QUANTUM LEPTOGENESIS II

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CP-asymmetry in vacuum decays

Leptogenesis is inherently quantum effect

$$\varepsilon_{CP} = \frac{\Gamma(N \to l\Phi) - \Gamma(N \to \bar{l}\Phi^c)}{\Gamma(N \to l\Phi) + \Gamma(N \to \bar{l}\Phi^c)} = \varepsilon_V + \varepsilon_S$$

- computation can be done in vacuum
- hierarchical spectrum:

$$\varepsilon_{V+S} = \frac{3}{16\pi} \frac{\mathrm{Im}(K_{12})^2}{K_{11}} \frac{M_1}{M_2}, \quad K = \lambda^\dagger \lambda$$

- quasidegenerate spectrum:

$$\varepsilon_S = \frac{1}{8\pi} \frac{\mathrm{Im}(K_{12})^2}{K_{11}} \frac{M^2 \Delta}{M^2 \Delta^2 + (\Gamma_2 - \Gamma_1)^2}, \quad \Delta \equiv \frac{M_2^2 - M_1^2}{M^2}$$

Lepton asymmetry

The decay rate is given by

$$\Gamma = (\lambda^\dagger\lambda)_{11}\frac{2}{\omega}\int_{\mathbf{q},\mathbf{p}}(2\pi)^4\delta^4(k+p-q)f_{l\phi}p\cdot k,$$

where

$$f_{l\phi} = f_l f_\phi - (1-f_l)(1+f_\phi) = 1 - f_l + f_\phi$$

Majorana neutrino density

$$f_N(t) = f_N^{eq}(1-e^{-\Gamma t})$$

$$\begin{aligned} f_{l-\bar{l}}(t;k) &= -\varepsilon_{CP}\frac{1}{k}\int_{\mathbf{q},\mathbf{p}}(2\pi)^4\delta^4(k+p-q)p\cdot k \times \\ & f_{l\phi}f_N^{eq}\frac{1}{\Gamma}\left(1-e^{-\Gamma t}\right) \end{aligned}$$

Lepton asymmetry is generated in thermal equilibrium! => RIS substraction procedure

Conceptual problems with BE

How BE are applied in leptogenesis?

- When computing number density of Majorana neutrino
- When computing the difference of leptons and antileptons number densities

$$\frac{\partial f_l}{\partial t} = -\frac{1}{2k}\int_{\mathbf{q},\mathbf{p}}(2\pi)^4\delta^4(k+q-p)[|M_{l\phi\to N}|^2 f_l(k)f_\phi(q)\times (1-f_N(t;\omega)) - |M_{N\to l\phi}|^2 f_N(t;\omega)(1-f_l(k))(1+f_\phi(q))].$$

- One uses vacuum relation between the amplitudes

$$|M_{l\phi\to N}|^2 = |M_0|^2(1-\varepsilon_{CP}),\ \ |M_{\bar{l}\phi^c\to N}|^2 = |M_0|^2(1+\varepsilon_{CP})$$
$$|M_{N\to l\phi}|^2 = |M_0|^2(1+\varepsilon_{CP}),\ \ |M_{N\to\bar{l}\phi^c}|^2 = |M_0|^2(1-\varepsilon_{CP}).$$

Refining with QBE

QBE are simplified version of KBE's

e.g. Buchmüller, Fredenhagen, Di Bari, Riotto, Simone, Raffelt, Blanchet etc

- Consider, e.g. equation on fermion spectral function

$$(i\gamma_0\partial_{t_1} - \mathbf{q}\gamma - M)S^-_{\mathbf{q}}(t_1, t_2) + \int_{t_1}^{t_2} \Sigma^-_{\mathbf{q}}(t_1 - t')S^-_{\mathbf{q}}(t', t_2) = 0$$

