DYNNLO

A fully exclusive parton level Monte Carlo code for the Drell-Yan process in NNLO QCD

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In collaboration with: S. Catani, L. Cieri, D. de Florian & M. Grazzini

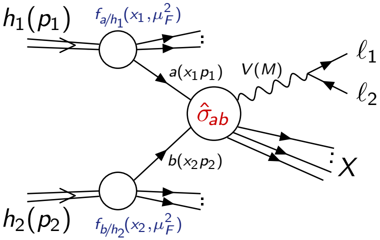
Outline

1. The Drell-Yan process
2. The Catani-Grazzini NNLO subtraction formalism
3. The code DYNNLO
4. Numerical Results
5. Conclusions

The Drell-Yan process

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; V(M) + X \;\rightarrow\; \ell_1 + \ell_2 + X$$

where $V = \gamma^*, Z^0, W^\pm$ and $\ell_1 \ell_2 = \ell^+\ell^-, \ell\nu_\ell$



According to the QCD factorization theorem:

$$d\sigma(p_1, p_2, \{y\}) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \, f_{a/h_1}(x_1, \mu_F^2) \, f_{b/h_2}(x_2, \mu_F^2) \, d\hat{\sigma}_{ab}(x_1 p_1, x_2 p_2, \{y\}; \mu_F^2) + \mathcal{O}\left(\frac{\Lambda^2}{M^2}\right).$$

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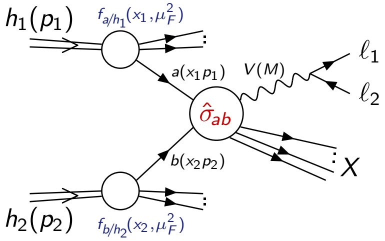
$\{y\} \equiv$ Infrared safe constraints on final states.

The Drell-Yan process



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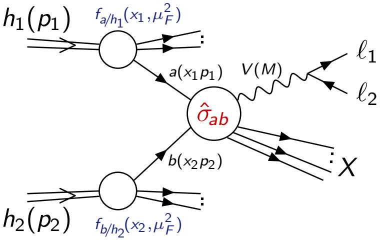

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The Catani-Grazzini NNLO subtraction formalism

- A NNLO extension of the subtraction formalism valid for the production of colourless high-mass system in hadron collisions was proposed by [Catani,Grazzini('07)] and applied for Higgs boson production in the parton level Monte Carlo code HNNLO.
- This method was used to perform a fully exclusive NNLO calculation for vector boson production which includes the γ-Z interference, finite-width effects, the leptonic decay of the vector bosons and the corresponding spin correlations [Catani, Cieri, G.F., de Florian, Grazzini('09)].
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The code DYNNLO

- The Fortran code of DYNNLO can be downloaded from:
  http://theory.fi.infn.it/grazzini/dy.html
- It has been tested on Linux and OSX Systems.
- Extract the main directory dynnlo/:
  ```
  $ tar xzvf dynnlo-v1.0.tgz
  ```
- Compile the code:
  ```
  $ cd dynnlo/
  $ make
  ```
- Run the executable:
  ```
  $ cd bin/
  $ ./dynnlo < infile
  ```

The structure of the main directory:

- `$dynnlo/bin/` The working directory.
- `$dynnlo/doc/` The directory containing a note.
- `$dynnlo/obj/` The directory containing the object files.
- `$dynnlo/src/` The directory containing the source files.

The structure of the working directory:

- `$dynnlo/bin/dynnlo` The executable file.
- `$dynnlo/bin/infile` The input file.
- `$dynnlo/bin/Pdfdata` The directory containing the PDFs grids.

The input file

This is a typical example of input file:

```
7d3 !  sroot   Double precision variable for CM energy (GeV).
1 1 !  ih1 ih2   Integers identifying the beam: (anti)proton=(−)1.
1 !  nproc   Vector boson produced: W+ → l+ν (1), W− → l−ν̄ (2), Z/γ* → l+l− (3).
80.419d0 80.419d0 !  mur, muf   Renorm. and factoriz. scales (GeV).
2 !  order   Order of calculation LO (0), NLO (1), NNLO (2).
'tota' !  part   String identifying the part of the calculation performed:
                  real (real), virtual (virt), total (tota)
15 1000000 !  itmx1, ncall1  # of iterations and calls to the Vegas grid.
30 8000000 !  itmx2, ncall2  # of iterations and calls to the Vegas run.
617 !  rseed   Random number seed.
92 0!  iset nset   Integers identifying the set and the error eigenvector for PDFs.
'nnlo' !  runstring   String for grid and output files.
```

