

# Radiative Corrections to Semileptonic Meson Decays

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14/01/2010



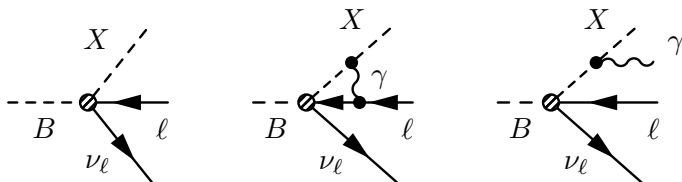
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- ➋ Methods
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# Motivation



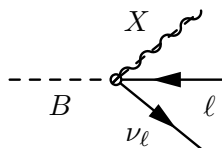
- Determination of  $V_{xb}$

$$\Gamma_{\text{measured}} = \eta_{\text{QCD}}^2 \eta_{\text{QED}}^2 |V_{xb}|^2 \Gamma'_{\text{Born}}$$

→ experiments extract rates which include radiative effects

- Corrections to measured kinematics
    - real and virtual photons alter momenta of involved particles
    - fits to kinematic variables can be very sensitive to slight changes
- [Phys. Rev. D 79, 012002\(2009\)](#)

# Model



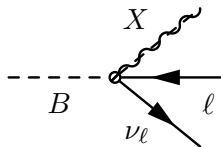
$$\mathcal{M} = \sqrt{2}G_F V_{xb} H_\mu L^\mu$$

$$L^\mu = \bar{u}_\nu P_R \gamma^\mu v_\ell$$

$$H^\mu = \langle X | V^\mu | B \rangle - \langle X | A^\mu | B \rangle$$

- 1 QED corrections can be adequately described at  $\mathcal{O}(\alpha)$
- 2 hadronic model fully characterises weak interaction
- 3 presence of QED radiation does not modify structure of form factors
- 4 momentum dependence of form factors negligible in loop integrations
- 5 scalar/vector QED is sufficient to describe meson-photon interaction

# Model



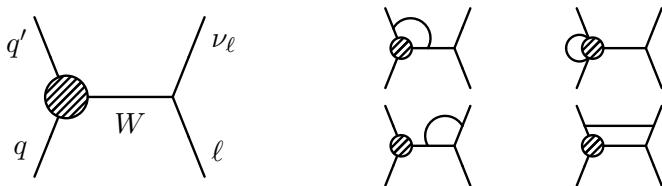
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- supplement phenomenological model with QED
  - scalar, fermion, vector QED
  - emission off phen. vertex through minimal coupling
  - fully gauge invariant effective theory, Ward identities are fulfilled
- renormalise analogous to QED
  - on-shell
  - $G_F$  renormalised in short distance picture
    - match to muon decay [Rev.Mod.Phys.50 \(1978\) 573](#)
  - long distance corrections need to be matched to short distance corrections

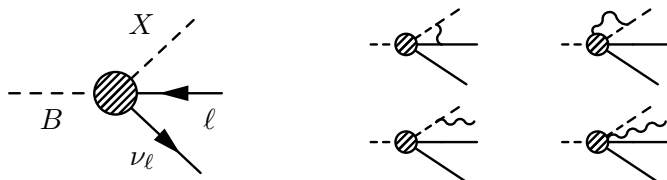
## $SD$ result – full EW SM



- full SM corrections  $\mathcal{O}(\alpha_{\text{QED}}^4)$  for partonic process
- needs an infrared cut-off  $\Lambda \ll m_W$  at hadronic scale
- photon receives mass  $\Lambda$   
 → no real emissions if  $\Lambda \geq \frac{1}{2}m_B$

$$\mathcal{M}_{0,SD}^1 = \frac{\alpha}{\pi} \ln \frac{m_Z^2}{\Lambda^2} \mathcal{M}_0^0$$

## $LD$ result – pheno- $LO \otimes QED$



- $m_\gamma = 0 \rightarrow$  real and virtual emissions
- effective vertex may emit photons (gauge invariance)
- UV-regularisation using Pauli-Villars method  
 $\rightarrow$  introduce heavy unphysical photon of mass  $M$  with negative propagator sign
- set  $M = \Lambda$  to match to  $SD$  results

$$\Gamma_{\text{measured}} = \eta_{\text{QCD}}^2 \eta_{\text{QED}}^2 |V_{xb}|^2 \Gamma'_{\text{Born}}$$

$$\eta_{\text{QED}}^2 = 1 + \delta_{\text{QED}}^{\text{SD}} + \delta_{\text{QED}}^{\text{LD}} = \frac{\Gamma [\mathcal{O}(\alpha G_F^2)]}{\Gamma [\mathcal{O}(G_F^2)]}$$

