FROM SOFT WALLS TO INFRARED BRANES

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Planck 2010

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Soft Wall models are generalizations of RS that do not possess an IR brane

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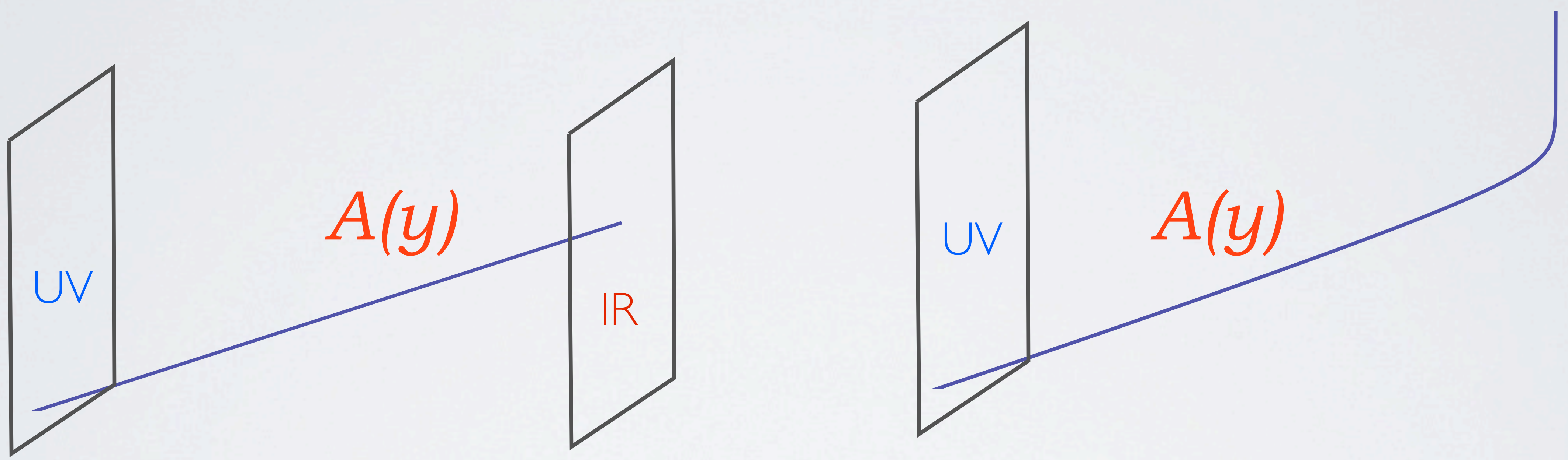

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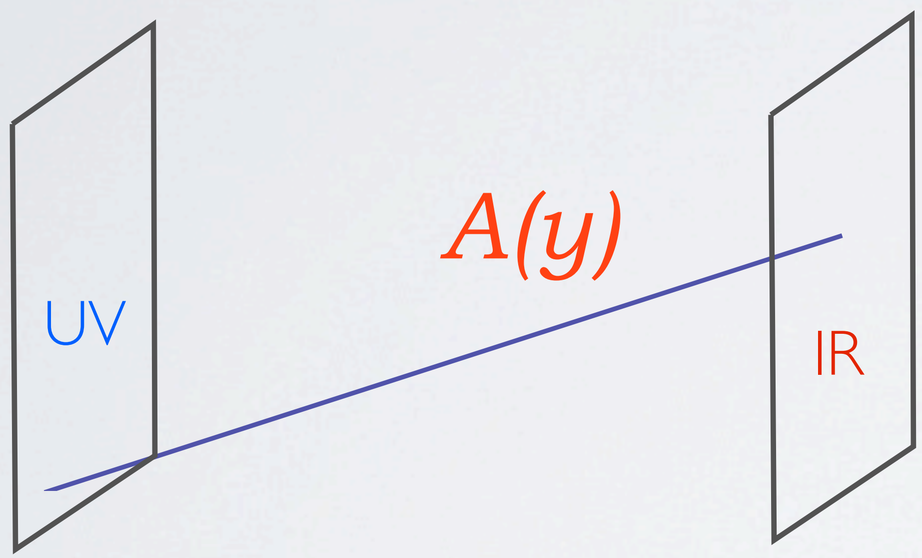

