A Simple UV-Completion of QED in 5D

Marco Serone, SISSA, Trieste

In collaboration with Roberto Iengo,
based on 1003.4430 [hep-th]

Planck 2010, CERN, June 3 2010

Constructing UV completions of genuine (i.e. not deconstructed) higher dimensional field theories is not an easy task, even in the limit in which gravity is neglected

Explicit string theory constructions very hard (and gravity cannot be decoupled). As a matter of fact, no simple UV completion known so far

Less motivated, but much simpler, UV completions can be obtained by considering **Lifshitz-like** Quantum Field Theories where the explicit breaking of Lorentz invariance allow for the construction of renormalizable theories

Such theories contain higher derivative quadratic operators leading to a better UV behavior with respect to standard theories

General properties of Lifshitz-like theories

Anisotropic scale invariance

$$t = \lambda^z t' \,, \qquad x^i = \lambda x^{i\prime} \,, \qquad \phi(x^i, t) = \lambda^{\frac{z-d}{2}} \phi'(x^{i\prime}, t')$$

It is useful to introduce weighted scaling dimensions so that

$$[t]_w = -z \,, \qquad [x^i]_w = -1 \,, \qquad [\phi]_w = \frac{d-z}{2}$$

Renormalizability by power-counting still holds, substituting standard scaling dimensions with weighted scaling dimensions

[Anselmi&Helat, 0707.2480]

Consider for instance QED in d+1 space-time dimensions

$$[\psi]_w = \frac{d}{2} \,, \qquad [A_0]_w = \frac{d+z-2}{2} \,, \qquad [A_m]_w = \frac{d-z}{2} \,.$$

$$[g]_w = \frac{2-d+z}{2}$$

The gauge coupling is effectively dimensionless in

$$d = z + 2$$

Take $z = 2$. Most general Lagrangian with (weighted) marginal and relevant couplings and invariant under a certain $\mathbf{Z}_2$ symmetry is

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}F_{m0}^2 - \frac{c_\gamma^2}{4}F_{mn}^2 - \frac{a_\gamma^2}{4\Lambda^2}(\partial_m F_{np})^2 + \\ &\quad \bar{\psi}(i\not{D}_0 - ic_\psi \not{D} - M)\psi - \frac{a_\psi}{\Lambda}|D_m\psi|^2 - \frac{i\lambda}{\Lambda^{3/2}}F_{mn}\bar{\psi}\gamma^{mn}\psi\end{aligned}$$

At low energies it reduces to usual QED in 5D, provided $c_\psi = c_\gamma$.

Let us study UV behavior of the theory, setting all relevant couplings to zero: $c_\psi = c_\gamma = M = 0$

Work in the Coulomb gauge $\partial_m A_m = 0$

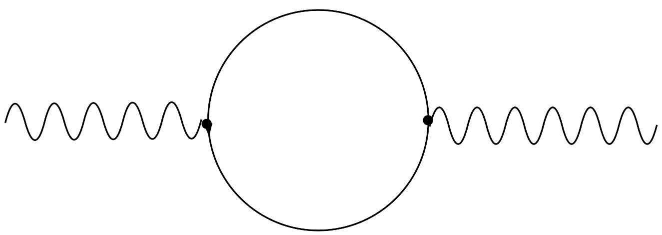
Marginal couplings are g, λ, a_ψ and a_λ

Fermion propagator:

$$-iG^0_\psi(\omega, p) = \frac{P_+}{\omega - a_\psi p^2 + i\epsilon} + \frac{P_-}{-\omega - a_\psi p^2 + i\epsilon}$$

$$P_\pm = \frac{1 \pm \gamma^0}{2}$$

All fermions loops vanish in this limit!

