# Multijet Matching and Merging

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# Outline

# Introduction

- We want to understand SM multijet events. They are interesting in their own right, and they are important backgrounds for almost any search for new physics.
- W+$n$-jets.
- We want to have as high precision as possible.
- We need to understand hadronization effects.
- We need to simulate fully exclusive hadronic final states.

# Simulating fully exclusive final states

- To get several hard jets we need to have tree-level matrix elements.
- To get precision, we need to have NLO calculations.
- To understand hadronization corrections we need parton showers and hadronization models.
- We need to combine all this to get reliable predictions.

# Nature is cheap

If we want to study W+2-jets, it's not enough to just use a parton-level NLO calculation.

It's expensive to produce two hard partons.
($\alpha_s$ is small, steaply falling $p_\perp$-spectrum)

Sometimes it's cheaper to produce lots of soft stuff and hope that things happens to lump together into jets which pass the trigger.

The phase space for emitting soft gluons at the LHC is huge.

At the LHC there are several semi-hard interactions per collision.

# Partons vs. hadrons

All reasonable hadronization models requires that the partonic final state is correctly modeled in the soft and collinear limits (the soft partons between the jets, and the partonic structure inside the jets).

For a given parton shower and a given hadronization model we can tune the parameters to get a reasonable predictivity for the hadronization process.

Here I will not care about hadrons. As long as the soft and collinear behavior of the parton shower is the same, the tuning should hold.

# The formalism

We start out with some Born-level process without QCD vertices, and we want to generate events according to the fully differential exclusive cross sections for additional partons above some resolution scale

$$d\sigma_{+n} = C_n(\Omega_n)\alpha_s^n\left[1 + c_{n,1}(\Omega_n)\alpha_s + c_{n,2}(\Omega_n)\alpha_s^2 + \ldots\right]d\Omega_n$$

where $\Omega_n = (q_1, \ldots, q_m; p_1, \ldots, p_n)$ is the phase space for an $m$-particle Born process with $n$ extra partons.

$$d\sigma_{+n} = C_n(\Omega_n)\alpha_s^n \left[1 + c_{n,1}(\Omega_n)\alpha_s + c_{n,2}(\Omega_n)\alpha_s^2 + \ldots\right] d\Omega_n$$

The resolution scale is normally given by the PS cutoff, $\rho_c$, and is tuned to the hadronization model and is $\mathcal{O}(1\ GeV)$.

$\alpha_s$ is not very small, and all the coefficients are divergent for small resolution scales, and we really need to resum the series to all orders to get a finite answer.

# The problem

We know the leading (and next-to leading) behavior of all coefficients in the soft and collinear region, so we can get the inter- and intra-jet structure right with parton shower algorithms so that we can use our hadronization model.

But we also want precision in the distribution of the jets themselves, so some coefficients need to be calculated exactly using three-level and NLO (or even higher order) matrix elements.

# Parton Showers

$$d\sigma_{+n}^{\mathrm{PS}} = C_n^{\mathrm{PS}}(\Omega_n^{\mathrm{PS}})\alpha_{\mathrm{s}}^n \Delta_S(\Omega_n^{\mathrm{PS}}, \rho_c) d\Omega_n^{\mathrm{PS}}$$

An iterative procedure, where each emission is described with some splitting function depending on the splitting variables, $\rho, z$. We keep adding emissions below some maximum scale $\rho_0$, with ever decreasing $\rho$ until we hit the cutoff, $\rho_c$.

$$\Omega_n^{\mathrm{PS}} = (q_1, \ldots, \mathrm{q}_m; \rho_1, z_1 \ldots, \rho_n, z_n)$$

$C_n^{\mathrm{PS}}$ is the Born cross section times a product of splitting functions.

$$\Delta_S(\Omega_n^{\mathrm{PS}}, \rho_c) = \prod_i \Delta_{S_i}(\rho_{i-1}, \rho_i)\Delta_{S_n}(\rho_n, \rho_c)$$

The Sudakov form factor $\Delta_{S_i}(\rho_i, \rho_{i+1})$ is the probability of there being no emissions between the scale $\rho_i$ and $\rho_{i+1}$.

$$\Delta_{S_i}(\rho_i, \rho_{i+1}) = \exp\left(-\int_{\rho_{i+1}}^{\rho_i} d\rho \int dz \alpha_{\mathrm{s}} P_n(\rho, z)\right)$$

This is easily expanded out in powers of $\alpha_{\mathrm{s}}$.

