{-m}}}{2}\Big[\ln(2)\,\text{sgn}(\phi_k)\cdot\left[2H(\lvert\phi^{(k)}\rvert - \frac{1}{2}) + 1\right]$ $-\sum_{n=1}^3 \ln\left(\frac{\Gamma(\lvert\phi^{(n)}\rvert)}{\Gamma(1-\lvert\phi^{(n)}\rvert)}\right)^{\text{sgn}(\phi^{(n)})}$ $-\sum_{n=1}^3 \ln\left(\frac{\Gamma(\phi^{(n)}+\frac{1}{2}-\text{sgn}(\phi^{(n)})\cdot H(\lvert\phi^{(n)}\rvert-\frac{1}{2}))}{\Gamma(\frac{1}{2}-\phi^{(n)}+\text{sgn}(\phi^{(n)})\cdot H(\lvert\phi^{(n)}\rvert-\frac{1}{2}))}\right)\Big]$

Lattice sum for relative **Wilson lines** t and **displacements** s depend on Kähler moduli v, for $s = t = 0$ also on $(\text{1-cycle length})^2 = V$

$$\Lambda(s,t,v;V) \equiv \ln \left| e^{-\pi s^2 v/4} \frac{\vartheta_1(\frac{t}{2} - i\frac{s}{2}\,v, iv)}{\eta(iv)} \right|^2 \qquad \Lambda(0,0,v;V) \equiv \ln\left(2\pi v V\,\eta^4(iv)\right)$$

Annulus: $\frac{\text{Angle}}{\pi}$	sector	tadpole	$\ln\left(\frac{M_{\text{string}}}{\mu}\right)^2$	Δ_{ab} contributing to $\Delta_{SU(N_a)}$
$(\phi, -\phi, 0)$	$\mathbf{1}$	$-\frac{N_b}{2} V_{ab}^{(3)} I_{ab}^{(1\cdot 2)}$	$-\frac{N_b}{2} \delta_{\sigma^3_{ab},0}\, \delta_{\tau^3_{ab},0}\, I_{ab}^{(1\cdot 2)}$	$\frac{N_b}{2} I_{ab}^{(1\cdot 2)}\, \Lambda(s^3_{ab}, t^3_{ab}, v_3; V_{ab}^{(3)})$
$(\phi, 0, -\phi)$	$\mathbb{Z}_2$	$-\frac{N_b}{2} V_{ab}^{(2)} I_{ab}^{\mathbb{Z}_2,(1\cdot 3)}$	$-\frac{N_b}{2} \delta_{\sigma^2_{ab},0}\, \delta_{\tau^2_{ab},0}\, I_{ab}^{\mathbb{Z}_2,(1\cdot 3)}$	$\frac{N_b}{2} I_{ab}^{\mathbb{Z}_2,(1\cdot 3)}\, \Lambda(s^2_{ab}, t^2_{ab}, v_2; V_{ab}^{(2)})$
$(0, \phi, -\phi)$	$\mathbb{Z}_2$	$-\frac{N_b}{2} V_{ab}^{(2)} I_{ab}^{\mathbb{Z}_2,(1\cdot 3)}$	–	$\frac{N_b}{2} I_{ab}^{\mathbb{Z}_2} \ln(2)\,(\text{sgn}(\phi) - 2\,\phi)$
Möbius: $(\phi, -\phi, 0)$	$\Omega\mathcal{R}\theta^{-m}$	$2\,\tilde{V}^{(3)}_{a\Omega\mathcal{R}\theta^{-m}}\, \tilde{I}_a^{\Omega\mathcal{R}\theta^{-m},(1\cdot 2)}$	$\delta_{\sigma^3_{aa'},0}\, \delta_{\tau^3_{aa'},0}\, \tilde{I}_a^{\Omega\mathcal{R}\theta^{-m},(1\cdot 2)}$	$-\tilde{I}_a^{\Omega\mathcal{R}\theta^{-m},(1\cdot 2)}\, \Lambda(s^3_{aa'}, t^3_{aa'}, \tilde{v}_3; 2\tilde{V}^3_{aa})$

- Total **gauge threshold** for $SU(N_a)$

$$\Delta_{SU(N_a)} = \sum_b \left(\Delta_{ab} + \Delta_{ab'}\right) + \Delta_{a,\Omega\mathcal{R}}$$

- For $SO(2M_c)$ and $Sp(2M_c)$

$$\Delta_{SO/Sp(2M_c)} = \sum_b \Delta_{cb} + \frac{1}{2}\Delta_{c,\Omega\mathcal{R}}$$