- Assume that system is close to thermal equilibrium
- Use Wigner coordinates $t = \frac{t_1+t_2}{2}$, $y = t_1 - t_2$
- Do Wigner transform and the derivative expansion

$$\int_{-\infty}^{\infty} e^{i\omega y} dy \int_{t_1}^{t_2} \Sigma^-_{\mathbf{q}}(t_1 - t')S^-_{\mathbf{q}}(t', t_2) \rightarrow$$

$$\Sigma^R_{\mathbf{q}}(t;\omega)S^A_{\mathbf{q}}(t;\omega) + \frac{i}{2}\{\Sigma^R_{\mathbf{q}}(t;\omega), S^A_{\mathbf{q}}(t;\omega)\}_{PB} + \ldots$$

Refining with QBE

Dropping Poisson brackets

- All *memory* is lost
- Two coupled 1st order ODE's $\sim$ 2nd order ODE
- Real and imaginary parts of 2nd QBE $\Rightarrow$
  - Equation on propagating frequency, i.e. dispersion relation
  - Usual BE for number density!

With Poisson brackets

- Short *memory* effects are kept
- Tiny correction to dispersion relation
- Tiny correction to the width

Possible issues

More corrections=>more precision?

- Secular terms

$$y'' + y + \varepsilon y''' = 0,$$

Solving perturbatively:

$$y_1'' + y_1 = -\varepsilon \sin(t),$$

Perturbation is in resonance with zero order solution

$$y_1 \sim \varepsilon t \sin(t).$$

Late time behaviour?

Full nonequilibrium treatment is still necessary.

Solutions for Fermions: small width

The width

$$\Sigma_{\mathbf{q}}(\omega) = a_{\mathbf{q}}(\omega)\not{q} + b_{\mathbf{q}}(\omega)\not{u}$$

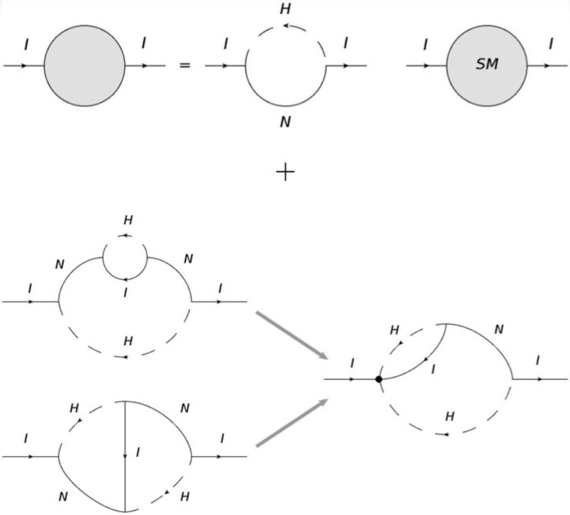
$$\Gamma \equiv \Gamma_{\mathbf{q}}(\omega) = 2\left(\mathrm{Im}[b(\omega_{\mathbf{q}})] + \frac{\mathrm{Im}[a(\omega_{\mathbf{q}})]M^2}{\omega_{\mathbf{q}}}\right)$$

Small width solutions

$$S^-(y) = \left(i\gamma_0 \cos[\omega_{\mathbf{q}} y] + \frac{M - \mathbf{q}\gamma}{\omega_{\mathbf{q}}}\sin[\omega_{\mathbf{q}} y]\right) e^{-\frac{\Gamma|y|}{2}}$$

$$S^+_{\mathbf{q}}(t,y) = -\left(i\gamma_0 \cos[\omega_{\mathbf{q}}(y)] - \frac{M - \mathbf{q}\gamma}{\omega_{\mathbf{q}}}\sin[\omega_{\mathbf{q}}(y)]\right) \times \left[\frac{\tanh\left(\frac{\beta\omega}{2}\right)}{2} e^{-\frac{\Gamma|y|}{2}} + f_N^{eq}(\omega) e^{-\Gamma t}\right]$$

Lepton Self-Energy



The Source of the Asymmetry

lepton self energy splits into a pure SM part and a part involving N :

$$\Pi^{\pm}_{\mathbf{k}ij}(t_1, t_2) = \Sigma^{\pm,\mathrm{SM}}_{\mathbf{k}ij}(t_1 - t_2) + \delta\Sigma^{\pm}_{\mathbf{k}ij}(t_1, t_2)$$

S^{+} can be split into a solution in the absence of N and a correction:

$$S^{\pm}_{\mathbf{k}ij}(t_1, t_2) = S^{\pm,\mathrm{SM}}_{\mathbf{k}ij}(t_1 - t_2) + \delta S^{\pm}_{\mathbf{k}ij}(t_1, t_2)$$

Only the correction can generate a non-zero leptonic charge!