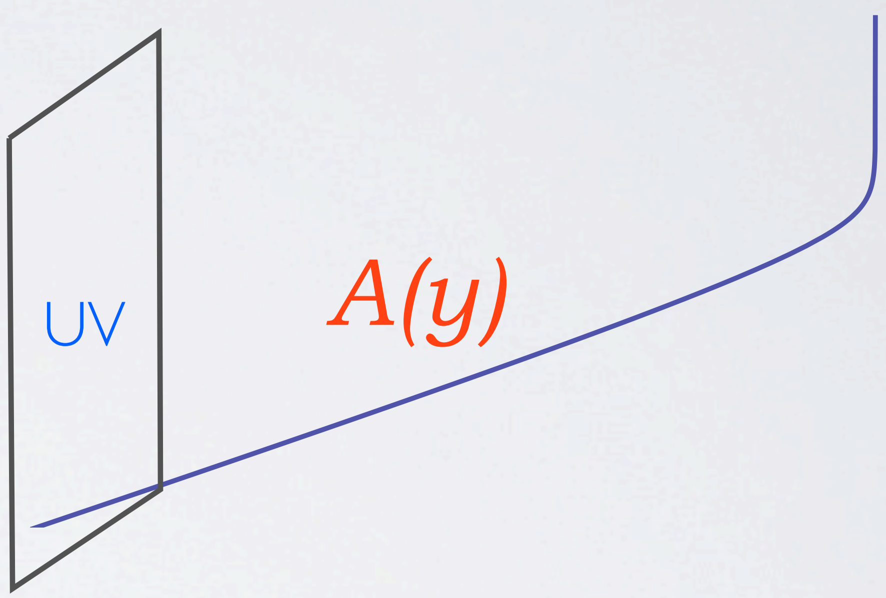

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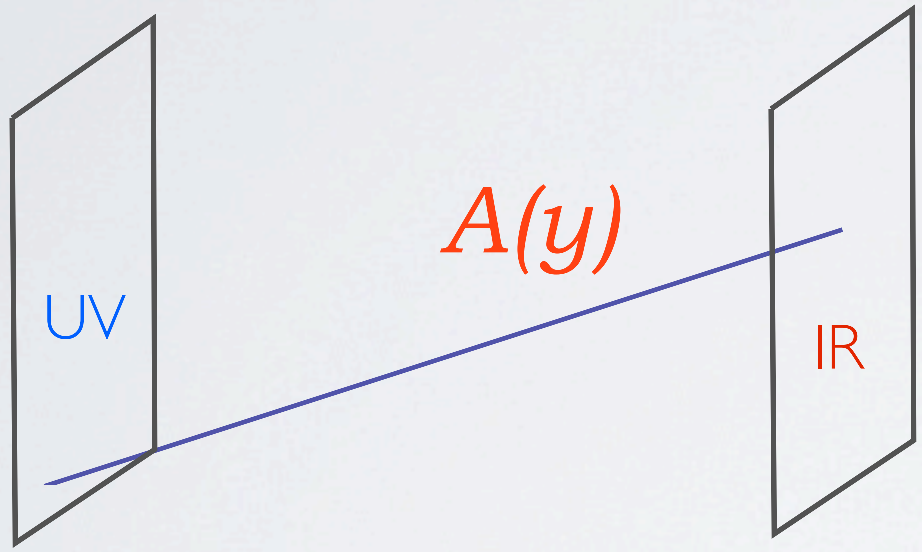

