Hard Radiation at a High Energy Collider

Radiative Corrections to (H/W/Z+)Jets

Jeppe R. Andersen

MC4LHC Readiness/V+jets
CERN, April 1, 2010

The Challenge, The Solution, Status

The Challenge (fka Problem), (in trivial statements)

Hard emission is less suppressed at **increasing energies**.
NLO gets the **one** hard emission right, but **one may not be sufficient**.
Parton shower does **many emissions**, but **not the hard ones**.
PS+matching is **good at Tevatron**, but **sufficient at LHC?**

The Solution

High Energy Jets (HEJ): What it is; what it is **not**

Status

What **HEJ** can do for you **now**
What $1fb^{-1}$@7TeV can tell us about our perturbative tools

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Prerequisites for the discussion

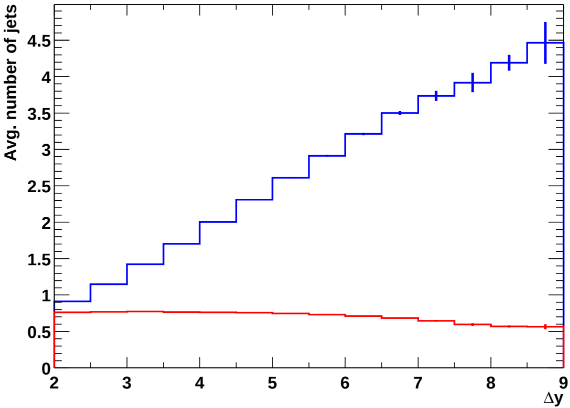
Jets: Will discuss mostly situations of **more than or equal to two hard jets** ($p_\perp > 40$GeV) since:

a) W/Z/H(+1 jet) described OK with existing perturbative tools
b) We can develop a framework for the leading, hard radiation in $2 \to 2$ coloured scattering (including leading virtual corrections).

Observables: Focus is on the final state in terms of jet count and configuration (but not jet substructure). Concentrate on a few possible observables, which capture the relationship between the **increasing phase space** and the **amount of hard radiation**:

1) Rapidity difference between most forward and most backward hard jet, Δy (**NOT!** just between the two hardest jets!)
2) $\frac{\sigma_{N+1}}{\sigma_N}$, $\langle \#\text{jets} \rangle$,... vs. Δy.

⟨#jets⟩ vs. Δy



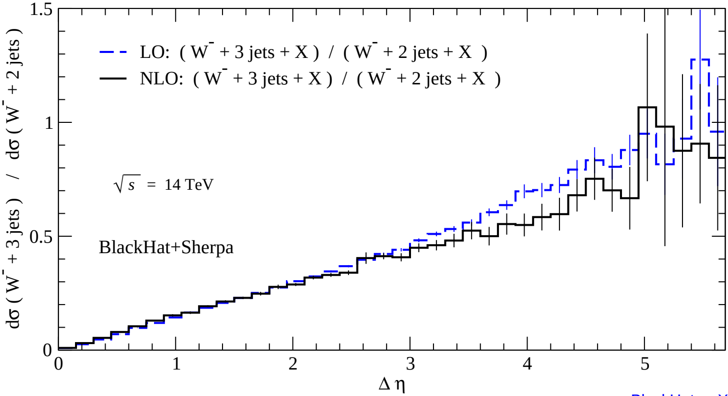
Red: Average number of central ($|y| < 1$) jets.

JRA, V. Del Duca, F. Maltoni, W.J. Stirling, hep-ph/0105146

Basic observation of increasing phase space for hard emissions with increasing Δy is the motivation for e.g. BFKL resummation.

However, don't just take *my* word for it. . .

