Anomaly Constraints on Monopoles and Dyons

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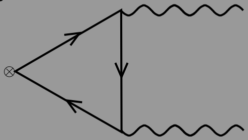
Introduction

- **Motivation:**
  Search for models with EWSB due to the monopole condensation (discussed by C. Csaki on Monday)
- Do theories with chiral monopoles/dyons exist?
  - Examples with SUSY theories with massless fermionic monopoles: Argyres-Douglas (see also Intriligator-Seiberg).
  - No known theories with chiral monopoles.
- Bottom-up approach: are there new consistency conditions on such models?
  - Dirac quantization conditions
  - New anomaly constraints:



  - Tools: $SL(2, Z)$ and Zwanziger's Lagrangian

$SL(2,Z)$ transformations

- A set of field redefinitions that leaves physics unchanged
- This is not a symmetry, does not leave Lagrangian invariant
- S-duality:
  - Exchanges magnetic and electric charges: $(q,g) \to (-g,q)$
  - Changes the coupling $e \to 1/e$
- T-duality:
  - Shifts electric charge: $(q,g) \to (q+g,g)$
  - In a theory with θ-parameter:

$$\theta \to \theta + 2\pi$$
$$(q,g) \to (q+g,g)$$

  Witten charge $q_{eff} = q + g\frac{\theta}{2\pi}$
- Together form an $SL(2,Z)$ group.
  For "holomorphic" coupling $\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$:

$$\tau \to \frac{a\tau+b}{c\tau+d}$$
$$ad - bc = 1$$

$SL(2,Z)$ transformations

- There is a basis where a dyon carries only electric charges

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} n \\ 0 \end{pmatrix}$$
$$n = \mathrm{gcd}(q,g)$$

- Assume that anomaly contributions of each field can be calculated independently.
- For each field calculate in the basis where it is an electron

$$\partial_\mu j^\mu_A = \frac{n^2}{16\pi^2} F' {}^*F'$$

- Transform back to original basis
- Need to find $SL(2,Z)$ transformations of the fields and currents

Axial anomaly

- Electric J^μ and magnetic K^μ currents transform

$$K^\mu \to aK'^\mu + cJ'^\mu, \qquad J^\mu \to bK'^\mu + dJ'^\mu$$

- Field strength

$$(F^{\mu\nu} + i\,{}^*\!F^{\mu\nu}) \to \frac{1}{c\tau^* + d}(F'^{\mu\nu} + i\,{}^*\!F'^{\mu\nu})$$

(From Maxwell equations $\frac{\mathrm{Im}\,\tau}{4\pi}\partial_\mu(F^{\mu\nu} + i\,{}^*\!F'^{\mu\nu}) = J^\mu + \tau K^\mu$)

Axial anomaly

- Axial anomaly

$$\partial_\mu j^\mu_A = \frac{n^2}{16\pi^2} F'{}^*F' = \frac{n^2}{16\pi^2}\mathrm{Im}(F' + i{}^*F')^2$$
$$= \frac{1}{16\pi^2}\left\{\left[\left(q + \frac{\theta}{2\pi}g\right)^2 - g^2\frac{16\pi^2}{e^4}\right] F^*F + \left[\left(q + \frac{\theta}{2\pi}g\right)g\right] F^2\right\}$$

- Related to β-function — possibility of the conformal fixed point
- CP-invariant theory: $\sum_i g_i(q_i + \frac{\theta}{2\pi}) = 0$
- No CP invariance: can rotate F^2 away but F^*F is not a total derivative and can serve as a kinetic term.
- Rotating θ away gives

$$\sum_i q_{Ai} q_i^2 = 0 \qquad \sum_i q_{Ai} q_i g_i = 0 \qquad \sum_i q_{Ai} g_i^2 = 0$$

Zwanziger Lagrangian

- To calculate gauge anomalies need to know transformations of gauge fields under $SL(2,Z)$.
- Zwanziger introduced local but non-Lorentz invariant Lagrangian with two gauge potentials A_μ and B_μ.
- Zwanziger Lagrangian generalized to include θ:

$$\begin{aligned}\mathcal{L} \;=\; & -\mathrm{Im}\frac{\tau}{8\pi n^2}\left\{[n\cdot\partial\wedge(A+iB)]\cdot[n\cdot\partial\wedge(A-iB)]\right\}\\ & -\mathrm{Re}\frac{\tau}{8\pi n^2}\left\{[n\cdot\partial\wedge(A+iB)]\cdot[n\cdot^{*}\partial\wedge(A-iB)]\right\}\\ & -J\cdot A-\frac{4\pi}{e^2}K\cdot B.\end{aligned}$$

- A_μ has a local coupling to electric current
- B_μ has a local coupling to magnetic current.
- Only two on-shell degrees of freedom.

Gauge anomalies

- $SL(2,Z)$ covariance of Zwanziger lagrangian implies that gauge potentials transform:

$$A + iB \to \frac{1}{c\tau^* + d}(A' + iB')$$

- Mixed anomalies in a theory with $SU(N) \times U(1)$
- Gravitational anomaly
- Mixed anomalies in a presence of additional $U(1)_X$
- Cubic anomaly

Gauge anomalies in $SU(N) \times U(1)$

- Anomalous transformation of the Lagrangian

$$\mathcal{L}_{\rm anom} = c\Omega G^* G$$
$$\Omega = \Omega_A + i\Omega_B$$

- Ω_A and Ω_B are gauge transformation parameters
- Ω transforms the same way as gauge potentials
- After $SL(2, Z)$ transformation

$$c\Omega' = \frac{nT(r)}{16\Pi^2}\Omega' \longrightarrow \left(q + \frac{\theta}{2\pi}g\right)\Omega_A + g\frac{2\pi}{e^2}\Omega_B$$

Gauge anomalies

- $SU(N)^2U(1)_{\rm m}$ anomaly: $\sum_i T(r_i)g_i = 0$
- Gravitational anomaly: $\sum g_i = 0$
- Mixed anomalies with $U(1)_X$ give several new conditions:

$$\sum_i q_{Xi} g_i^2 = 0 \quad \sum_i q_{Xi} q_i g_i = 0 \quad \sum_i q_{Xi}^2 g_i = 0$$

- Cubic anomaly

$$\mathcal{L}_{\rm anom} = \frac{n^3}{16\pi^2} \Omega'_A F'^* F'$$

- New conditions

$$\sum_i q_i^2 g_i = 0, \quad \sum q_i g_i^2 = 0 \quad \sum_i g_i^3 = 0$$

Conclusions

- Bottom-up approach allows to impose some consistency constraints on theories with chiral monopoles and dyons
- There are 8 new anomaly conditions
- These constraints can be applied to building EWSB models (discussed by Csaba Csaki's on Monday)