Kadanoff-Baym Equation for δS^+

To leading order

$$
\begin{aligned}
(i\gamma_0\partial_{t_1} - \mathbf{k}\gamma)\,\delta S^+_{\mathbf{k}ij}(t_1,t_2) \quad &- \quad \int_0^{t_1} dt' \Sigma^{-SM}_{\mathbf{k}ij}(t_1 - t')\delta S^+_{\mathbf{k}ij}(t',t_2) \\
&= \quad \zeta^1_{\mathbf{k}ij}(t_1,t_2) + \zeta^2_{\mathbf{k}ij}(t_1,t_2) + \zeta^3_{\mathbf{k}ij}(t_1,t_2)
\end{aligned}
$$

Kadanoff-Baym Equation for δS^+

To leading order

$$\begin{aligned}(i\gamma_0\partial_{t_1} - \mathbf{k}\gamma)\,\delta S^+_{\mathbf{k}ij}(t_1,t_2) &- \int_0^{t_1} dt' \Sigma^{-SM}_{\mathbf{k}ij}(t_1-t')\delta S^+_{\mathbf{k}ij}(t',t_2) \\ &= \zeta^1_{\mathbf{k}ij}(t_1,t_2) + \zeta^2_{\mathbf{k}ij}(t_1,t_2) + \zeta^3_{\mathbf{k}ij}(t_1,t_2)\end{aligned}$$

The l.h.s. of the above equation is a homogeneous equation for $\delta S^+_{\mathbf{k}ij}$, and the sources are given by

$$\zeta^1_{\mathbf{k}ij}(t_1,t_2) = \int_0^{t_1} dt' \delta\Sigma^-_{\mathbf{k}ij}(t_1,t') S^{+SM}_{\mathbf{k}ij}(t'-t_2),$$

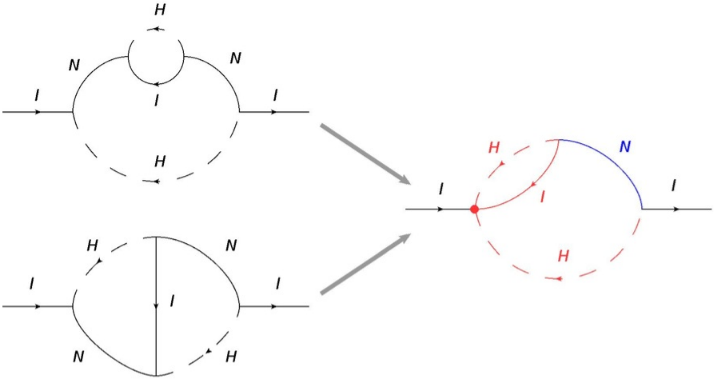
$$\zeta^2_{\mathbf{k}ij}(t_1,t_2) = -\int_0^{t_2} dt' \delta\Sigma^+_{\mathbf{k}ij}(t_1,t') S^{-SM}_{\mathbf{k}ij}(t'-t_2),$$

$$\zeta^3_{\mathbf{k}ij}(t_1,t_2) = -\int_0^{t_2} dt' \Sigma^{+SM}_{\mathbf{k}ij}(t_1-t')\delta S^-_{\mathbf{k}ij}(t',t_2)$$

Solution for δS^{+}

$$
\begin{aligned}
\delta S^{+}_{\mathbf{k}ij}(t_1, t_2) = \\
& \int_0^{t_1} dt' \int_0^{t_2} dt'' \, S^{-,F}_{\mathbf{k}ij}(t_1 - t') \delta\Sigma^{+}_{\mathbf{k}ij}(t', t'') S^{-,F}_{\mathbf{k}ij}(t'' - t_2) \\
- & \int_0^{t_1} dt' \int_0^{t'} dt'' \, S^{-,F}_{\mathbf{k}ij}(t_1 - t') \delta\Sigma^{-}_{\mathbf{k}ij}(t', t'') S^{+,F}_{\mathbf{k}ij}(t'' - t_2) \\
- & \int_0^{t_2} dt'' \int_0^{t''} dt' \, S^{+,F}_{\mathbf{k}ij}(t_1 - t'') \delta\Sigma^{+}_{\mathbf{k}ij}(t'', t') S^{-,F}_{\mathbf{k}ij}(t' - t_2)
\end{aligned}
$$