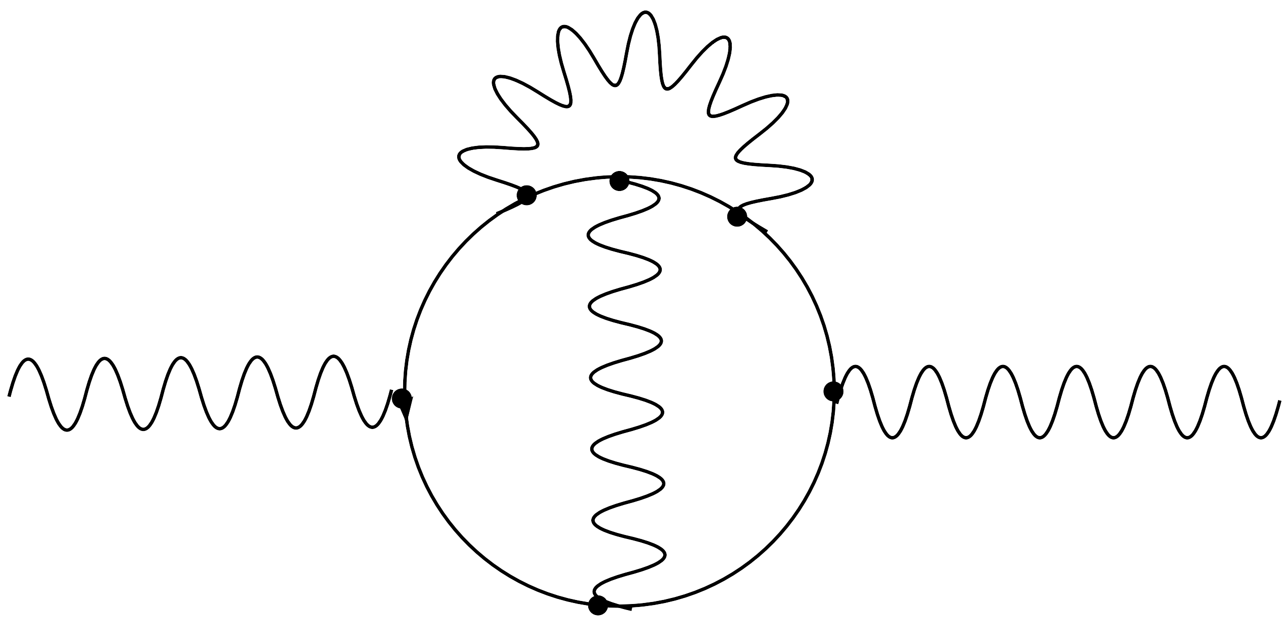


$$\sim \int d\omega d^d q \frac{P_+}{(\omega + \omega_p - a_\psi (q+p)^2 + i\epsilon)(\omega - a_\psi p^2 + i\epsilon)}$$

$$+(P_+ \to P_-) = 0$$

(residue theorem, close the contour in the upper half-plane)

Actually vacuum polarization vanish at all orders in perturbation theory!



$$= 0$$

$$\Rightarrow \quad \beta_g = \beta_{a_\gamma} = \gamma_{A_0} = \gamma_{A_m} = 0$$

Remaining couplings a_ψ and λ are renormalized

$$\beta_{a_\psi} = \frac{3a_\psi^2}{16\pi^2}\left(\frac{g^2}{a_\gamma(a_\gamma + a_\psi)} + \frac{4\lambda^2}{a_\gamma(a_\gamma + a_\psi)^3}\right)$$

$$\beta_\lambda = \frac{\lambda^3}{\pi^2 a_\gamma(a_\gamma + a_\psi)^2}$$

In the deep UV the effective couplings of the theory are

$$\alpha_0 \equiv \frac{g^2}{a_\gamma}, \qquad \beta_0 \equiv \frac{\lambda^2}{a_\gamma a_\psi^2}$$

Along the RG flow

$$\alpha_0(t) = \alpha_0(0)$$

$$\beta_0(t) = e^{-\frac{3g^2 t}{8\pi^2}}\beta_0(0)\left[1 + \frac{4\beta_0(0)}{3g^2}(e^{-\frac{3g^2 t}{8\pi^2}} - 1)\right]^{-1}$$

$\boxed{\beta_0 \text{ is UV free and the theory is UV safe}}$

IR behavior of the theory : $E \ll \Lambda$

Switch on relevant couplings c_ψ and c_γ

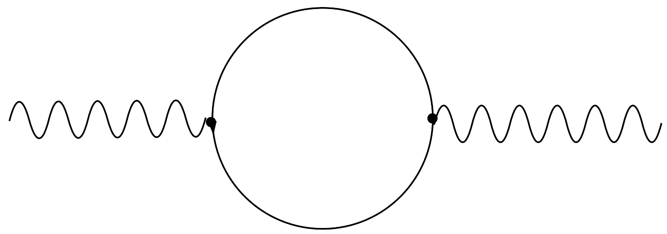
Focus on energy behaviour of the gauge coupling constant

$$g \to g_5, \qquad g_5^2(E) = 2\pi R g_4^2(E)$$

Recall that no divergences appear and the energy dependence is completely finite

It should reproduce the IR dominated terms in the usual RG evolution of the coupling in QED