Infrared cuts on final states

Infrared cuts on final states can be set in the `src/User/cuts.f` file.
For instance:

```
pt3=dsqrt(pjet(3,1)**2+pjet(3,2)**2)
pt4=dsqrt(pjet(4,1)**2+pjet(4,2)**2)
eta3=etarap(3,pjet)
eta4=etarap(4,pjet)
C Cuts in GeV
if(pt3.lt.25d0) cuts=.true.
if(pt4.lt.25d0) cuts=.true.
if(dabs(eta3).gt.1d0) cuts=.true.
if(dabs(eta4).gt.1d0) cuts=.true.
```

Input parameters and setup file

In the calculation we use the so called G_μ scheme (G_F, m_Z, m_W).
The values of input parameters are:
$G_F = 1.16637 \times 10^{-5}$ GeV^{-2} $m_W = 80.398$ GeV, $\Gamma_W = 2.141$ GeV, $m_Z = 91.1876$ GeV, $\Gamma_Z = 2.4952$ GeV, $V_{ud} = 0.97419$, $V_{us} = 0.2257$, $V_{ub} = 0.00359$, $V_{cd} = 0.2256$, $V_{cs} = 0.97334$, $V_{cb} = 0.0415$ ([PDG ('08)]).
Important features can be set in the `src/Need/setup.f` file:

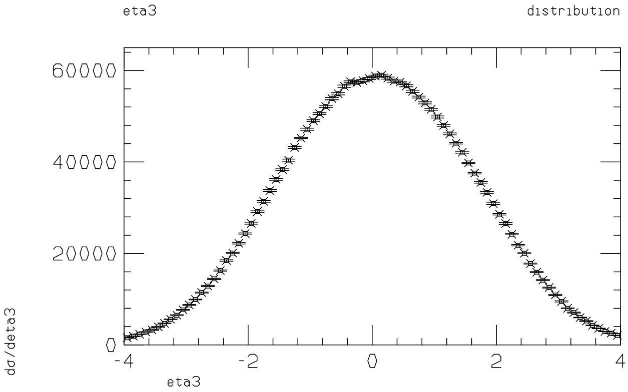
```
CC Narrow width approximation
zerowidth=.false.
CC Branching ratio
removebr=.false.
CC Lepton isolation is set in src/User/isolation.f
isolation=.true.
CC Jets are reconstructed according to the kT algorithm
CC Parameters used to define jets
ptjetmin=0d0
etajetmin=0d0
etajetmax=20d0
Rcut=0.4d0
```

Plotted distributions

Desired distributions in the form of bin histograms can be set in the `src/User/plotter.f` file.
A Topdrawer file will be generated.

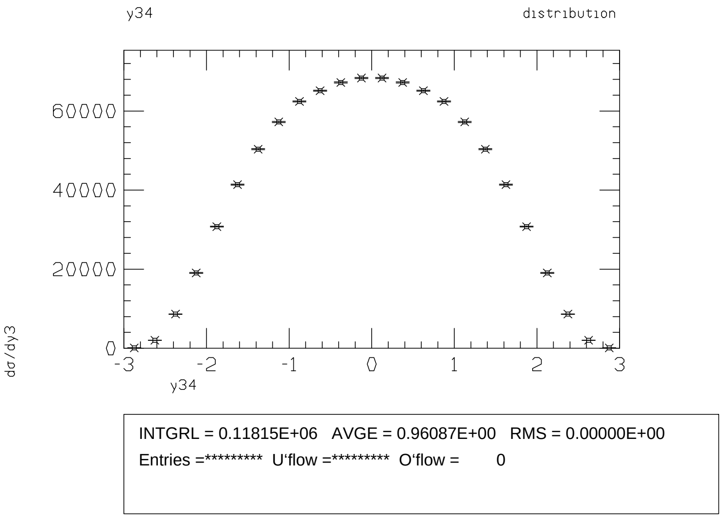
Let's consider for instance Z production at the Tevatron:

```
eta3=etarap(3,p)
pt3=pt(3,p)
y34=yraptwo(3,4,p)
pt34=pttwo(3,4,p)
CC
n=1
call bookplot(n,tag,'eta3',eta3,wt,-4d0,4d0,0.1d0,'lin')
n=n+1
call bookplot(n,tag,'y34',y34,wt,-3d0,3d0,0.25d0,'lin')
n=n+1
call bookplot(n,tag,'pt34',pt34,wt,0d0,100d0,2d0,'log')
n=n+1
```