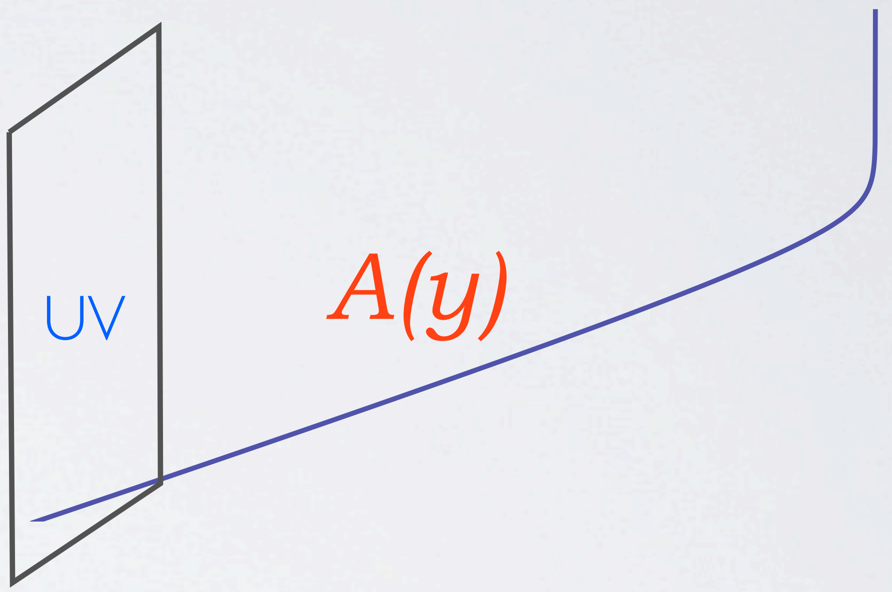

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Singularities are "good" if

$$e^{-A(y)} \to 0 \text{ faster than } (y_s - y)^{\frac{1}{4}}$$

Gubser'02, Forste et al '02
Cabrer, GG & Quirós '09

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- Soft Walls give "resolution" of IR brane

CLASSES OF SOFT WALLS

Soft Walls are rather restricted by demanding:

- A Gap in the spectrum should exist
- The singularity should be a "good"

Then, asymptotics near singularity fall into 2 classes

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Proper Length coordinates	Conformally flat coordinates
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SW2