$\alpha_{\mathrm{s}}$ is running with the transverse momentum of the emissions.

# Matching vs. Merging

$$d\sigma_{+n} = C_n(\Omega_n)\alpha_s^n \left[1 + c_{n,1}(\Omega_n)\alpha_s + c_{n,2}(\Omega_n)\alpha_s^2 + \dots\right] d\Omega_n$$

We know how to generate few-parton final states according to exact tree-level matrix elements (and some times to NLO).

How can we add parton showers to these so that the first coefficients are still exact, and all others are the ones given by the parton shower?

Matching: modify the matrix elements and/or the parton shower to fit them together.

Merging: define a merging scale $k_{\perp MS}$, above which we generate with exact matrix elements, but reweight to get the parton shower coefficients correctly. Then add parton shower below.

# Tree-level matching and merging

There are automated matrix element generators (e.g. MadGraph) where we can generate any process with up to $\sim 7$ final state particles.

For each jet multiplicity we only have the leading order in $\alpha_{\rm s}$, and we get the inclusive cross section for *at least n* extra jets. Hence we cannot just add different multiplicities and add parton showers below $k_{\perp MS}$.

# Matching the hardest jet

This has been around since the eighties (Sjöstrand).

Change the splitting function for the first parton shower emission

$$P(\rho_1, z_1) \to C_1^{\mathrm{ME}}(\Omega_1^{\mathrm{PS}})/C_0^{\mathrm{ME}}(\Omega_0^{\mathrm{PS}})$$

This will still have the correct soft and collinear limits and it is easily exponentiated in the Sudakov form factor.

Easy if the first emission is also the hardest (i.e. if $\rho \sim k_\perp$)

Difficult to generalize to higher jet multiplicities (c.f. Vincia).

[Bengtsson, Sjöstrand, *Phys. Lett.* **B185** (1987) 435]
[Seymour, *Comp. Phys. Commun.* **90** (1995) 95]
[Skands et al., *Phys. Rev.* **D78** (2008) 014026]

# Multijet merging

We use a matrix element generator to get $+n$-parton states, with $0 \leq n \leq N$, using the $k_{\perp MS}$ scale to regularize divergencies.

$$d\sigma_{+n} = C_n^{\mathrm{ME}}(\Omega_n)\alpha_{\mathrm{s}}^n(k_{\perp MS})d\Omega_n$$

$k_{\perp MS}$ is typically defined in terms of a jet measure $\sim k_\perp$.

If we assume that our parton shower is also ordered in $k_\perp$, we need to find a *parton shower history* ($\sim$ using a jet clustering algorithm)

$$\Omega_n \mapsto \Omega_n^{\mathrm{PS}} = (q_1, \ldots, q_m; \rho_1, z_1 \ldots, \rho_n, z_n)$$

We use the reconstructed scales to calculate Sudakov form factors, and we reweight the generated +$n$-parton states with these and the $\alpha_{\mathrm{s}}$-values the parton shower would have used.

$$d\sigma_{+n} = C_n^{\mathrm{MS}}(\Omega_n^{\mathrm{PS}})\prod_i \alpha_{\mathrm{s}}^n(\rho_i)\Delta_{S_i}(\rho_{i-1},\rho_i)\Delta_{S_n}(\rho_n,k_{\perp MS})d\Omega_n^{\mathrm{PS}}$$

We can then add a parton shower with emissions below $k_{\perp MS}$, and we get what we want.

(We need to treat the case $n = N$ separately to allow more than $N$ jets above $k_{\perp MS}$.)

What if the evolution scale in the shower is not the one used to define $k_{\perp MS}$? (HERWIG: angle, PYTHIA: virtuality)

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# Catani-Krauss-Kuhn-Webber

- Uses the $k_\perp$-algorithm to define $k_{\perp MS}$ and to reconstruct clustering scales, $k_{\perp 1}, \ldots, k_{\perp n}$.
- Uses these scales to calculate *analytical* NLL Sudakov form factors, and to get the running $\alpha_s$.
- Adds a parton shower, ordered in e.g. angle, starting from the maximum possible $\rho$-scale, but with all emissions with $k_\perp > k_{\perp MS}$ *vetoed*.
- (Sherpa and HERWIG++ uses *truncated, vetoed* shower, which is more correct.)
- If the shower is correct to NLL, the dependence on $k_{\perp MS}$ vanishes to NLL accuracy.