W+Multiple Jets @ NLO



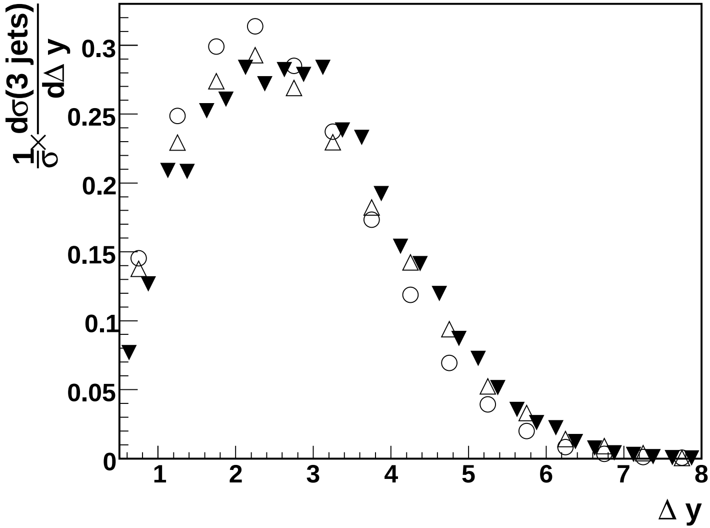
BlackHat, arXiv:0912.4927

The inclusive 3-jet rate is large compared to the inclusive 2-jet rate, even for normal rapidity spans obviously, the inclusive 3-jet rate "ought to" be smaller than the inclusive 2-jet rate.

All calculational methods and processes will agree on the opening of phase space as Δy increases; but the mechanism for emission differs.

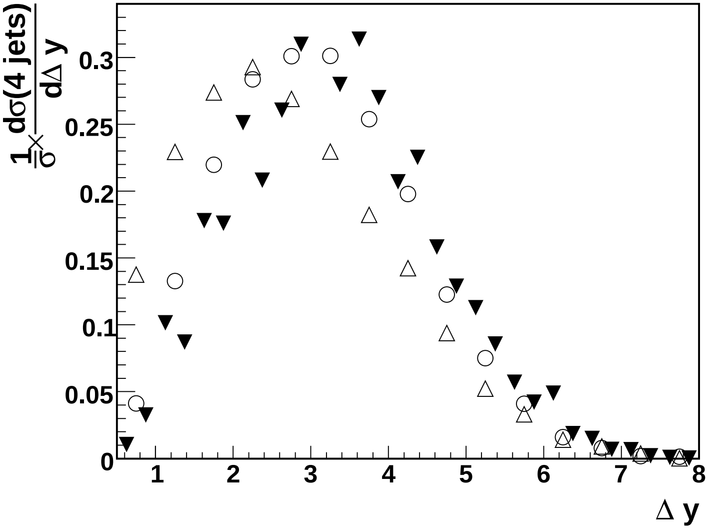
$1/\sigma\ d\sigma/d\Delta y$

JRA, M. Campanelli, J. Campbell, V. Ciulli, J. Huston, P. Lenzi, R. Mackeprang, arXiv:1003.1241



$\Delta y \approx 2 - 3$ (where σ_{3j}/σ_{2j} is already very large) is not "tail of distribution"!

$1/\sigma \, d\sigma/d\Delta y$



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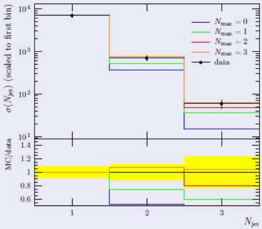
What about PS+Matching?

Z^0+jets at Tevatron: jet multiplicities

Jet rates and -spectra improved compared to pure PS simulation

- Example: DY-pair production $\sigma_{e^+e^- + N_{jet}}$

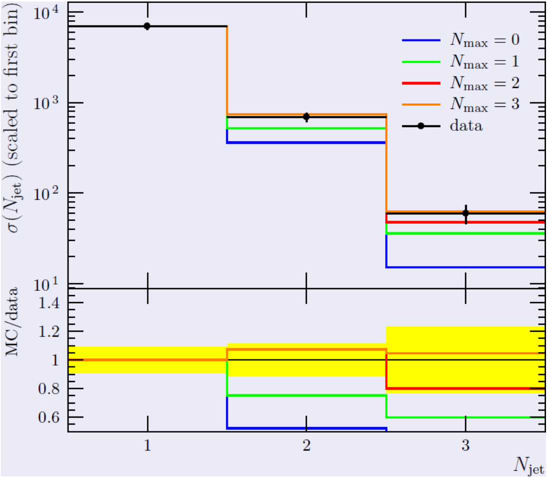
CDF Data: PRL **100** (2008) 102001



$Q_{cut} = 30$ GeV

F. Krauss' talk this Monday

What about PS+Matching?



