Flavor Violation in Split UED

or

"How much freedom does Flavor allow in the sUED KK Spectrum"

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Outline

- UED review
- Split UED
- Flavor violation in sUED
- Implications for the sUED mass spectrum
- Conclusions and Outlook

UED: The basic setup

- UED models are models with flat, compact extra dimensions in which *all* fields propagate. [Appelquist, Cheng, Dobrescu,(2001)]
- The Standard Model (SM) particles are identified with the lowest-lying modes of the respective Kaluza-Klein (KK) towers.
- Compactification on S^1/Z_2 allows for boundary conditions on the fermion and gauge fields such that
  - half of the fermion zero mode is projected out $\Rightarrow$ chiral fermions
  - $A_5^{(0)}$ is projected out $\Rightarrow$ no additional massless scalar
- The presence of orbifold fixed points breaks 5D translational invariance.

  $\Rightarrow$ KK-number conservation is violated, *but*

  a discrete Z_2 parity (KK-parity) remains.

  $\Rightarrow$ The lightest KK mode (LKP) is stable.

UED basics: The action

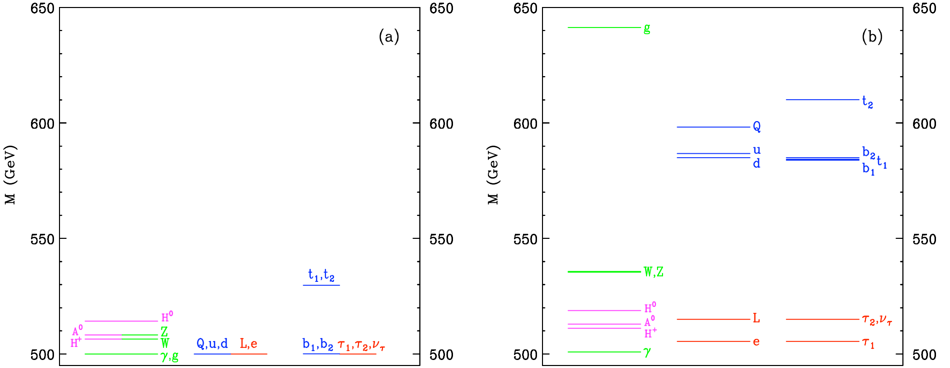
- UED action

$$S_{UED,bulk} = S_g + S_H + S_f,$$

with

$$\begin{aligned}
S_g &= \int d^5x \left\{ -\frac{1}{4\hat{g}_3^2} G^A_{MN} G^{AMN} - \frac{1}{4\hat{g}_2^2} W^I_{MN} W^{IMN} - \frac{1}{4\hat{g}_Y^2} B_{MN} B^{MN} \right\}, \\
S_H &= \int d^5x \left\{ (D_M H)^\dagger (D^M H) + \hat{\mu}^2 H^\dagger H - \hat{\lambda} (H^\dagger H)^2 \right\}, \\
S_f &= \int d^5x \left\{ i\overline{\psi}\gamma^M D_M \psi + \left(\hat{\lambda}_E \overline{L} E H + \hat{\lambda}_U \overline{Q} U \tilde{H} + \hat{\lambda}_D \overline{Q} D H + \text{h.c.} \right) \right\}.
\end{aligned}$$

UED - the spectrum



[Cheng, Matchev, Schmaltz, PRD **66** (2002) 036005, hep-ph/0204342]

UED - Constraints

- Phenomenological constraints on R^{-1}
  - lower bound $\Rightarrow$ $R^{-1} \gtrsim 600\ \mathrm{GeV}$
    - no detection of KK-modes [Appelquist *et al.* (2001); Rizzo (2001); Macesanu *et al.* (2002)]
    - FCNCs [Buras, Weiler *et al.* (2003); Weiler, Haisch (2007)]
    - Electroweak Precision Constraints [Appelquist, Yee (2002)]
  - upper bound: preventing over closure of the Universe by $B^{(1)}$ dark matter
    $\Rightarrow$ $R^{-1} \lesssim 1.5\mathrm{TeV}$ [Servant, Tait (2002); Matchev, Kong (2005); Burnell, Kribs (2005)]
- for an extensive review see [Hooper,Profumo (2007)]

Why are the bounds on UED so weak?