[Catani et al., *JHEP* **11** (2001) 063]
[Hoeche et al., *JHEP* **05** (2009) 053]

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# Catani-Krauss-Kuhn-Webber

- Reconstruct a proper parton shower history, with complete on-shell kinematics of intermediate states.
- Use the fact that Sudakov form factors are *no-emission probabilities*. Make a trial emission from each reconstructed state and throw away the whole event if the emission is above $k_{\perp i}$.
  Equivalent to reweighting with $\Delta_{S_i}(\rho_i, \rho_{i+1})$.
- Reweight with the $\alpha_s$ the shower would have used.
- For the *n*-parton state, $S_n$, keep the trial emission and continue if below $k_{\perp MS}$.
  (For $n = N$, always keep the trial emission and continue.)

[Lönnblad, *JHEP* **05** (2002) 046]

# Catani-Krauss-Kuhn-Webber-and-me

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- For $k_\perp$-ordered shower, it is formally equivalent to CKKW if used with a NLL-correct shower.
- The $N$ first shower emissions are ME-corrected if above $k_{\perp MS}$.
- Kinematical effects are taken into account in the Sudakovs in the same way as in the shower (true no-emission probability).
- Requires a shower with on-shell intermediate states (ARIADNE, $k_\perp$-ordered PYTHIA).

# Parton densities

For initial-state splittings in a backwards evoluted parton shower, the Sudakov form factors are not the same as no-emission probabilities.

$$\Delta_{S_i}(\rho_i, \rho_{i+1}) = \frac{f(x, \rho_i)}{f(x, \rho_{i+1})} \times \mathcal{P}_{no-em}(\rho_i, \rho_{i+1})$$

So, CKKW stays the same, but CKKW-L gets an extra reweighting with PDF ratios for each intermediate state.

[Krauss, *JHEP* **08** (2002) 015]
[Lavesson, Lönnblad, *JHEP* **07** (2005) 054]

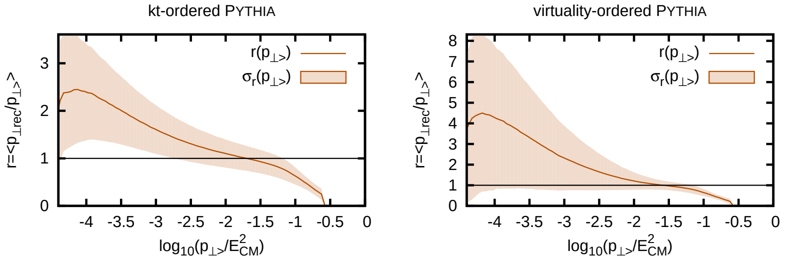
# Alternative: Mrennas Pseudo-shower

Similar to CKKW-L, but uses the $k_\perp$-algorithm to define a $k_\perp$-ordered shower history.

Instead of trial emissions to get the no-emission probabilities, perform a full vetoed shower shower from each intermediate state, and cluster back with the $k_\perp$-algorithm, to get the first (hardest) emission in $k_\perp$.

[Mrenna, Richardson, *JHEP* **05** (2004) 040]

It is difficult to recluster a shower in this way.




[Lavesson, Lönnblad, *JHEP* **04** (2008) 085]

# Michelangelos alternative

Just add shower, and cluster back to $k_{\perp MS}$.

The probability that we have approximately the same jets as the $n$ partons we started with $\sim$ the Sudakov form factor.

Simple, and much used. And it gives reasonable results.

Sensitive to how parton shower handles un-ordered emissions and initial conditions.

Difficult to get precision.

[Mangano et al., *JHEP* **01** (2007) 013]

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# NLO matching and merging

Tree-level matching/merging can never give us precision.

Scale dependencies are large, and we only have leading order cross sections.

For shapes of distributions we can still do a reasonable job.

May be enough if we just want to estimate hadronization corrections.

To obtain overall precision we need to go to next-to-leading order. (or beyond).