GG '10

PLAN

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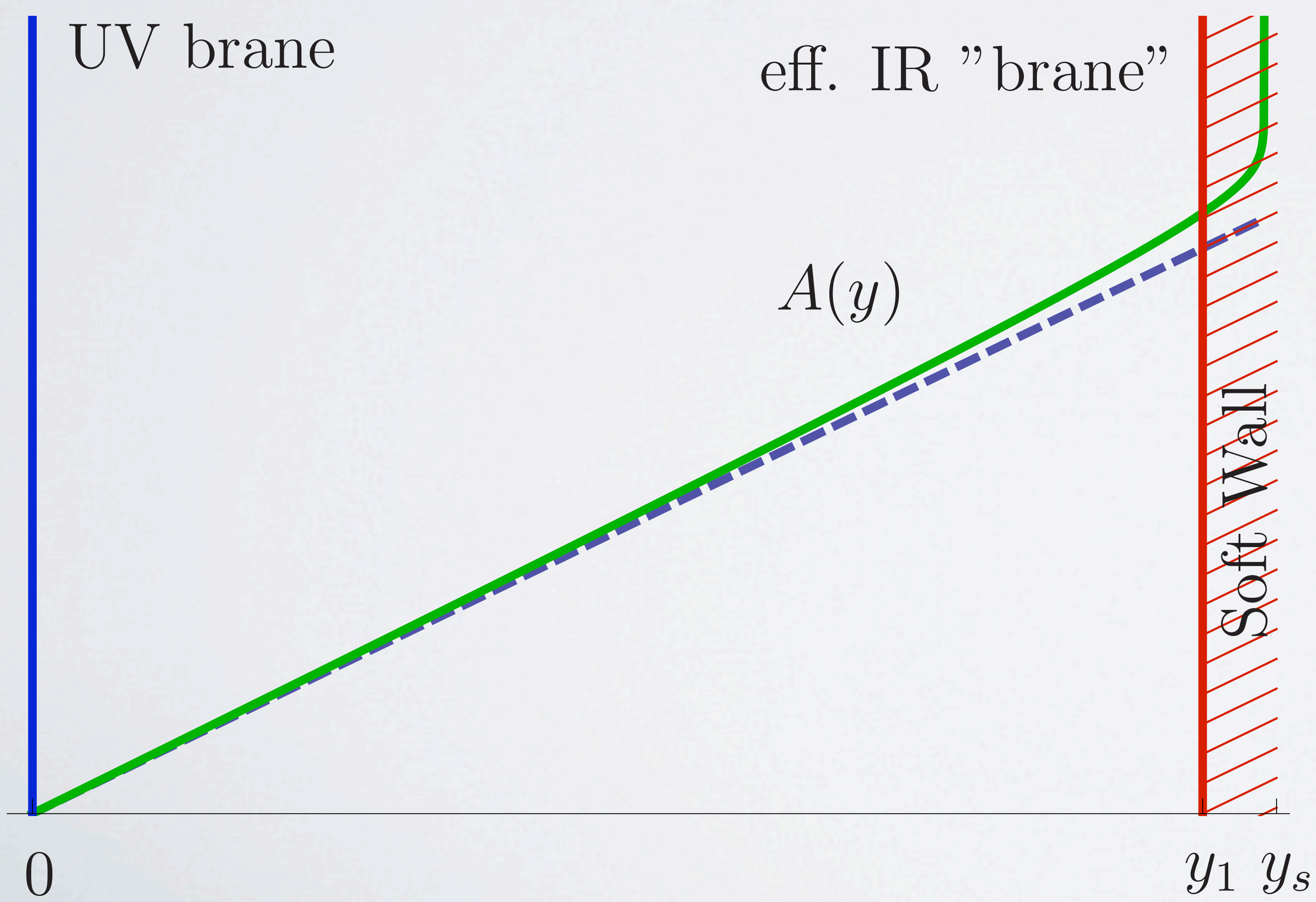
Integrate over the region near the singularity

$$S = \int_0^{y_s} d^5x \, \mathcal{L}_{\rm bulk} \longrightarrow S = \int_0^{y_1} d^5x \, [\mathcal{L}_{\rm bulk} + \mathcal{L}_{\rm SW} \, \delta(y - y_1)]$$