	$1 + \delta_{\text{QED}}^{\text{LD}}$	stat. error	$\eta_{\text{QED}}$	error
$B^0 \rightarrow D^{\mp} e^{\pm} \nu_e$	1.00817	0.00025	1.0110	0.001
$B^0 \rightarrow D^{\mp} \mu^{\pm} \nu_e$	1.00757	0.00023	1.0108	0.001
$B^{\pm} \rightarrow D^0 e^{\pm} \nu_e$	0.99679	0.00023	1.0054	0.0005
$B^{\pm} \rightarrow D^0 \mu^{\pm} \nu_e$	0.99633	0.00021	1.0052	0.0005
$B^0 \rightarrow \pi^{\mp} e^{\pm} \nu_e$	1.04476	0.00079	1.0290	0.002
$B^0 \rightarrow \pi^{\mp} \mu^{\pm} \nu_e$	1.04544	0.00081	1.0292	0.002
$B^{\pm} \rightarrow \pi^0 e^{\pm} \nu_e$	0.99718	0.00046	1.0055	0.001
$B^{\pm} \rightarrow \pi^0 \mu^{\pm} \nu_e$	0.99598	0.00048	1.0049	0.001

→ QED breaks isospin symmetry



## Resummation Effects – YFS-scheme

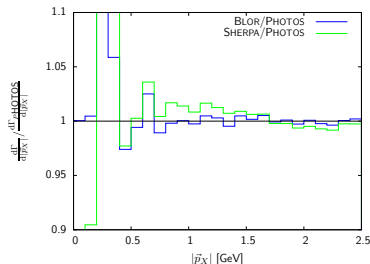
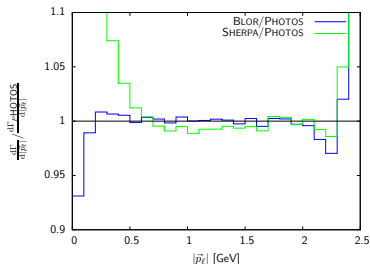
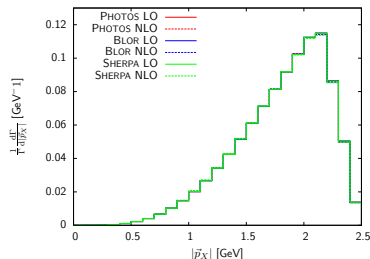
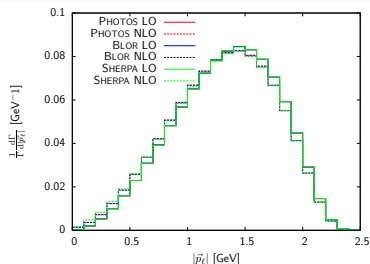
- resummation of soft limit ( $k \rightarrow 0$ )
- separation of soft limit of real and virtual amplitudes  
→ YFS form factor  $\exp[Y(\Omega)]$
- reorganisation of perturbative series  
→ series in  $e$  with infrared divergent amplitudes into series in  $\alpha$  with infrared subtracted squared amplitudes

$$\left| \mathcal{M}_0^0 + \mathcal{M}_1^{\frac{1}{2}} + \mathcal{M}_0^1 + \dots \right|^2$$

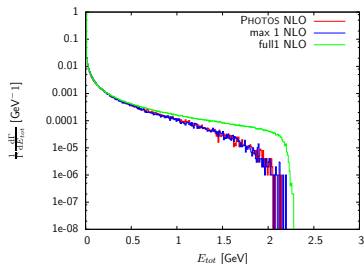
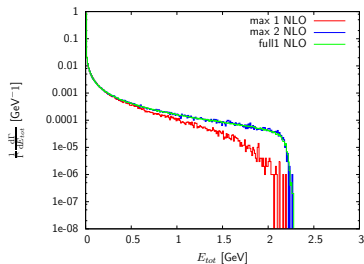
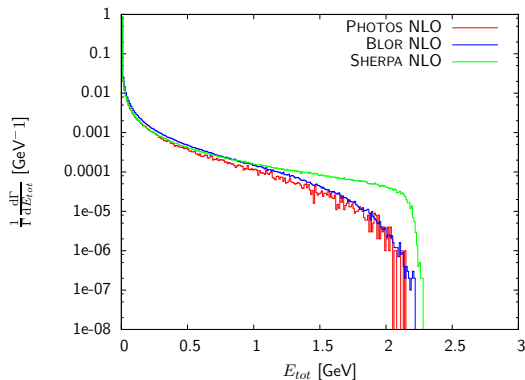
$$= e^{Y(\Omega)} \prod_0^i \tilde{S}(k_i, \Omega) \left( \tilde{\beta}_0^0 + \tilde{\beta}_0^1 + \sum_i \frac{\tilde{\beta}_1^1(k_i)}{\tilde{S}(k_i)} + \dots \right)$$