INTGRL = 0.23318E+06   AVGE = 0.78763E-01   RMS = 0.00000E+00
Entries =********* U‘flow =242017840 O‘flow =264462160

Z production at the Tevatron: electron rapidity distribution.



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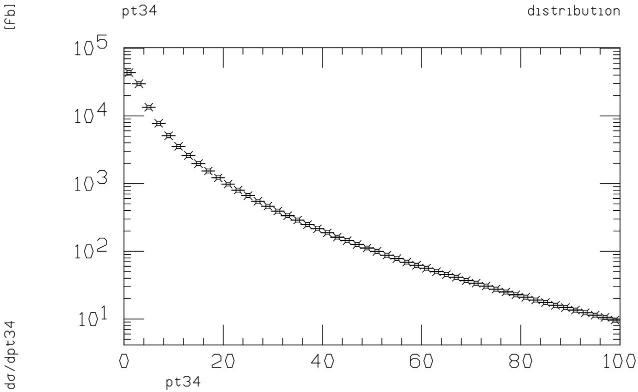


INTGRL = 0.23466E+06 AVGE = 0.57931E+01 RMS = 0.00000E+00
Entries =152189516 U'flow = 0 O'flow =409990100

Z production at the Tevatron: Z transverse momentum distribution.

Parton Distribution Functions

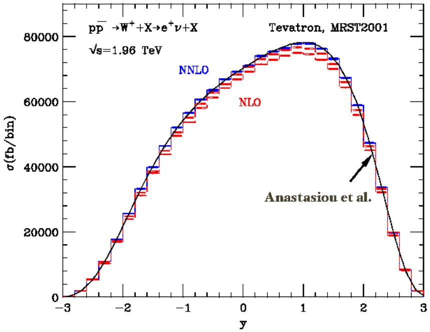
Desired PDFs sets can be implemented in the files `src/Need/pdf.f` and `src/Need/pdfset.f`. Adding the corresponding PDFs grids in the `bin/Pdfdata` directory. List of PDFs sets and their corresponding `iset` and values of $\alpha_S(M_Z)$ is:

iset	Pdf set	$\alpha_S(M_Z)$
1	CTEQ4 LO	0.132
2	CTEQ4 Standard NLO	0.116
11	MRST98 NLO central gluon	0.1175
12	MRST98 NLO higher gluon	0.1175
13	MRST98 NLO lower gluon	0.1175
14	MRST98 NLO lower α_S	0.1125
15	MRST98 NLO higher α_S	0.1225
16	MRST98 LO	0.125
21	CTEQ5M NLO Standard Msbar	0.118
22	CTEQ5D NLO DIS	0.118
23	CTEQ5L LO	0.127
24	CTEQ5HJ NLO Large-x glu. enhanc.	0.118
25	CTEQ5HQ NLO Heavy Quark	0.118
28	CTEQ5M1 NLO Improved	0.118
29	CTEQ5HQ1 NLO Improved	0.118
30	MRST99 NLO	0.1175
31	MRST99 higher gluon	0.1175
32	MRST99 lower gluon	0.1175
33	MRST99 lower α_S	0.1125
34	MRST99 higher α_S	0.1225
41	MRST2001 NLO central gluon	0.119
42	MRST2001 NLO lower α_S	0.117
43	MRST2001 NLO higher α_S	0.121

iset	Pdf set	$\alpha_S(M_Z)$
44	MRST2001 NLO better jet data fit	0.121
45	MRST2001 NNLO	0.1155
46	MRST2001 NNLO fast evolution	0.1155
47	MRST2001 NNLO slow evolution	0.1155
48	MRST2001 NNLO better jet data fit	0.1180
51	CTEQ6L LO	0.118
52	CTEQ6L1 LO	0.130
53	CTEQ6M NLO	0.118
55	CTEQ6.6M NLO	0.118
49	MRST2002 LO	0.130
61	MRST2002 NLO	0.1197
62	MRST2002 NNLO	0.1154
65	GJR08VF LO	0.1263
66	GJR08VF NLO	0.1145
67	JR09VF NNLO	0.1124
71	MRST2004 NLO	0.1205
72	MRST2004 NNLO	0.1167
75	A06 NNLO	0.1128
85	ABKM09 NNLO	0.1129
90	MSTW2008 LO	0.13939
91	MSTW2008 NLO	0.12018
92	MSTW2008 NNLO	0.11707

Numerical Results

Rapidity distribution



Rapidity distribution for W^+ production at the Tevatron (no cuts).

