TOWARDS A FULL QUANTUM THEORY OF LEPTOGENESIS

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Thermal Leptogenesis

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_R \partial\!\!\!/ \nu_R - \bar{l}_L \tilde{\Phi} \lambda \nu_R - \frac{1}{2} \bar{\nu}_R^c M \nu_R + h.c.$$

- see-saw mechanism explains small neutrino masses
- complex phases violate CP
- singlet fermions are out of equilibrium

$\Rightarrow$ CP violating decay of $N \approx \nu_R + \nu_R^c$ creates lepton asymmetry

- sphaleron processes can transfer asymmetry to baryonic sector

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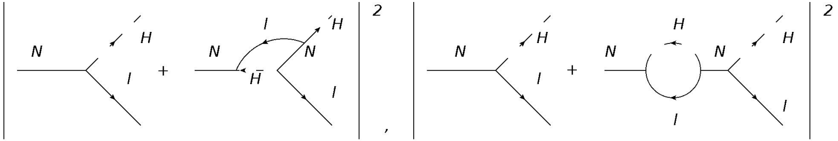
- sphaleron processes can transfer asymmetry to baryonic sector

⇒ baryon asymmetry can be generated

Quantum Genesis

- usually studied in terms of semiclassical Boltzmann equations (classical particle numbers , collision terms from vacuum S-matrix)
- creation (of matter) from interference is a quantum effect



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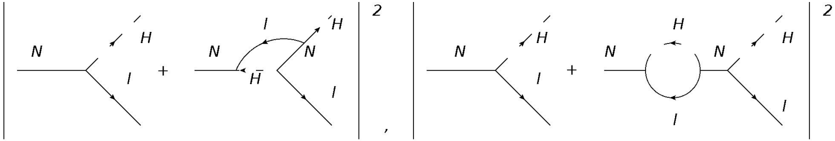


conceptual problems in the semi-classical description

- non-Markovian / memory effects
- no asymptotic states / particle number in omnipresent plasma
- off-shell effects
- flavour effects: coherent oscillations, quantum zeno effect...
- modified spectrum (quasiparticles, collective excitations...)

Methods

- Boltzmann equations (BE)
- quantum Boltzmann equations (QBE)
- (effective) kinetic equations for reduced density matrices
- Kadanoff-Baym equations (KBE)

Is a Quantum Treatment possible?

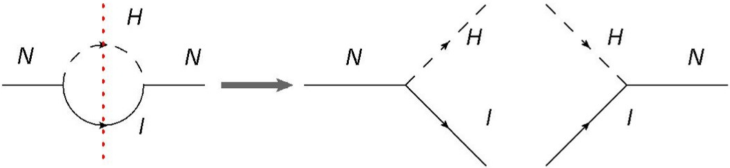
- spacial homogeneity
- weak coupling $\Rightarrow$ perturbative
- background plasma is in equilibrium
- backreaction can be neglected

'Weak coupling to a thermal bath'

Boltzmann vs Kadanoff-Baym Equations

- initial value problem for density matrix $\rho(t)\ldots$
- $\ldots$or for correlation functions $\langle\ldots\rangle = \mathrm{tr}(\rho\ldots)$
- KBE contain full quantum mechanics

$$\text{particle numbers} \Leftrightarrow \text{correlation functions}$$
$$\text{collision term} \Leftrightarrow \text{self energies}$$



Statistical and Spectral Propagators

$$\begin{aligned}
\Delta^+(x_1, x_2) &= \frac{1}{2}\langle\{\Phi(x_1), \Phi(x_2)\}\rangle_c \\
\Delta^-(x_1, x_2) &= i\langle[\Phi(x_1), \Phi(x_2)]\rangle_c
\end{aligned}$$

$$\begin{aligned}
S^+_{\alpha\beta}(x_1, x_2) &= \frac{1}{2}\langle[\Psi(x_1)_\alpha, \bar{\Psi}_\beta(x_2)]\rangle_c \\
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Equilibrium: KMS-relations, e.g. $\Delta^+_{\mathbf{q}}(\omega) = -i\left(\frac{1}{2} + f^{eq}_\phi(\omega)\right)\Delta^-_{\mathbf{q}}(\omega)$

$\Rightarrow$ equilibrium propagators not independent, Bose/Fermi statistics

Kadanoff Baym Equations

$$
\begin{aligned}
C(i\not\partial_1 - m)G^-(x_1,x_2) &= -\int d^3\mathbf{x}' \int_{t_2}^{t_1} dt' \Sigma^-(x_1,x') G^-(x',x_2) \\
C(i\not\partial_1 - m)G^+(x_1,x_2) &= \int d^3\mathbf{x}' \int_{t_i}^{t_1} dt' \Sigma^-(x_1,x') G^+(x',x_2) \\
&\quad + \int d^3\mathbf{x}' \int_{t_i}^{t_2} dt' \Sigma^+(x_1,x') G^-(x',x_2)
\end{aligned}
$$