- improved predictions

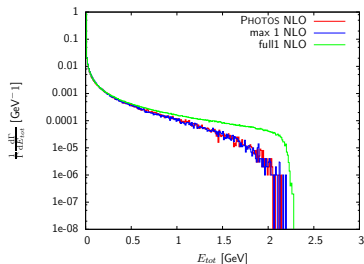
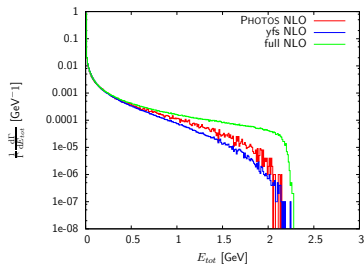
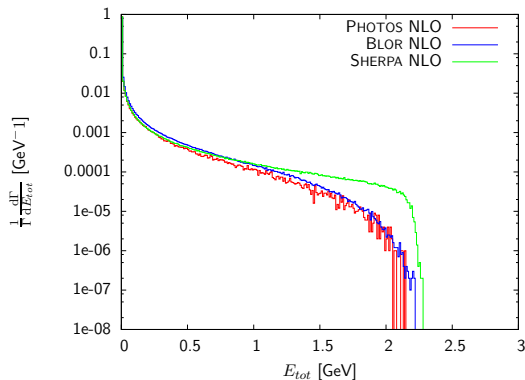
# Differential distributions – $B^0 \rightarrow D^- e^+ \nu_e$



# Origin of differences – $B^0 \rightarrow D^- e^+ \nu_e$



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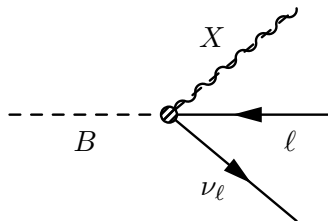


# Conclusion

- corrections and smaller uncertainties to correction factor  $\eta_{\text{QED}}$  in  $V_{xb}$
- some phase space regions receive much larger corrections  
→ can only be adequately described using both fixed order ME and resummation
- quantify systematic uncertainties due to matching of  $SD$  and  $LD$
- TODO: implement integration using YFS-improved calculation

# Appendix

# Phenomenological LO decay – $\mathcal{O}(G_F)$



$$\mathcal{M} = \sqrt{2}G_F V_{xb} H_\mu L^\mu$$

$$L^\mu = \bar{u}_\nu P_R \gamma^\mu v_\ell$$

$$H^\mu = \langle X | V^\mu | B \rangle - \langle X | A^\mu | B \rangle$$

$$\text{FS scalar} \quad \langle X | V^\mu | B \rangle = f_+(t)(p_B + p_X)^\mu + f_-(t)(p_B - p_X)^\mu$$

$$\langle X | A^\mu | B \rangle = 0$$

$$\text{FS vector} \quad \langle X | V^\mu | B \rangle = 2ig(t)\epsilon^{\mu\nu\rho\sigma}\epsilon_\nu^* p_{B\rho} p_{X\sigma}$$

$$\begin{aligned} \langle X | A^\mu | B \rangle &= f(t)\epsilon^{\mu*} + a_+(t)(p_B + p_X)^\mu (p_B - p_X)^\nu \epsilon_\nu^* \\ &\quad + a_-(t)(p_B - p_X)^\mu (p_B + p_X)^\nu \epsilon_\nu^* \end{aligned}$$

## Advantages over LL Factorization

		phen. model	PHOTOS
V	$t$ dependency in FF	approximate	none
V	corrections on kinematics	yes	none
V	vertex emission	yes	none
V	contribution to decay rate	yes	none
R	$t$ dependency in FF	yes	none
R	full photon emission coupling	yes	resummed soft limit
R	vertex emission	yes	none
R	parton level emission	none	none
R	emission ration at first order	yes	approximate
RV	total rate change	yes/no	none