- No cuts are applied on final states.
- The error bars in the histograms refer to the Monte Carlo numerical errors.
- NNLO result compared with the NLO band (obtained by varying $m_Z/2 \le \mu_F = \mu_R \le 2\, mZ$) and with the NNLO analytical result by [Anastasiou et al.('03)].
- Results from DYNNLO MC code agree with known analytical results within the numerical error.

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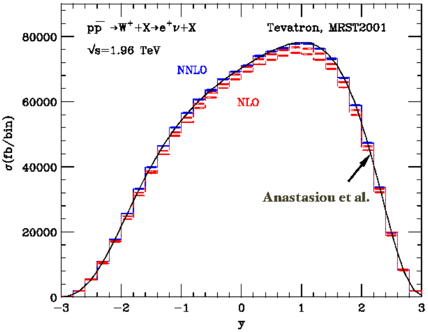

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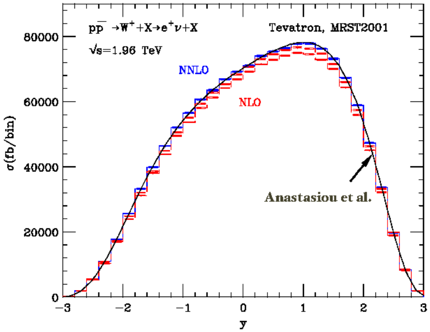

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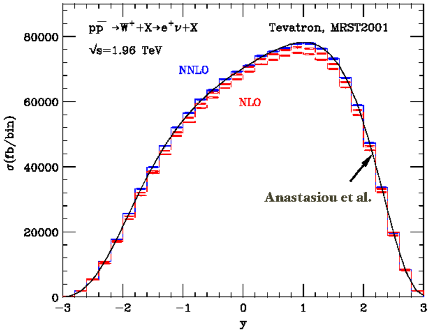

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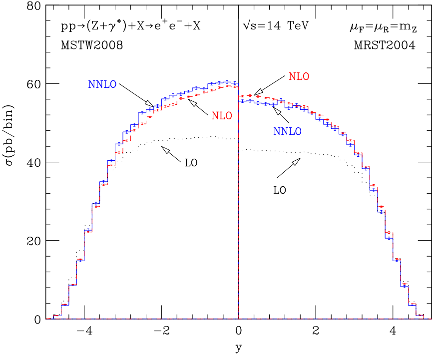

Rapidity distribution for Z production at the LHC (no cuts).

- No cuts are applied on final states.
- The error bars in the histograms refer to the Monte Carlo numerical errors.
- Left panel: MSTW 2008 PDFs:
  $\sigma_{NLO} = 2.030 \pm 0.001$ nb,
  $\sigma_{NNLO} = 2.089 \pm 0.003$ nb.
- Right panel: MRST 2004 PDFs:
  $\sigma_{NLO} = 1.992 \pm 0.001$ nb,
  $\sigma_{NNLO} = 1.954 \pm 0.003$ nb.
- σ_{NNLO} scale variations:
  -1.7% for $\mu_R = \mu_F = m_Z/2$,
  $+1.5\%$ for $\mu_R = \mu_F = 2\,m_Z$.
- Typical computing time for smooth distributions with a percent level accuracy on a standard PC: LO: few minutes, NLO: few hours, NNLO: three days.
  The computing time for cross sections is reduced by a factor two.

