Towards a Model for Minimum Bias Events in Sherpa

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Outline

1. Introduction: A single eikonal model
2. Implementing the KMR model
3. First Results
4. Conclusions & Outlook

Preliminaries

s-channel unitarity and cross sections

- Optical theorem relates total cross section $\sigma_{\rm tot}$ with elastic scattering amplitude $\mathcal{A}(s,t)$ through

$$\sigma_{\rm tot}(s) = \frac{1}{s}\,{\rm Im}[\mathcal{A}(s,t=0)]$$

- Rewriting $\mathcal{A}(\int,\sqcup)$ as $a(s,b)$ in impact parameter space

$$\mathcal{A}(s,t=-\vec{q}_\perp^{\,2}) = 2s\int {\rm d}^2b_\perp e^{i\vec{q}_\perp\cdot\vec{b}_\perp} a(s,\vec{b}_\perp)$$

  yields

$$\begin{array}{rcl}\sigma_{\rm tot}(s) & = & 2\int {\rm d}^2 b_\perp {\rm Im}[a(s,\vec{b}_\perp)]\\ \sigma_{\rm el}(s) & = & 2\int {\rm d}^2 b_\perp |a(s,\vec{b}_\perp)|^2\\ \sigma_{\rm inel}(s) & = & \sigma_{\rm tot}(s)-\sigma_{\rm el}(s)\end{array}$$

- N.B.: real part of $a(s,\vec{b}_\perp)$ vanishing

Parametrisations of total cross section

- Good old strong interaction physics (Regge poles): cross section given by sum over Regge exchanges
- Leading Regge pole (Pomeron) yields

$$\sigma_{\rm tot} = \sigma_0 \left(\frac{s}{s_0}\right)^{\alpha_P - 1}$$

- Donnachie-Landshoff:

$$\sigma_0 = 21.7 \text{ mb},\ s_0 = 1 \text{ GeV}^2,\ \alpha_P = 1.0808.$$

- CDF: $\sigma_0 = 24.36$ mb, rest like DL.
- Two-pomeron fit:

$$\sigma_{\rm tot} = 0.0139{\rm mb} \left(\frac{s}{s_0}\right)^{0.452} + 24.22{\rm mb} \left(\frac{s}{s_0}\right)^{0.0667}$$

Parametrisations of total cross section (cont'd)

- Predictions for $\sigma_{\rm tot}(14{\rm TeV})$:

$$\sigma_{\rm tot}(14{\rm TeV}) = \begin{cases} 101.5\,{\rm mb} & ({\rm DL}) \\ 114.0{\rm mb} & ({\rm CDF}) \\ 164.4{\rm mb} & (\text{two-pom}) \end{cases}$$

- Common problem: all violate unitarity (because $\alpha_P > 1$)
- Possible solution: use parametrisation for (see next slide) eikonal rather than for cross section

$$\Omega(s) \simeq \left(\frac{s}{s_0}\right)^{\alpha_P - 1}$$

- N.B.: pomeron intercept t-dependent

$$\alpha_P(t) = \alpha_P + \alpha'_P t \qquad (\alpha' = 0.25\ {\rm GeV}^{-2})$$

Eikonals

- Common parametrisation of amplitude though eikonal Ω

$$a(s,\,\vec{b}_\perp) = \frac{e^{-\Omega(s,\vec{b}_\perp)}-1}{2i}$$

Then:

$$\begin{aligned}\sigma_{\rm tot}(s) &= 2\int {\rm d}^2b_\perp[1-e^{-\Omega(s,\vec{b}_\perp)}]\\ \sigma_{\rm el}(s) &= \int {\rm d}^2b_\perp[1-e^{-\Omega(s,\vec{b}_\perp)}]^2\\ \sigma_{\rm inel}(s) &= \int {\rm d}^2b_\perp[1-e^{-2\Omega(s,\vec{b}_\perp)}]\end{aligned}$$

A useful relation

- Consider elastic slope B

$$B(s, t=0) = \left[\frac{\mathrm{d}}{\mathrm{d}t} \ln \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}t}\right]_{t=0}$$