Convincing at the Tevatron, where there is a large suppression for each extra jet. What about the LHC? Should we start worrying not just about total rates, but also the impact of "hard" radiation also below $Q_{\rm cut}$. For Z+4-5j?

HEJ (High Energy Jets)

- What is this HEJ?
- What is it **not**

HEJ (High Energy Jets)

Goal (inspired by the great Fadin & Lipatov)

Sufficiently **simple** model for hard radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)

but. . .

Sufficiently **accurate** that the description is relevant

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit → **eikonal approximation** → enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider…

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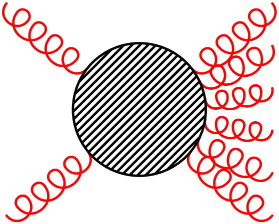
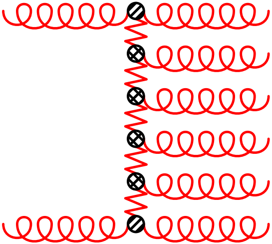


High Energy Limit

$$|\hat{t}| \quad \text{fixed}, \quad \hat{s} \to \infty$$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2}\left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)}\frac{V^{J_i}(q_i,q_{i+1})}{q_i^2 q_{i+1}^2}\right) e^{\omega(q_{n+1})(y_n-y_{n+1})}\frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|}\right)$$

to all orders in α_s.

At LL only gluon production; at NLL also quark–anti-quark pairs produced. Approximation of any-jet rate possible.

Comparison of 3-jet scattering amplitudes

Universal behaviour of scattering amplitudes in the HE limit:

$$\forall i \in \{2, \ldots, n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i, j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}^{MRK}_{gg\to g\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2 - 1}\,\frac{g^2\,C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2}.$$

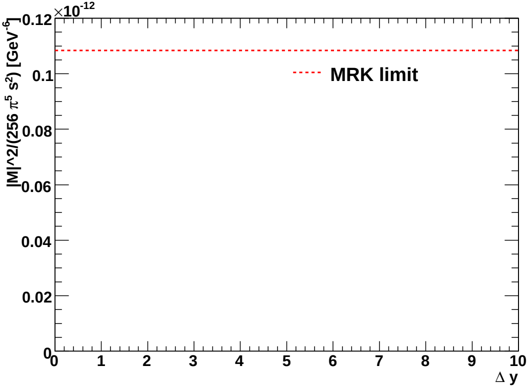
$$\left|\overline{\mathcal{M}}^{MRK}_{qg\to qg\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2 - 1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2},$$

$$\left|\overline{\mathcal{M}}^{MRK}_{qQ\to qg\cdots Q}\right|^2 = \frac{4\,s^2}{N_C^2 - 1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (BFKL equation).
However, how well does this actually approximate the amplitude?

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:



JRA, J.M. Smillie, arXiv:0908.2786

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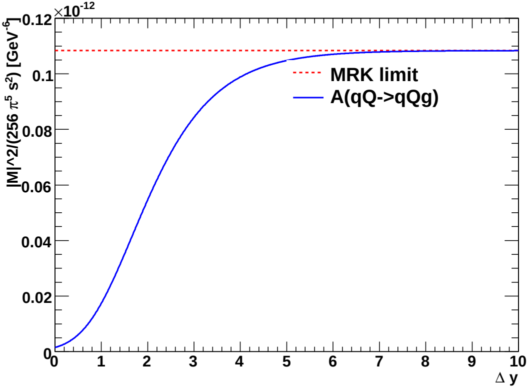

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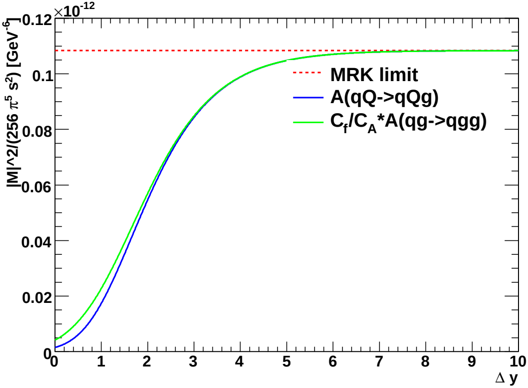