- In "standard" UED, the KK basis $\{f_n(y)\}$ is identical *for all fields*.

  $\Rightarrow$ no triple-vertices of two zero modes with a higher KK mode exist.

  $\Rightarrow$ KK mode exchange does not influence 4-fermi operators at tree-level.

  Relevant for

  Quark flavor physics: E.g. no $s\bar{d} \to \bar{s}d$

  Lepton flavor physics: E.g. no $\mu \to e e \bar{e}$

  Electroweak Precision Tests: No $\mu\bar{\nu}_\mu \to e\bar{\nu}_e$ or $\mu\bar{\mu} \to e\bar{e}$ corrections.

- The UED GIM mechanism guarantees that loop contributions to FCNCs are not only loop- but also CKM-suppressed.

Extensions of UED with minimal field content

Even without extending the field content, the spectrum and/or the interactions of the (M)UED model can by modified by the inclusion of additional operators. Three classes are

1. Bulk mass terms for fermions (dimension 4 operators),
2. kinetic and mass terms at the orbifold fixed points (dimension 5; radiatively induced in MUED),
3. bulk or boundary localized interactions (dimension 6 or higher)

The former two modify the free field equations and thereby the spectrum and the KK bases $\{f_n^{\psi}(y)\}$.

Today, we focus on bulk mass terms: "split UED"

Bulk mass terms for fermions

[Park, Shu, *et al.* (2009); for earlier work, see Csaki (2003)]

A plain bulk mass term for fermions of the form

$$S \supset \int d^5x \; - M\overline{\Psi}\Psi$$

is forbidden by KK parity, but

it can be allowed if realized by a KK-parity odd background field

$$S \supset \int d^5x - \lambda\Phi\overline{\Psi}\Psi,$$

where $\Phi(-y) = -\Phi(y)$

(Orbifold fixed points are at $\pm L = \pm\pi R/2$)

In the simplest case $\lambda\Phi(y) = m^{\psi}\theta(y)$

(similar to the bulk fermion mass term in Randall-Sundrum models)

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Bulk mass terms for fermions

Structural consequences: [Park, Shu, *et al.* (2009); Kong, Park, Rizzo (2010)]

- The chiral fermion zero modes remain massless,

  but the profile of the zero mode becomes exponentially localized towards

  to or away from the orbifold fixed points:

$$f_0^{\psi_L,R} = \sqrt{\frac{\pm m^\psi}{1 - e^{\mp 2m^\psi L}}} e^{\mp 2m^\psi |y|}$$

- The KK mode masses are $m^{(n)} = \sqrt{(m^\psi)^2 + k_n^2}$ with k_n^2 determined from

  $0 = \cot(k_n \pi R/2)$ for even-numbered modes and

  $(m^\psi)^2 = k_n^2 \cot(k_n \pi R/2)$ for odd-numbered modes.

- Wave functions of the fermion and gauge KK modes are not orthogonal:

$$g_{002n} = g^{SM} \mathcal{F}^{\psi\psi}_{002m}(m^\psi L) = g^{SM} \frac{(m^\psi L)^2(-1 + (-1)^n e^{2m^\psi L}(\coth(m^\psi L) - 1)}{\sqrt{2(1 + \delta_{0n}}((m^\psi L)^2 + n^2\pi^2/4)}$$

sUED - known bounds

Bounds studied so far (all for uniform $m^Q = m^U = m^D = M \times \mathbb{1}$):

- Modifications of di-quark channels at Tevatron:

$$R^{-1} \gtrsim .6 \text{ TeV for } m^\psi L = 10$$

[Park, Shu, *et al.* (2009)]

- Modifications of contact interactions at LEP2:

$$R^{-1} \gtrsim .75 \text{ TeV for } m^\psi L = 10$$

[Kong, Park, Rizzo (2010)]