- In simple pomeron parametrisation:

$$B \longrightarrow 2\alpha'_P \ln \frac{s}{s_0} + B_0$$

leading to (CDF parameters)

$$\begin{aligned} B &= \left[\ln \frac{\sqrt{s}}{1.8\mathrm{TeV}} + (17 \pm 0.25)\right] \mathrm{GeV}^{-2} \\ &= \left[\ln \frac{\sqrt{s}}{14\mathrm{TeV}} + (19 \pm 0.25)\right] \mathrm{GeV}^{-2} \end{aligned}$$

- Also calculable from eikonal form:

$$B = \frac{1}{\sigma_{\mathrm{tot}}} \int \mathrm{d}^2 b_\perp b_\perp^2 [1 - e^{-\Omega(s, \vec{b}_\perp)}]$$

Multi-channel eikonals

Motivation

- Impossible to describe "diffractive excitation" (like e.g. $p \longrightarrow N(1440)$) with one eikonal only:
  such processes are a consequence of the
  internal structure of the colliding hadrons
- For description employ high-energy limit:
  in this limit the Fock states of the hadrons are "frozen",
  (lifetime of fluctuations $\tau = E/m^2$ large)
  and each component can interact separately, destroying coherence of the colliding hadrons

Good-Walker states

- Introduce Good-Walker states (diffractive eigenstates):
$$|p\rangle = \sum_i a_i|\phi_i\rangle,\ \text{where}\ \langle\phi_i|\phi_k\rangle = \delta_{ik}\ \text{and}\ \sum_i |a_i|^2 = 1$$
- These states diagonalise the $\mathcal{T}$-matrix:
$$\langle\phi_i|\mathrm{Im}\mathcal{T}|\phi_k\rangle = T_k^D\delta_{ik}$$
- Therefore only "elastic scattering" of these states
- N.B.: use two states (more later),
$$|p,\ N^*\rangle = \frac{1}{\sqrt{2}}\left[|\phi_1\rangle \pm |\phi_2\rangle\right],$$
related to two different form factors,
$$F_{1,2}(q_\perp) = \beta_0^2(1\pm\kappa)\frac{\exp\left[-\frac{(1\pm\kappa)\xi q_\perp^2}{\Lambda^2}\right]}{\left[1+\frac{(1\pm\kappa)q_\perp^2}{\Lambda^2}\right]^2}$$

Cross sections with Good-Walker states

- Decompose incoming state $|j\rangle = a_{jk}|\phi_k\rangle$ and write
$$\langle j|\mathrm{Im}\mathcal{T}|j\rangle = \sum_k |a_{jk}|^2 T_k \equiv \langle T\rangle$$
- Allows to write cross sections as
$$\begin{aligned}
\frac{\mathrm{d}\sigma_{\mathrm{tot}}}{\mathrm{d}^2 b} &= 2\mathrm{Im}\langle j|\mathcal{T}|j\rangle = 2\langle T\rangle \\
\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2 b} &= |\langle j|\mathcal{T}|j\rangle|^2 = \langle T\rangle^2 \\
\frac{\mathrm{d}\sigma_{\mathrm{el+SD}}}{\mathrm{d}^2 b} &= |\langle \phi_k|\mathcal{T}|j\rangle|^2 = \sum_k |a_{jk}|^2 T_k^2 = \langle T^2\rangle \\
\frac{\mathrm{d}\sigma_{\mathrm{SD}}}{\mathrm{d}^2 b} &= \langle T^2\rangle - \langle T\rangle^2
\end{aligned}$$
- Single diffraction given by statistical dispersion of absorption probabilities of diffractive eigenstates

Comments

- If all components of j, i.e. all eigenstates ϕ_k, experienced same absorption:
  $\longrightarrow$ dispersion vanishes
  $\longrightarrow$ diffractive production xsec vanishes
- This happens in the black disc limit ($T_k = 1$), at small $b_\perp$
- Consequence: already at Tevatron energies diffractive production processes due to large $b_\perp$
- This region is responsible for small t components
- Also: there eikonal (equivalent to optical density, opacity) is small
  $\longrightarrow$ large rapidity gap survival probability

(due to small density, only few scatters)