# Leading $\alpha_s$ correction

The standard Catani-Seymour subtraction scheme

$$
\begin{aligned}
d\sigma^{\mathrm{CS}}_{+0} &= \left[C^{\mathrm{ME}}_0(\Omega_0) + \alpha_s\left(C^{\mathrm{loop}}_{0,1}(\Omega_0) + \int\left\{\frac{d\Omega_1}{d\Omega_0}\right\} C^{\mathrm{CS}}_1(\Omega_1)\right)\right] d\Omega_0 \\
d\sigma^{\mathrm{CS}}_{+1} &= \left[\alpha_s C^{\mathrm{ME}}_1(\Omega_1) - \alpha_s C^{\mathrm{CS}}_1(\Omega_1)\right] d\Omega_1
\end{aligned}
$$

Adding together gives the correct NLO cross section.

$C^{\mathrm{CS}}_1$ must have the same soft and collinear poles as $C^{\mathrm{ME}}_1$

[Catani, Seymour, *Phys. Lett.* **B378** (1996) 287]

# MC@NLO

Since the splitting functions in the shower has the correct soft and collinear behavior, we can use $C_1^{\mathrm{CS}} \to C_1^{\mathrm{PS}}$.

We can then add the parton shower, and we get the total cross section right to NLO.

We also get the correct ME behavior for the first emission.

But not necessarily for the *hardest* emission.

May give negative weights, which can be dealt with but are a bit ugly.

[Frixione, Webber, *JHEP* **06** (2002) 029]

# POWHEG

Implement the first shower emission as in the old tree-level matching, assuming a $k_\perp$-ordered shower, but also reweight with the correct NLO cross section.

Continue with any $k_\perp$-ordered shower, or with any shower properly *truncated* and *vetoed*.

- No negative weights!
- Hardest emission corrected with ME
- Hence the name.

[Nason, *JHEP* **11** (2004) 040]
[Frixione, Nason, Oleari, *JHEP* **11** (2007) 070]

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# Multijets with loops

But how do we get higher jet multiplicities correct to NLO?

We have NLO calculations of e.g. W+1jet to NLO, so we can start with W+1jet state and add a shower using POWHEG.

We need to regularize the W+1jet Born cross section with a jet cutoff.

But how do we treat parton-shower resummed effects from W+0jet where all emissions are below the cutoff, but anyway gets clustered together into a jet above?

How do we combine different jet multiplicities (as in CKKW) and keep NLO accuracy?

# Multi-jet merging (NL$^3$)

Assume we have a tree-level matrix element generator giving us a Born process with up to $N$ extra partons above some merging scale $k_{\perp MS}$.

$$d\sigma_{+n}^{\text{tree}} = C_n(\Omega_n)\alpha_s^n(\mu)d\Omega_n$$

Assume also that we have a NLO generator which can generate up to $N-1$ partons according to the exact *exclusive* NLO cross section, using the same $k_{\perp MS}$.

$$d\sigma_{+n}^{\text{loop}} = C_n(\Omega_n)\alpha_s^n(\mu)\left[1 + c_{n,1}(\Omega_n)\alpha_s(\mu)\right]d\Omega_n$$

[Lavesson, Lönnblad, *JHEP* 12 (2008) 070]

Now we do the CKKW(-L) treatment on the tree-level states

- $\sigma_{+n}^{\mathrm{CKKW-L}}$ gives exclusive n-jet states approximately correct (as far as the PS is correct) to all orders in $\alpha_{\mathrm{s}}$.
- $\sigma_{+n}^{\mathrm{loop}}$ gives exclusive n-jet states exactly correct to the leading two orders in $\alpha_{\mathrm{s}}$.

In both cases we can add a shower below $k_{\perp MS}$.
(Assume for now that we have a $k_{\perp}$-ordered shower)

We want to add these two samples together, but in order not to double-count, the CKKW(-L) sample needs to have the two first terms in its $\alpha_{\mathrm{s}}$-expansion removed.

We have

$$d\sigma_{+n}^{\mathrm{CKKW-L}} = C_n(\Omega_n)\alpha_s^n(\mu) K \prod_{i=1}^{n} \frac{\alpha_s^{\mathrm{PS}}(\rho_i)}{\alpha_s(\mu)} \prod_{i=0}^{n} \Delta_{S_i}(\rho_i, \rho_{i+1}) d\Omega_n$$
$$= C_n(\Omega_n)\alpha_s^n(\mu) \left[1 + \alpha_s(\mu) B^{\mathrm{PS}} + \mathcal{O}\left(\alpha_s^2(\mu)\right)\right] d\Omega_n$$

We want

$$d\sigma_{+n}^{\mathrm{PScorr}} = C_n(\Omega_n)\alpha_s^n(\mu) \left[K \prod_{i=1}^{n} \frac{\alpha_s^{\mathrm{PS}}(\rho_i)}{\alpha_s(\mu)} \prod_{i=0}^{n} \Delta_{S_i}(\rho_i, \rho_{i+1}) - 1 - \alpha_s(\mu) B^{\mathrm{PS}}\right] d\Omega_n$$

# K-Factors

The parton shower approximation never influences the total cross section, while the NLO cross section does.