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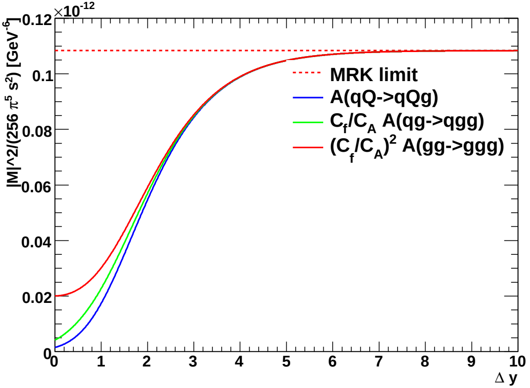

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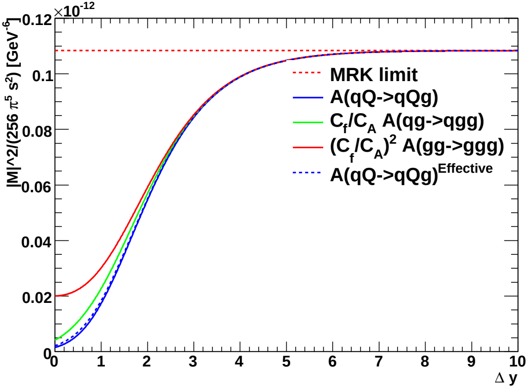

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Conclusion from Study of Partonic Cross Sections

- Correct limit is obtained - but outside LHC phase space. Limit alone irrelevant.
- Universality obtained before limit is reached.

Will build frame-work which has the right MRK limit (i.e. reproduces full QCD) but also retains correct behaviour at smaller rapidities (i.e. is relevant for the LHC)

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$:

$$M_{q^-Q^-\to q^-Q^-} = \langle 1|\mu|a\rangle \frac{g^{\mu\nu}}{t} \langle 2|\nu|b\rangle$$

t**-channel factorised**: Contraction of (local) currents across t-channel pole

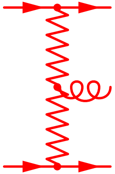
$$\begin{aligned}\left|\overline{\mathcal{M}}^t_{qQ\to qQ}\right|^2 = &\frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2\\ &\cdot \left(g^2\ C_F\ \frac{1}{t_1}\right)\\ &\cdot \left(g^2\ C_F\ \frac{1}{t_2}\right).\end{aligned}$$

Extend to $2 \to n \ldots$

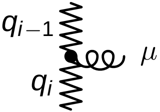
J.M.Smillie and JRA: arXiv:0908.2786

Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

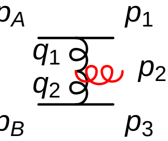


$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

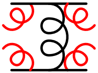


$$V^\mu(q_{i-1}, q_i)$$

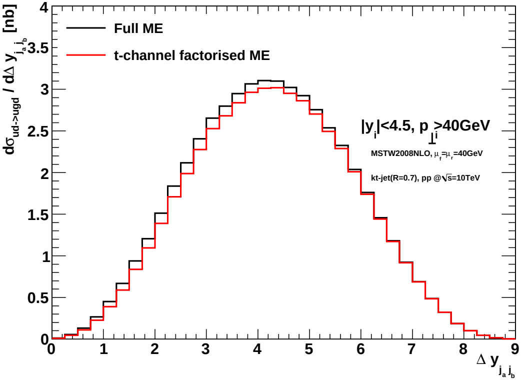
$$j^\nu = \overline{\psi}\gamma^\nu\psi$$



$$\begin{aligned} V^\rho(q_1, q_2) = & -(q_1 + q_2)^\rho \\ & + \frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2 \cdot p_A} + \frac{p_2 \cdot p_B}{p_A \cdot p_B} + \frac{p_2 \cdot p_n}{p_A \cdot p_n}\right) + p_A \leftrightarrow p_1 \\ & - \frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2 \cdot p_B} + \frac{p_2 \cdot p_A}{p_B \cdot p_A} + \frac{p_2 \cdot p_1}{p_A \cdot p_1}\right) - p_B \leftrightarrow p_3. \end{aligned}$$