Developing a partonic picture

Basic idea: Regge physics rules

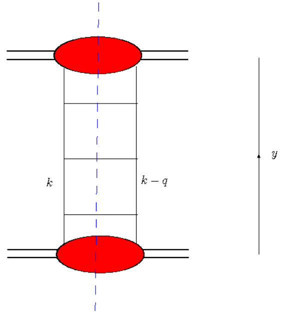
- Want to use Regge physics, in particular pomeron exchange, for amplitudes
- Naively write $A(s, \vec{b}_\perp) \propto (s/s_0)^{\alpha_P(t)}$
- Want to connect Regge picture to QCD
- Unitarisation (naive): identify amplitude with eikonal
  eikonal related to sum over all possible pomeron-exchanges between hadronic states

The pomeron in QCD - trivialised

- Pomeron as the sum of ladder-type diagrams, i.e. a sum over all (inelastic) $2 \to N$ processes

- Treat diagram (right) as amplitude squared for the production of N particles, homogeneously distributed in rapidity y with $y \in [-Y/2, Y/2]$ and $Y = \ln s/m_p^2$:

$$\sigma_{2\to N} = A_{2\to N}(Y) A^*_{2\to N}(Y)$$



The pomeron in QCD - trivialised (cont'd)

- The amplitude then reads

$$A(Y) = \sum_n \frac{1}{n!} \prod_{i=1}^{N} \int_{-Y/2}^{Y/2} \mathrm{d}y_i \alpha = \sum_n \frac{(\alpha Y)^n}{n!} = e^{\alpha Y} = s^\alpha$$

- with the $k_\perp$-integral over each cell of the ladder:

$$\alpha(q_\perp^2) = \frac{g^2}{16\pi^2} \int \frac{\mathrm{d}^2 k_\perp}{[k_\perp^2 - m^2][(k_\perp - q_\perp)^2 - m^2]}$$

- Alternative way to obtain result:

$$\frac{\mathrm{d}A(y)}{\mathrm{d}y} = \alpha A(y)$$

- Similar to DGLAP equation, but for evolution in y - α plays role of splitting kernel

(describing with uniform emission probability)

Parameters

- Typically written as recursion from the amplitude for the $n-1$ to the n particles final state:

$$f_n(x,k_\perp) = \frac{N_c\alpha_s}{\pi}\int_x^1 \frac{\mathrm{d}x'}{x'}\int\frac{\mathrm{d}^2k'_\perp}{\pi}K(k_\perp,k'_\perp)f_{n-1}(x',k'_\perp)$$

- Rewrite recursively

$$\frac{\mathrm{d}f}{\mathrm{d}\ln 1/x} = \frac{N_c\alpha_s}{\pi}K\otimes f$$

  Therefore $f\propto x^{-\Delta_0}$, where $\alpha_P = 1+\Delta$ and

$$\Delta_0 = \frac{N_c\alpha_s}{\pi}\langle K\rangle = \frac{N_c\alpha_s}{\pi}4\ln 2 \approx 0.5$$

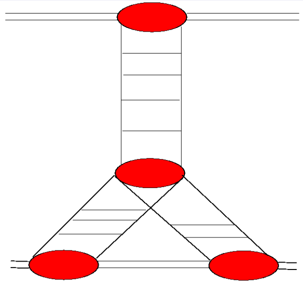
- Including higher order effects,

$$\Delta \approx \Delta_0\exp(-6\frac{N_c\alpha_s}{\pi}) \approx 0.3$$

Rescattering

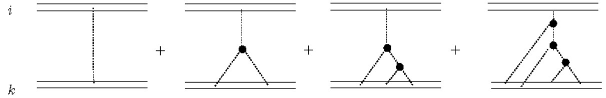
Motivation

- High-density, strong coupling regime: rescattering effects important
- In other words: large triple pomeron vertex g_{3P}
- Physical effect: high-mass dissociation through diagrams like the one ot the right
- Note: other cuts apart from naive vertical possible



Fan diagrams and the Schwimmer model

- If large size differences of colliding particles (e.g. γp, pZ etc.): can resum "fan diagrams" (pomeron as dashed line)