CP-violating Part of the Lepton Self Energy



$$L_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\, 8\pi \int_{\mathbf{q},\mathbf{q}'} \frac{k\cdot k'}{|\mathbf{k}||\mathbf{k}'|\omega} f_{l\phi}(k,q) f_{l\phi}(k',q') f_N^{eq}(\omega)\times$$

$$\times\, \frac{\frac{1}{2}\Gamma}{((\omega-k-q)^2+\frac{\Gamma^2}{4})((\omega-|\mathbf{k}'|-|\mathbf{q}'|)^2+\frac{\Gamma^2}{4})} \times \Big\{$$

$$\left(e^{-\frac{\Gamma t}{2}} - \cos[(|\mathbf{k}|+|\mathbf{q}|-\omega)t]\right)\left(e^{-\frac{\Gamma t}{2}} - \cos[(|\mathbf{k}'|+|\mathbf{q}'|-\omega)t]\right)$$

$$-\sin[(\omega-|\mathbf{k}|-|\mathbf{q}|)t]\,\sin[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t]\Big\}\,,$$

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$$-\sin[(\omega-|\mathbf{k}|-|\mathbf{q}|)t]\sin[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t]\Big\},$$

with $\mathbf{p} = \mathbf{q}+\mathbf{k} = \mathbf{q}'+\mathbf{k}'$

$$\int_{\mathbf{p}} \cdots = \int \frac{d^3p}{(2\pi)^3 2\omega_{\mathbf{p}}} \cdots$$

$$f_{l\phi}(k,q) = f_l(k) f_\phi(q) + (1-f_l(k))(1+f_\phi(q))$$
$$= 1 - f_l(k) + f_\phi(q)$$

Comparison to Boltzmann Result

$$L_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\,\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{q}'}\frac{k\cdot k'}{|\mathbf{k}'|\omega} f_{l\phi}(k,q) f_N^{eq}(\omega) f_{l\phi}(k',q')$$

$$\times \frac{\frac{1}{4}\Gamma}{((\omega-k-q)^2+\frac{\Gamma^2}{4})((\omega-|\mathbf{k}'|-|\mathbf{q}'|)^2+\frac{\Gamma^2}{4})}\Big\{$$

$$\times\left(e^{-\frac{\Gamma t}{2}}-\cos[(|\mathbf{k}|+|\mathbf{q}|-\omega)t]\right)\left(e^{-\frac{\Gamma t}{2}}-\cos[(|\mathbf{k}'|+|\mathbf{q}'|-\omega)t]\right)$$

$$-\sin[(\omega-|\mathbf{k}|-|\mathbf{q}|)t]\sin[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t]\Big\}\,,$$

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'} k\cdot k' f_{l\phi}(k,q) f_N^{eq}(\omega)$$

$$\times\frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p)$$

$$\times\left(1-e^{-\Gamma t}\right)$$

On-Shell Approximation

$$L^{os}_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\frac{16\pi}{k}\int_{\mathbf{q},\mathbf{q}',\mathbf{p},\mathbf{k}'} k\cdot k' f_{l\phi}(k,q) f_N^{eq}(\omega) f_{l\phi}(k',q') \times \frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p) \times \left(1-e^{-\frac{\Gamma t}{2}}\right)^2$$

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On-Shell Approximation: right way

- when t is arbitrarily large oscillations are important:

$$\int \frac{F(X)}{X^2 + \frac{\Gamma^2}{4}} \to \frac{2}{\Gamma} F(0), \quad \text{vs.} \quad \int \frac{F(X)\cos(tX)}{X^2 + \frac{\Gamma^2}{4}} \to \frac{2}{\Gamma} e^{-\frac{\Gamma t}{2}} F(0)$$

- Phase space for internal loop allows on-shell prescription
- Using correct on-shell prescription $\Rightarrow$ yield zero!