- $SO(2M_c)$ and $Sp(2M_c)$ are correctly identified via their β function (intersections): no need for Chan-Paton matrices!
  This applies in particular to **hidden gauge groups**

Gauge threshold for a massless U(1)

- **massless U(1)s** are **linear combinations** from various branes

$$U(1)_X = \sum_b x_b U(1)_b$$

- field theoretically computed β **function**

$$b_{U(1)_X} = \sum_b x_b^2 b_{U(1)_b} + 2 \sum_{a<b} N_a N_b x_a x_b (\varphi^{ab'} - \varphi^{ab})$$

- **tadpoles cancel** if the U(1) is massless

$$\sum_{a,b} (N_a x_a)(N_b x_b)(\kappa_{ab} - \kappa_{ab'}) = 0 \qquad \Leftrightarrow \qquad \Pi_X \star [\Pi_X - \Pi_X'] = 0$$

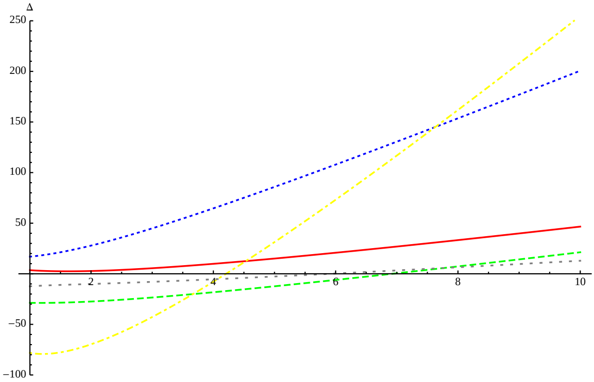
- **gauge thresholds** for a massless $U(1)_X$

$$\Delta_{U(1)_X} = \sum_b x_b^2 \Delta_{U(1)_b} + 4 \sum_{a<b} x_a x_b (\Delta_{ab'} - \Delta_{ab})$$

This applies in particular to the **hyper charge** $U(1)_Y$

A numerical example: the SM on $T^6/\mathbb{Z}_6$

- at **tree level** $\frac{1}{\alpha_{SU(3)}} = \frac{1}{\alpha_{SU(2)}} = 0.38 \cdot \frac{M_{\rm Pl}}{M_{\rm s}}$
- **threshold corrections** for SU(3), SU(2), $U(1)_Y$, $U(1)_{B-L}$, $Sp(2)$ depending on the universal Kähler modulus $v = \frac{1}{2}\left(g_{\rm s}\ \frac{M_{\rm Pl}}{M_{\rm s}}\right)^{2/3}$



- **Numerical example:** set $M_{\rm s} = M_{\rm GUT}$ and $g_{\rm s} = g_{\rm GUT}$ $\Rightarrow v \sim 30$ (in units of α')

$$\frac{1}{\alpha_{SU(3),{\rm tree}}} + \frac{\Delta_{SU(3)}}{4\pi} \sim 228{+}16 = 244, \qquad \frac{1}{\alpha_{SU(2),{\rm tree}}} + \frac{\Delta_{SU(2)}}{4\pi} \sim 228{+}15 = 243$$

- for this example $b_{SU(3)} = 21 + 9_m$, $b_{SU(2)} = 20 + 7_m$
- numerical values obviously don't fit with $1/\alpha_{\rm GUT} = 24$ and $(b_{SU(3)}, b_{SU(2)})_{MSSM} = (-3, 1)$

Conclusions

- **Full CFT computation** of the **gauge couplings** for (bulk, fractional, rigid) intersecting D6-branes **at 1-loop**
  - including all SUSY combinations (non)-vanishing angles
  - moduli dependence in the string frame
  - with all values for Wilson lines & displacement moduli
- **Numerical analysis** performed for some SM-like vacua on $T^6/\mathbb{Z}_6$ and $T^6/\mathbb{Z}_6'$

Outlook

- Further explore the **generic field theory** for fractional, rigid D6-branes ...future work
  - separation into *holomorphic* gauge kinetic function & 1-loop redefined moduli, *non-holomorphic* Kähler metrics
  - extend to perturbative Yukawa & higher order couplings using scattering amplitudes
  - instantonic couplings (gauge thresholds re-appear in exp.)
- **Need improved** explicit **SM examples** with **rigid D6-branes** where the field theory can be applied ...work in progress on orbifolds with torsion with S. Förste, G. Sukumaran & C. Timirgaziu