3 Jets @ 10 TeV



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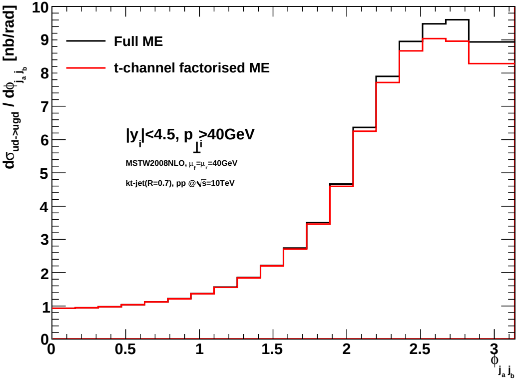


J.M.Smillie and JRA: arXiv:0908.2786

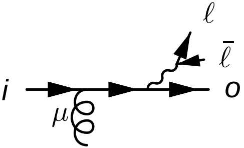
Building Blocks for an Amplitude

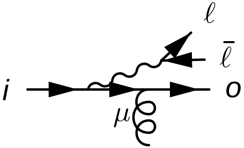
The approximation for $qQ \to qg \cdots gQ$ is given by

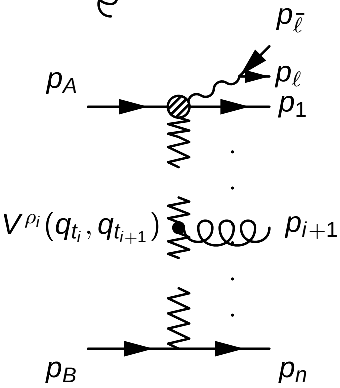
$$\left|\overline{\mathcal{M}}^t_{qQ\to qg\dots gQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\ C_F\ \frac{1}{t_1}\right)\cdot\ \left(g^2\ C_F\ \frac{1}{t_{n-1}}\right) \cdot \prod_{i=1}^{n-2}\left(\frac{-g^2 C_A}{t_i t_{i+1}}\ V^\mu(q_i,q_{i+1})V_\mu(q_i,q_{i+1})\right),$$

W+Jets

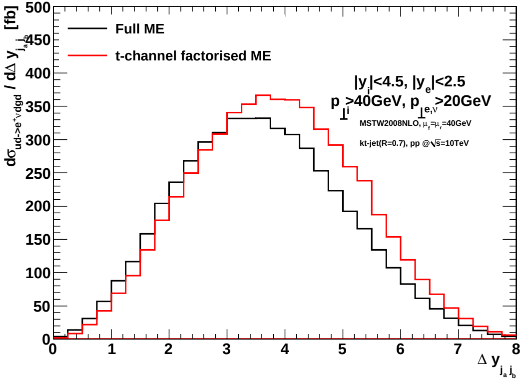
Two currents to calculate for $W + jets$:







W+ 3 Jets @ LHC



J.M.Smillie and JRA: arXiv:0908.2786

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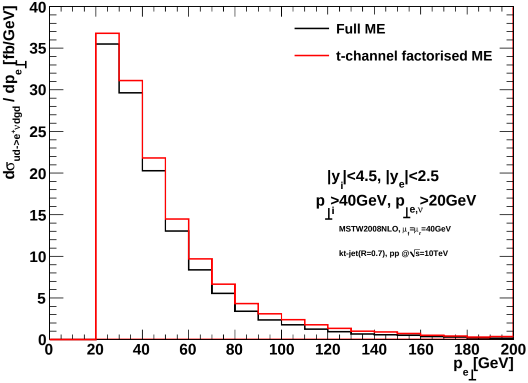