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- An accident: if one does not assume hierarchy $M_2 \gg M_1$ the result is not zero
- Including expansion of the Universe will break this cancellation as well
- Including widths for lepton and Higgs will also break this cancellation

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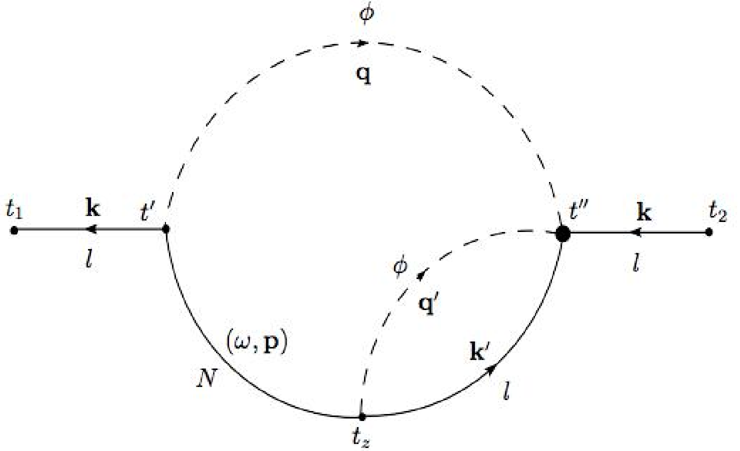
- But very good example! Surprises are very likely !

CP-violating Part of the Lepton Self Energy



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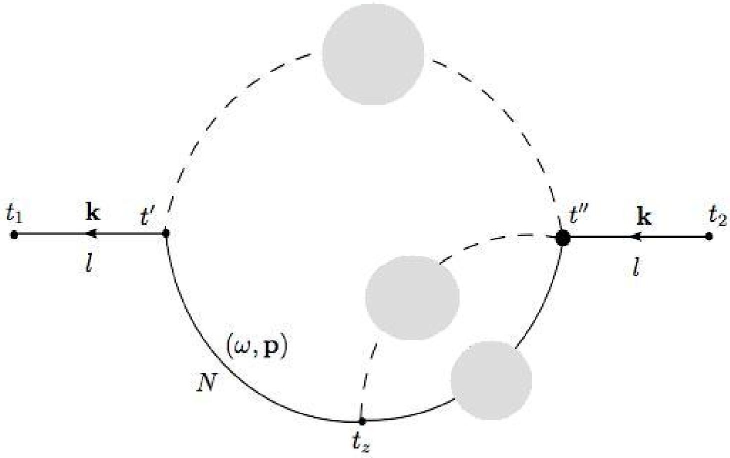

Inclusion of SM widths

$$\tilde{L}_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\,\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{q}'}\frac{k\cdot k'}{|\mathbf{k}'|\omega} f_{l\phi}(k,q) f_N^{eq}(\omega) f_{l\phi}(k',q')$$

$$\times\,\frac{1}{\Gamma}\,\frac{\frac{1}{4}\Gamma_{l\phi}\Gamma_{\phi}}{((\omega-k-q)^2+\frac{1}{4}\Gamma_{l\phi}^2)((\omega-k'-q')^2+\frac{1}{4}\Gamma_{\phi}^2)}$$

$$\left(1-e^{-\Gamma t}\right)$$

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'} k\cdot k' f_{l\phi}(k,q) f_N^{eq}(\omega)$$

$$\times\,\frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p)$$

$$\times\left(1-e^{-\Gamma t}\right)$$

This is not yet a consistent treatment of gauge interactions!!!

Conclusions

Framework

- Static Universe/constant temperature bath
- SM widths and masses of leptons and Higgs are neglected/equlibrium propagators used
- Remarkable resemblance to BE and at the same time drastic deviation from the BE result is observed.

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Does leptogenesis actually work?

- The consistent inclusion of all SM corrections, especially the ones for on-shell lepton legs!
- Expansion of the Universe will make things more local(but not exactly!) $\Rightarrow$ closer to BE result
- Clarifying these issues is urgent!