Normally we multiply the parton shower result with a global K-factor calculated from the integrated full NLO cross section.

$$K = 1 + k_1\alpha_s(\mu)$$

We need to include this factor to compare $\sigma_{+n}^{\mathrm{CKKW-L}}$ with $\sigma_{+n}^{\mathrm{loop}}$.

# $\alpha_s$ considerations

- Note that $\alpha_s(\mu) \neq \alpha_s^{\mathrm{PS}}(\mu)$.
- $\alpha_s^{\mathrm{PS}}$ is typically a one- or two-loop $\alpha_s$ with $\Lambda_{\mathrm{QCD}}$ a free parameter fitted reproduce event shapes at LEP.
- $\alpha_s(\mu)$ is here just a fixed number corresponding to the "world-average" $\alpha_s(M_Z)$ running with $\Lambda_{\overline{MS}}$

Typically $\alpha_s^{\mathrm{PS}}(M_Z) > \alpha_s(M_Z)$ because the parton shower underestimates the hard emission probabilities, and needs to *boost* the probabilities to fit the data.

Rather than tuning $\Lambda_{\mathrm{QCD}}$, we could say we are using $\Lambda_{\bar{MS}}$ and instead tune a scale factor, $\alpha_s^{\mathrm{PS}}(\rho) = \alpha_s(b\rho)$.

Hence, we can write

$$\frac{\alpha_s^{\mathrm{PS}}(\rho)}{\alpha_s(\mu)} = 1 - \frac{\log\frac{b\rho}{\mu}}{\alpha_0}\alpha_s(\mu) + \mathcal{O}\left(\alpha_s^2(\mu)\right)$$

# Expanding the Sudakov form factor

$$\Delta_{S_i}(\rho_i, \rho_{i+1}) = \exp\left( - \int_{\rho_{i+1}}^{\rho_i} d\rho \alpha_s(\rho) \Gamma_{S_i}(\rho) \right)$$

We know how to generate this from the shower:

- Start the shower from the state $S_i$, with $\rho_i$ as the maximum scale.
- Generate one emission giving a scale $\rho$.
- The probability that $\rho < \rho_{i+1}$ is exactly $\Delta_{S_i}(\rho_i, \rho_{i+1})$.

- If $\rho > \rho_{i+1}$, restart from $S_i$ and generate again one emission starting from $\rho$ as maximum scale.
- Continue until we find a $\rho < \rho_{i+1}$.
- Count the number of emissions $n_{acc}$ before going below $\rho_{i+1}$.

$$\langle n_{acc} \rangle = -\log \Delta_{S_i}(\rho_i, \rho_{i+1}) = \int_{\rho_{i+1}}^{\rho_i} d\rho \alpha_s(\rho) \Gamma_{S_i}(\rho)$$

(add a trick to get $\alpha_s(\rho) \to \alpha_s(\mu)$)

# Parton densities

So far we only have this working for $\mathrm{e}^+\mathrm{e}^- \to$jets

We are working on W+jets, which will also mean expanding out ratios of PDFs. C.f.

$$\Delta_{S_i}(\rho_i, \rho_{i+1}) = \frac{f(x, \rho_i)}{f(x, \rho_{i+1})} \times \mathcal{P}_{no-em}(\rho_i, \rho_{i+1})$$

# The algorithm

Now we have everything we need:

- Generate events with $0 \leq n \leq N$ extra jets according to the tree-level ME cut off at $k_{\perp MS}$.
- Generate events with $0 \leq n < N$ extra jets according to the exclusive NLO ME cut off at $k_{\perp MS}$.
- Reconstruct $\Omega_n \mapsto \Omega_n^{\mathrm{PS}}$.
- For one-loop events, add PS below $k_{\perp MS}$.
- For tree-level events, with $n = N$, reweight with

$$K \prod_{i=1}^{n} \frac{\alpha_{\mathrm{s}}^{\mathrm{PS}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu)} \prod_{i=0}^{n-1} \Delta_{S_i}(\rho_i, \rho_{i+1})$$

and continue below $\rho_N$.