- Evolution equation:

$$\frac{\mathrm{d}f(y)}{\mathrm{d}y} = \Delta f(y) - g_{3P} f(y)$$

- Total cross section becomes

$$\sigma_{\rm tot} = \frac{g_i g_k e^{\Delta Y}}{1+\epsilon(e^{\Delta y}-1)} \xrightarrow{Y\to\infty} \frac{g_i g_k}{\epsilon},$$

where $\epsilon = g_{3P} g_k/\Delta$ is maximal density of pomerons

- Problem: too low xsec for large-mass dissociation

(Re-) Interpretation of the amplitude

- Start with the evolution equation for the bare pomeron contribution to the elastic amplitude:

$$\frac{\mathrm{d}f(y)}{\mathrm{d}y} = \Delta f(y)$$

- This can also be regarded as the evolution equation for the parton density of hadron i (entering at $y = -Y/2$) in the presence of hadron k:

$$\frac{\mathrm{d}\Omega_{i(k)}(y)}{\mathrm{d}y} = \Delta\Omega_{i(k)}(y) \Longleftrightarrow \Omega_{i(k)}(y) = g_i^2 e^{\Delta(y+Y/2)}$$

- g_i is fixed by the initial condition, i.e. the interaction probability or the parton density at $y = -Y/2$

(given by the Fourier transform related to the eigenstate ϕ_i)

Summing all rescattering contributions

- Multi-pomeron contributions (rescattering) arise from absorption of intermediate s-channel partons (the rungs in the ladder during evolution
- Allowing for all possible rescatterings/absorptions, must sum over all possible ladder configurations:

$$\frac{\mathrm{d}f(y)}{\mathrm{d}y} = \exp[-\lambda f(y)]\Delta f(y)\,,$$

where $\lambda \propto g_{3P} \approx 0.25g$, the pomeron-nucleon coupling

(in contrast Schwimmer model: $\mathrm{d}f(y)/\mathrm{d}y = \Delta f(y) - g_{3P}f(y)$)

The Khoze-Martin-Ryskin model (naive version)

- Construct eikonal from individual parton densities:

$$\Omega(\vec{b}_\perp) = \frac{1}{2\beta_0^2}\int \mathrm{d}^2 b_\perp^{(1)} \mathrm{d}^2 b_\perp^{(2)} \delta^2(\vec{b}_\perp - \vec{b}_\perp^{(1)} - \vec{b}_\perp^{(2)}) \cdot \Omega_{i(k)}(\vec{b}_\perp^{(1)}, \vec{b}_\perp^{(2)}, y)\Omega_{(i)k}(\vec{b}_\perp^{(1)}, \vec{b}_\perp^{(2)}, y)\,,$$

(The value of the integral will not depend on y)

- where the parton densities fulfil the evolution equations

$$\frac{\mathrm{d}\ln\Omega_{i(k)}(y)}{\mathrm{d}y} = +\exp\left\{-\frac{\lambda}{2}\left[\Omega_{i(k)}(y) + \Omega_{(i)k}(y)\right]\right\}\Delta$$

$$\frac{\mathrm{d}\ln\Omega_{(i)k}(y)}{\mathrm{d}y} = -\exp\left\{-\frac{\lambda}{2}\left[\Omega_{i(k)}(y) + \Omega_{(i)k}(y)\right]\right\}\Delta$$

The Khoze-Martin-Ryskin model (naive version)

- The boundary conditions read:

$$\Omega_{i(k)}(\vec{b}_\perp^{(1)}, \vec{b}_\perp^{(2)}, -Y/2) = F_i(b_\perp(1)^2)$$

$$\Omega_{(i)k}(\vec{b}_\perp^{(1)}, \vec{b}_\perp^{(2)}, +Y/2) = F_k(b_\perp(2)^2),$$

  i.e. the parton densities at maximal rapidities are given by the parton densities of the hadrons, as modelled by the form factors.
- Form factor parameters:
  $\beta_0^2 \approx 30$ mb, $\kappa \approx 0.5$, $\Lambda^2 \approx 1.5$ GeV^2, $\xi \approx 0.2$
- Evolution parameters: $\Delta \approx 0.3$, $\lambda \approx 0.25$