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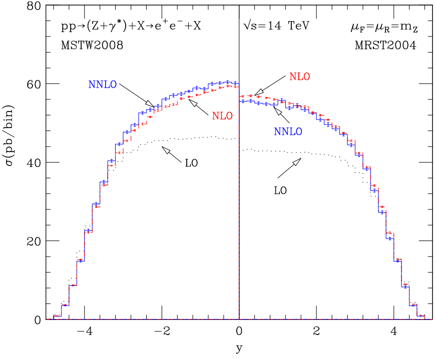

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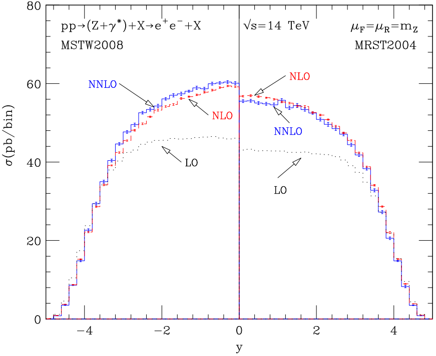

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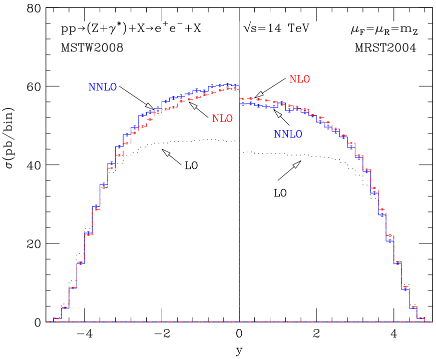

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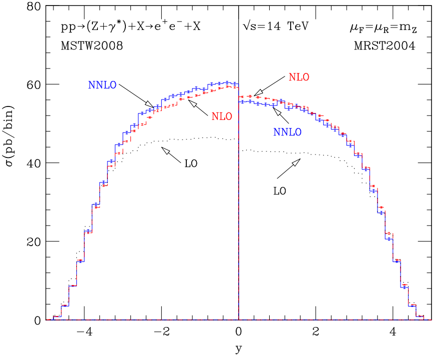

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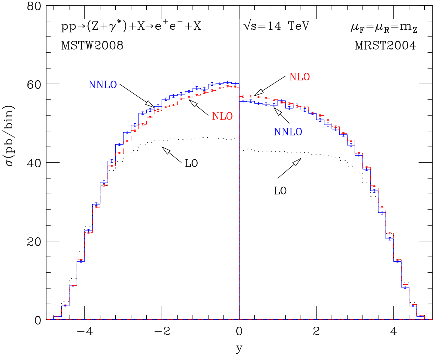

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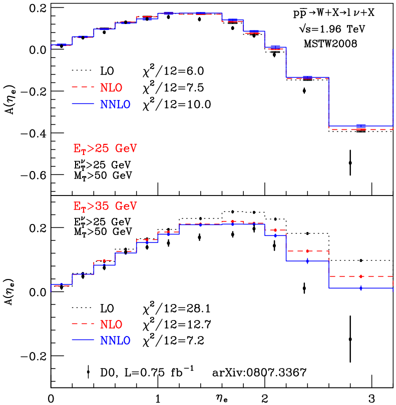
  The computing time for cross sections is reduced by a factor two.

Lepton charge asymmetry: $A(y_l) = \frac{d\sigma(l^+)/dy_l \; - \; d\sigma(l^-)/dy_l}{d\sigma(l^+)/dy_l \; + \; d\sigma(l^-)/dy_l}$



The electron charge asymmetry in LO, NLO and NNLO QCD with MSTW08 PDFs at wide (top) and high (bottom) E_T, compared with D0 data.

- D0 data on electron charge asymmetry [arXiv:0807.3367].
- Selection cuts on final states: $E_T^\nu > 25\ GeV$, $M_T > 50\ GeV$, $E_T > 25\ GeV$ (top) and $E_T > 35\ GeV$ (bottom).
- Lepton isolation requirements: $E_T^{\rm iso}/E_T < 0.15$. Where $E_T^{\rm iso}$ is the hadronic (partonic) transverse energy in a cone along the direction of the lepton momentum with radius $R = 0.4$ in the lepton $(\eta\text{–}\phi)$ space
- NNLO corrections can be larger than experimental errors while NNLO scale dependence $(M_W/2 \le \mu_F = \mu_R \ge 2M_W)$ comparable to the Monte Carlo numerical error.
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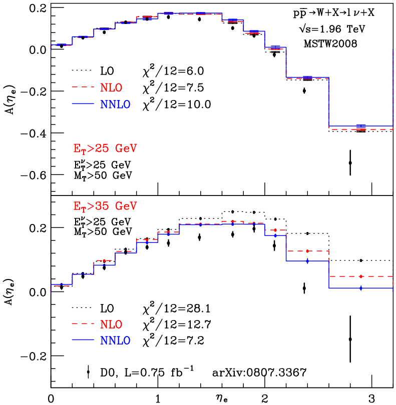