- For tree-level events with $n < N$, reweight with

$$K \prod_{i=1}^{n} \frac{\alpha_s^{\mathrm{PS}}(\rho_i)}{\alpha_s(\mu)} \prod_{i=0}^{n} \Delta_{S_i}(\rho_i, \rho_{i+1})$$

$$-1$$

$$-\alpha_s(\mu)k_1$$

$$+\alpha_s(\mu)\sum_{i=1}^{n} \frac{\log \frac{b\rho_i}{\mu}}{\alpha_0}$$

$$+\alpha_s(\mu)\sum_{i=0}^{n} \int_{\rho_{i+1}}^{\rho_i} d\rho \Gamma_{S_i}(\rho)$$

and add PS below $k_{\perp MS}$

All weights are positive as long as

- $k_{\perp MS}$ is large enough for the loop ME to be positive
- $\mu < b\rho_i$

The net result is events generated so that all $n$-jet observables (above the merging scale and $n < N$) will be correct to NLO with a PS-simulated resummation. And $N$-jet observables will correct to LO+PSresum.

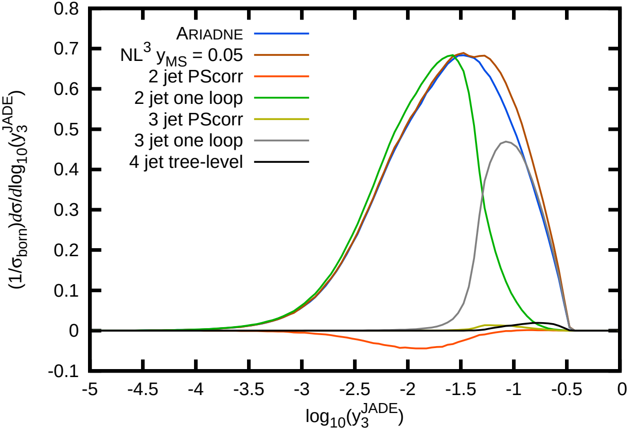
$$d\sigma_{+n} = C_n(\Omega_n)\alpha_s^n \left[1 + c_{n,1}(\Omega_n)\alpha_s + c_{n,2}^{\mathrm{PS}}(\Omega_n)\alpha_s^2 + \ldots\right] d\Omega_n$$

Why combine several different jet multiplicities?

If we are looking at three-jet observables, why generate two-jet events? Isn't it enough to generate 3-jet loop + 4-jet tree?

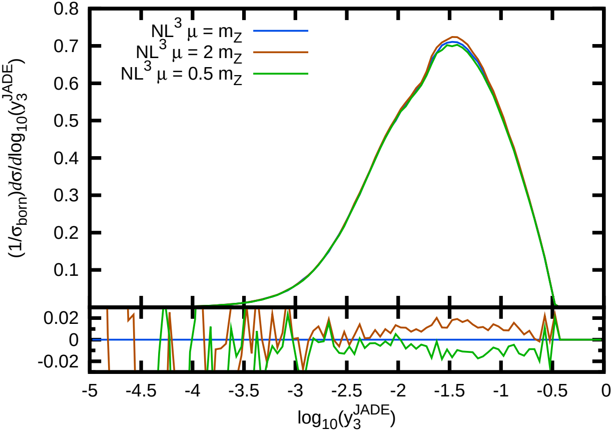
In our proof-of concept paper we used invariant mass cuts on the MEs (JADE) and generated $\mathrm{e^+e^-} \to 2, 3$ jets to NLO and $\mathrm{e^+e^-} \to 2, 3, 4$ jets to LO, combined with ARIADNE ($k_\perp$-ordered).

Look at the distribution in $y_3^{\mathrm{JADE}}$, the scale at wich a 4-jet event is clustered into 3 jets in the JADE algorithm.




(Note, this is without the extra $\alpha_s$-scale, otherwise ARIADNE is almost identical to NLO.)

Scale dependencies in the NLO are reduced by resummation.



# Outlook

- Event generators are entering the precision era.
- Tree-level ME generators are already automated.
- NLO calculations are getting there.
- Matching and merging with parton showers is working but also needs to be automated.

# Multijet Matching and Merging

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