Equivalent description of SW in terms of IR brane



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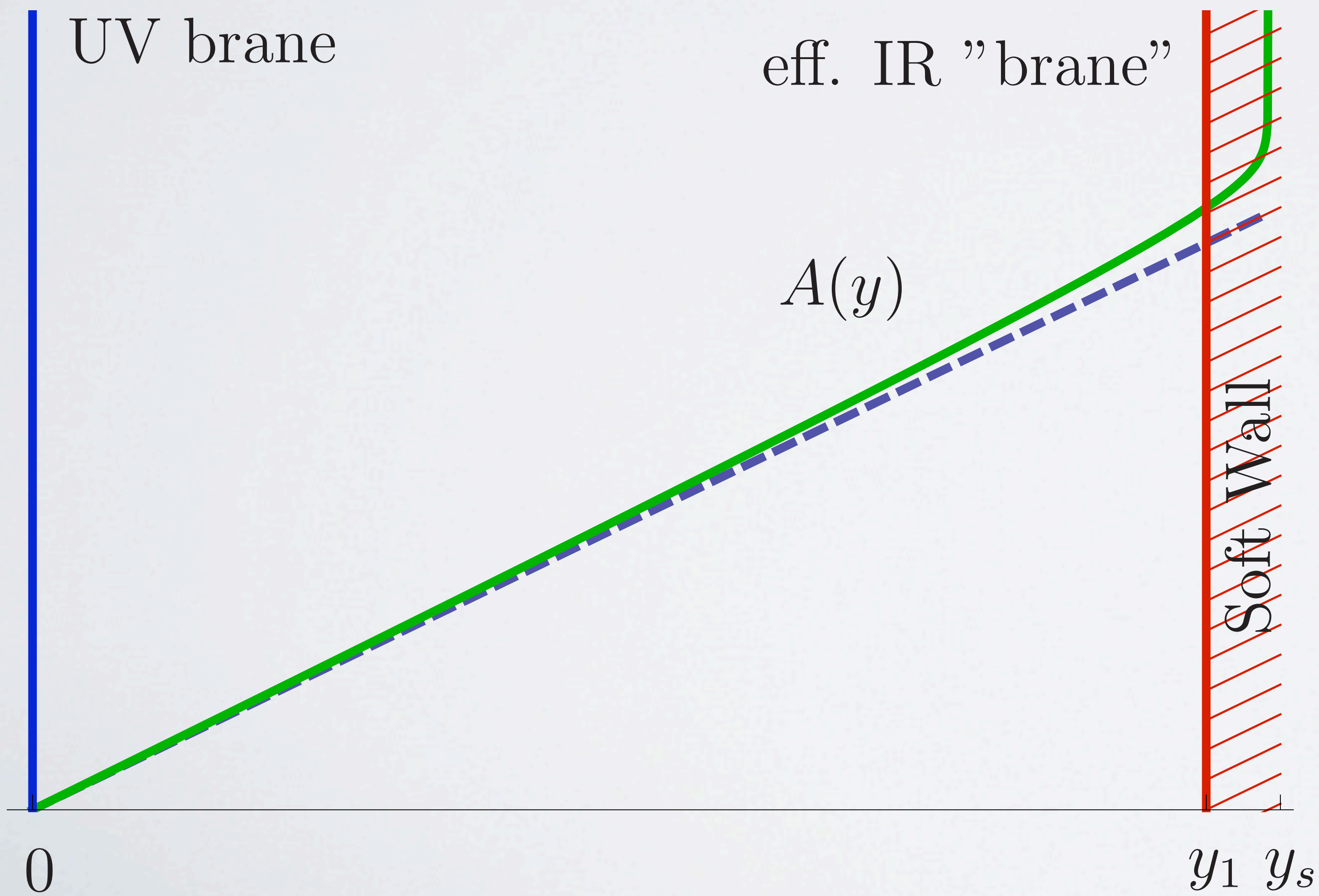

- For y_1 close to y_s close IR Lagrangian "universal"
- Facilitates comparison with standard 2 brane compactif.
- IR Lagrangian makes sense even for p>TeV
- Useful approximation scheme: Approximate "new" bulk by RS metric

GG '10

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Witten '98

$$K(x,x';y_1) = \delta(x-x') \qquad \psi(x,y) = \int d^4x' K(x,x';y)\psi(x',y_1)$$

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$$\mathcal{F}(p) = K'(p,y_1)$$

$$\mathcal{S}_{\rm SW} = \frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\sqrt{-g(y)}\,\mathcal{F}(p)\,\psi(-p,y)\psi(p,y)\,\delta(y-y_1)$$

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SW2 $$\mathcal{F}(p^2) = e^{A(z_1)}\sqrt{-p^2}\tan\left(c_\beta\left[-\frac{p^2}{\rho^2}\right]^{\frac{1}{4\beta}}\right) \qquad \beta = \frac{\sigma-1}{2\sigma}$$