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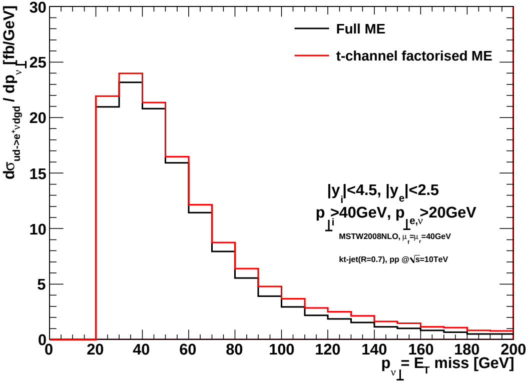
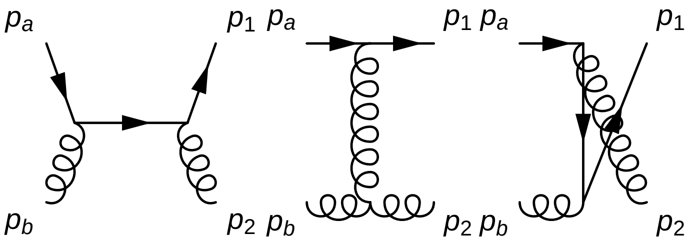


J.M.Smillie and JRA: arXiv:0908.2786

Quark-Gluon Scattering

"What happens in $2 \to 2$-processes with gluons? Surely the t-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \to q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p_b^-}{p_2^-}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p_2^-}{p_b^-}} \right) \langle b|\sigma|2\rangle \ \times \langle 1|\sigma|a\rangle.$$

Complete t-channel factorisation!

J.M.Smillie and JRA

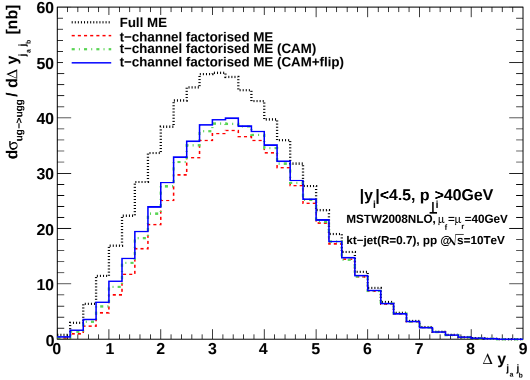
Quark-Gluon Scattering

For the helicity choices where a qQ-channel exists, the t-channel current generated by a gluon in qg scattering is that of a quark, but with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

instead of C_F. Tends to C_A in MRK limit.

Quark-Gluon Scattering



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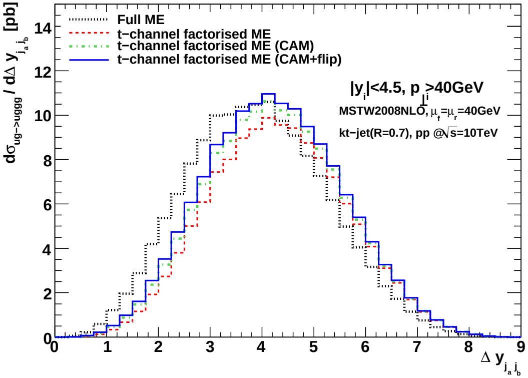


J.M.Smillie and JRA

All-Orders and Regularisation

- Have prescription for $2 \to n$ matrix element, including virtual corrections
- Organisation of cancellation of IR (soft) divergences easy
- Can calculate the sum over the n-particle phase space explicitly ($n \sim 25$) to get the all-order corrections
- Match to n-jet tree-level where known

J.M. Smillie, JRA arXiv:0908.2786, arXiv: 0910:5113

What HEJ is NOT

- **Small-x evolution of pdfs**. x isn't even small. And we are using standard collinear factorisation - which allows for a stringent comparison with standard PT!
- **BFKL**
  - We have no approximation of kinematic invariants. $q_\perp^2 \neq -t$ at LHC energies. Try for yourself. It's orders of magnitude off!
  - No evolution equation
  - No kernel
  - No impact factors
  - . . . but we do have gauge invariance. Everywhere in phase space. Not just asymptotically.