Weak Coupling to a thermal Bath

- spectral propagators Δ^-, S^-, G^- are time translation invariant
- KBE are equivalent to a stochastic Langevin equation
- KBE can be solved analytically up to a *memory integral*

Spectral Propagator

$$G^{-}_{\mathbf{q}}(t_1 - t_2) \;=\; i\int \frac{d\omega}{2\pi} e^{-i\omega(t_1-t_2)}\rho_{\mathbf{q}}(\omega)$$

$$\rho_{\mathbf{q}}(\omega) \;=\; \left(\frac{i}{\not{q} - m - C\Sigma^{R}_{\mathbf{q}}(\omega) + i\not{u}\epsilon} - \frac{i}{\not{q} - m - C\Sigma^{A}_{\mathbf{q}}(\omega) - i\not{u}\epsilon} \right) C^{-1}$$

- (quasi)poles of ρ give spectrum of resonances
- determined by retarded self energy Σ^R
- $\Sigma^R = \Sigma^R|_{T=0} + \delta\Sigma^R(T)$ has a vacuum part and a correction due to the medium
- rich phenomenology (flavour structure, collective excitations...) is encoded in Σ^R

Statistical Propagator

$$
\begin{aligned}
G^{+}_{\mathbf{q}}(t_1, t_2) \;=\; & -G^{-}_{\mathbf{q}}(t_1) C\gamma^0 G^{+}_{\mathbf{q}}(0,0)\gamma^0 C^{-1} G^{-}_{\mathbf{q}}(-t_2) \\
& + \int_0^{t_1} dt' G^{-}_{\mathbf{q}}(t_1 - t') \int_0^{t_2} dt'' C^{-1} \Sigma^{+}_{\mathbf{q}}(t' - t'') G^{-}_{\mathbf{q}}(t'' - t_2)
\end{aligned}
$$

- no restriction on the size of initial deviation from equilibrium!
- no a priori parameterisation of the propagators by distribution functions!

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$$

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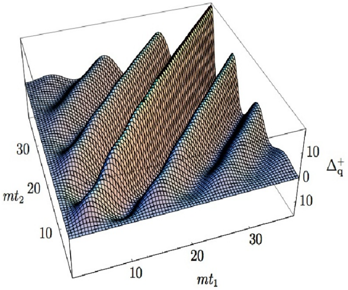
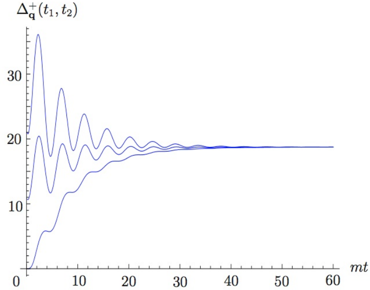
In the narrow width limit for vanishing initial particle number:

$$
\begin{aligned}
G^{+}_{\mathbf{q}}(t; y) = &- \left(i\gamma_0 \sin(\omega_{\mathbf{q}} y) - \frac{M - \mathbf{q}\gamma}{\omega_{\mathbf{q}}} \cos(\omega_{\mathbf{q}} y) \right) \\
&\times \left(\frac{1}{2} \tanh(\beta\omega_{\mathbf{q}}/2) e^{-\Gamma_{\mathbf{q}}|y|/2} + f_N^{eq}(\omega_{\mathbf{q}}) e^{-\Gamma_q t} \right) C^{-1}
\end{aligned}
$$

with $y = t_1 - t_2$, $t = (t_1+t_2)/2$ and $\Gamma \propto \mathrm{disc}\Sigma$

The Statistical Propagator





- depends on two time arguments
- equilibrates independent of initial conditions after characteristic time $\tau \sim {}^1\!/_\Gamma$
- oscillates with plasma frequency

Lepton Asymmetry

- description in terms of correlation functions without reference to number densities
- observables can be computed from correlation functions
- knowledge of the propagators allows to compute Feynman diagrams
- additional complication due to explicit time dependence

Lepton Number Matrix

$$L_{\mathbf{k}ij}(t_1, t_2) = -\mathrm{tr}[\gamma^0 S^+_{\mathbf{k}ij}(t_1, t_2)].$$

- $L_{\mathbf{k}ii}(t, t)$ gives asymmetry in flavour i

Since leptogenesis comes from a LO-NLO interference, we need the dressed NLO nonequilibrium lepton propagator!

Conclusion

- leptogenesis is a nonequilibrium quantum process
- semiclassical methods suffer from severe conceptional problems
- KBE allow full quantum treatment
- we solved KBE for scalars and fermions weakly coupled to a thermal bath
- solutions can be used for full quantum treatment of leptogenesis and other freezeout processes if T changes adiabatically
- we computed an expression for the asymmetry for the type I seesaw model with hierarchical masses
  $\Rightarrow$ see subsequent talk by Alexey Anisimov