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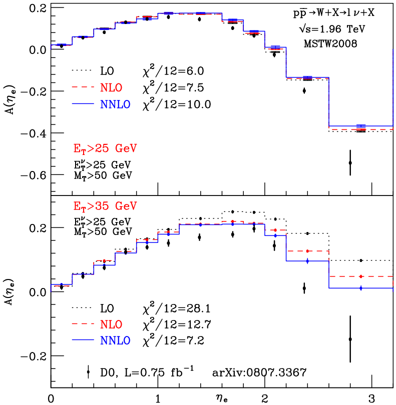

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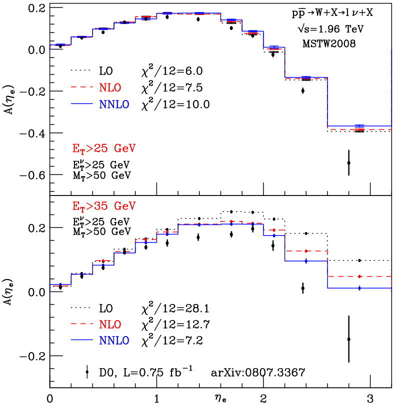

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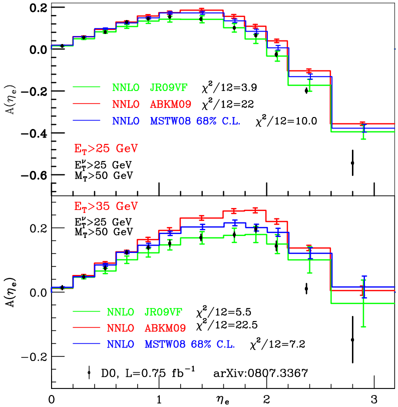

The electron charge asymmetry in NNLO QCD with MSTW08,ABKM09,JR09VF PDFs (with errors) at wide (top) and high (bottom) E_T, compared with D0 data.

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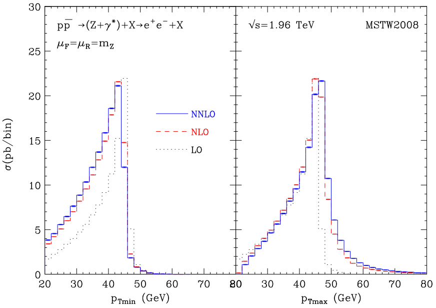
Minimum and maximum p_T distribution



Minimum (left) and maximum (right) lepton p_T distribution for Z production at the Tevatron.

- Cuts: $p_{T\,min} \geq 20\ GeV$; $|\eta| < 2$; $70\ GeV \leq m_{e^+e^-} \leq 110\ GeV$
- At LO the distributions are kinematically bounded by $p_T < (m_{e^+e^-})_{max}/2 = 55\,GeV$
- The NNLO corrections make the $p_{T_{min}}$ distribution softer, and the $p_{T_{max}}$ distribution harder.
- Accepted cross sections (errors refer to Monte Carlo numerical errors):
  $\sigma_{LO} = 103.37 \pm 0.04\ pb$,
  $\sigma_{NLO} = 140.43 \pm 0.07\ pb$,
  $\sigma_{NNLO} = 143.86 \pm 0.12\ pb$.
- σ_{NNLO} scales variation:
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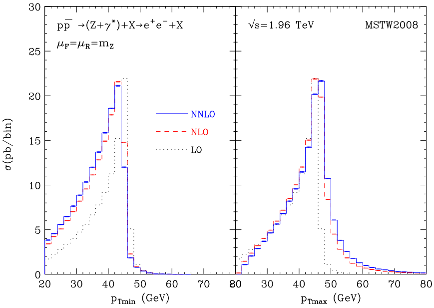

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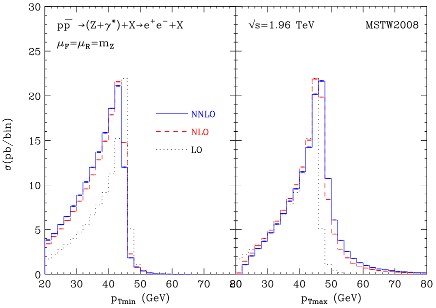