- logarithmic behavior for $E < 1/R$
- linear behavior for $E > 1/R$
- "Lifshitz" (UV) behavior for $E > \Lambda \gg 1/R$



$$= \Pi_{00} = i c_\psi^2 p^2 f(\omega, p^2)$$

f is responsible for a finite renormalization of the photon kinetic term.

$$A \to \frac{A}{\sqrt{1+f}} \qquad \Rightarrow g_5^2(\omega, p^2) = \frac{g_5^2(\omega_0, p_0^2)}{1 + f(\omega^2, p^2) - f(\omega_0^2, p_0^2)}$$

At low energies, with $c_\psi = c_\gamma$, theory is Lorentz-invariant, so enough to study flow with energy, $g_5 = g_5(E)$

$$f_4(E) \equiv g_4^{-2} f(iE, 0) = \frac{1}{3\pi^2} \sum_{n=-\infty}^{\infty} \int_0^\infty \frac{s^2(3\tilde{s}^4 + 3\tilde{s}^2 - s^2) ds}{\tilde{s}^3 (1+\tilde{s}^2)^{3/2} (4\tilde{s}^2 + 4\tilde{s}^4 + \mu^2)}$$

$$\mu = \frac{a_\psi E}{c_\psi^2}, \qquad s = \frac{q a_\psi}{c_\psi}, \qquad \tilde{s}^2 = s^2 + \frac{a_\psi^2 n^2}{c_\psi^2 R^2}$$

At *any* scale, $f_4(E)$ gives the one-loop energy behaviour of the coupling

$$g_4^{-2}(E) = g_4^{-2}(E_0) + f_4(E) - f_4(E_0)$$

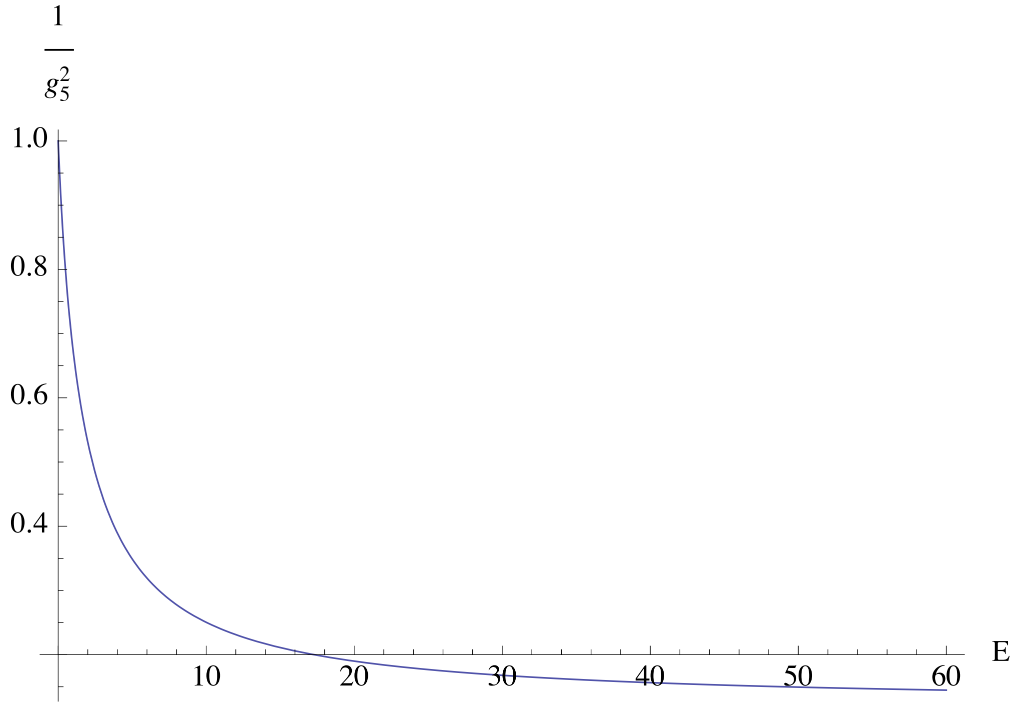
$$f_4(E) - f_4(E_0) \simeq \frac{2}{3\pi^2}\int_0^\infty \frac{s(\mu_0^2-\mu^2)ds}{(4s^2+\mu^2)(4s^2+\mu_0^2)} = -\frac{1}{6\pi^2}\log\frac{E}{E_0}\,, \quad E, E_0 \ll \frac{1}{R}\,,$$