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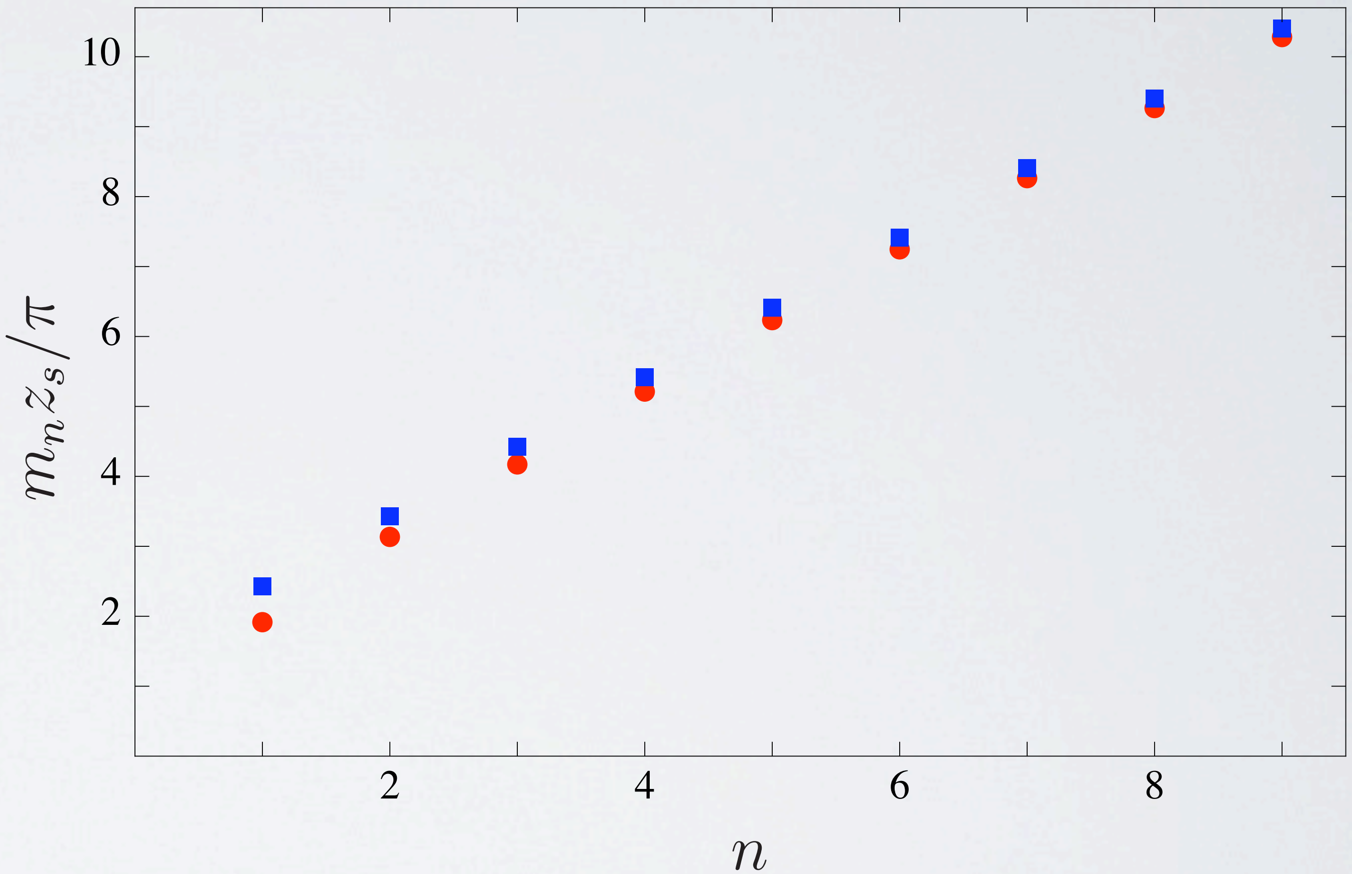