The MC realisation

Selecting the mode

- Select elastic vs. inelastic processes according to

$$\sigma^{pp}_{\rm tot} = 2\int {\rm d}^2 b_\perp \sum_{i,k=1}^{S} \left\{ |a_i|^2 |a_k|^2 \left[1 - e^{-\Omega_{ik}(b_\perp)}\right] \right\}$$

$$\sigma^{pp}_{\rm inel} = \int {\rm d}^2 b_\perp \sum_{i,k=1}^{S} \left\{ |a_i|^2 |a_k|^2 \left[1 - e^{-2\Omega_{ik}(b_\perp)}\right] \right\}$$

$$\sigma^{pp}_{\rm el} = \int {\rm d}^2 b_\perp \left\{ \sum_{i,k=1}^{S} \left[|a_i|^2 |a_k|^2 \left(1 - e^{-\Omega_{ik}(b_\perp)}\right)\right] \right\}^2$$

Selecting gross features of scattering mode

- If elastic is chosen, fix t according to

$$\frac{\mathrm{d}\sigma_{\rm el}}{\mathrm{d}t} = \frac{1}{4\pi}\left\{\int \mathrm{d}^2 b_\perp e^{i\vec{q}_\perp\cdot\vec{b}_\perp} \sum_{i,k}\left[|a_i|^2\,|a_k|^2\left(1-e^{-\Omega_{ik}(\vec{b}_\perp)}\right)\right]\right\}^2 .$$

- If inelastic is chosen, fix $\{ik\}$ according to partial contribution and $\vec{b}_\perp$ according to integrand,

$$\mathcal{P}_{ik}(b_\perp)\pi b_\perp\left(1-e^{-2\Omega_{ik}(b_\perp)}\right)$$

Inelastic scattering: generating ladders

- Assume no correlations between ladders - will be included a posteriori
- Select (naive) number of ladders to be exchanged according to Poissonian:

$$\mathcal{P}_{n=N_{\rm naive}-1} = \frac{[2\Omega_{ik}(b_\perp)]^n}{n!} \exp[-2\Omega_{ik}(b_\perp)]$$

- For each ladder, fix $\vec{b}_\perp^{(1,2)}$ with $\vec{b}_\perp = \vec{b}_\perp^{(1)} + \vec{b}_\perp^{(2)}$:

$$\frac{{\rm d}^2\Omega_{ik}(b_\perp)}{{\rm d}^2 b_\perp^{(1)}} = \frac{1}{2}\Omega_{i(k)}(\vec{b}_\perp^{(1)}, \vec{b}_\perp^{(2)})\Omega_{(i)k}(\vec{b}_\perp^{(1)}, \vec{b}_\perp^{(2)})$$

- After each ladder, check momentum of incoming hadrons
  if E_1 or E_2 exhausted terminate exchanging ladders
  therefore $N_{\rm ladders} \leq N_{\rm naive}$

Inelastic scattering: generating emissions

- Emit $n_{\rm gluons}$ gluons in each ladder, with $\langle n_{\rm gluon}\rangle = \int\limits_{-Y/2}^{Y/2} {\rm d}y\rho(y)$ and y-distribution according to ρ.
- $\rho(y) = \exp\left\{-\frac{\lambda}{2}\left[\Omega_{i(k)}(y) + \Omega_{(i)k}(y)\right]\right\}\Delta$
- Select IS and FS “beam-particles” according to PDF
- For each t-channel propagator select colour

(assume only colour singlet and octet exchange)

$$\mathcal{P}_1 \propto \left\{1 - \exp\left[-\frac{\Omega_{i(k)}(y_{i+1})}{\Omega_{i(k)}(y_i)}\right]\right\}^2$$

$\mathcal{P}_8 \propto 1 - \mathcal{P}_1$ – this projects all non-singlet colours on an octet.