The sUED fermion action

The most general action for fermions reads

$$S = \int d^5 x \mathcal{L}_f + \mathcal{L}_{Yuk}$$

with

$$\begin{aligned}\mathcal{L}_f &= \sum_{ij} \left\{ \frac{i}{2}\delta_{ij} \left(D_M \overline{\Psi}_i \Gamma^M \Psi_j - \overline{\Psi}_\Gamma^M D_M \Psi_j \right) - m_{ij}^{\Psi}(y) \overline{\Psi}_i \Psi_j \right\}, \\ \mathcal{L}_{Yuk} &= \sum_{ij} \left\{ \lambda_{ij}^U \overline{Q}_i \tilde{H} U_j + \lambda_{ij}^D \overline{Q}_i H D_j + \lambda_{ij}^E \overline{L}_i H E_j \right\} + \text{h.c..}\end{aligned}$$

$m^{Q,u,d,L,e}$ are 3×3 hermitian matrices in flavor space,
$\lambda^{U,D,E}$ are 3×3 matrices in flavor space.

Calculating the 4D effective action

With a set of $m^\psi, \lambda^{U,D,E}$, we perform the KK decomposition and then integrate out the heavy modes.

Via field redefinitions, the mass matrices m^ψ can be diagonalized, and the fermion zero mode Lagrangian in the zero mode approximation reads

$$
\begin{aligned}
\mathcal{L}_{kin} &= \overline{\psi}^{(0)} i\gamma^\mu \partial_\mu \psi^{(0)} \\
\mathcal{L}_{f,g} &= \sum_{n=0} \left[\overline{\psi}^{(0)} i\gamma^\mu (D_\mu - \partial_\mu)^{(2n)} \psi^{(0)} \mathcal{F}^{\psi,\psi}_{002n} \right] \\
\mathcal{L}_{Yuk} &= \overline{u^q}^{(0)}_{L,i} \frac{\lambda'^U_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{q^i_L, u^j_R}_{000} u^{(0)}_{R,j} + \overline{d^q}^{(0)}_{L,i} \frac{\lambda'^D_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{q^i_L, d^j_R}_{000} d^{(0)}_{R,j} + \overline{e^l}^{(0)}_{L,i} \frac{\lambda'^E_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{l^i_L, e^j_R}_{000} u^{(0)}_{R,j} \\
&\quad + \text{h.c.}
\end{aligned}
$$

The basis in which the m^ψ are diagonal thus signifies the gauge eigenbasis.

Transformation to the quark mass eigenbasis by bi-unitary transformations:

$$u_L = S_u^\dagger (u^Q)_L^{(0)}, \quad d_L = S_d^\dagger (d^Q)_L^{(0)}, \quad e_L = S_e^\dagger (e^E)_L^{(0)}$$
$$u_R = T_u^\dagger u_R^{(0)}, \quad d_R = T_d^\dagger d_R^{(0)}, \quad e_R = T_e^\dagger e_R^{(0)}.$$

In the fermion mass eigenbasis, the couplings to the KK gluons read

$$\begin{aligned}\mathcal{L}_q \quad \subset \quad & g_3 \sum_n \Big[\overline{u}_{L,i} \lambda^A \gamma^\mu G_\mu^{(2n)A} (S_u^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_u)_{ij} u_{L,j} + \overline{u}_{R,i} \lambda^A \gamma^\mu G_\mu^{(2n)A} (T_u^\dagger \mathcal{F}_{002n}^{u_R,u_R} T_u)_{ij} u_{R,j} \\ & + \overline{d}_{L,i} \lambda^A \gamma^\mu G_\mu^{(2n)A} (S_d^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_d)_{ij} d_{L,j} + \overline{d}_{R,i} \lambda^A \gamma^\mu G_\mu^{(2n)A} (T_d^\dagger \mathcal{F}_{002n}^{d_R,d_R} T_d)_{ij} d_{R,j} \Big] \\ & \equiv \sum_{n=0} g_3 G^{(2n)A\mu} J_{q\mu}^{(2n)A}\end{aligned}$$

and analogous for the electroweak gauge bosons.