For $E > 1/R$

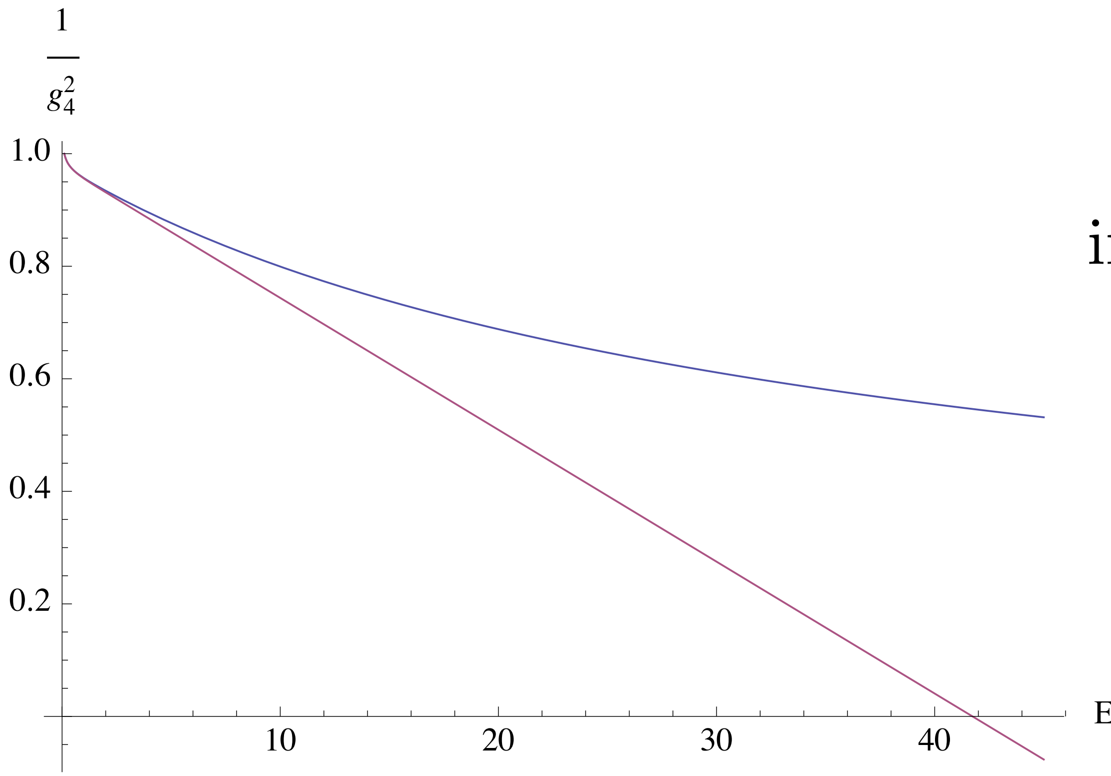
$$f_5(E) \simeq \frac{4\pi + 6\mu - (8+3\mu^2)\arctan\left(\frac{2}{\mu}\right)}{128\pi^2 a_\psi \mu}$$

$$f_5(E) - f_5(E_0) = -\frac{3}{256\pi}(E - E_0) + \mathcal{O}(E^2)\,, \quad E \ll 1$$

$$f_5(E) = \frac{1}{32\pi a_\psi^2}\frac{1}{E} + \mathcal{O}\left(\frac{1}{E^2}\right), \quad E \gg 1$$



in units of Λ



in units of $1/R$

Compute the maximum allowed gap between compactification scale and cut-off

$$g^{-2}(E=\infty) \geq 0 \quad \text{or} \quad \frac{1}{4\pi}$$

$$\Lambda R \gtrsim \frac{48\pi}{5}\left(\frac{1}{g_4^2} - \frac{1}{g_{4,\infty}^2}\right)$$

roughly as predicted by (correct) NDA using 4D, rather than 5D, loop factor

Price to be paid by breaking Lorentz invariance:

- Severe tuning to enforce $c_\psi = c_\gamma$ at low energies

[Collins et al; Iengo, Russo and MS]

- Strong constraints on Λ

Example: Fermi (probably not the most stringent)

$$\Lambda \gtrsim 10^9 \text{ GeV}$$

Conclusions

A surprisingly simple UV completion of QED in 5D is possible by breaking Lorentz invariance

Generalizations to more complicated and realistic 5D models straightforward

Although such UV completion is not well motivated (and strongly constrained phenomenologically), it can be used as a "toy" laboratory to quantify UV effects in theories with extra dimensions