Inelastic scattering: generating emissions (cont'd)

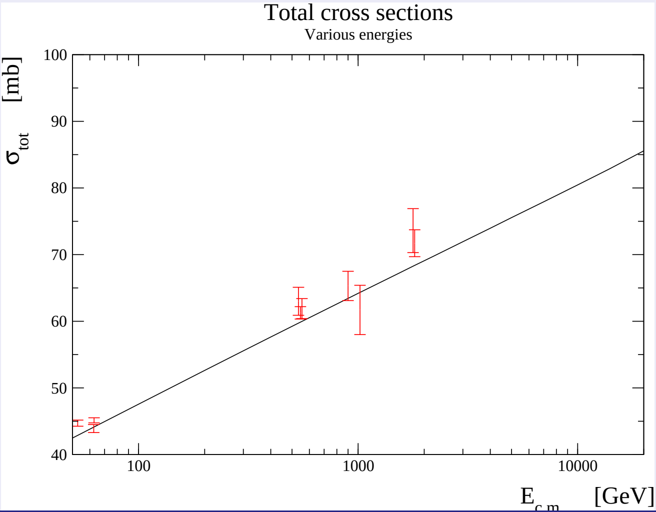
- Generate t-channel momenta $q_\perp$ according to

$$\begin{aligned}\mathcal{P}_{\rm sing}(q^2_{\perp,i}) &= \frac{\alpha_s(k^2_{\perp,i})}{(q^2_{\perp,i}+\mu_1^2)}\\ \mathcal{P}_{\rm oct}(q^2_{\perp,i}) &= \frac{\alpha_s(k^2_{\perp,i})}{(q^2_{\perp,i}+\mu_8^2)^{1+\frac{N_c\alpha_S}{\pi}(y_{i+1}-y_i)}}\end{aligned}$$

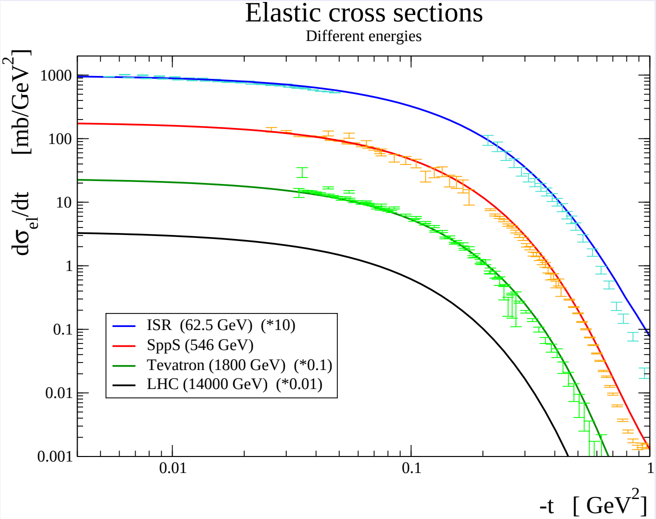
- Reconstruct emitted $\vec{k}_{\perp,i} = \vec{q}_{\perp,i+1} - \vec{q}_{\perp,i}$ and $k_i^\mu = k_{\perp,i}(\cosh y_i, \cos\phi_i, \sin\phi_i, \sinh y_i)$
- Accept ladder with probability given by ME expression for hardest $2 \to 2$ interaction.

First results

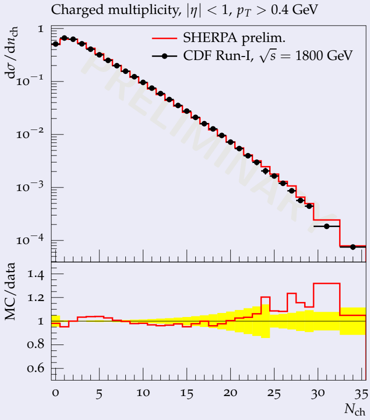
Total Cross Section

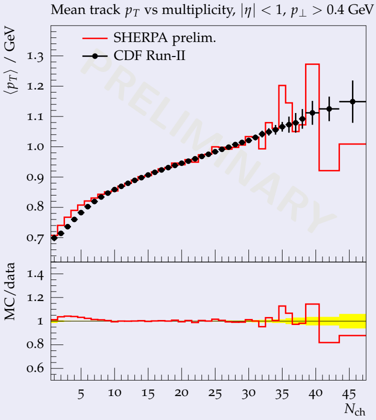


Elastic Cross Section



Minimum Bias from CDF ($p\bar{p}$ @ 1800 GeV)