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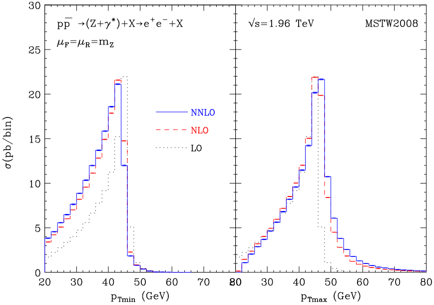

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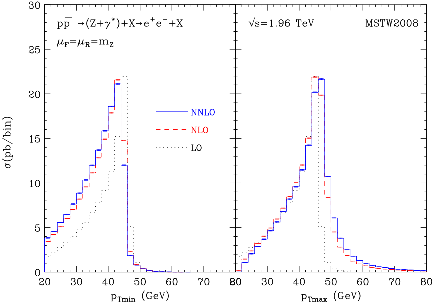

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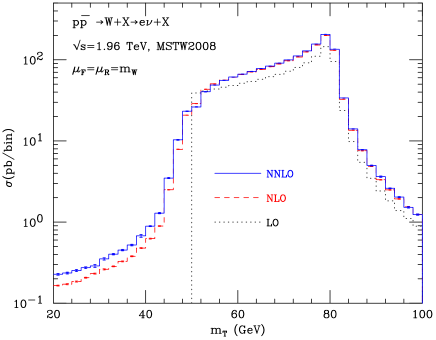
Transverse mass distribution



Transverse mass distribution for W production at the Tevatron:

$$m_T = \sqrt{2p_T^l p_T^{miss}(1-\cos\phi_{l\nu})}$$

- Cuts: $p_T{}^{miss} \geq 25\ GeV$; $|\eta| < 2$; $p_T{}^l \geq 20\ GeV$
- LO distribution bounded at $m_T = 50$ GeV. Around the bound there are perturbative instabilities from LO to NLO and to NNLO (Sudakov shoulder [Catani,Webber('97)]).
- Below the boundary, the $\mathcal{O}(\alpha_S^2)$ corrections are large (e.g. +40% at $m_T \sim 30$ GeV). Not unexpected: in this region the $\mathcal{O}(\alpha_S^2)$ result is only a NLO calculation.
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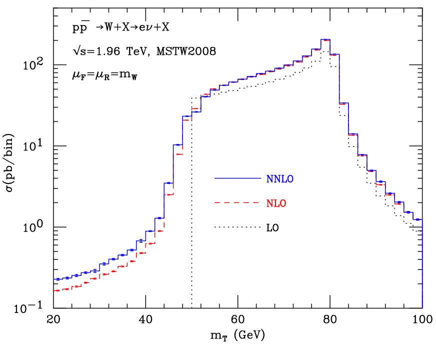

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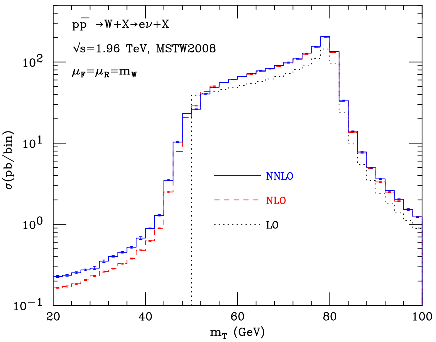

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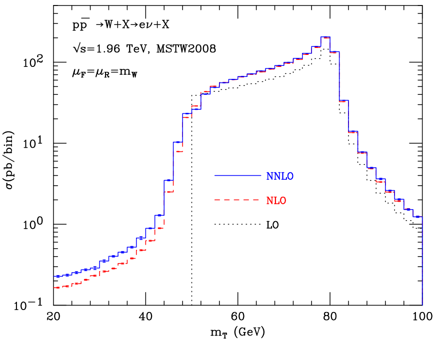

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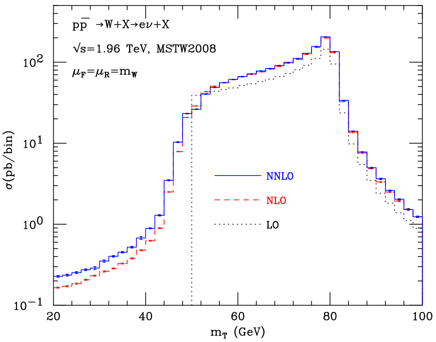