KK gauge boson exchange induces FCNCs *unless*

$S_u^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_u$, $S_d^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_d$, $T_u^\dagger \mathcal{F}_{002n}^{u_R,u_R} T_u$ and $T_d^\dagger \mathcal{F}_{002n}^{d_R,d_R} T_d$ are flavor diagonal.

Implications for the sUED mass spectrum

A two-family example:

- Start with mass matrices $m^{Q,U,D}$ which determine the KK decomposition and all overlap factors $\mathcal{F}^{\psi\psi}_{002n}$.
- Find all Yukawa matrices which are consistent with the observed quark masses and the CKM matrix.
- Check whether the associated S_u, S_d, T_u, T_d together with the $\mathcal{F}^{\psi\psi}_{002n}$ only lead to flavor-diagonal corrections.

One set of consistent Yukawa couplings can be constructed as follows:

- Choose $\lambda^d = diag(m_d, m_s)/v \Rightarrow S_d = \mathbb{1} = T_d$.
  $V_{CKM} = S_u^\dagger S_d \Rightarrow$ must choose $S_u^\dagger = V_{CKM}$.
- Choosing $T_u = \mathbb{1}$ determines λ^u because
$$diag(m_u, m_c) = \frac{v}{\sqrt{2}} \times S_u \left(\mathcal{F}_{000}^{q_L^i, u_R^j} \lambda_{ij}^U \right).$$
- Absence of FCNCs in this set means that $S_u^\dagger \mathcal{F}_{002n}^{q_L, q_L} S_u$ is diagonal
  $\Rightarrow \mathcal{F}_{002n}^{q_L, q_L} \propto \mathbb{1} \Rightarrow m^Q \propto \mathbb{1}$.

All other solutions can be obtained by performing the transformations
$S_d \to A_q$, $T_d \to B_d$, $S_u \to A_q V_{CKM}$ and $T_u = B_u$.

- These transformations leave the couplings to the zero mode gauge bosons invariant, but modify the couplings to the KK modes.
- For all A_q, B_u, B_d, the requirement of $M_Q \propto \mathbb{1}$ remains. For $T_u \neq \mathbb{1}$ and/or $T_d \neq \mathbb{1}$ one has to demand $M^U \propto \mathbb{1}$ and/or $M^D \propto \mathbb{1}$, as well.

Estimate for bounds on the allowed mass splitting

In the above, we assumed that FCNCs have to be strictly absent.

Experimentally, FCNCs are present but strongly constraint, which can be parameterized via $\Delta F = 2$ effective operators. [see Bona *et al.*. [UTfit] (2007)]

The constraints translate into bounds on the mass splitting $(m_{11}^Q - m_{22}^Q)$.

For a typical compactification scale of 1 TeV we find for the (least constraint) aligned solution:

$$\frac{m_{11}^Q - m_{22}^Q}{m_{11}^Q + m_{22}^Q} \lesssim \begin{cases} .01 & \text{for} \quad \mu L < 5 \\ .05 & \text{for} \quad \mu L < 10 \end{cases}$$

Conclusions

- Fermionic bulk mass terms are thought to arise from couplings of the fermions to a KK-odd background field.
  In the absence of a flavor symmetry, there is no reason to assume the 5D mass matrices to be uniform.
- We showed that the absence of FCNCs implies that the 5D matrix of the $SU(2)$ charged quarks, m^Q, must be approximately flavor blind.
  This implies a set of 4 nearly mass degenerate fermionic states at the 1st KK mode level.
- We showed that first KK modes of the right handed quarks must also come in two nearly mass degenerate sets
  unless λ^U and λ^D are aligned with the 5D mass matrices.

Outlook

- The results on FCNCs via KK gauge boson exchange are about to be published.
- The analysis of the third family quarks is more involved as for the top, KK Higgs exchanges are to be included (work in progress).
- So far, all results are calculated at tree-level, only.
- Generic m^L, m^E, λ^E induce lepton flavor violation which deserves further attention.
- The analysis presented here can be extended to nUED models with boundary localized kinetic and mass terms (work in progress)