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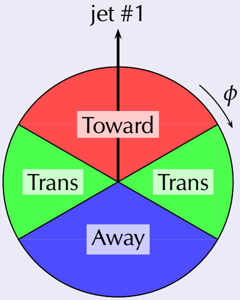

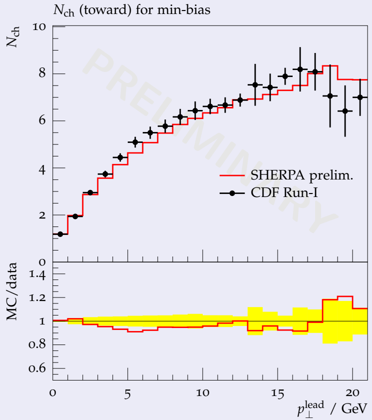

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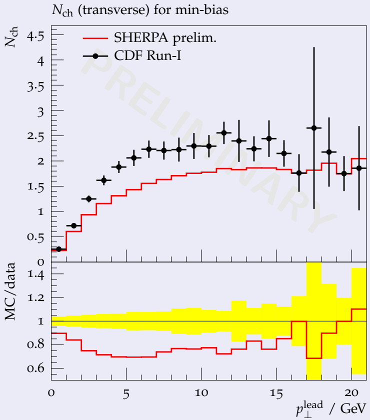

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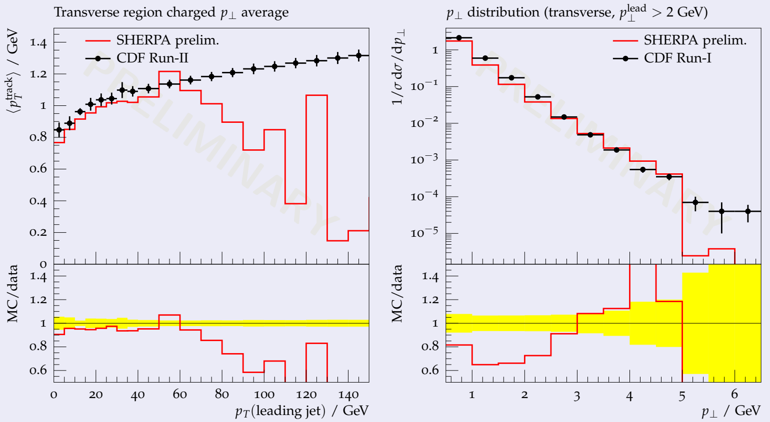
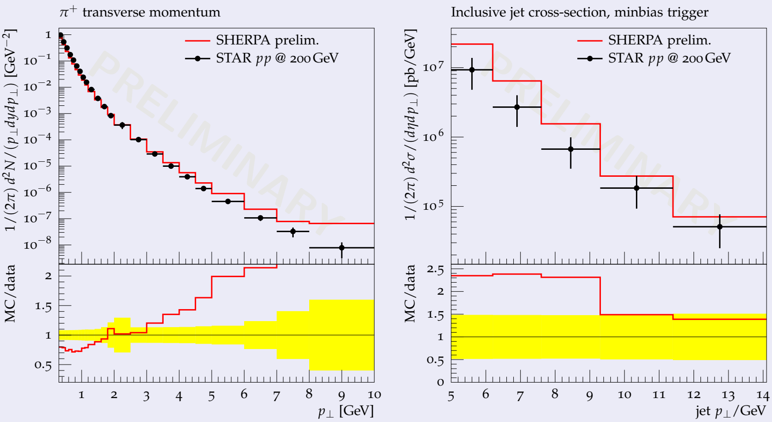


Pion Spectrum and Jet cross section at STAR (pp @ 200 GeV)



Conclusions

- Implemented KMR model in naive form
- Ambitious model: Tries to incorporate elastic, diffractive and inelastic scattering on identical footing
- MC implementation of these scattering modes straightforward
- However: Very inclusive in its original formulation –
  is being substantially enhanced through MC implementation
- Important contribution missed yet: Rescattering of ladders
  "Pomeron in pomeron" – maybe dominant
- Improved modelling of energy-momentum conservation under way –
  use parton luminosities to fix c.m.-energy of ladders
  will have to continue PDFs for $Q^2 \to 0$ – impact should be small.