What HEJ can do for you

Describe the hard multi-jet environment for several processes (all matched):

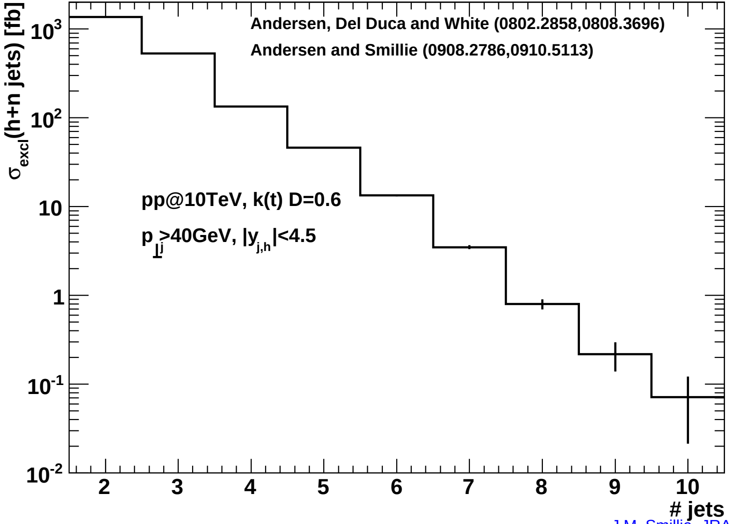
NOW

- Released code: H+jets
- root n-tuples: W+jets (or ask nicely and you will get the code)

soon

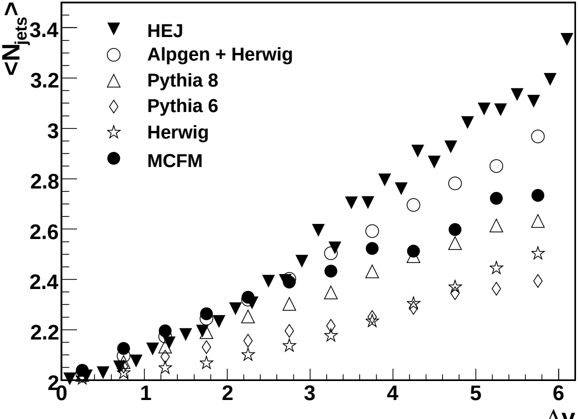
Z+jets, jets. . .

What HEJ can do for you



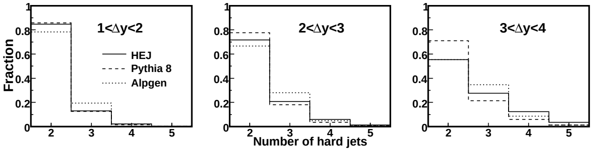
J.M. Smillie, JRA arXiv:1001.4463

What can $1fb^{-1}$ tell us about our perturbative tools



W+dijets, JRA, M. Campanelli, J. Campbell, V. Ciulli, J. Huston, P. Lenzi, R. Mackeprang, arXiv:1003.1241

$1fb^{-1}$@7TeV could be enough to tell the predictions apart!
[agreement here is not necessarily agreement in e.g. $\langle cos(\phi_{jj})\rangle$ vs. Δy.]
Obviously, similar results for pure dijets with much less data



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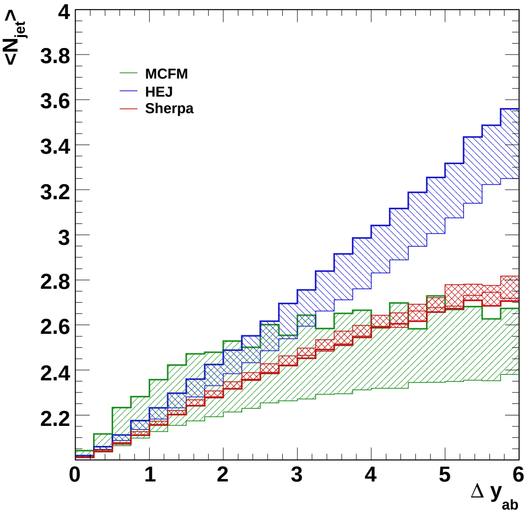
Similarities to H+dijets



W+jets can help in investigating jet veto efficiencies, . . .

JRA, J. Campbell, S. Höche,

arXiv:1003.1241

Summary

1. HEJ is a new perturbative tool for the description of multi-jet events at high energy colliders
   1. Simplify pert. corrections by concentrating on widely separated emissions
   2. Filling in the details of each jet (soft, collinear) is a job left for a parton shower
2. Even the 1st $fb^{-}1$@7TeV will shed light on the multi-jet environment in the new high energy domain.