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- We have implemented the calculation in the parton level Monte Carlo code DYNNLO. The program allows the user to apply arbitrary kinematical cuts on the final state and on the associated jet activity computing the required distributions in the form of bin histograms.
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- If you use this program please quote:
  S. Catani, L. Cieri, G. Ferrera, D. de Florian and M. Grazzini, Phys. Rev. Lett. **103** (2009) 082001;
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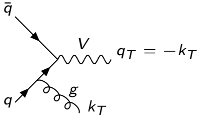
Back up slides

A NNLO extension of the subtraction method

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; V(M, q_T) + X$$

V is one or more colourless particles (vector bosons, leptons, photons, Higgs bosons,...) [Catani,Grazzini('07)].



- Key point I: at LO the q_T of the V is exactly zero.

$$d\sigma^{V}_{(N)NLO}|_{q_T \neq 0} = d\sigma^{V+\mathrm{jets}}_{(N)LO} \quad ,$$

for $q_T \neq 0$ the NNLO IR divergences cancelled with the NLO subtraction method.

- The only remaining NNLO singularities are associated with the $q_T \rightarrow 0$ limit.
- Key point II: treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from q_T resummation formalism [Catani,de Florian,Grazzini('00)].

$$d\sigma^{V}_{N^nLO} \stackrel{q_T \to 0}{\longrightarrow} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M) dq_T^2 = d\sigma^{V}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} \, d^2 q_T$$

$$d\sigma^{CT} \stackrel{q_T \to 0}{\longrightarrow} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M) dq_T^2$$

The final result is:

$$d\sigma^{V}_{(N)NLO} = \mathcal{H}^{V}_{(N)NLO} \otimes d\sigma^{V}_{LO} + \left[d\sigma^{V+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right] ,$$

$$\textit{where} \quad \mathcal{H}^{V}_{NNLO} = \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} \right]$$

- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT} \xrightarrow{q_T \to 0} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M) dq_T^2$. Note that $\Sigma(q_T/M)$ is universal.
- $d\sigma^{CT}$ regularizes the $q_T = 0$ singularity of $d\sigma^{V+\text{jets}}$: *double real* and *real-virtual* NNLO contributions, while *two-loops virtual* correction are contained in $\mathcal{H}^{V}_{NNLO}$.
- Final state partons only appear in $d\sigma^{V+\text{jets}}$ so that NNLO IR cuts are included in the NLO computation: observable-independent NNLO extension of the subtraction formalism.
- NLO calculation requires $d\sigma^{V+\text{jets}}_{LO}$ and $\mathcal{H}^{V(1)}$ [de Florian, Grazzini('01)].
- At NNLO we need also $d\sigma^{V+\text{jets}}_{NLO}$ [Giele et al.('93), MCFM] and $\mathcal{H}^{V(2)}$.

- The general relation between $\mathcal{H}^{V(2)}$ and the IR finite part of the two-loops correction to a generic process is unknown. We explicit computed it for the DY process with the following method.

$$\sigma^{V,tot}_{NNLO} = \int_0^\infty dq_T^2 \, \frac{d\sigma^V_{NLO}}{dq_T^2}.$$

- We decompose the q_T distribution as following:

$$\frac{d\sigma^V_{NLO}}{dq_T^2} = \frac{d\sigma^{V,(res.)}_{NLO}}{dq_T^2} + \frac{d\sigma^{V,(fin.)}_{NLO}}{dq_T^2},$$

where the first term on the r.h.s. contains all the the logarithmically-enhanced contributions at small q_T while the second term is free of such contributions.

- Following the [Bozzi, Catani, de Florian, Grazzini('06)] formalism we can then write

$$\sigma^{V,tot}_{NNLO} = \sigma^V_{LO} \, \mathcal{H}^V_{NNLO} + \int_0^\infty dq_T^2 \, \frac{d\sigma^{V,(fin.)}_{NLO}}{dq_T^2}.$$

- This formula allows us to analytically compute $\mathcal{H}^V_{NNLO}$ from the knowledge of the NNLO total cross section and the NLO q_T distribution.