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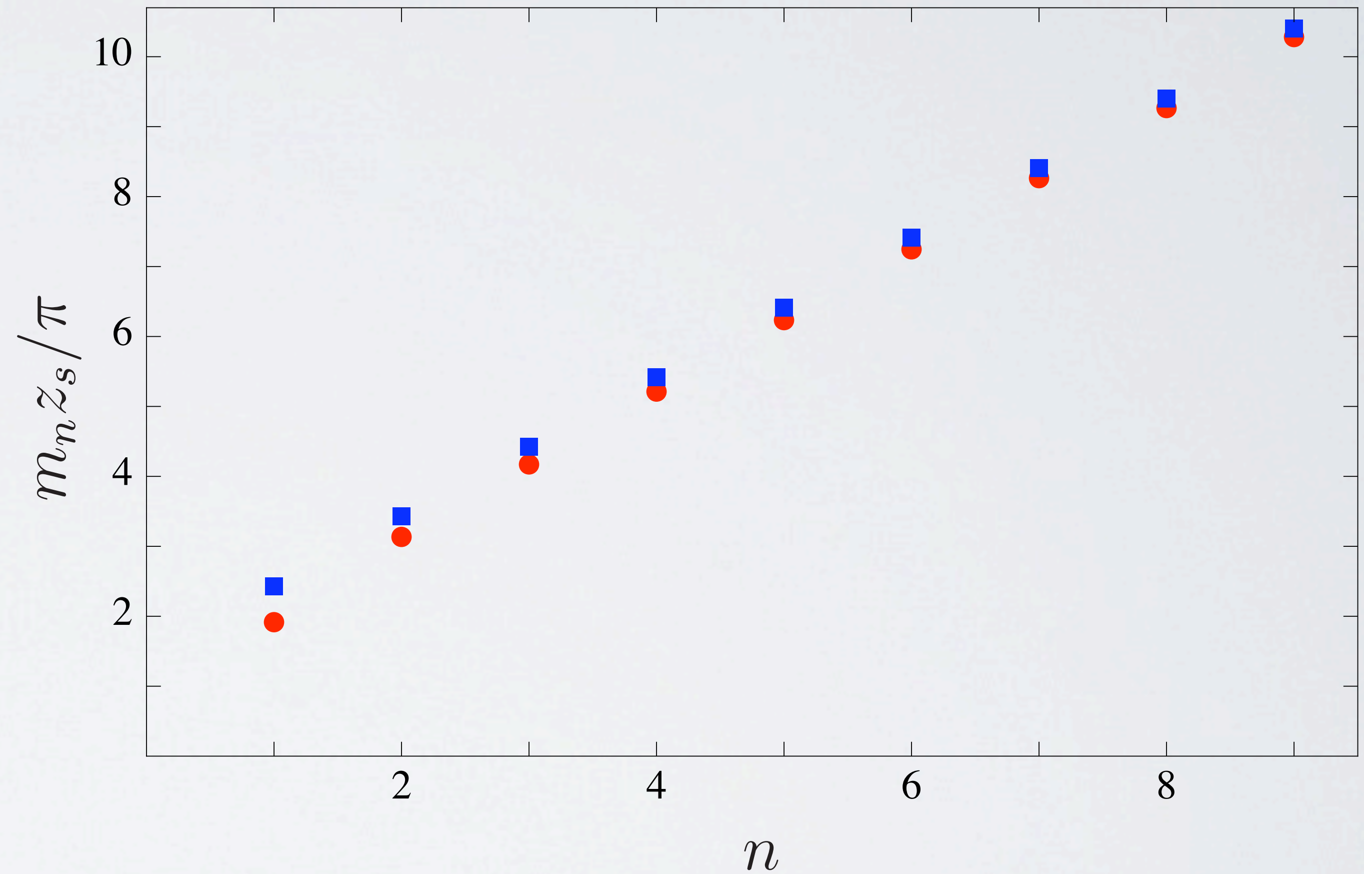

Asymptotic Spectrum (n large)

Approximation $\quad m_n z_s/\pi = n + 0.34$

Numerical $\quad m_n z_s/\pi = (10.40, \ 50.37, \ 100.36, \ 500.35)$

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Compare with exact solution (analytic only for $\beta = \frac{1}{4}$)

Exact spectrum is $m_n = \sqrt{12\, n}\, \rho$

Conicides with previous method since $c_{\frac{1}{4}} = \frac{\pi}{12}$

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- Future work: higher spin, symmetry breaking...